ECE 342
Electronic Circuits

Lecture 4
Diode Models

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The Diode

• **Diode Properties**
  – Two-terminal device that conducts current freely in one direction but blocks current flow in the opposite direction.
  – The two electrodes are the anode which must be connected to a positive voltage with respect to the other terminal, the cathode in order for current to flow.
Ideal Diode Characteristics

- For $V < 0$, $I > 0$ (OFF)
- For $V > 0$, $I > 0$ (ON)

Actual diode behavior deviates from the ideal diode behavior at low voltages due to the finite barrier height and thermal effects.
Ideal Diode Characteristics
Diode Models

Exponential

Piecewise Linear

Constant-Voltage-Drop
Diode Models

**Ideal-diode**

**Small-signal**

\[
\text{Slope} = \frac{1}{r_d}
\]
Piecewise-Linear Model

\[ \text{for } v_D \leq V_{D0} : \quad i_D = 0 \]

\[ \text{for } v_D \geq V_{D0} : \quad i_D = \frac{1}{r_D}(v_D - V_{D0}) \]
Piecewise-Linear Model
Constant-Voltage-Drop Model

$$\text{for } i_D > 0: \quad v_D = 0.7 \, V$$
Constant-Voltage-Drop Model
Diodes Logic Gates

OR Function

\[ Y = A + B + C \]

AND Function

\[ Y = A \cdot B \cdot C \]
**Diode Circuit Example 1**

**IDEAL Diodes**

Assume both diodes are on; then

\[ V_B = 0 \quad \text{and} \quad V = 0 \]

\[ I_{D_2} = \frac{10 - 0}{10} = 1 \text{ mA} \]

At node \( B \)

\[ I + 1 = \frac{0 - (-10)}{5} \Rightarrow I = 1 \text{ mA}, V = 0 \text{ V} \]

\( D_1 \) is conducting as originally assumed
Diode Circuit Example 2

Assume both diodes are on; then

\[ V_B = 0 \quad \text{and} \quad V = 0 \]

\[ I_{D2} = \frac{10 - 0}{5} = 2 \text{ mA} \]

At node \( B \)

\[ I + 2 = \frac{0 - (-10)}{10} \Rightarrow I = -1 \text{ mA} \Rightarrow \text{wrong} \]

original assumption is not correct … assume \( D_1 \) is off and \( D_2 \) is on

\[ I_{D2} = \frac{10 - (-10)}{15} = 1.33 \text{ mA} \]

\[ V_B = -10 + 10 \times 1.33 = +3.3 \text{ V} \]

\( D_1 \) is reverse biased as assumed
Example

The diode has a value of $I_S = 10^{-12}$ mA at room temperature ($300^\circ$ K)
(a) Approximate the current $I$ assuming the voltage drop across the
diode is 0.7V
(b) Calculate the accurate value of $I$
(c) If $I_S$ doubles for every $6^\circ$ C increase in temperature, repeat part
   (b) if the temperature increases by $40^\circ$ C

(a) The resistor will have an approximate
   voltage of 6-0.7 = 5.3 V. Ohm’s law then
gives a current of
   \[ I = \frac{5.3}{2} = 2.65 \text{ mA} \]
(b) The current through the resistor must
   equal the diode current; so we have
   \[ I = \frac{6-V}{2} \text{ (resistor current)} \quad I = I_S e^{V/V_T} \text{ (diode current)} \]
Example (cont’d)

\[
\frac{6 - V}{2} = 10^{-12} e^{V/0.026}
\]

Nonlinear equation \(\rightarrow\) must be solved iteratively
Solution: \(V = 0.744 V\)

Using this value of the voltage, we can calculate the current

\[
I = \frac{6 - V}{2} = \frac{6 - 0.744}{2} = 2.63 \text{ mA}
\]

When the temperature changes, both \(I_s\) and \(V_T\) will change. Since \(V_T = kT/q\) varies directly with \(T\), the new value is:

\[
V_T(340) = V_T(300) \times \frac{340}{300} = 0.0295
\]
Example (cont’d)

The value of $I_s$ doubles for each 6$^\circ$ C increase, thus the new value of $I_s$ is

$$I_s(340) = I_s(300) \times 2^{40/6} = 1.016 \times 10^{-10} \ mA$$

The equation for $I$ is then

$$I = \frac{6-V}{2} = 1.016 \times 10^{-10} \times e^{V/0.0295}$$

Solving iteratively, we get

$$V = 0.640 \ V \quad \text{and} \quad I = 2.68 \ mA$$
Example

Two diodes are connected in series as shown in the figure with $I_{s1} = 10^{-16}$ A and $I_{s2} = 10^{-14}$ A. If the applied voltage is 1 V, calculate the currents $I_{D1}$ and $I_{D2}$ and the voltage across each diode $V_{D1}$ and $V_{D2}$.

The diode equations can be written as:

$$I_{D1} = I_{s1} e^{V_{D1}/V_T} \quad I_{D2} = I_{s2} e^{V_{D2}/V_T} \quad \frac{I_{s1}}{I_{s2}} e^{V_{D1}/V_T} = \frac{I_{D1}}{I_{D2}} = 1$$

from which $V_{D1} - V_{D2} = -V_T \ln \left( \frac{I_{s1}}{I_{s2}} \right) = -0.12$

Using KVL, we get $V_{D1} + V_{D2} = 1$ from which $V_{D2} = 0.44$ V and $V_{D1} = 0.56$ V

$$I_{D1} = 10^{-16} e^{0.56/0.026} = 0.22 \ \mu A = I_{D2}$$
Small Signal Model

Approximation - valid for small fluctuations about bias point

\[ i_D = I_D e^{v_d/nV_T} \]

\[ r_d = \frac{1}{\frac{\partial i_D}{\partial v_D}} = \frac{nV_T}{I_D} \]

\[ i_D = I_D + i_d \]

Total DC applied (small)

\[ v_D = V_D + v_d \]
Diode Circuits

\[
V_{out} = V_D \\
I_D = I_S \left(e^{V_D/V_T} - 1\right) \\
V_S = RI_D + V_D = RI_D(V_D) + V_D
\]

Nonlinear transcendental system ➔ Use graphical method

Solution is found at intersection of load line characteristics and diode characteristics
Diode Circuits – Iterative Methods

Newton-Raphson Method

Wish to solve $f(x)=0$ for $x$

Use: $x_{k+1} = x_k - f'(x_k)^{-1} f(x_k)$

$x^{(k+1)} = x^{(k)} - f'(x^{(k)})^{-1} f(x^{(k)})$

$f(V_D) = \frac{V_D - V_S}{R} + I_S \left( e^{V_D/V_T} - 1 \right) = 0$

$f'(V_D) = \frac{1}{R} + \frac{I_S}{V_T} e^{V_D/V_T}$

\[
V_D^{(k+1)} = V_D^{(k)} - \frac{V_D^{(k)} - V_S}{R} + I_S \left( e^{V_D^{(k)}/V_T} - 1 \right) - \frac{1}{R} + \frac{I_S}{V_T} e^{V_D^{(k)}/V_T}
\]

Where $V_D^{(k)}$ is the value of $V_D$ at the $k$th iteration

Procedure is repeated until convergence to final (true) value of $V_D$ which is the solution. Rate of convergence is quadratic.