ECE 342
Electronic Circuits

Lecture 21
Transfer Functions - 2

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Transfer Function Representation

In general, the transfer function of an amplifier can be expressed as

$$F_H(s) = a_m \frac{(s - Z_1)(s - Z_2)\ldots(s - Z_m)}{(s - P_1)(s - P_2)\ldots(s - P_m)}$$

$Z_1, Z_2, \ldots Z_m$ are the **zeros** of the transfer function

$P_1, P_2, \ldots P_m$ are the **poles** of the transfer function

s is a complex number $s = \sigma + j\omega$

3dB Frequency Determination

$$A(s) \equiv A_M F_H(s)$$

- Designer is interested in midband operation
- However needs to know upper 3-dB frequency
- In many cases some conditions are met:
  - Zeros are infinity or at very high frequencies
  - One of the poles (ω_{P1}) is at much lower frequency than other poles (➔dominant pole)
- If the conditions are met then $F_H(s)$ can be approximated by:

$$F_H(s) \equiv \frac{1}{1 + s/\omega_{P1}} \quad \text{and we have} \quad \omega_H \cong \omega_{P1}$$

3dB Frequency Determination

If the lowest frequency pole is at least 4 times away from the nearest pole or zero, it is a **dominant pole**

If there is no dominant pole, the 3-dB frequency ω_H can be approximated by:

$$\omega_H \cong 1/\sqrt{\left(\frac{1}{\omega_{P1}^2}+\frac{1}{\omega_{P2}^2}+\ldots\right)-2\left(\frac{1}{\omega_{Z1}^2}+\frac{1}{\omega_{Z2}^2}+\ldots\right)}$$

Open-Circuit Time Constants

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \ldots + a_n s^n}{1 + b_1 s + b_2 s^2 + \ldots b_n s^n}$$

The coefficients *a* and *b* are related to the frequencies of the zeros and poles respectively.

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \ldots + \frac{1}{\omega_{pn}}$$

b_1 can be obtained by summing the individual time constants of the circuit using the *open-circuit time constant method*

Open-Circuit Time Constant Method

- The time constant of each capacitor in the circuit is evaluated. It is the product of the capacitance and the resistance seen across its terminals with:
  - All other internal capacitors open circuited
  - All independent voltage sources short circuited
  - All independent current sources opened

- The value of b_1 is computed by summing the individual time constants

$$b_1 = \sum_{i=1}^{n} C_i R_{io}$$

Open-Circuit Time Constant Method

- **An approximation can be made by using the value of b_1 to determine the 3dB upper frequency point ω_H**
- **If the zeros are not dominant and if one of the poles P1 is dominant, then**

$$b_1 \simeq \frac{1}{\omega_{P1}}$$

Assuming that the 3-dB frequency will be approximately equal to ω_{P1}

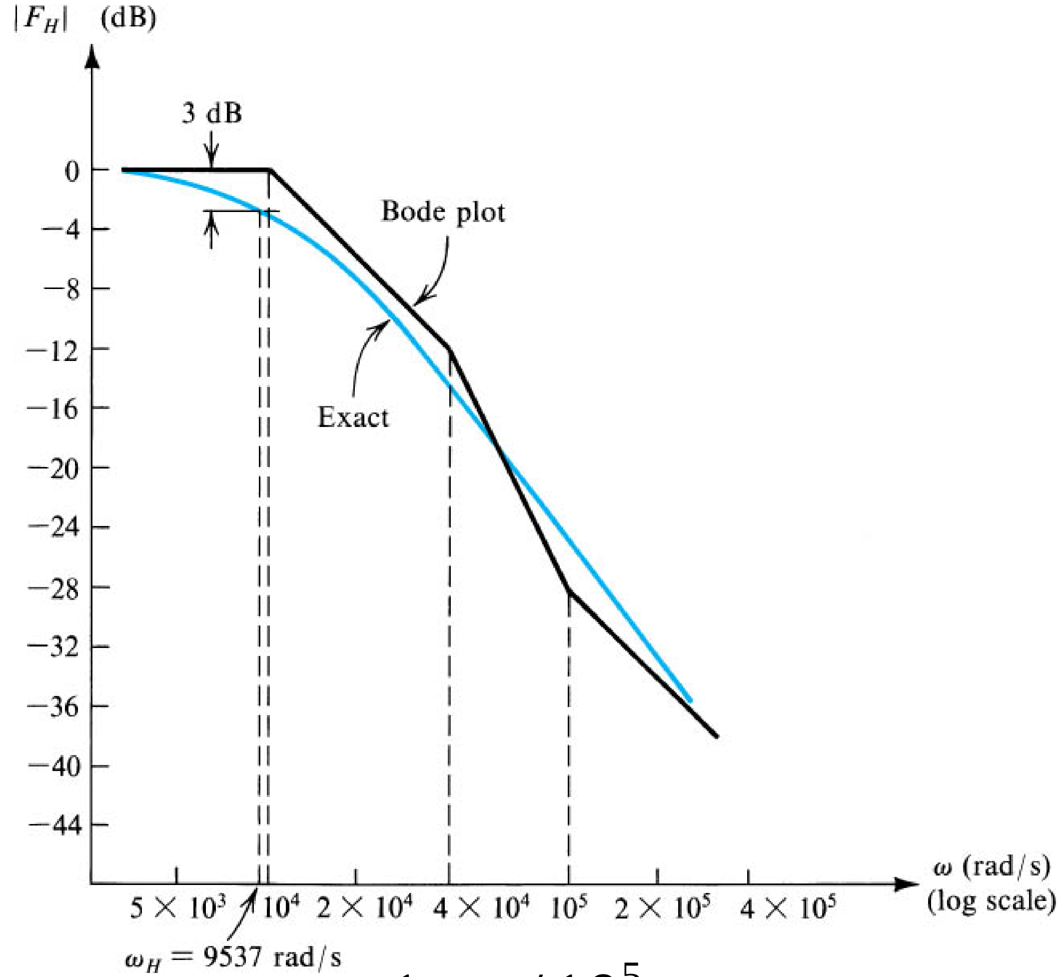
$$\omega_H \simeq \frac{1}{b_1} = \frac{1}{\sum_i C_i R_{io}}$$

Bandwidth of Multistage Amplifier

- The poles of a multistage amplifier are difficult to obtain analytically

- An approximate value for the 3dB upper frequency point ω_{3dB} can be obtained by assigning an open circuit time constant τ_{io} to each capacitor C_i

High-Frequency Behavior - Example



$$F_H(s) \equiv \frac{1 - s/10^5}{\left(1 + s/10^4\right)\left(1 + s/4\times10^4\right)}$$