ECE 342
Electronic Circuits

Lecture 7
DC Analysis of MOSFET

Jose E. Schutt-Aine
Electrical & Computer Engineering
University of Illinois
jesa@illinois.edu
MOS – Active Region

- **Saturation**
  - Channel is pinched off
  - Increase in $V_{DS}$ has little effect on $i_D$
  - Square-law behavior wrt $(V_{GS} - V_T)$
  - Acts like a current source
Diode-Connected Transistor

When the drain and gate of a MOSFET are connected together the result is a two-terminal device known as a diode-connected transistor.

\[ V_{GD} \leq V_{T} \]

for saturation region. Since \( V_{GD} \) is zero, then the device is always in the saturation region.
Diode-Connected Transistor

\[ i_D = i = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2 \]

If we replace \( V_{GS} \) by \( V \) and use \( k' = \frac{1}{2} k_n' \)

\[ i = k' \frac{W}{L} (V - |V_t|)^2 \]

incremental resistance

\[ r = \left( \frac{\partial i}{\partial V} \right)^{-1} = \frac{1}{2k' \frac{W}{L} (V - |V_t|)} = \frac{1}{k_n \frac{W}{L} V_{ov}} \]

\[ V = |V_t| + V_{ov} \]
Example

An MOS process technology has $L_{\text{min}} = 0.4$ $\mu$m, $t_{ox} = 8$ nm, $\mu = 450$ cm$^2$/V.s, $V_T = 0.7$V

(a) Find $C_{ox}$ and $k_n' = \mu_n C_{ox}$

(b) $W/L = 8$ $\mu$m/0.8$\mu$m. Calculate $V_{GS}$, $V_{DS_{\text{min}}}$ for operation in saturation with $I_D = 100$ $\mu$A

(c) Find $V_{GS}$ for the device in (b) to operate as a 1 k$\Omega$ resistor for small $v_{DS}$
Example - Solution

\[
C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.45 \times 10^{-11}}{8 \times 10^{-9}} = 4.32 \times 10^{-3} \text{ F/m}^2 = 4.32 \text{ fF/\mu m}^2
\]

\[C_{ox} = 4.32 \text{ fF/\mu m}^2\]

\[k'_n = \mu_n C_{ox} = 450 \text{ cm}^2/\text{V.s} \times 4.32 \text{ fF/\mu m}^2 = 194 \mu A/V^2\]

For operation in saturation region

\[i_D = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_T)^2\]

\[100 = \frac{1}{2} \times 194 \times \frac{8}{0.8} (V_{GS} - 0.7)^2 \Rightarrow V_{GS} - 0.7 = 0.32 V \Rightarrow V_{GS} = 1.02 V\]

\[V_{DS_{min}} = V_{GS} - V_T = 0.32 V\]

\[V_{DS_{min}} = 0.32 V\]
Triode region with $v_{DS}$ very small

$$r_{DS} = \left. \frac{v_{DS}}{i_D} \right|_{\text{small } v_{DS}} = \frac{1}{k_n \frac{W}{L} (V_{GS} - V_T)}$$

$$100 = \frac{1}{194 \times 10^{-6} \times 10 (V_{GS} - 0.7)}$$

$$V_{GS} - 0.7 = 0.52 \text{ V}$$

$$V_{GS} = 1.22 \text{ V}$$
Body Effect

- **The body effect**
  - $V_T$ varies with bias between source and body
  - Leads to modulation of $V_T$

**Potential on substrate affects threshold voltage**

\[
V_T(V_{SB}) = V_{To} + \gamma \left[ \left( 2|\phi_F| + V_{SB} \right)^{1/2} - \left( 2|\phi_F| \right)^{1/2} \right]
\]

\[
|\phi_F| = \left( \frac{kT}{q} \right) \ln \left( \frac{N_a}{n_i} \right)
\]

Fermi potential of material

\[
\gamma = \left( \frac{2qN_a \varepsilon_s}{C_{ox}} \right)^{1/2}
\]

Body bias coefficient
With depletion layer widening, the channel length is in effect reduced from \( L \) to \( L - \Delta L \) \( \Rightarrow \) Channel-length modulation.

This leads to the following I-V relationship

\[
i_D = \frac{1}{2} k_n \frac{W}{L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})
\]

Where \( \lambda \) is a process technology parameter.
Channel-Length Modulation

Channel-length modulation causes $i_D$ to increase with $v_{DS}$ in saturation region.
Problem

A MOSFET has $V_T = 1$ V with measured data:

<table>
<thead>
<tr>
<th>$V_{GS}(V)$</th>
<th>$V_{DS}(V)$</th>
<th>$I_D(\mu A)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>80</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>91</td>
</tr>
</tbody>
</table>

Find $\lambda$

(a) $V_{GS} > V_T \quad V_{DS} = V_{GS} - V_T \Rightarrow Pinchoff$

(b) $V_{GS} > V_T \quad V_{DS} > V_{GS} - V_T = 1V \Rightarrow Active\ region$
Problem (cont’)

Find $i_D$ at pinchoff $V_{DSP} = V_{GS} - V_T = 1\, \text{V}$

$$I_D = \frac{1}{2} k_n \frac{W}{L} (V_{GS} - V_T)^2 \left(1 + \lambda V_{DS}\right)$$

$$I_{D1} = \frac{1}{2} k_n \frac{W}{L} (V_{GS1} - V_T)^2 \left(1 + \lambda V_{DS1}\right)$$

$$I_{D2} = \frac{1}{2} k_n \frac{W}{L} (V_{GS2} - V_T)^2 \left(1 + \lambda V_{DS2}\right)$$
Problem (cont’)

\[
R = \frac{1 + \lambda V_{DS1}}{1 + \lambda V_{DS2}} = \frac{91}{80} = 1.1375
\]

\[
1 + \lambda V_{DS2} = R + R\lambda V_{DS1}
\]

\[
\lambda (V_{DS2} - RV_{DS1}) = R - 1
\]
NMOS – IV Characteristics

characteristics for a device with \( k'_n (W/L) = 1.0 \text{ mA/V}^2 \).
NMOS IV Curves
The MOSFET in the circuit shown has $V_t = 1V$, $k_n' = 100 \mu A/V^2$ and $\lambda = 0$. Find the required values of $W/L$ and of $R$ so that when $v_I = V_{DD} = +5 V$, $r_{DS} = 50 \Omega$ and $v_o = 50 mV$.

\[ v_I = V_{GS} = 5 V, \quad v_o = V_{DS} = 0.05 V \]

\[ r_{DS} = 50 \Omega = \frac{V_{DS}}{I_D} \Rightarrow I_D = \frac{0.05}{50} = 0.001 A = 1 mA \]

\[ R = \frac{V_{DD} - v_o}{I_D} = \frac{5 - 0.05}{1} = 4.95 k\Omega \]
MOSFET Circuit at DC – Problem 1 (cont’)

\[ V_{DS} < V_{GS} - V_t \implies \text{triode region} \]

\[ I_D = k'_n \frac{W}{L} \left[ (V_{GS} - V_t)V_{DS} - \frac{V_{DS}^2}{2} \right] \]

\[ 1 = 100 \times 10^{-3} \frac{W}{L} \left[ (5 - 1) \times 0.05 - \frac{0.05^2}{2} \right] \]

\[ \frac{W}{L} = 50 \]
The NMOS transistors in the circuit shown have $V_t = 1\text{V}$, $\mu_nC_{ox} = 120\mu\text{A/V}^2$, $\lambda = 0$ and $L_1=L_2=1\mu\text{m}$. Find the required values of gate width for each of $Q_1$ and $Q_2$ and the value of $R$, to obtain the voltage and current values indicated.

$V_{GS1} = 1.5\text{V}$

Using $I_D = \frac{1}{2}k_n\frac{W}{L}(V_{GS} - V_t)^2$

$120\ \mu\text{A} = \frac{1}{2} \times 120 \times \frac{W_1}{1} (1.5 - 1)^2 \Rightarrow W_1 = 8\mu\text{m}$
MOSFET Circuit at DC – Problem 2

Using \( I_D = \frac{1}{2} k_n \frac{W}{L} (V_{GS} - V_t)^2 \)

\( V_{GS2} = 3.5 - 1.5 = 2V \)

\[ 120 \mu A = \frac{1}{2} \times 120 \frac{W^2}{1} (2 - 1)^2 \Rightarrow W_2 = 2 \mu m \]

\[ R = \frac{5 - 3.5}{0.120} = 12.5 \text{ k}\Omega \]