ECE 342
Electronic Circuits

Lecture 7
DC Analysis of MOSFET

Jose E. Schutt-Aine
Electrical & Computer Engineering
University of Illinois
jesa@Illinois.edu
MOS – Active Region

• **Saturation**
  – Channel is pinched off
  – Increase in $V_{DS}$ has little effect on $i_D$
  – Square-law behavior wrt $(V_{GS} - V_T)$
  – Acts like a current source
Diode-Connected Transistor

When the drain and gate of a MOSFET are connected together the result is a two-terminal device known as a diode-connected transistor

\[ V_{GD} \leq V_T \]

for saturation region. Since \( V_{GD} \) is zero, then the device is always in the saturation region.
Diode-Connected Transistor

\[ i_D = i = \frac{1}{2} k_n \frac{W}{L} (V_{GS} - V_t)^2 \]

If we replace \( V_{GS} \) by \( V \) and use \( k' = \frac{1}{2} k_n \)

\[ i = k' \frac{W}{L} (V - |V_t|)^2 \]

incremental resistance

\[ r = \left( \frac{\partial i}{\partial V} \right)^{-1} = \frac{1}{2k' \frac{W}{L} (V - |V_t|)} = \frac{1}{k_n \frac{W}{L} V_{ov}} \]

\[ V = |V_t| + V_{ov} \]
Example

An MOS process technology has $L_{min} = 0.4 \ \mu m$, $t_{ox} = 8 \ \text{nm}$, $\mu = 450 \ \text{cm}^2/\text{V.s}$, $V_T = 0.7V$

(a) Find $C_{ox}$ and $k_n' = \mu_n C_{ox}$

(b) $W/L = 8 \ \mu m/0.8 \ \mu m$. Calculate $V_{GS}$, $V_{DSmin}$ for operation in saturation with $I_D = 100 \ \mu A$

(c) Find $V_{GS}$ for the device in (b) to operate as a 1 k$\Omega$ resistor for small $V_{DS}$
Example - Solution

\[ C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.45 \times 10^{-11}}{8 \times 10^{-9}} = 4.32 \times 10^{-3} \text{ F/m}^2 = 4.32 \text{ fF/\mu m}^2 \]

\[ C_{ox} = 4.32 \text{ fF/\mu m}^2 \]

\[ k'_n = \mu_n C_{ox} = 450 \text{ cm}^2/\text{V.s} \times 4.32 \text{ fF/\mu m}^2 = 194 \text{ \mu A/V}^2 \]

For operation in saturation region

\[ i_D = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_T)^2 \]

\[ 100 = \frac{1}{2} \times 194 \times \frac{8}{0.8} (V_{GS} - 0.7)^2 \Rightarrow V_{GS} - 0.7 = 0.32 \text{ V} \Rightarrow V_{GS} = 1.02 \text{ V} \]

\[ V_{DS_{\text{min}}} = V_{GS} - V_T = 0.32 \text{ V} \]

\[ V_{DS_{\text{min}}} = 0.32 \text{ V} \]
Example – (con’t)

Triode region with \( v_{DS} \) very small

\[
r_{DS} = \frac{v_{DS}}{i_D} \bigg|_{\text{small } v_{DS}} = \frac{1}{k' \frac{W}{L} (V_{GS} - V_T)}
\]

1000 = \[
\frac{1}{194 \times 10^{-6} \times 10 (V_{GS} - 0.7)}
\]

\[V_{GS} - 0.7 = 0.52 \, V\]

\[V_{GS} = 1.22 \, V\]
Body Effect

• The body effect
  – $V_T$ varies with bias between source and body
  – Leads to modulation of $V_T$

Potential on substrate affects threshold voltage

$$V_T (V_{SB}) = V_{To} + \gamma \left[ (2|\phi_F| + V_{SB})^{1/2} - (2|\phi_F|)^{1/2} \right]$$

$$|\phi_F| = \left( \frac{kT}{q} \right) \ln \left( \frac{N^a}{n_i} \right)$$

Fermi potential of material

$$\gamma = \left( \frac{2qN^a \varepsilon_s}{C_{ox}} \right)^{1/2}$$

Body bias coefficient
With depletion layer widening, the channel length is in effect reduced from \( L \) to \( L-\Delta L \) \( \Rightarrow \) Channel-length modulation

This leads to the following I-V relationship

\[
i_D = \frac{1}{2} k_n \frac{W}{L} \left( V_{GS} - V_T \right)^2 \left( 1 + \lambda V_{DS} \right)
\]

Where \( \lambda \) is a process technology parameter.
Channel-Length Modulation

Channel-length modulation causes $i_D$ to increase with $v_{DS}$ in saturation region
Problem

A MOSFET has $V_T = 1$ V with measured data:

<table>
<thead>
<tr>
<th>$V_{GS}(V)$</th>
<th>$V_{DS}(V)$</th>
<th>$I_D(\mu A)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>80</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>91</td>
</tr>
</tbody>
</table>

Find $\lambda$

(a) $V_{GS} > V_T \quad V_{DS} = V_{GS} - V_T \implies Pinchoff$

(b) $V_{GS} > V_T \quad V_{DS} > V_{GS} - V_T = 1$ V $\implies Active$ region
Problem (cont’)

Find $i_D$ at pinchoff $V_{DSP} = V_{GS} - V_T = 1V$

\[ I_D = \frac{1}{2} k_n \frac{W}{L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) \]

\[ I_{D1} = \frac{1}{2} k_n \frac{W}{L} (V_{GS1} - V_T)^2 (1 + \lambda V_{DS1}) \]

\[ I_{D2} = \frac{1}{2} k_n \frac{W}{L} (V_{GS2} - V_T)^2 (1 + \lambda V_{DS2}) \]
Problem (cont’)

\[ R = \frac{1 + \lambda V_{DS1}}{1 + \lambda V_{DS2}} = \frac{91}{80} = 1.1375 \]

\[ 1 + \lambda V_{DS2} = R + R\lambda V_{DS1} \]

\[ \lambda (V_{DS2} - RV_{DS1}) = R - 1 \]

\[ \lambda = \frac{R - 1}{V_{DS2} - RV_{DS1}} = \frac{1.1375 - 1}{8 - 1.1375} = 0.02 V^{-1} \]
NMOS – IV Characteristics

characteristics for a device with $k'_n (W/L) = 1.0 \text{ mA/V}^2$. 

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NMOS IV Curves

$i_D$ vs $V_{DS}$

$V_{GS} = \ldots$

$Q_1$

$Q_2$

$V_{DD}$
MOSFET Circuit at DC – Problem 1

The MOSFET in the circuit shown has $V_t = 1\text{V}$, $k_n' = 100 \mu\text{A/V}^2$ and $\lambda = 0$. Find the required values of $W/L$ and of $R$ so that when $v_I = V_{DD} = +5\text{ V}$, $r_{DS} = 50\ \Omega$ and $v_o = 50\ \text{mV}$.

$v_I = V_{GS} = 5\text{ V}$, $v_o = V_{DS} = 0.05\text{ V}$

$r_{DS} = 50\ \Omega = \frac{V_{DS}}{I_D} \implies I_D = \frac{0.05}{50} = 0.001\ A = 1\ \text{mA}$

$R = \frac{V_{DD} - v_o}{I_D} = \frac{5 - 0.05}{1} = 4.95\ k\Omega$
MOSFET Circuit at DC – Problem 1 (cont’)

\[ V_{DS} < V_{GS} - V_t \Rightarrow \text{triode region} \]

\[ I_D = k_n' \frac{W}{L} \left[ (V_{GS} - V_t) V_{DS} - \frac{V_{DS}^2}{2} \right] \]

\[ 1 = 100 \times 10^{-3} \frac{W}{L} \left[ (5 - 1) \times 0.05 - \frac{0.05^2}{2} \right] \]

\[ \frac{W}{L} = 50 \]
The NMOS transistors in the circuit shown have $V_t = 1V$, $\mu_nC_{ox} = 120\mu A/V^2$, $\lambda = 0$ and $L_1=L_2=1\mu m$. Find the required values of gate width for each of $Q_1$ and $Q_2$ and the value of $R$, to obtain the voltage and current values indicated.

$V_{GS1} = 1.5V$

Using $I_D = \frac{1}{2}k_n\frac{W}{L}(V_{GS} - V_t)^2$

$120 \mu A = \frac{1}{2} \times 120 \frac{W_1}{1} (1.5 - 1)^2 \Rightarrow W_1 = 8 \mu m$
MOSFET Circuit at DC – Problem 2

Using $I_D = \frac{1}{2} k_n \frac{W}{L} (V_{GS} - V_t)^2$

$V_{GS2} = 3.5 - 1.5 = 2V$

$120 \mu A = \frac{1}{2} \times 120 \frac{W^2}{1} (2 - 1)^2 \Rightarrow W_2 = 2 \mu m$

$R = \frac{5 - 3.5}{0.120} = 12.5 \ k\Omega$