ECE 342
Electronic Circuits

Lecture 8
MOSFET Small Signal Model

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Biasing of MOS Transistors

• **Bias Characteristics**
  – Operation in saturation region
  – Stable and predictable drain current

\[
I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)^2
\]
Common Source MOSFET Amplifier

Small-Signal Equivalent Circuit for MOS (device only)

\[ I_D = \frac{1}{2} k_n \frac{W}{L} (V_{GS} - V_T)^2 \]

\[ g_m = \frac{\partial I_D}{\partial V_{GS}} \bigg|_{V_{GS}=V_{GSQ}} = \frac{2I_D}{V_{eff}} \]

where \( V_{GS} - V_T = V_{eff} \)

Which leads to

\[ g_m = \sqrt{2k_n'} \sqrt{W/L} \sqrt{I_D} \]

\( g_m \) is proportional to \( \sqrt{W/L} \)
MOSFET Output Impedance

To calculate $r_{ds}$, account for $\lambda$

$$r_{ds} = \left. \frac{\partial V_{DS}}{\partial I_D} \right|_{V_{GS}=V_{GSQ}} = \frac{1}{\lambda \mu \frac{W}{2L} C_{ox} [V_{GS} - V_T]^2} = \frac{1}{\lambda I_{DP}}$$

$$I_{DP} = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_T)^2$$

$r_{ds}$, accounts for channel width modulation resistance.
Body Effect

• The body effect
  – $V_T$ varies with bias between source and body
  – Leads to modulation of $V_T$

Potential on substrate affects threshold voltage

$$V_T(V_{SB}) = V_{To} + \gamma \left[ \left(2|\phi_F| + V_{SB}\right)^{1/2} - \left(2|\phi_F|\right)^{1/2} \right]$$

$$|\phi_F| = \left(\frac{kT}{q}\right) \ln \left(\frac{N_a}{n_i}\right)$$

Fermi potential of material

$$\gamma = \left(\frac{2qN_a\varepsilon_s}{C_{ox}}\right)^{1/2}$$

Body bias coefficient
Body Effect – (Con’t)

Define \( g_{mb} \) as the body transconductance

\[
g_{mb} = \left. \frac{\partial I_D}{\partial V_{BS}} \right|_{V_{GS}=\text{constant}} \left. \frac{V_{GS}}{V_{DS}=\text{constant}} \right.
\]

Can show that \( g_{mb} = \chi g_m \)

\[
\text{where } \chi = \frac{\partial V_T}{\partial V_{SB}} = \frac{\gamma}{2\sqrt{\phi_F + V_{SB}}}
\]