CS Amplifier with Source Degeneration

The presence of $R_s$ in the source-side leads to modifications (benefits) to the CS amp. properties.

These benefits are in proportion to the SD (source degeneration) factor:

$$1 + g_m R_s \rightarrow "SD\ factor"$$

Benefits include:
1. improved (increased) input linear range
2. increased drain resistance $R_d$
3. gain being independent of $g_m$.
A drawback is loss in equivalent $g_m$. 
SSM of CS-SD Amplifier:

\[ v_i = v_{gs} + g_m v_{gs} R_s \]

\[ \Rightarrow v_{gs} = \frac{v_i}{1 + g_m R_s} \quad \text{improved input linearity} \]

Input linear range of a CS amp:

\[ |v_{gs}| < 2V_{ov} \Rightarrow |v_i| < 2V_{ov} (1 + g_m R_s) \]

Input linearity increased by a factor \((1 + g_m R_s)\) compared to a CS amp.

\[ A_{2e}: \quad v_o = -g_m v_{gs} R_D \]

\[ = -g_m R_D \frac{v_i}{1 + g_m R_s} \]
\[ A_v = \frac{v_o}{v_i} = -\frac{g_m R_D}{1 + g_m R_S} \quad \text{gain of a CS Amp} \]

Gain is reduced by a factor \((1 + g_m R_S)\).

**Note:** As \( R_S \gg g_m R_S \),

\[ A_v \approx -\frac{g_m R_D}{g_m R_S} = -\frac{R_D}{R_S} \]

\( A_v \) is relatively independent of \( g_m \). This is a good property as \( g_m \), being a transistor parameter, varies with process. \( R_D \) & \( R_S \) can be realized with tighter tolerance than \( g_m \).

\[ G_m : \]

\[ i_0 = g_m v_{gs} = \frac{g_m v_i}{1 + g_m R_S} \quad \Rightarrow \quad G_m = \left| \frac{I_0}{V_i} \right| = \frac{g_m}{V_i} \mid_{V_i=0} = \frac{g_m}{1 + g_m R_S} \]
\[ G_m = g_m \text{ for a CS amplifier.} \]

\[ G_m = \frac{g_m}{1 + g_m R_s} \quad \text{loss in transconductance (tax)} \]

\[ R_0 : \]

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**Note:** \( V_s = I_d R_s \); \( V_{gs} = -V_s \)

We wish to calculate \( R_d \).

\[ R_d = \frac{V_T}{I_d} \]
KCL at 1:

\[ l_d = g_m v_{gs} + \frac{v_T - v_s}{r_{ds}} \]

\[ = -g_m v_s + \frac{v_T - v_s}{r_{ds}} \]

\[ = -g_m l_d R_s + \frac{v_T - l_d R_s}{r_{ds}} \]

\[ l_d \left[ 1 + g_m R_s + \frac{R_s}{r_{ds}} \right] = \frac{v_T}{r_{ds}} \]

\[ \frac{v_T}{l_d} = R_d = R_s + r_{ds} (1 + g_m R_s) \]

As \( r_{ds} \gg R_s \)

\[ R_d \approx r_{ds} (1 + g_m R_s) \]
Summary:

\[ \text{transconductance} \]

\[ \begin{align*}
\text{CS stage} & : \quad L_d \rightarrow R_d \\
\text{CS-SD transconductance} & : \quad L_d \rightarrow R_d
\end{align*} \]

\[ v_{gs} = v_i \quad \leftrightarrow \quad v_{gs} = \frac{v_i}{1 + g_m R_s} \]

\[ G_m = g_m \quad \leftrightarrow \quad G_m = \frac{g_m}{1 + g_m R_s} \]

\[ I_d = G_m v_i = g_m v_{gs} \quad \leftrightarrow \quad I_d = \frac{g_m v_i}{1 + g_m R_s} \]

\[ R_d = r_{ds} \quad \leftrightarrow \quad R_d = R_s + r_{ds} (1 + g_m R_s) \]

\[ \approx r_{ds} (1 + g_m R_s) \]