1. Given $V_{BE(on)} = 0.7V$, $\beta = 50$, $V_A = -20V$, determine the voltage gain $A_v = \frac{v_{out}}{v_{in}}$. Assume $V_T = 25mV$ and $C_E \to \infty$.

Solution:

a) \[
\frac{V_{CC}-V_B}{R_1} = \frac{V_B}{R_2} + I_B, \text{ where } I_B = \frac{I_E}{\beta+1} = \frac{V_B-0.7}{R_E(\beta+1)}
\]

$V_B \approx 2.954V$, $V_E = V_B - 0.7 = 2.254V$

$\frac{\beta}{\beta+1} \frac{V_E}{R_E} \approx 1.105mA$

$V_C = V_{CC} - R_C I_C = 7.79V$

Since $V_{CB} > 0V$, the transistor is in FA mode

b) \[
\begin{align*}
\text{Diagram here}
\end{align*}
\]
\[ r_0 = \left| \frac{V_A}{I_{C}} \right| \approx 18.1k\Omega, \quad g_m = \frac{I_C}{V_T} = 44.2mA/V, \quad r_\pi = \frac{\beta}{g_m} \approx 1.131k\Omega \]

By doing KVL and KCL analysis, we have

\[ R_i = R_{SRC} + R_1||R_2||r_\pi \]
\[ R_o = R_L||R_C||r_0 \]
\[ v_{be} = v_{in} \times \frac{R_1||R_2||r_\pi}{R_{SRC} + R_1||R_2||r_\pi} \]
\[ G_m = \frac{v_{be} \times g_m}{v_{in}} = \frac{R_1||R_2||r_\pi}{R_{SRC} + R_1||R_2||r_\pi} \times g_m \]
\[ A_v = -G_m R_o = \frac{(R_1||R_2||r_\pi)(R_L||R_C||r_0)}{R_{SRC} + R_1||R_2||r_\pi} \times g_m \]

c) \[ R_i \approx 1.235k\Omega, \quad R_o \approx 0.643k\Omega, \quad G_m \approx 26.3067mA/V, \quad A_v \approx -16.9152, \quad v_{be} \approx 2.976mV \]

d) \[ i_C = v_b \times g_m \times \left( 1 - \frac{R_L||R_C}{R_L||R_C + r_o} \right) \approx 213.15\mu A \]
2. Repeat Problem 1 but this time assume $C_E = 0$. Compare the results of part b)-d) with that of Problem 1.

Solution:

Note: The final result of this question could look quite complicated, but steps are straightforward and can be solved by inspection.

a) Same as in Problem 1, removing the bypass capacitor does not change the DC

b) Small signal model:

\[ R_B = R_{SRC} || R_1 || R_2, \quad V_{DR} = \frac{R_1 || R_2}{R_1 || R_2 + R_{SRC}} \]

$v_{be}$:

\[ R_{out-emitter} = R_E || r_\pi \left( \frac{1}{g_m} \right) \left( 1 + \frac{R_C || R_L}{r_o} \right) \]
Having the input resistance of the base of BJT, we can find $v_b$ in terms of $v_{in}$:

\[
v_b = v_{in} \cdot VDR \cdot \frac{R_{in-base}}{R_{in-base} + R_B}
\]

\[
v_{be} = v_b \cdot \left(1 - \frac{v_e}{v_b}\right)
\]

$R_i$:

\[
R_i = R_{SRC} + R_1 || R_2 || R_{in-base}
\]

$G_M$:

\[
R_{in-base-GM} = r_\pi + (\beta + 1) \cdot R_E || r_o
\]

\[
G_M = \frac{1}{1 + g_m(R_E || r_o)} \cdot VDR \cdot \frac{R_{in-base-GM}}{(R_{in-base-GM} + R_B)}
\]

$R_o$:
Note that the small-signal model shown on left and right are equivalent, where:

\[ g_{m-new} = g_m \star \frac{r_\pi}{R_B + r_\pi} \]

Then \( R_{out} \) can be expressed as:

\[ R_o = g_{m-new}r_o\left(R_E \parallel (r_\pi + R_B)\right) + r_o + R_E \parallel (r_\pi + R_B) \]

\( A_v \):

\[ A_v = -G_M R_o \]

c) Plug in the numbers into the expressions in b), we have

\[ v_{be} = 50.47 \mu V, R_i = 2.552 k\Omega, G_M = 388.55 \mu S, \]

\[ R_o = 665.83 \Omega, A_v = -0.2587 \]

d) Note that \( i_c = i_{R_L} \), we have already calculated the gain from \( v_{in} \) to \( v_{out} \), take out the attenuation factor due to \( R_{SRC}, R_1 \) and \( R_2 \), then we have

\[ A_{vb} = \frac{v_{out}}{v_{in}} = \frac{A_v}{VDR \times \frac{R_{in-base}}{R_{in-base} + R_B}} = -0.32174 \]

\[ i_c = \frac{v_b A_{vb}}{R_L} = 804 nA \]
3. Determine expressions for $G_m, R_i$, and $R_o$ for the following circuits by parsing the circuit appropriately and using the concept of BJT driving point (terminal) resistances. Assume $\beta \gg 1$ and FA-mode.

![Circuit Diagrams](image)

Solution:

a) Notice that the above circuit is a common emitter amplifier therefore the current gain

$$G_m = g_{m2} = \frac{\beta}{r_{\pi2}}$$

You can obtain this also by connecting $v_{out}$ to ground in the small signal model and calculating the current.

The input resistance $R_i = r_{\pi2}$, obtained by applying KCL at the input node on the small signal model.

To calculate the output resistance, short the input and apply a test voltage at the output node in the ssm.

Applying KCL at node $v_{c1}$

$$\frac{v_{c1}}{R_c} = g_{m1}v_{test} + \frac{v_{test} - v_{c1}}{r_{o1}}$$

$$v_{c1} = \frac{g_{m1} + r_{o1}^{-1}}{R_c^{-1} + r_{o1}^{-1}}v_{test}$$

Applying KCL at $v_{out}$

$$i_{test} = (r_{o2}^{-1} + g_{m1} + r_{\pi1}^{-1})v_{test} + \frac{v_{test} - v_{c2}}{r_{o1}}$$

$$i_{test} = (r_{o2}^{-1} + g_{m1} + r_{\pi1}^{-1} + R_L^{-1})v_{test} + \frac{R_c^{-1} - g_{m1}}{r_{o1}(R_c^{-1} + r_{o1}^{-1})}v_{test}$$
If we assuming $r_o = \infty$

$$R_o = \frac{v_{\text{test}}}{i_{\text{test}}} = \frac{1}{g_m + r_{n1}^{-1} + R_L}$$

Since $\beta \gg 1$

$$R_o = \frac{v_{\text{test}}}{i_{\text{test}}} = \frac{1}{r_{o2}^{-1} + g_m + R_L^{-1}}$$

b) The input resistance given by

$$R_i = R_B + (\text{Input resistance of Q1})||(\text{Input resistance of Q2})$$

Input resistance of Q1 = $r_{n1}$

Input resistance of Q2 = $r_{n2} + (1 + \beta)(R_E||r_{o2})$

Therefore (assuming $\beta \gg 1$)

$$R_i = R_B + r_{n1}||(r_{n2} + \beta(R_E||r_{o2}))$$

The current gain at the output is given by

$$G_m = \frac{v_b g_{m1}}{v_i}$$

where, $v_b$ is the ssm base voltage of Q1
\[ v_b = \frac{r_{\pi 1}||(r_{\pi 2} + \beta (R_E || r_{o 2}))}{r_{\pi 1}||(r_{\pi 2} + \beta (R_E || r_{o 2}))} + R_B - v_i \]

Therefore

\[ G_m = \frac{r_{\pi 1}||(r_{\pi 2} + \beta (R_E || r_{o 2}))}{g_m 1}[\frac{\beta}{r_{\pi 1}||(r_{\pi 2} + \beta (R_E || r_{o 2})) + R_B r_{\pi 1}}] \]

The output resistance is rather straightforward

\[ R_o = R_c || r_{o 1} \]

\[ c) \] The input resistance

\[ R_i = (\text{input resistance of Q3}) || (\text{input resistance of Q2}) \]

\[ R_i = r_{\pi 2} || \frac{1}{1 - \frac{v_{e 2}}{v_{in}}} \]

where \( \frac{v_{e 2}}{v_{in}} = \frac{1}{r_{\pi 2}} || r_{o 2} || r_{\pi 2} || \frac{1}{g_m 2} || \left( 1 + \frac{r_{o 1} r_{B 1} + R_{B 2} + r_{\pi 1}}{r_{o 2}} \right) \). Note: refer to Problem 2 part b, \( v_{be} \)

To calculate \( G_m \) short the output node and measure current through the node as shown

KCL at \( v_x \):

\[ g_{m 3} v_{in} + \frac{v_x}{r_{o 3}} + \frac{v_x}{r_{o 2}} = \frac{v_{in} - v_x}{r_{\pi 2}} + g_{m 2} (v_{in} - v_x) \]

\[ v_x = \frac{(g_{m 2} - g_{m 3}) + r_{\pi 2}^{-1}}{r_{o 1}^{-1} + r_{o 2}^{-1} + r_{\pi 2}^{-1} + g_{m 2}} v_{in} \]
The current through the output node is
\[ g_{m2}(v_{in} - v_x) - \frac{v_x}{r_{o2}} \]
\[ g_{m2} + \left( g_{m2} - g_{m3} + r_{\pi 2}^{-1} \right) \left( g_{m2} + r_{o2}^{-1} \right) \]
\[ v_{in} \]

Therefore the current gain \( G_m \)
\[ G_m = \left( g_{m2} - \left( g_{m2} - g_{m3} + r_{\pi 2}^{-1} \right) \left( g_{m2} + r_{o2}^{-1} \right) \right) \]

The output resistance \( R_o \) is given by \( R_o = R_{up} || R_{down} \)
\[ R_{up} = \left( \frac{(R_{B1} || R_{B2}) + r_{\pi 1}}{1 + \beta} \right) \]
\[ R_{down} = r_{o2} + (1 + g_{m2} r_{o2})(r_{o3} || r_{\pi 2}) \]

Total output impedance is
\[ R_o = \left( \frac{(R_{B1} || R_{B2}) + r_{\pi 1}}{1 + \beta} \right) \left( r_{o1} \right) \left( r_{o2} + (1 + g_{m2} r_{o2})(r_{o3} || r_{\pi 2}) \right) \]