Two-port Networks

Recall: Any linear circuit can be simplified into a 2-element circuit when viewed from two arbitrarily chosen terminals. There are two types of such 2-element circuits — Thevenin and Norton equivalent circuits.

e.g. original circuit

Using the Thevenin or Norton equivalent circuits, it is straightforward to calculate the voltage and current flowing through an external load element.
\[ I_L = \frac{3.59}{1.282} = 2.8 \text{mA}; \]

\[ V_L = \frac{1}{1.282} \times 3.59 = 2.8 \text{V}; \]

This same \( I_L \) and \( V_L \) will occur if the original circuit were to be analyzed with the 1kΩ load.

In other words, the Thevenin and Norton equivalent circuits provide an abstraction of a complex linear circuit when seen between two arbitrarily chosen terminals.
Most circuits have inputs and outputs. Therefore, one can create abstraction of the circuit as seen from input terminals and the output terminals.

This way we can analyze the response of the original circuit to an arbitrary source driving the input terminals when loaded by an arbitrary load across its output terminals. → two port network model

2-port Network

Input port → linear Network (N) ← output port

How to abstract a complicated circuit?
2-port network:

1. Model the input port with an input resistance (impedance) between the input terminals.
   This input resistance $R_i = R_f$ seen between the input terminals.

2. Model the output port using either Thevenin or Norton equivalent circuit.

3. Relate the input to the output using controlled sources.

\[ \begin{align*}
  & \text{Input voltage} \\
  & \text{Output voltage} \\
  & \text{Input - voltage} \\
\end{align*} \]
\[ R_i : \text{Thevenin resistance seen at the input port} \rightarrow \text{input resistance} \]

\[ R_o : \text{Thevenin resistance seen at the output port} \rightarrow \text{output resistance} \]

\[ A_{vo} = \frac{V_o}{V_i} |_{\text{open-ckt}} = \text{voltage gain}. \]

Therefore, a complex network can be abstracted down to 3 elements:

\[ R_i, R_o, \text{ and } A_{vo}. \]

Note: The source can be either a voltage or a current source. Also, the output port can be modeled using either Thevenin or Norton equivalent ckt.

\[ \text{there exists four different 2-port networks for any given circuit.} \]
Previously, we had found the Thevenin and Norton equivalent ckt. for the output port.

Find $R_i$ the Thevenin resistance between $C\to D$ terminals.

\[
R_i = \frac{5V}{I_1} \quad \text{(Treat } V_i=5V \text{ as a test voltage source } V_{\text{test}})\]

\[
I_1 = \frac{5 - V_{oc}}{20k} = \frac{5 - 3.59}{20k} = 70.5\,\mu\text{A}
\]

\[
R_i = \frac{5}{70.5\,\mu\text{A}} = 71\,\text{G} \Omega \text{ (very large input resistance)}
\]
\[ R_0 = 282.5 \Omega \]

\[ R_i = 71.6 \Omega \]

\[ A_{vo} = \frac{v_o}{v_i} \]

Output is open-ckt

\[ \frac{V_{oc}}{5} = \frac{3.59}{5} = 0.718 \text{ V/V} \]

\[ = 0.72 \text{ V/V} \]

Thus, \( R_i = 71.6 \Omega \) \( R_0 = 282 \Omega \) and \( A_{vo} = 0.72 \text{ V/V} \) used in the 2-port network above completely defines the behavior of the network when seen from the input & output ports.

The original circuit may have 100's or 1000's of elements. As long as the circuit is linear, we can abstract it down to a 3-element 2-port network.

Note: If \( A_{vo} \) were > 1.0 then the original circuit is called an amplifier. Most of the future lectures will focus on amplifiers and calculating their \( R_i, R_0, \& \text{ gains} \).
Four types of 2-port networks:

1. **VCVS (Voltage Controlled Voltage Source)**

   ![Diagram of VCVS](image)

   \[ A_{v_o} = \left| \frac{V_o}{V_i} \right| I_o = 0 \]

2. **VCCS (Voltage Controlled Current Source)**

   ![Diagram of VCCS](image)

   \[ G_{m_o} = \left| \frac{I_o}{V_i} \right| V_o = 0 \]

3. **CCVS (Current Controlled Voltage Source)**

   ![Diagram of CCVS](image)

   \[ R_{m_o} = \left| \frac{V_o}{I_i} \right| I_o = 0 \]

4. **CCCS (Current Controlled Current Source)**

   ![Diagram of CCCS](image)

   \[ A_{i_o} = \left| \frac{I_o}{I_i} \right| V_o = 0 \]
Real voltage & current sources will be used to drive the input ports. Real sources have finite, non-zero output resistances.

**Ideal VS**

\[ V_s \quad (R_s = 0) \]

**Real VS**

\[ V_s \quad R_s \]

**Ideal CS**

\[ I_s \quad (R_s = \infty) \]

**Real CS**

\[ I_s \quad \frac{1}{R_s} \]

When a real voltage (current) source is applied across a load resistor, only a part of the voltage (current) is seen by the load, there is a "loss".

\[ V_L = \frac{R_s}{R_s + R_L} V_s \]

\[ I_L = \frac{R_s}{R_s + R_L} I_s \]
Real sources driving Real network

There is loss at the input port and loss at the output port.

Overall gain:

\[ A_v = \frac{V_o}{V_i} = \frac{V_o}{V_i} \times \frac{V_i}{V_s} \]

loss at output

\[ V_o = \frac{R_L}{R_L + R_o} \times A_v \frac{V_i}{V_i} = \frac{R_L}{R_L + R_o} \times A_v \]

\[ V_i = \frac{R_i}{R_s + R_i} \frac{V_s}{V_s} = \frac{R_i}{R_s + R_i} \]

loss at input

Applied \( V_s \) volts but only a fraction \( \frac{R_i}{R_s + R_i} \) appeared at the amplifier input terminals \( \odot \) and \( \odot \). The amplifier provided a gain of \( A_v \) but only a fraction \( \frac{R_L}{R_L + R_o} \) was realized.

For \( A_v \rightarrow A_v \odot \Rightarrow R_i \gg R_s \quad \text{and} \quad R_L \gg R_o \) (Design strategy)