ECE 342
Electronic Circuits

Lecture 21
Transfer Functions - 2

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Transfer Function Representation

In general, the transfer function of an amplifier can be expressed as

\[ F_H(s) = a_m \frac{(s - Z_1)(s - Z_2) \ldots (s - Z_m)}{(s - P_1)(s - P_2) \ldots (s - P_m)} \]

\( Z_1, Z_2, \ldots Z_m \) are the zeros of the transfer function

\( P_1, P_2, \ldots P_m \) are the poles of the transfer function

\( s \) is a complex number \( s = \sigma + j\omega \)
3dB Frequency Determination

\[ A(s) \equiv A_M F_H(s) \]

• Designer is interested in midband operation
• However needs to know upper 3-dB frequency
• In many cases some conditions are met:
  - Zeros are infinity or at very high frequencies
  - One of the poles \( (\omega_{P1}) \) is at much lower frequency than other poles (dominant pole)
• If the conditions are met then \( F_H(s) \) can be approximated by:

\[ F_H(s) \equiv \frac{1}{1 + s / \omega_{P1}} \quad \text{and we have} \quad \omega_H \simeq \omega_{P1} \]
3dB Frequency Determination

If the lowest frequency pole is at least 4 times away from the nearest pole or zero, it is a dominant pole.

If there is no dominant pole, the 3-dB frequency $\omega_H$ can be approximated by:

$$
\omega_H \cong 1/ \sqrt{\left(1/\omega_{P1}^2 + 1/\omega_{P2}^2 + ... \right) - 2 \left(1/\omega_{Z1}^2 + 1/\omega_{Z2}^2 + ... \right)}
$$
Open-Circuit Time Constants

\[ F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \ldots + a_n s^n}{1 + b_1 s + b_2 s^2 + \ldots b_n s^n} \]

The coefficients \( a \) and \( b \) are related to the frequencies of the zeros and poles respectively.

\[ b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \ldots + \frac{1}{\omega_{pn}} \]

\( b_1 \) can be obtained by summing the individual time constants of the circuit using the open-circuit time constant method.
Open-Circuit Time Constant Method

• The time constant of each capacitor in the circuit is evaluated. It is the product of the capacitance and the resistance seen across its terminals with:
  - All other internal capacitors open circuited
  - All independent voltage sources short circuited
  - All independent current sources opened

• The value of $b_1$ is computed by summing the individual time constants

$$b_1 = \sum_{i=1}^{n} C_i R_{io}$$
Open-Circuit Time Constant Method

• An approximation can be made by using the value of $b_1$ to determine the 3dB upper frequency point $\omega_H$

• If the zeros are not dominant and if one of the poles $P_1$ is dominant, then

$$b_1 \approx \frac{1}{\omega_{P_1}}$$

Assuming that the 3-dB frequency will be approximately equal to $\omega_{P_1}$

$$\omega_H \approx \frac{1}{b_1} = \sum_i \frac{1}{C_i R_{io}}$$
Bandwidth of Multistage Amplifier

• The poles of a multistage amplifier are difficult to obtain analytically

• An approximate value for the 3dB upper frequency point $\omega_{3dB}$ can be obtained by assigning an open circuit time constant $\tau_{io}$ to each capacitor $C_i$
High-Frequency Behavior - Example

\[ F_H(s) \equiv \frac{1 - s/10^5}{(1 + s/10^4)(1 + s/4 \times 10^4)} \]