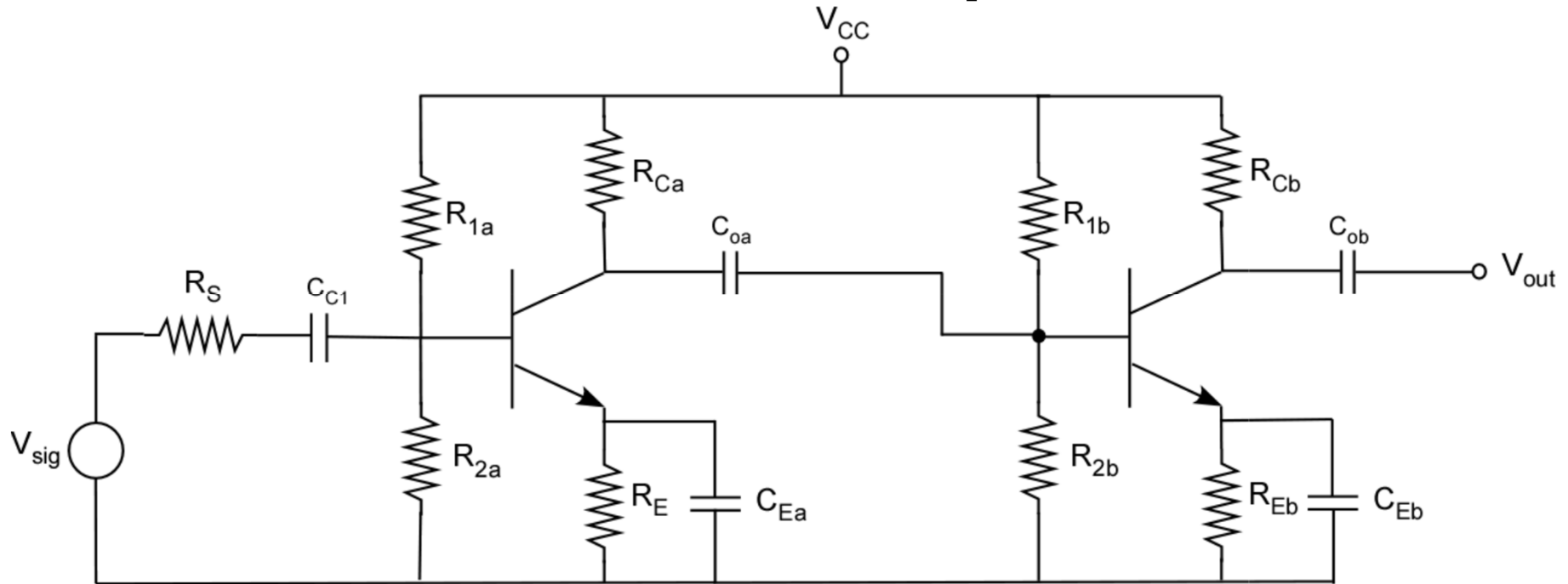# ECE 342
Electronic Circuits

Lecture 26
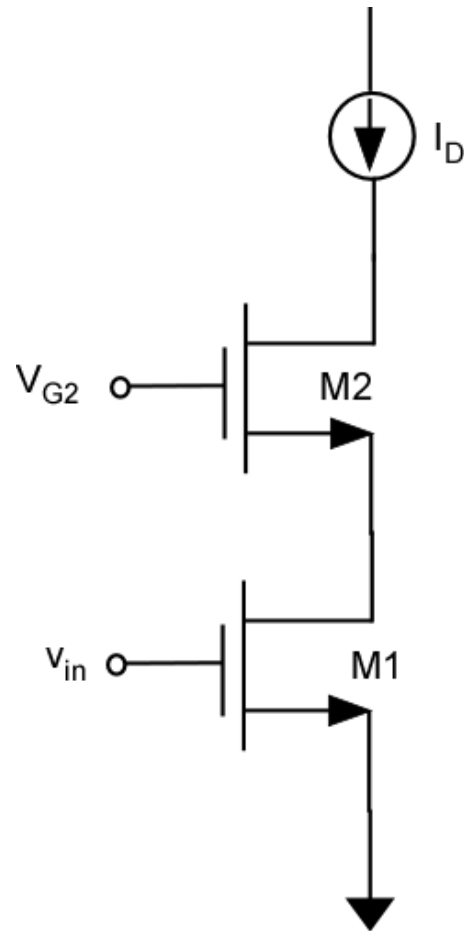# Frequency Response of Cascaded Amplifiers

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CE Cascade Amplifier

> ➤ **Exact analysis too tedious ➜ use computer**
> ➤ **CE cascade has low upper-cutoff frequency**

MOS Cascode Amplifier

Common source amplifier, followed by common gate stage – G_2 is an incremental ground

$$Define \ g_{o1} = \frac{1}{r_{o1}}$$

$$g_{o2} = \frac{1}{r_{o2}}$$

$$G_L = \frac{1}{R_L}$$

$$R_L = current \ source \ impedance$$

MOS Cascode Amplifier

- **CS cacaded with CG ➔ Cascode**
  - ➢ **Very popular configuration**
  - ➢ **Often considered as a single stage amplifier**
- **Combine high input impedance and large transconductance in CS with current buffering and superior high frequency response of CG**
- **Can be used to achieve equal gain but wider bandwidth than CS**
- **Can be used to achieve higher gain with same GBW as CS**

MOS Cascode Incremental Model
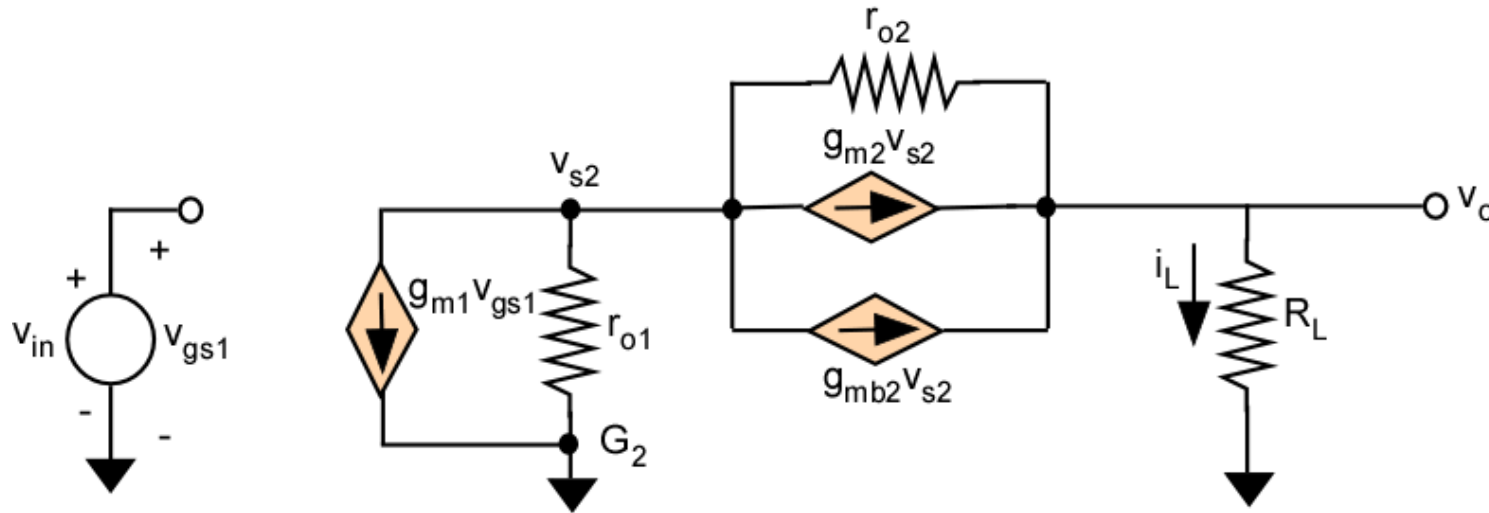
$$v_o = \frac{i_L}{G_L}$$

$$i_L = g_{mb2}v_{s2} + g_{m2}v_{s2} + (v_{s2} - v_o)g_{o2}$$

$$i_L = v_{s2}(g_{mb2} + g_{m2}) + v_{s2}g_{o2} - \frac{i_L}{G_L}g_{o2}$$

MOS Cascode Analysis

$$i_L \left(1 + \frac{g_{o2}}{G_L} \right) = v_{s2} \left(g_{mb2} + g_{m2} + g_{o2} \right)$$

$$v_{s2} = \frac{i_L \left(1 + g_{o2} / G_L \right)}{g_{mb2} + g_{m2} + g_{o2}}$$

KCL at v_{s2}

$$g_m v_{gs1} + v_{s2} g_{o1} + g_{m2} v_{s2} + g_{mb2} v_{s2} + g_{o2} (v_{s2} - v_o) = 0$$

MOS Cascode Analysis

$$g_{m1}v_{gs1} + \left[\frac{g_{o1}\left(1 + g_{o2}/G_L\right)}{g_{o2} + g_{m2} + g_{mb2}} + 1 \right] i_L = 0$$

$$i_L = \frac{-g_{m1}v_{gs1}}{\left[1 + \dfrac{g_{o1}\left(1 + g_{o2}/G_L\right)}{g_{o2} + g_{m2} + g_{mb2}} \right]}$$

$$\frac{v_o}{v_{in}} = \frac{-g_{m1}}{\left[G_L + \dfrac{g_{o1}\left(G_L + g_{o2}\right)}{g_{o2} + g_{m2} + g_{mb2}} \right]}$$

➔ **Two cases**

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MOS Cascode Analysis

CASE 1

$$Case\ 1:\ If\ G_L \approx 0$$

The voltage gain becomes

$$\frac{v_o}{v_{in}} = -g_{m1}\left(g_{o2} + g_{m2} + g_{mb2}\right)r_{o1}r_{o2}$$

$$A_{MB} = -\left[g_{m1}g_{o2} + g_{m1}g_{m2} + g_{m1}g_{mb2}\right]r_{o1}r_{o2}$$

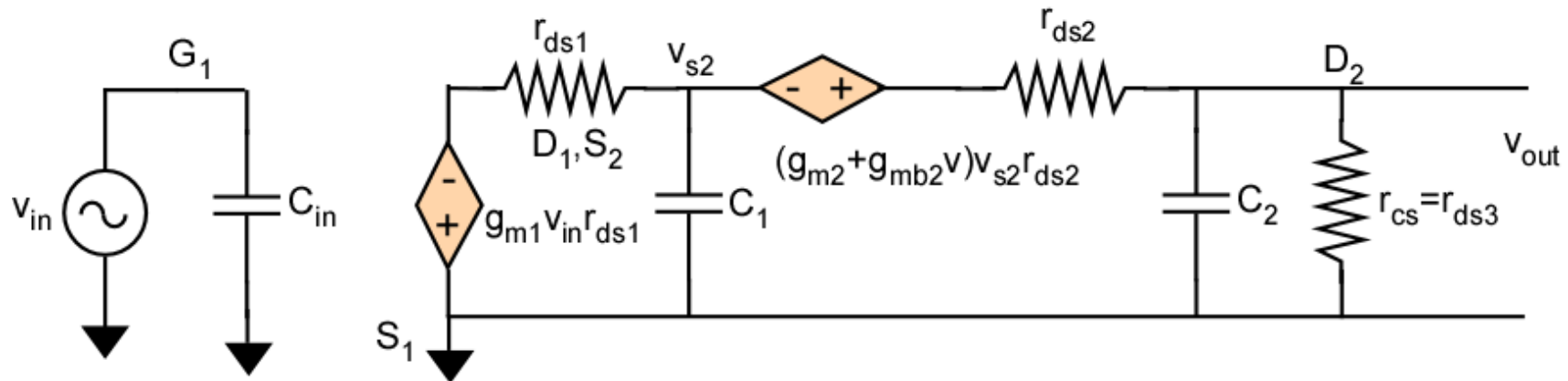$$A_{MB} \simeq -g_{m1}r_{o1}g_{m2}r_{o2}$$

MOS Cascode Analysis

CASE 2

$$Case\ 2 : If\ G_L \gg \frac{g_{o1}(G_L + g_{o2})}{g_{o2} + g_{m2} + g_{mb2}}$$

The voltage gain becomes

$$\frac{v_o}{v_{in}} = A_{MB} = \frac{-g_{m1}}{G_L}$$

MOS Cascode at High Frequency

The upper corner frequency of the cascode can be approximated as:

$$f_{2o} \approx \frac{1}{2\pi r_{ds3} C_2} \quad \text{where } C_2 = C_{db2} + C_{gd2} + C_{db3} + C_{gd3}$$

MOS Cascode at High Frequency

- Capacitance C_{gs1} sees a resistance R_{sig}
- Capacitance C_{gd1} sees a resistance R_{gd1}
- Capacitance $(C_{db1}+C_{gs2})$ sees resistance R_{d1}
- Capacitance (C_L+C_{gd2}) sees resistance $(R_L||R_{out})$

$$R_{gd1} = \left(1 + g_{m1}R_{d1}\right)R_{sig} + R_{d1} \qquad R_{d1} = r_{o1} \mathbin{/\!/} \left[\frac{1}{g_{m2} + g_{mb2}} + \frac{R_L}{A_{vo2}}\right]$$

$$\tau_H = C_{gs1}R_{sig} + C_{gd1}\left[\left(1 + g_{m1}R_{d1}\right)R_{sig} + R_{d1}\right]$$

$$+\left(C_{db1} + C_{gs2}\right)R_{d1} + \left(C_L + C_{gds2}\right)\left(R_L \mathbin{/\!/} R_{out}\right)$$

$$f_H \cong \frac{1}{2\pi\tau_H}$$
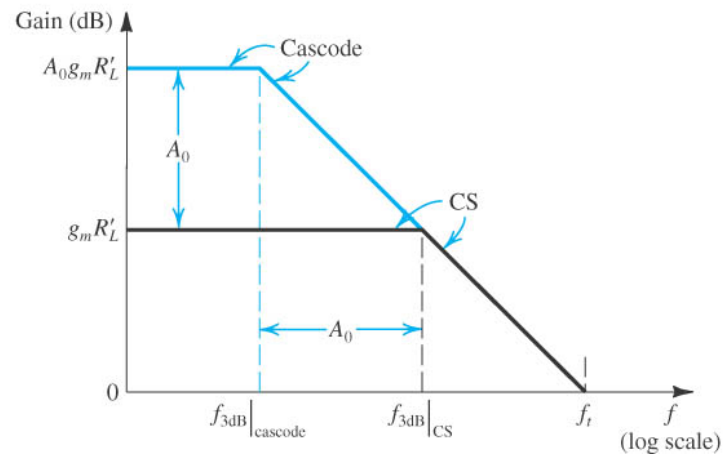
MOS Cascode at High Frequency

If R_{sig} is large, to extend the bandwidth we must lower R_L. This lowers R_{d1} and makes the Miller effect insignificant

If R_{sig} is small, there is no Miller effect. A large value of R_L will give high gain
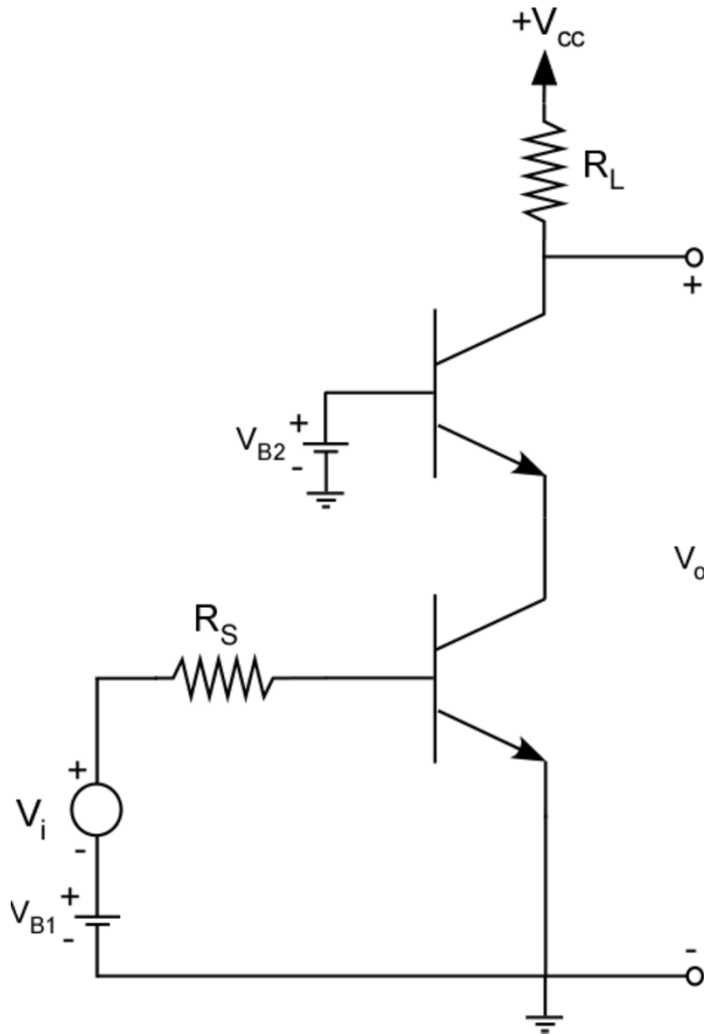
Effect of Cascoding
(when $R_{sig}=0$)

	Common Source	Cascode
Circuit	$R'_L = R_L \| r_o$	
DC Gain	$-g_m R'_L$	$-A_0 g_m R'_L$
f_{3dB}	$\dfrac{1}{2\pi(C_L + C_{gd})R'_L}$	$\dfrac{1}{2\pi(C_L + C_{gd})A_0 R'_L}$
f_t	$\dfrac{g_m}{2\pi(C_L + C_{gd})}$	$\dfrac{g_m}{2\pi(C_L + C_{gd})}$

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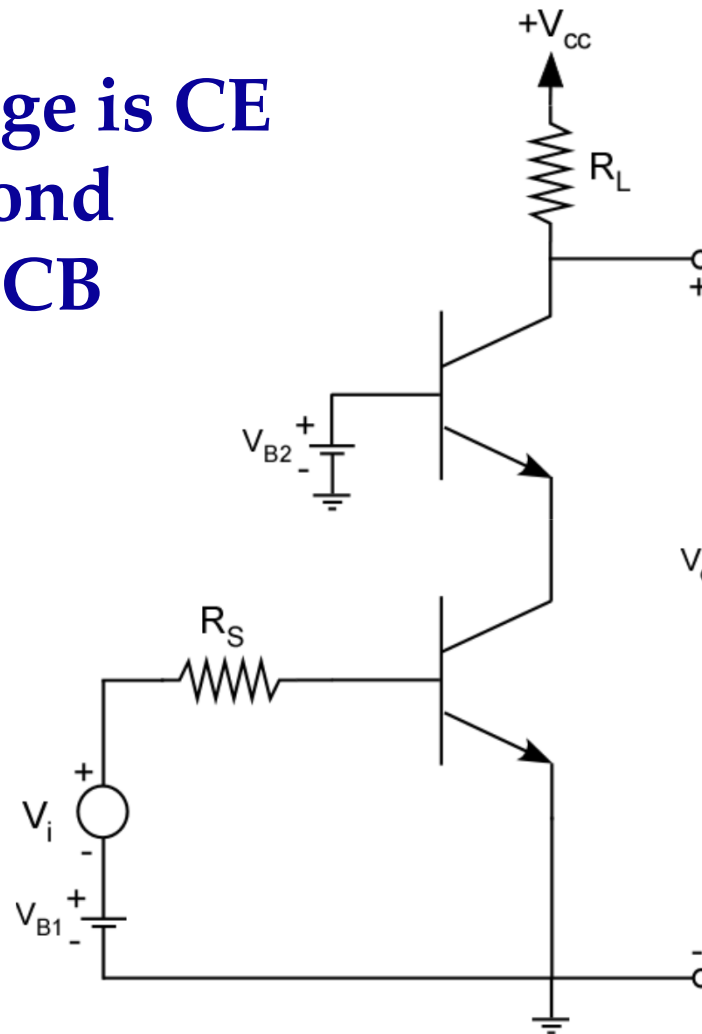University of Illinois at Urbana-Champaign

BJT Cascode Amplifier

Common emitter amplifier, followed by common base stage – Base of Q_2 is an incremental ground
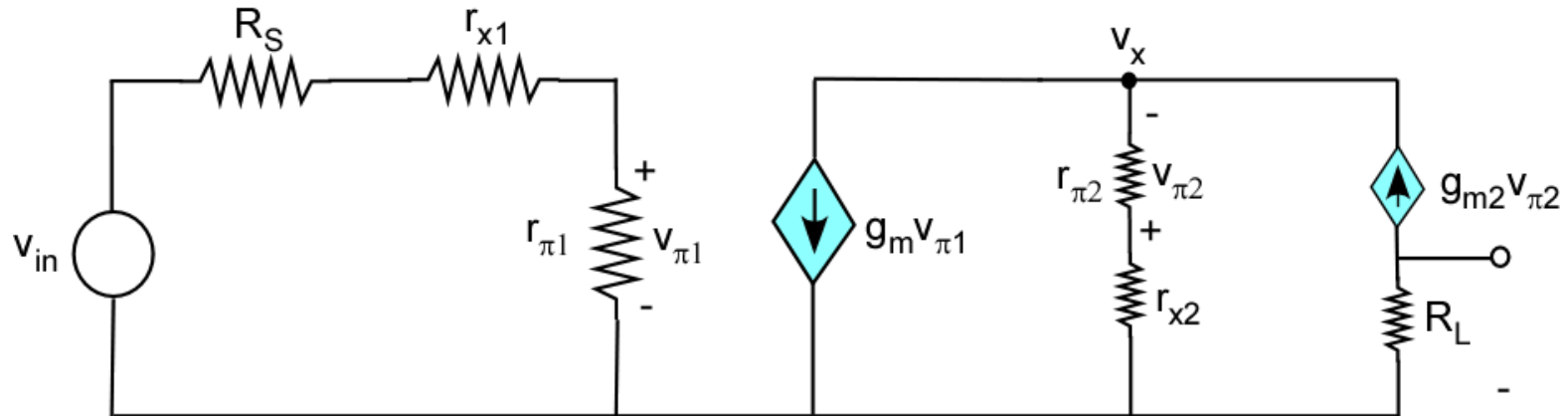
Cascode Amplifier

First stage is CE and second stage is CB

If $R_s \ll r_{\pi 1}$, the voltage gain can be approximated by

$$A_v = -g_{m1}\alpha_2 R_L$$

BJT Cascode Incremental Model

$$v_o = -g_{m2}R_L v_{\pi 2}$$

$$v_{\pi 2} = -\left(\frac{v_x r_{\pi 2}}{r_{x2} + r_{\pi 2}}\right) \Rightarrow v_x = \left(1 + \frac{r_{x2}}{r_{\pi 2}}\right)v_{\pi 2}$$

BJT Cascode Analysis

$$\frac{v_x}{r_{\pi 2} + r_{x2}} + g_{m1}v_{\pi 1} = -g_{m2}v_{\pi 2}$$

Ignoring r_{x2}

$$-\frac{v_{\pi 2}}{r_{\pi 2}} + g_{m1}v_{\pi 1} = -g_{m2}v_{\pi 2}$$

$$g_{m1}v_{\pi 1} = v_{\pi 2}\left(g_{m2} + \frac{1}{r_{\pi 2}}\right)$$

BJT Cascode Analysis

$$v_{\pi 1} = v_{in} \frac{r_{\pi 1}}{R_s + r_{x1} + r_{\pi 1}}$$

$$v_{\pi 2} = \frac{g_{m1} r_{\pi 1} v_{in}}{R_s + r_{x1} + r_{\pi 1}} \cdot \frac{1}{\left(g_{m2} + \dfrac{1}{r_{\pi 2}} \right)}$$

$$v_o = \frac{-g_{m2} R_L g_{m1} r_{\pi 1} v_{in}}{R_s + r_{x1} + r_{\pi 1}} \cdot \frac{1}{\left(g_{m2} + \dfrac{1}{r_{\pi 2}} \right)}$$

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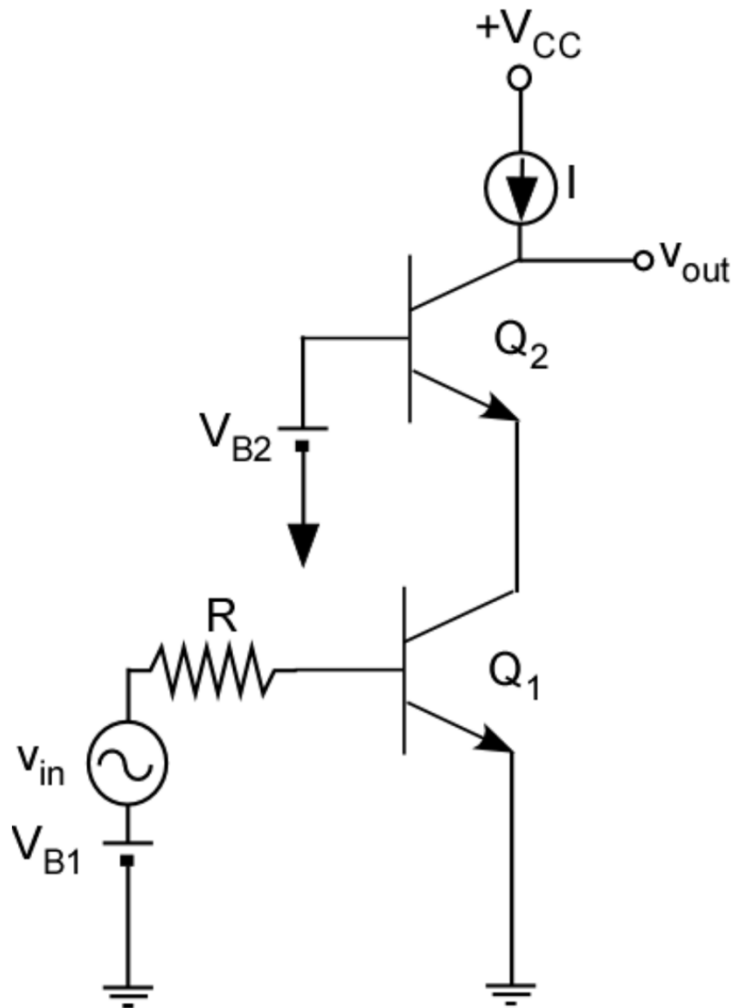Electrical and Computer Engineering
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BJT Cascode Analysis

$$\frac{v_o}{v_{in}} = \frac{-g_{m2}r_{\pi2}g_{m1}r_{\pi1}R_L}{\left(R_s + r_{x1} + r_{\pi1}\right)\left(1 + g_{m2}r_{\pi2}\right)}$$

$$\frac{v_o}{v_{in}} = \frac{-\beta_1\beta_2 R_L}{\left(R_s + r_{x1} + r_{\pi1}\right)\left(\beta_2 + 1\right)}$$

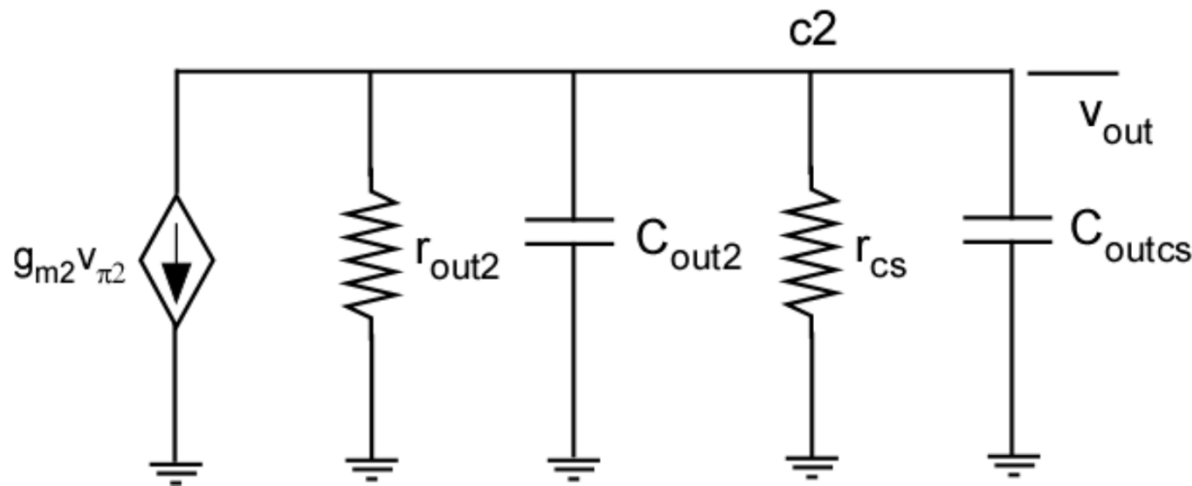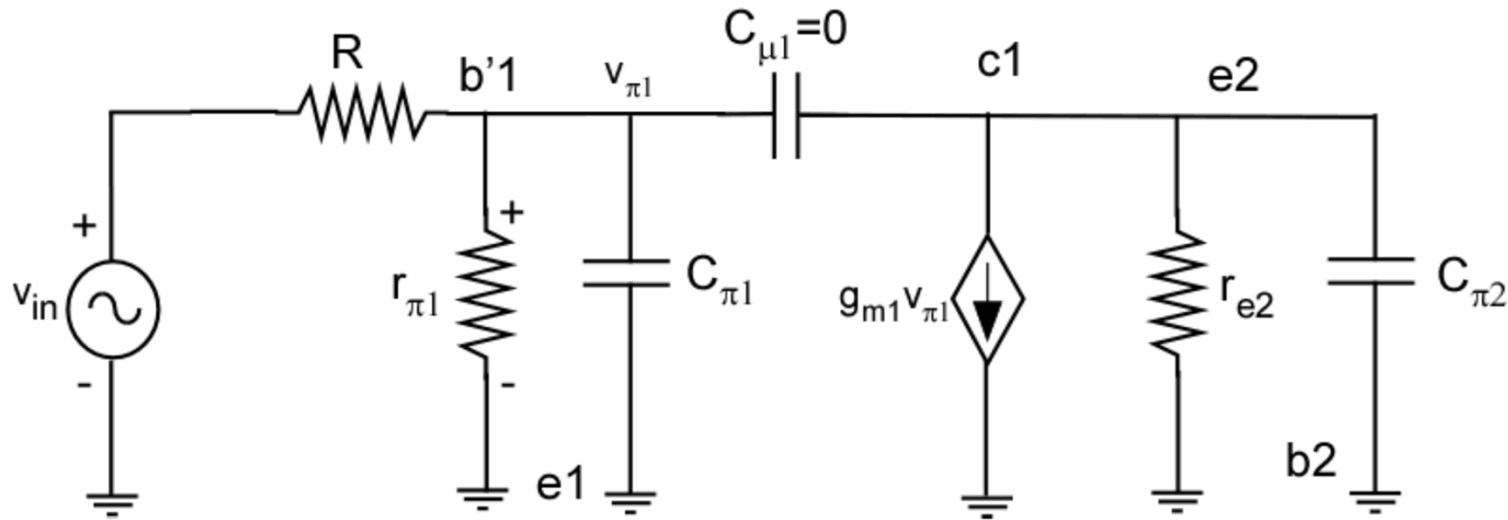If $R_s \ll r_{x1} + r_{\pi1}$, the voltage gain can be approximated by

$$A_v = -g_{m1}\alpha_2 R_L$$

BJT Cascode at High Frequency

Define r_{cs} as the internal impedance of the current source. R includes generator resistance and base resistance r_{x1} base of Q1

BJT Cascode at High Frequency

There is no Miller multiplication from the input of Q2 (emitter) to the output (collector).

BJT Cascode at High Frequency

$$Define\ R_3 = r_{out2} \parallel r_{cs}$$

$$A = A_{MB} \frac{1}{1 + j\omega C_{\pi 1}\left(r_{\pi 1} \parallel R\right)} \frac{1}{1 + j\omega r_{e2} C_{\pi 2}} \frac{1}{1 + j\omega R_3 C_{out}}$$

$$f_2 = \frac{1}{2\pi r_{e2} C_{\pi 2}} \qquad f_{in-high} = \frac{1}{2\pi\left(r_{\pi 1} \parallel R\right) C_{\pi 1}}$$

$$A = A_{MB} \frac{1}{1 + j\dfrac{f}{f_{in-high}}} \frac{1}{1 + j\dfrac{f}{f_{out-high}}}$$
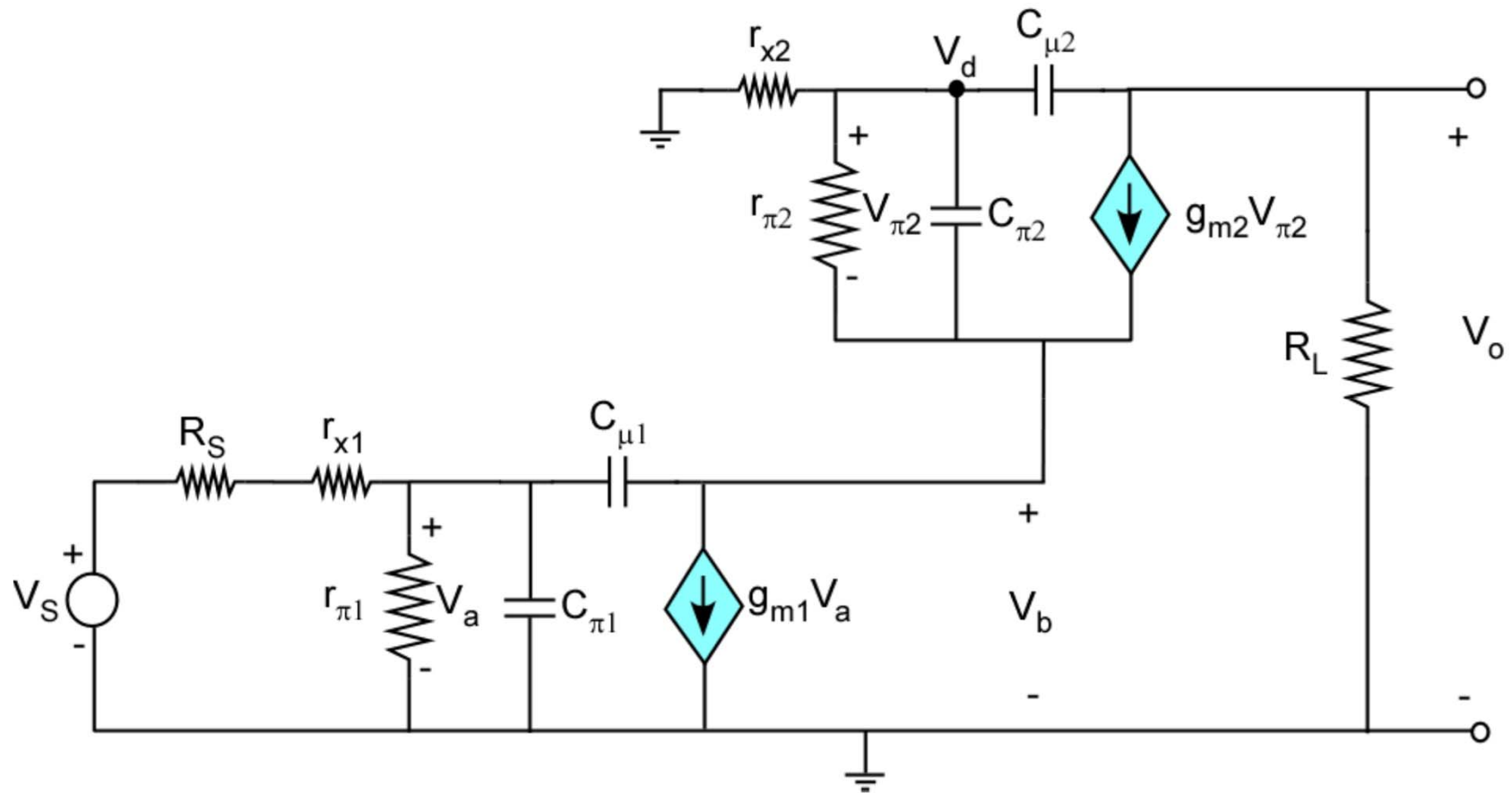
BJT Cascode at High Frequency

$$f_{out-high} = \frac{1}{2\pi C_{out} R_3}$$

$$C_{out} = C_{out2} + C_{outcs}$$

$$C_{outcs} = \text{current source output capacitance}$$

Cascode Amplifier – High Frequency

High-frequency incremental model

Cascode Amplifier – High Frequency

Applying Kirchoff's current law to each node:

$$G_s' V_i = \left[G_s' + g_{\pi 1} + s\left(C_{\pi 1} + C_{\mu 1} \right) \right] V_a - s C_{\mu 1} V_b$$

$$0 = \left(g_{m1} - s C_{\mu 1} \right) V_a + \left[g_{\pi 2} + g_{m2} + s\left(C_{\pi 2} + C_{\mu 1} \right) \right] V_b - \left(g_{\pi 2} + g_{m2} - s C_{\pi 2} \right) V_d$$

$$0 = -\left(g_{\pi 2} - s C_{\pi 2} \right) V_b + \left[g_{x2} + g_{\pi 2} + s\left(C_{\pi 2} + C_{\mu 2} \right) \right] V_d - s C_{\mu 2} V_o$$

$$0 = -g_{m2} V_b + \left(g_{m2} - s C_{\mu 2} \right) V_d + \left(G_L + s C_{\mu 2} \right) V_o$$

Find solution using a computer

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Cascode Amplifier – High Frequency

As an example use:

g_m=0.4 mhos β=100

r_π=250 ohms r_x=20 ohms

C_π=100 pF C_μ=5 pf

G_L=5 mmhos G_S'=4.5 mmhos

ZEROS (nsec^{-1})   POLES (nsec^{-1})

s_a=8.0 s_d=-0.0806

s_b=-2.02 + j5.99 s_e=-0.644

s_c=-2.02 – j5.99 s_f=-4.05

s_g=-16.45

Cascode Amplifier – High Frequency

If one pole is at a much lower frequency than the zeros and the other poles, (dominant pole) we can approximate ω_{3dB}

$$\omega_{3dB} \simeq 0.0806 \times 10^9 \, rad \, / \, sec$$

$$f_{3dB} \simeq 12.9 \, MHz$$

For the same gain, a single stage amplifier would yield:

$$\omega_{3dB} \simeq 0.0169 \times 10^9 \, rad \, / \, sec$$

$$f_{3dB} \simeq 2.7 \, MHz$$

Second stage in cascode increases bandwidth