ECE 342 Electronic Circuits

Lecture 23 Miller Effect

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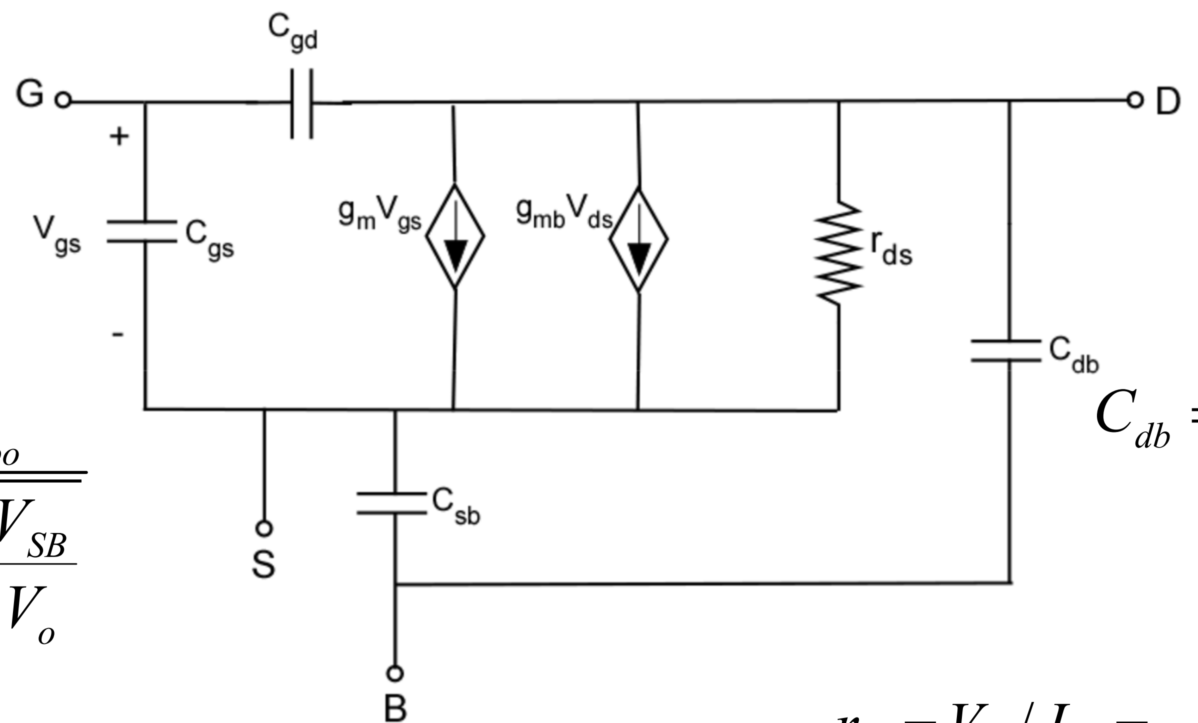
MOSFET High-Frequency Model



$$C_{sb} = \frac{C_{sbo}}{\sqrt{1+\frac{V_{SB}}{V_o}}}$$

$$C_{db} = \frac{C_{dbo}}{\sqrt{1+\frac{V_{DB}}{V_o}}}$$

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{eff} = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \frac{2I_D}{V_{eff}}$$

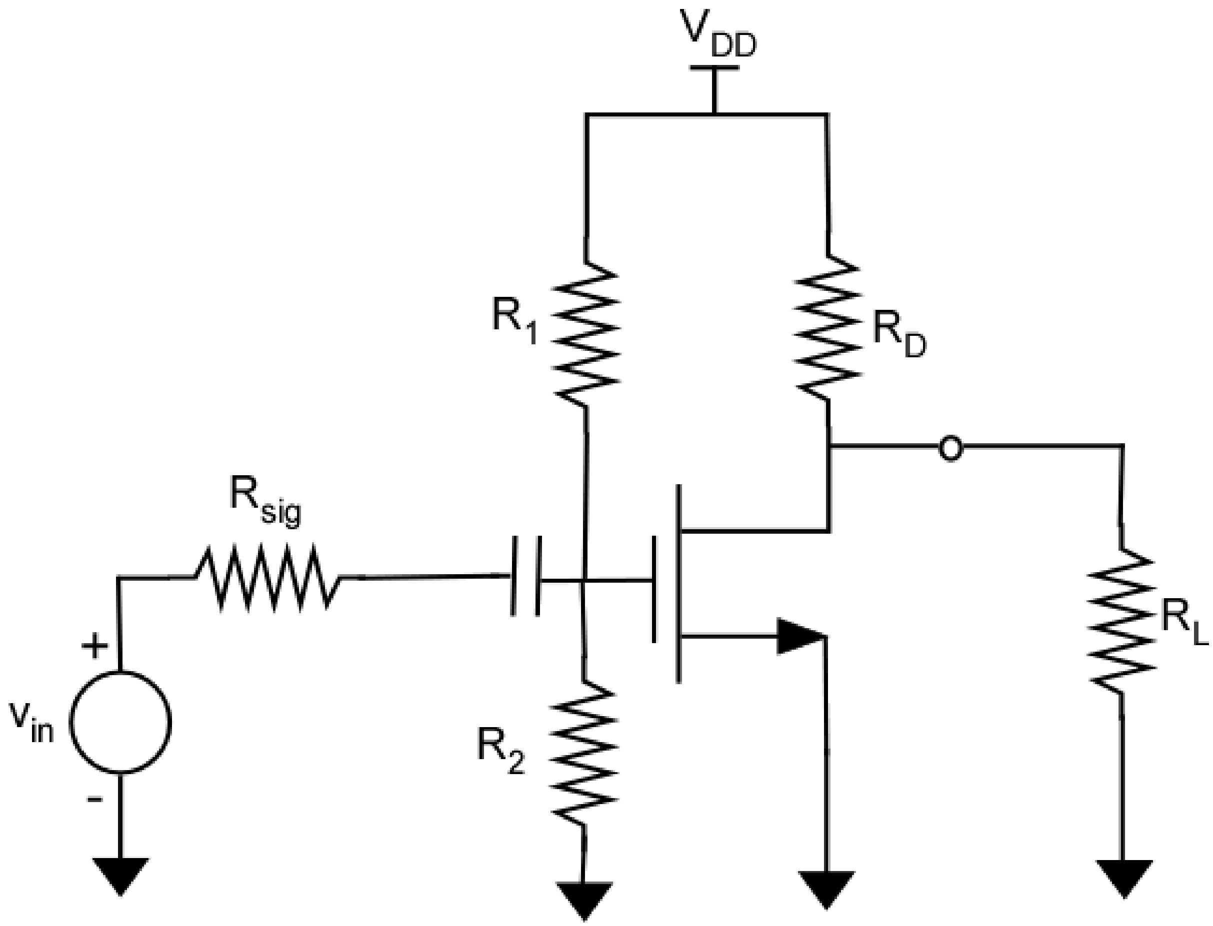
$$g_{mb} = \chi g_m = \frac{\gamma}{2\sqrt{2\phi_F + V_{sb}}} g_m$$

$$r_{ds} = V_A / I_D = \frac{1}{\lambda I_D}$$

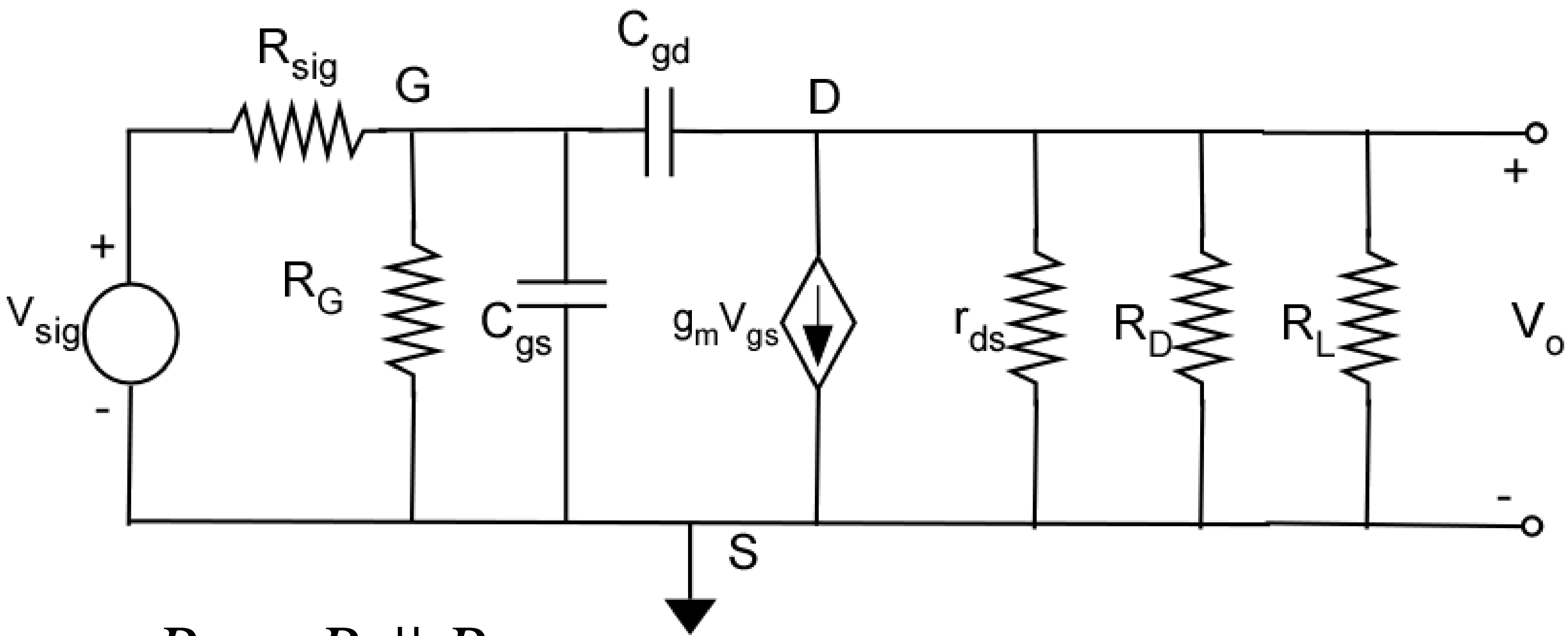
$$C_{gs} = \frac{2}{3} WLC_{ox} + WL_{ov}C_{ox}$$

$$C_{gd} = WL_{ov}C_{ox}$$

CS - High-Frequency Response



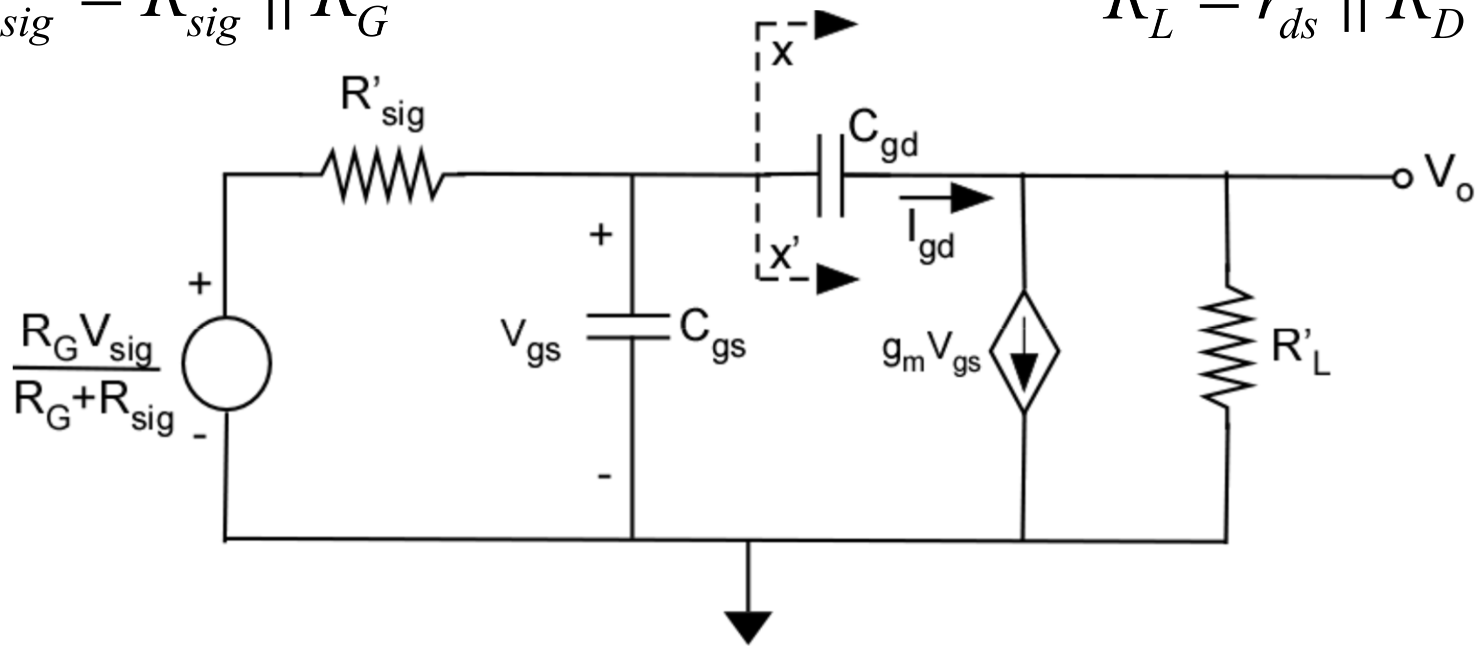
CS - High-Frequency Response



$$R_G = R_1 \parallel R_2$$

CS - High-Frequency Response

$$R_{sig}^{'} = R_{sig} \,||\, R_G \qquad R_L^{'} = r_{ds} \,||\, R_D \,||\, R_L$$



$$I_{gd} = sC_{gd}\left(V_{gs} - V_o\right) = sC_{gd}\left[V_{gs} - \left(-g_m R_L^{'} V_{gs}\right)\right]$$

$$I_{gd} = sC_{gd}\left(1 + g_m R_L^{'}\right)V_{gs}$$

CS – Miller Effect

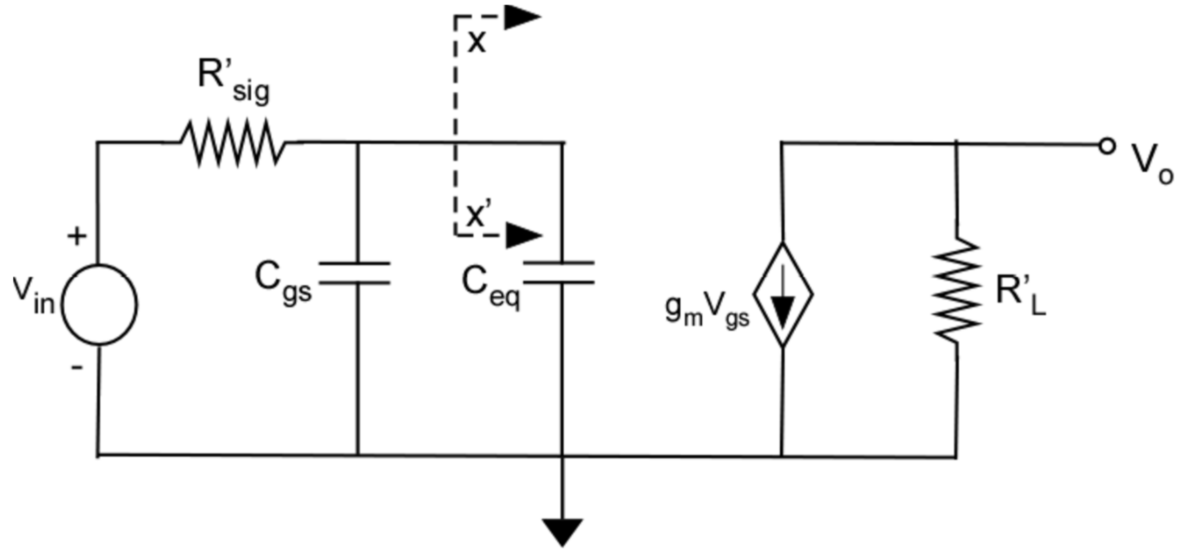
Define C_{eq} such that

$$sC_{eq}V_{gs} = sC_{gd}\left(1 + g_m R_L^{'}\right)V_{gs}$$

$$V_o = -g_m R_L^{'} V_{gs}$$

$$C_{eq} = C_{gd}\left(1 + g_m R_L^{'}\right) = \text{Miller Capacitance}$$

$$v_{in} = \frac{R_g V_{sig}}{R_g + R_{sig}}$$



CS – Miller Effect

$$V_{gs} = \left(\frac{R_G V_{sig}}{R_G + R_{sig}} \right) \frac{1}{1 + jf / f_o}$$

f_o is the corner frequency of the STC circuit

$$f_o = \frac{1}{2\pi C_{in} R_{sig}^{'}}$$

$$C_{in} = C_{gs} + C_{eq} = C_{gs} + \overbrace{C_{gd}\left(1 + g_m R_L^{'}\right)}^{Miller}$$

CS – Miller Effect

$$\frac{V_o}{V_{sig}} = -\left(\frac{R_G}{R_G + R_{sig}}\right) g_m R_L^{'} \frac{1}{1 + jf / f_o}$$

$$\frac{V_o}{V_{sig}} = \frac{A_M}{1 + jf / f_H}$$

$$f_H = f_o = \frac{1}{2\pi C_{in} R_{sig}^{'}}$$

Example

R_{sig} = 100 kΩ, R_G=4.7 MΩ, R_D =15 kΩ, g_m=1mA/V, r_{ds}=150 kΩ, R_L=10 kΩ, C_{gs}=1 pF and C_{gd}=0.4 pF

$$R_L^{'} = r_{ds} \,||\, R_D \,||\, R_L = 150 \,||\, 15 \,||\, 15 = 7.14\ k\Omega$$

$$A_M = -\frac{R_G}{R_G + R_{sig}} g_m R_L^{'} = -\frac{4.7}{4.7 + 0.1} \times 1 \times 7.14 = -7$$

$$Miller\ Cap : C_{eq} = C_M = \left(1 + g_m R_L^{'}\right) C_{gd}$$

$$C_M = 0.4 \times \left(1 + 7.14\right) = 3.26\ pF$$

Example (cont')

$$C_{in} = 1.0 + 3.26 = 4.26\ pF$$

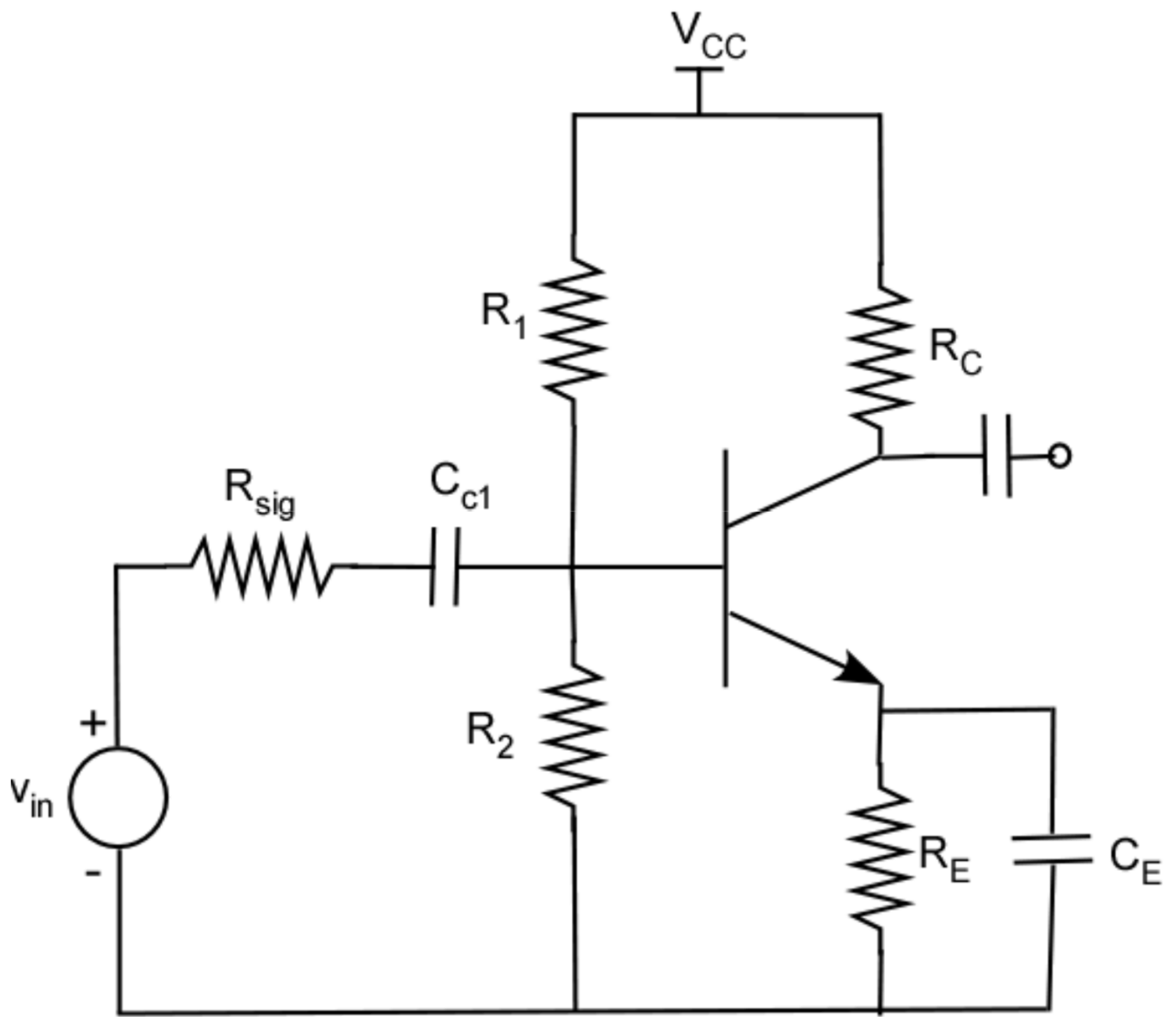
Upper 3 dB frequency is at:

$$f_H = \frac{1}{2\pi C_{in}\left(R_{sig} \,||\, R_G\right)}$$

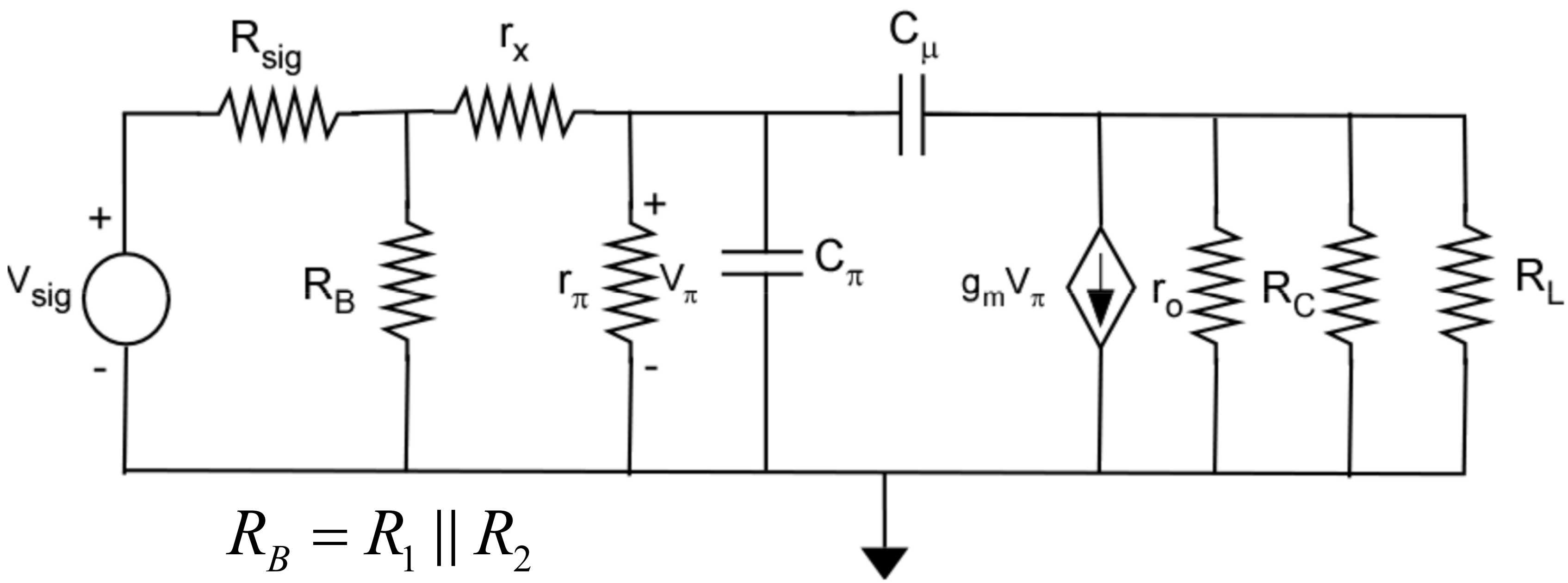
$$f_H = \frac{1}{2\pi \times 4.26 \times 10^{-12} \times \left(0.1 \,||\, 4.7\right) \times 10^{6}} = 3.82\ kHz$$

$$f_H = 3.82\ kHz$$

CE High-Frequency Model

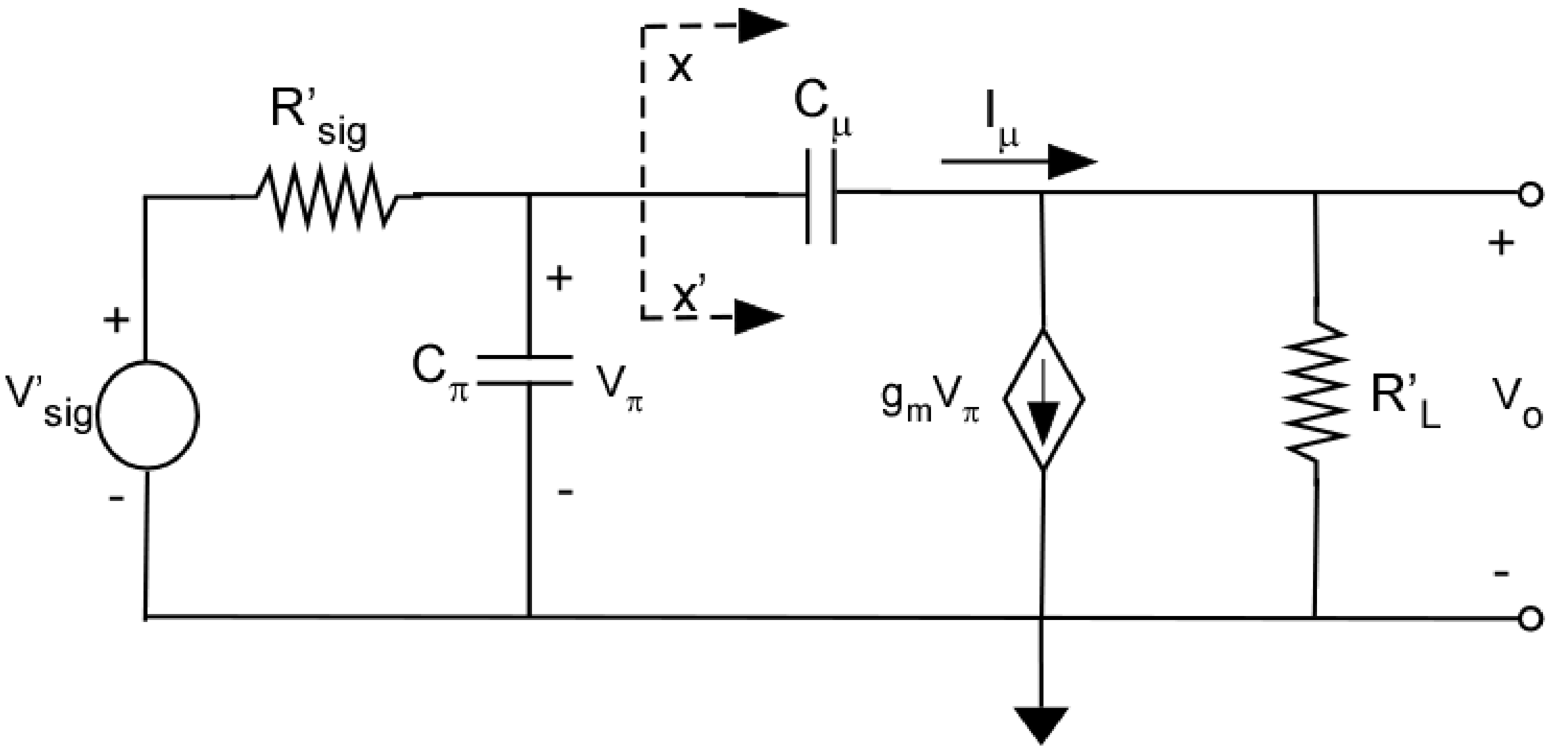


CE High-Frequency Model



$$R_B = R_1 \| R_2$$

CE High-Frequency Model



$$V_{sig}^{'} = V_{sig} \cdot \frac{R_B}{R_B + R_{sig}} \cdot \frac{r_\pi}{r_\pi + r_x + \left(R_{sig} \parallel R_B\right)}$$

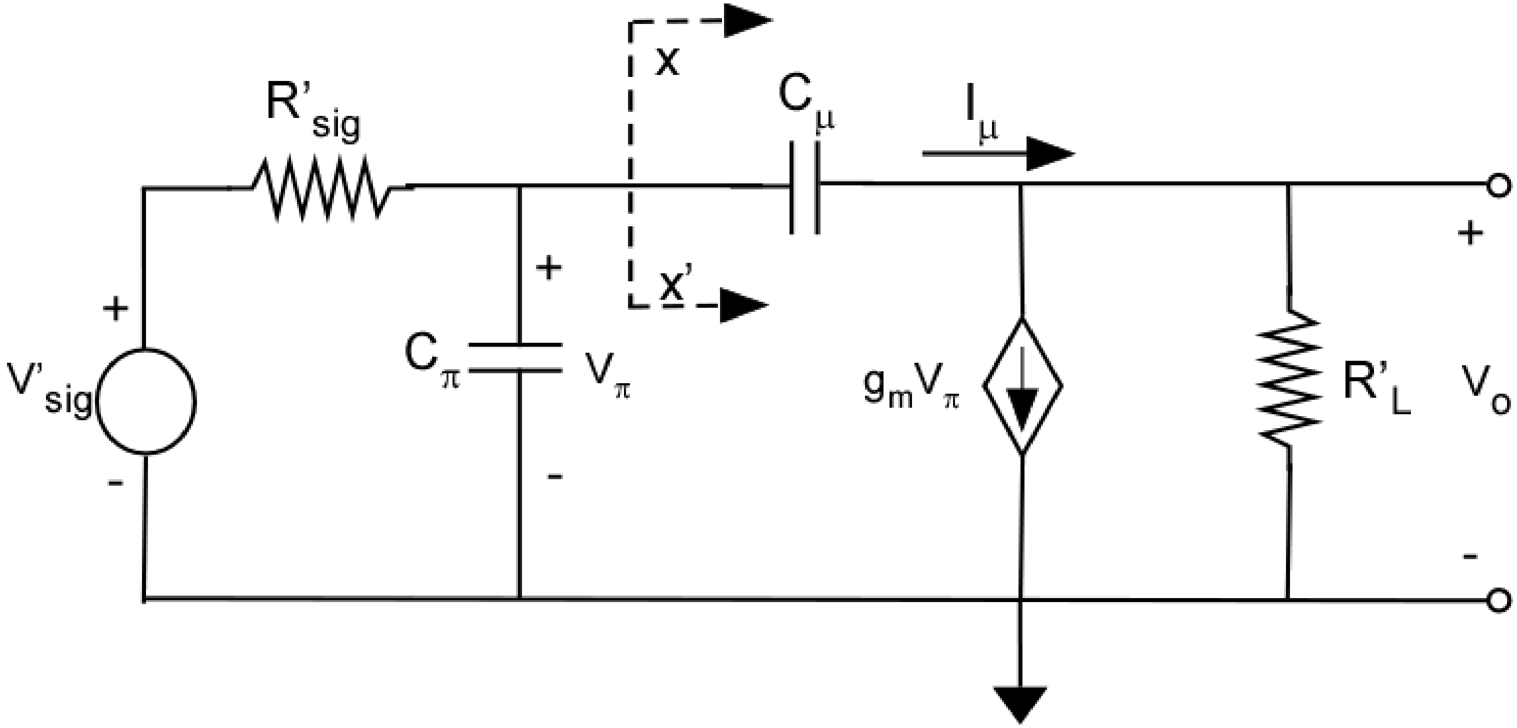
CE High-Frequency Model

$$R_{sig}^{'} = r_\pi \, || \left[r_x + \left(R_{sig} \, || \, R_B \right) \right]$$

$$R_L^{'} = r_o \, || \, R_C \, || \, R_L$$

$$V_o \simeq -g_m v_\pi R_L^{'}$$

Bipolar Miller Effect



The left hand side of the circuit at XX' knows the existence of C_μ only through the current I_μ ➔ replace C_μ with C_{eq} from base to ground

Bipolar Miller Effect

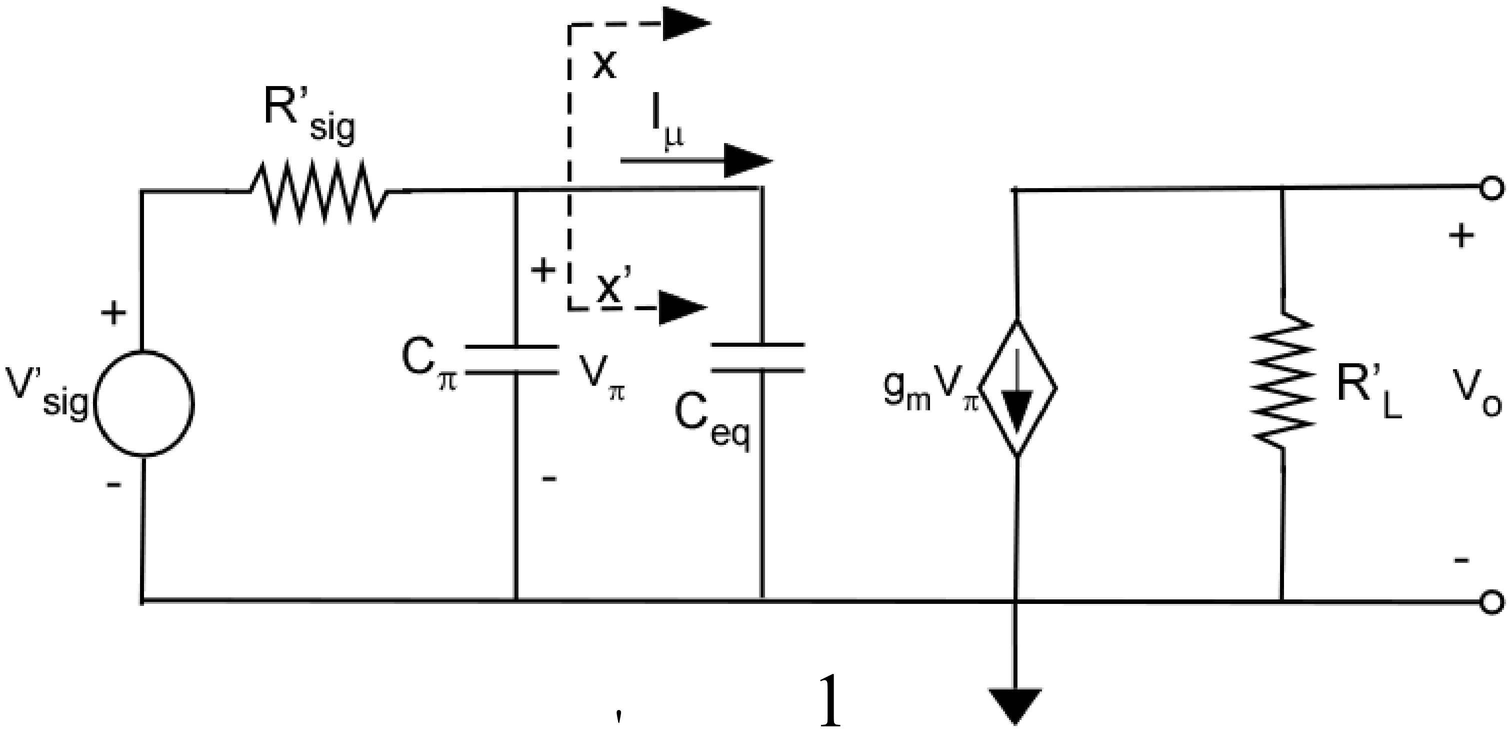
$$I_\mu = sC_\mu\left(v_\pi - v_o\right) = sC_\mu\left[v_\pi - \left(-g_m R_L^{'} v_\pi\right)\right]$$

$$I_\mu = sC_\mu\left(1 + g_m R_L^{'}\right)v_\pi$$

$$sC_{eq}v_\pi = I_\mu = sC_\mu\left(1 + g_m R_L^{'}\right)v_\pi$$

$$C_{eq} = C_\mu\left(1 + g_m R_L^{'}\right), \text{ Miller capacitance for BJT}$$

Bipolar Miller Effect



$$v_{\pi} = v_{sig}^{'} \frac{1}{1 + jf / f_o}$$

$$f_o = \frac{1}{2\pi C_{in} R_{sig}^{'}}$$

Bipolar Miller Effect (cont')

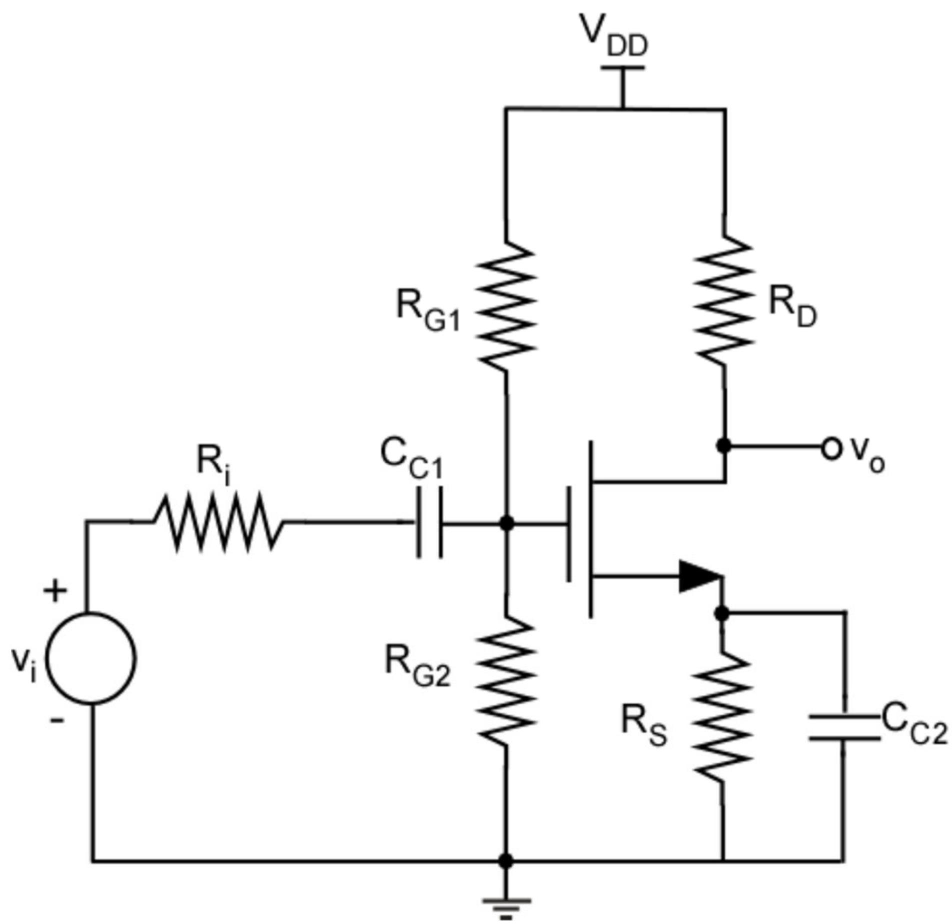
$$where\ C_{in} = C_\pi + C_{eq} = C_\pi + C_\mu\left(1 + g_m R_L^{'}\right)$$

$$\frac{V_o}{V_{sig}} = \left[\frac{R_B}{R_B + R_{sig}} \cdot \frac{r_\pi g_m R_L^{'}}{r_\pi + r_x + \left(R_{sig} \| R_B\right)}\right]\left[\frac{1}{1 + jf/f_o}\right]$$

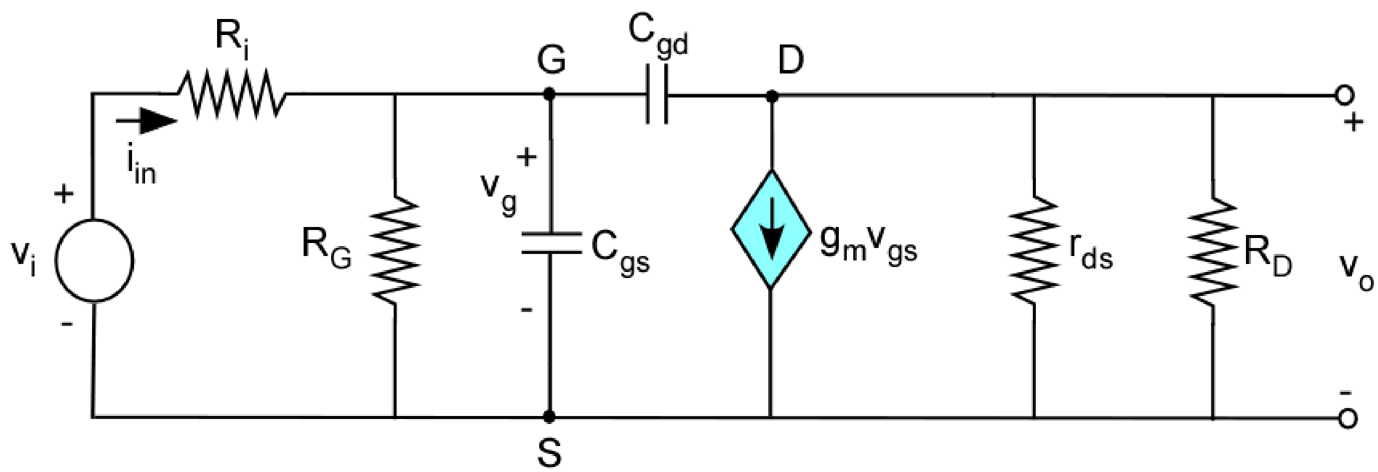
$$\frac{V_o}{V_{sig}} = A_M \frac{1}{1 + jf/f_o}$$

$$f_H = f_o = \frac{1}{2\pi C_{in} R_{sig}^{'}}$$

CS – Miller Effect – Exact Analysis



CS – Miller Effect – Exact Analysis



$$G_i = \frac{1}{R_i} \qquad G_D = \frac{1}{R_D} \qquad G_g = \frac{1}{R_g} \qquad g_{ds} = \frac{1}{r_{ds}}$$

$$R_D' = R_D \,//\, r_{ds} = \frac{1}{G_D + g_{ds}}$$

$$\frac{v_o}{v_i} = -\frac{G_i R_D'\left(g_m - sC_{gd}\right)}{G_i + G_g + s\left[C_{gs} + C_{gd}\right] + sC_{gd}R_D'\left[G_i + G_g\right] + sC_{gd}g_m R_D' + s^2 C_{gd}C_{gs}R_D'}$$

CS – Miller Effect – Exact Analysis

We neglect the terms in s^2 since

$$\left|s^2 C_{gd} C_{gs} R_D'\right| \ll \left|s C_{gd} g_m R_D'\right| \quad or \quad \left|s C_{gs}\right| \ll \left|g_m\right|$$

$$\frac{v_o}{v_i} = -\frac{G_i R_D' \left(g_m - s C_{gd}\right)}{G_i + G_g + s\left[C_{gs} + C_{gd}\left(1 + g_m R_D'\right) + C_{gd} R_D'\left(G_i + G_g\right)\right]}$$

Miller

If we multiply through by $R_i = \dfrac{1}{G_i}$

CS – Miller Effect – Exact Analysis

$$\frac{v_o}{v_i} = -\frac{R_D'\left(g_m - sC_{gd}\right)}{1 + R_i G_g + s\left\{R_i\left[C_{gs} + C_{gd}\left(1 + g_m R_D'\right)\right] + C_{gd}R_D'\left(1 + R_s G_g\right)\right\}}$$

From which we extract the 3-dB frequency point

$$f_H = \frac{1 + R_i G_g}{2\pi\left\{R_i\left[C_{gs} + C_{gd}\left(1 + g_m R_D'\right)\right] + C_{gd}R_D'\left(1 + R_i G_g\right)\right\}}$$

CS – Miller Effect – Exact Analysis

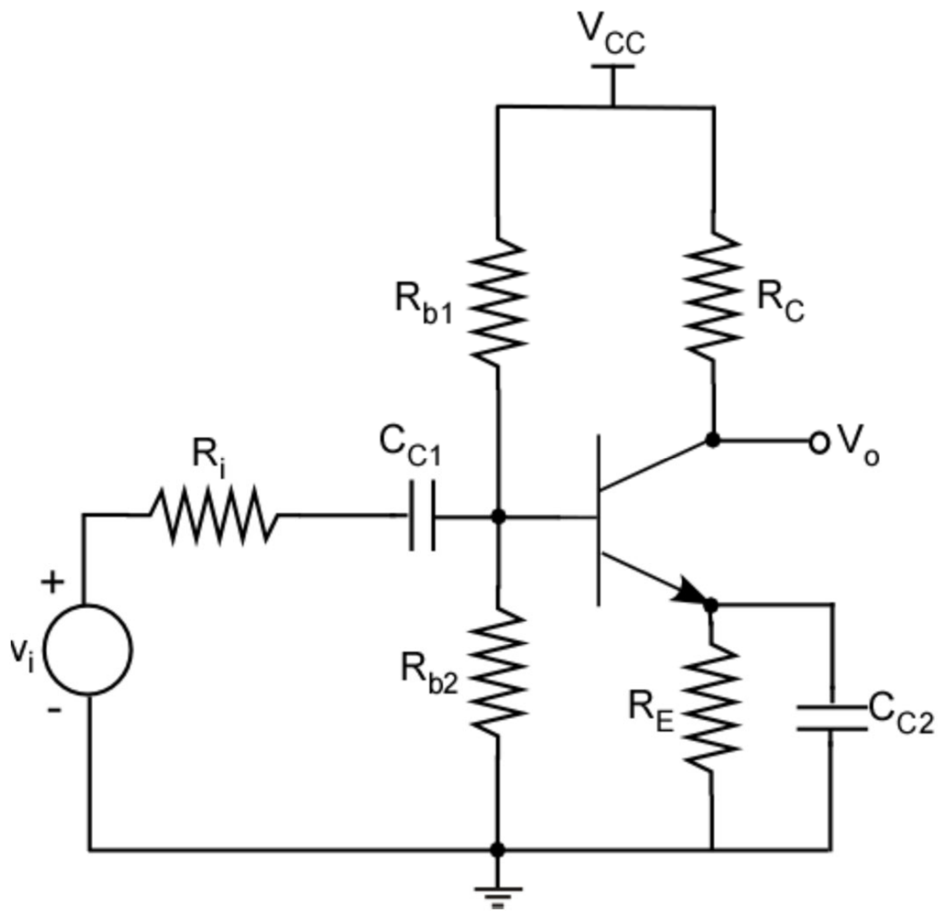
If G_g is negligible

$$f_H \simeq \frac{1}{2\pi\left\{R_i\left[C_{gs}+C_{gd}\left(1+g_m R_D'\right)\right]+C_{gd}R_D'\right\}}$$

If R_i =0

$$f_H \simeq \frac{1}{2\pi C_{gd} R_D'}$$

BJT-CE – Miller Effect – Exact Analysis

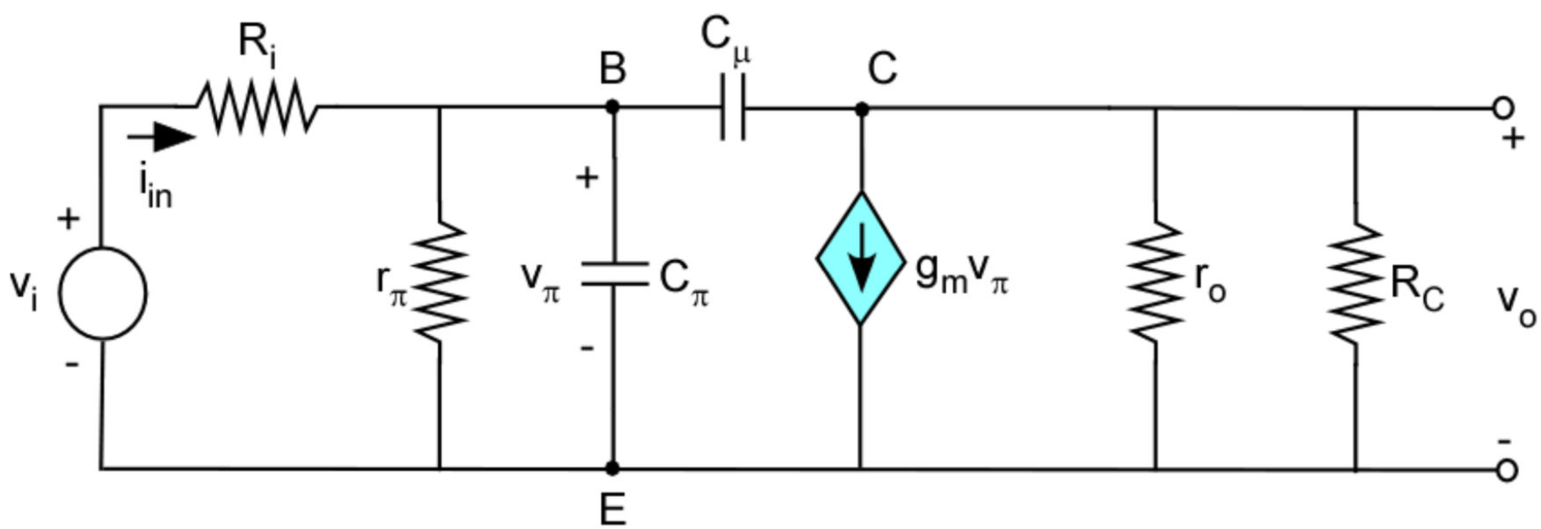


$$G_i = \frac{1}{R_i} \qquad G_C = \frac{1}{R_C}$$

$$g_\pi = \frac{1}{r_\pi} \qquad g_o = \frac{1}{r_o}$$

$$R_C^{'} = R_C \, // \, r_o = \frac{1}{G_C + g_o}$$

BJT-CE – Miller Effect – Exact Analysis



$$\frac{v_o}{v_i} = -\frac{G_s R_C' \left(g_m - sC_\mu\right)}{G_i + g_\pi + s\left[C_\pi + C_\mu\right] + sC_\mu R_C'\left[G_i + g_\pi\right] + sC_\mu g_m R_C' + s^2 C_\mu C_\pi R_C'}$$

BJT-CE – Miller Effect – Exact Analysis

We neglect the terms in s^2 since

$$\left|s^2 C_\mu C_\pi R_C'\right| \ll \left|s C_\mu g_m R_C'\right| \quad or \; \left|s C_\pi\right| \ll \left|g_m\right|$$

$$\frac{v_o}{v_i} = -\frac{G_i R_C'\left(g_m - sC_\mu\right)}{G_i + g_\pi + s\left[C_\pi + C_\mu\left(1 + g_m R_C'\right) + C_\mu R_C'\left(G_i + g_\pi\right)\right]}$$

Miller

If we multiply through by $R_i = \frac{1}{G_i}$

$$\frac{v_o}{v_i} = -\frac{R_C'\left(g_m - sC_\mu\right)}{1 + R_i g_\pi + s\left\{R_i\left[C_\pi + C_\mu\left(1 + g_m R_D'\right)\right] + C_\mu R_C'\left(1 + R_i g_\pi\right)\right\}}$$

BJT-CE – Miller Effect – Exact Analysis

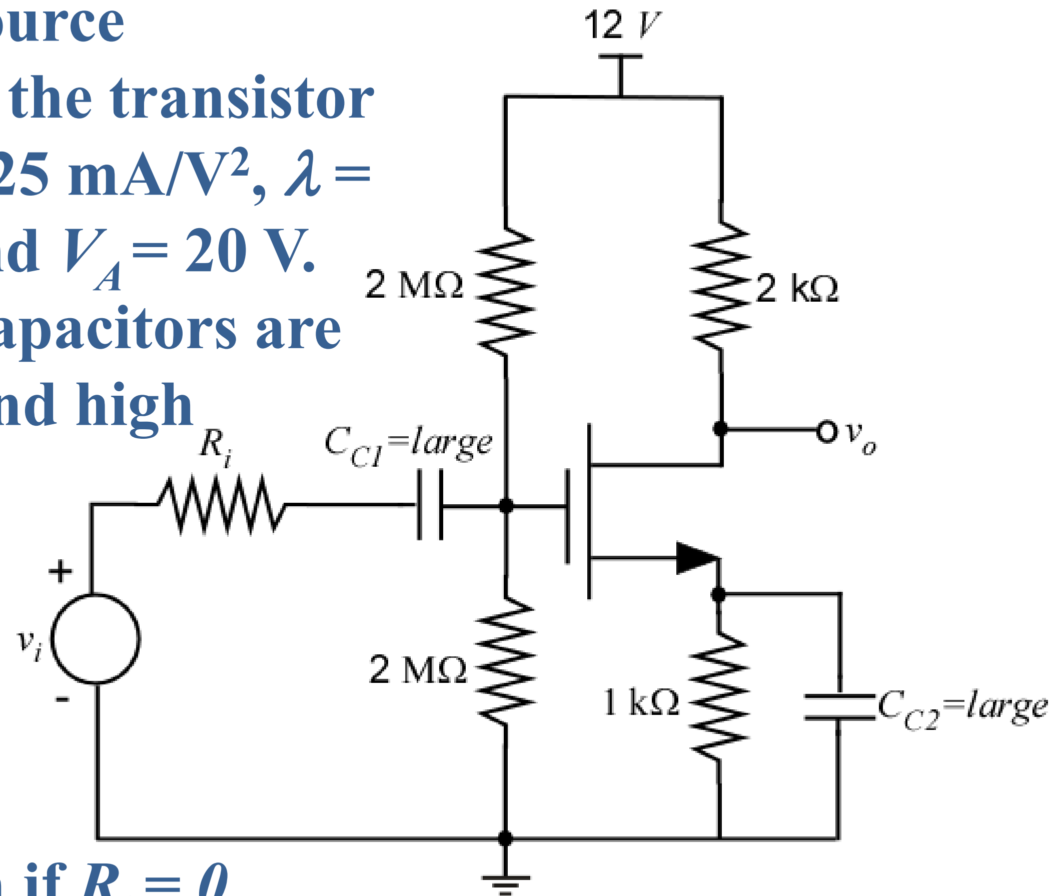
$$f_H = \frac{1 + R_i g_\pi}{2\pi \left\{ R_i \left[C_\pi + C_\mu \left(1 + g_m R_C' \right) \right] + C_\mu R_C' \left(1 + R_i g_\pi \right) \right\}}$$

If $R_i = 0$

$$f_H \simeq \frac{1}{2\pi C_\mu R_C'}$$

Example

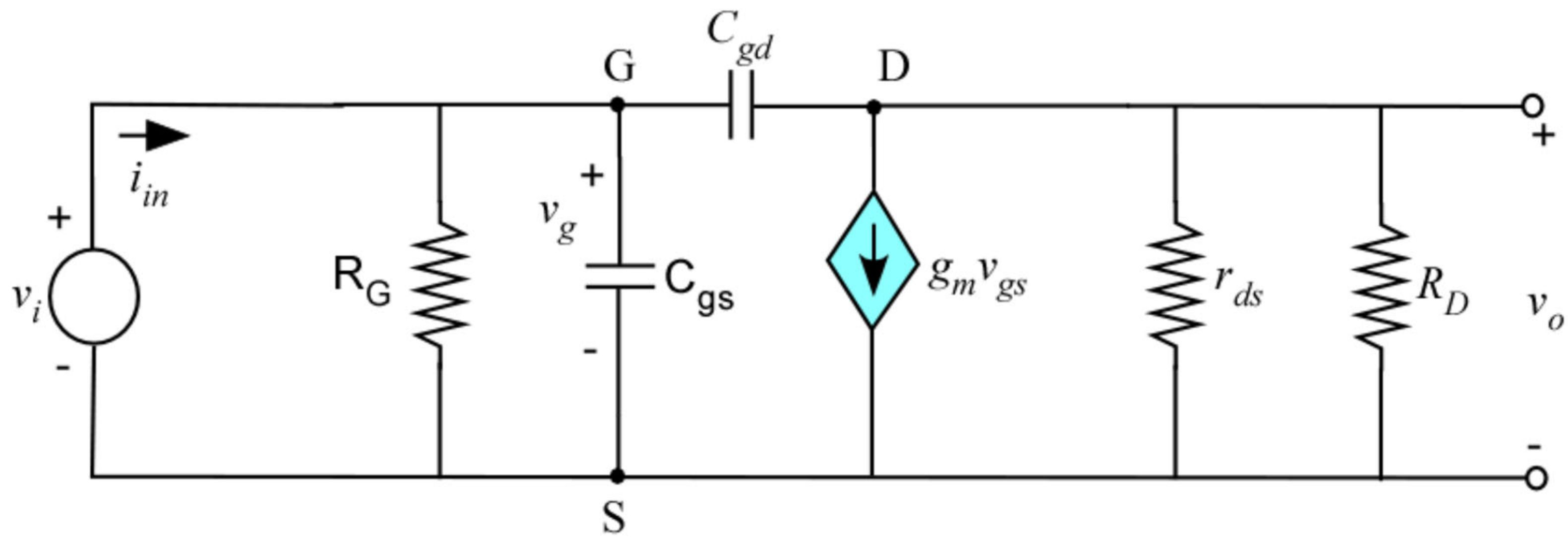
For the discrete common-source MOSFET amplifier shown, the transistor has $V_T = 1V$, $\mu C_{ox}(W/L) = 0.25\ mA/V^2$, $\lambda = 0$, $C_{gs} = 3\ pF$, $C_{gd} = 2.7\ pF$ and $V_A = 20\ V$. Assume that the coupling capacitors are short circuits at midband and high frequencies.



(a) Find the 3dB bandwidth if $R_i = 0$

(b) Find the 3dB bandwidth if $R_i = 50\ k\Omega$

Example – Part (a)

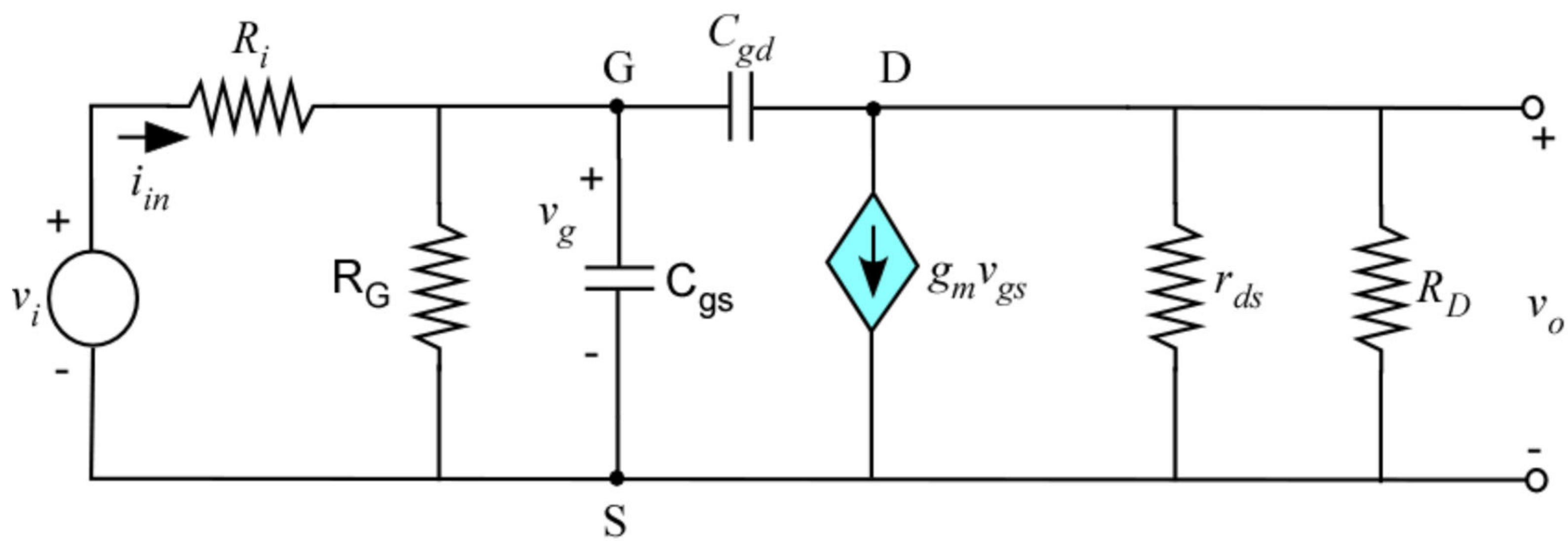


If $R_i = 0$, $\quad f_{3dB} = \dfrac{1}{2\pi C_{gd} R_D^{'}}$ $\qquad r_{ds} = \dfrac{|V_A|}{I_D} = \dfrac{20}{1.516} = 13\ k\Omega$

$$R_D^{'} = R_D \,||\, r_{ds} = 13 \,||\, 2 = 1.736\ k\Omega$$

$$f_{3dB} = \frac{1}{2\pi 2.7\times 10^{-12} \times 1.736\times 10^{3}} = 33.95\ MHz$$

Example Part (b)



If R_i =50 kΩ, $g_m R_D' = 0.870 \times 1.736 = 1.51$

$$f_H \simeq \frac{1}{2\pi\left\{R_i\left[C_{gs} + C_{gd}\left(1 + g_m R_D'\right)\right] + C_{gd} R_D'\right\}}$$

$$f_H \simeq \frac{1}{2\pi\left\{50\left[3 + 2.7\left(1 + 1.51\right)\right] + 2.7 \times 1.736\right\}} = 32.27\ kHz$$