ECE 342
Electronic Circuits

Lecture 29
Feedback Examples

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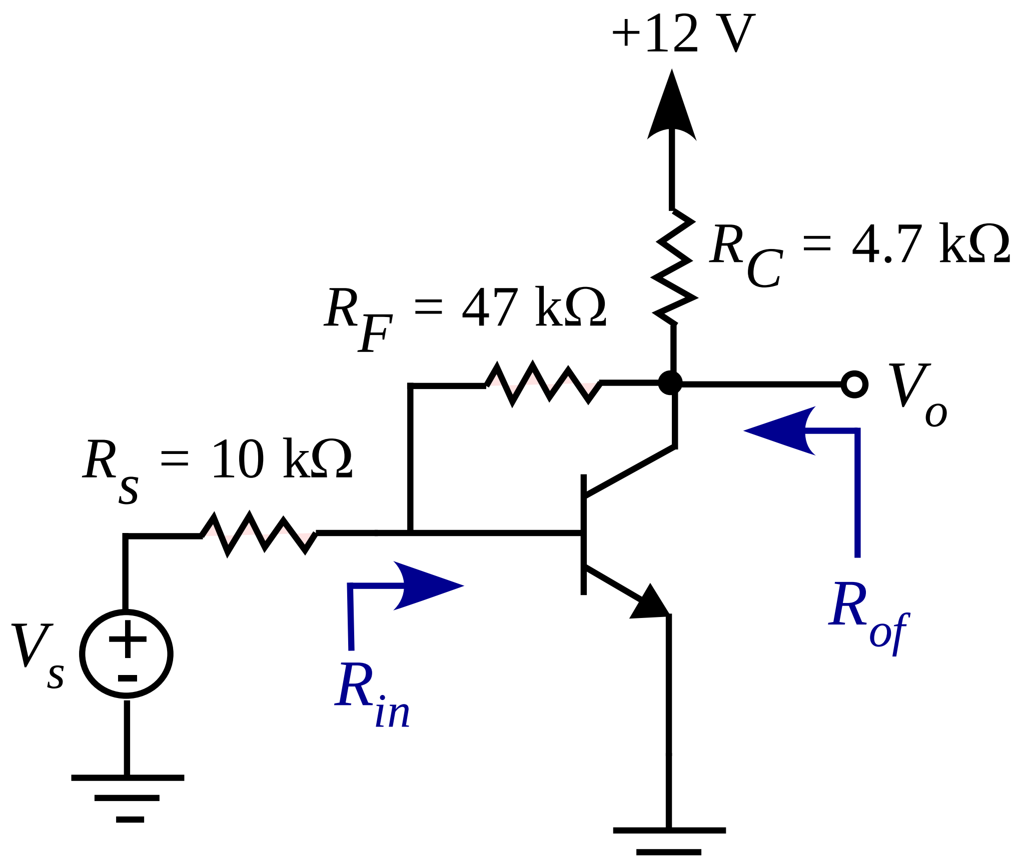
Single-Stage Amplifier with Feedback

We want to determine the small-signal voltage gain V_o/V_s, the input resistance and the output resistance $R_{out}=R_{of}$. The transistor has $\beta = 100$

Model as shunt-shunt



Single-Stage – DC Analysis

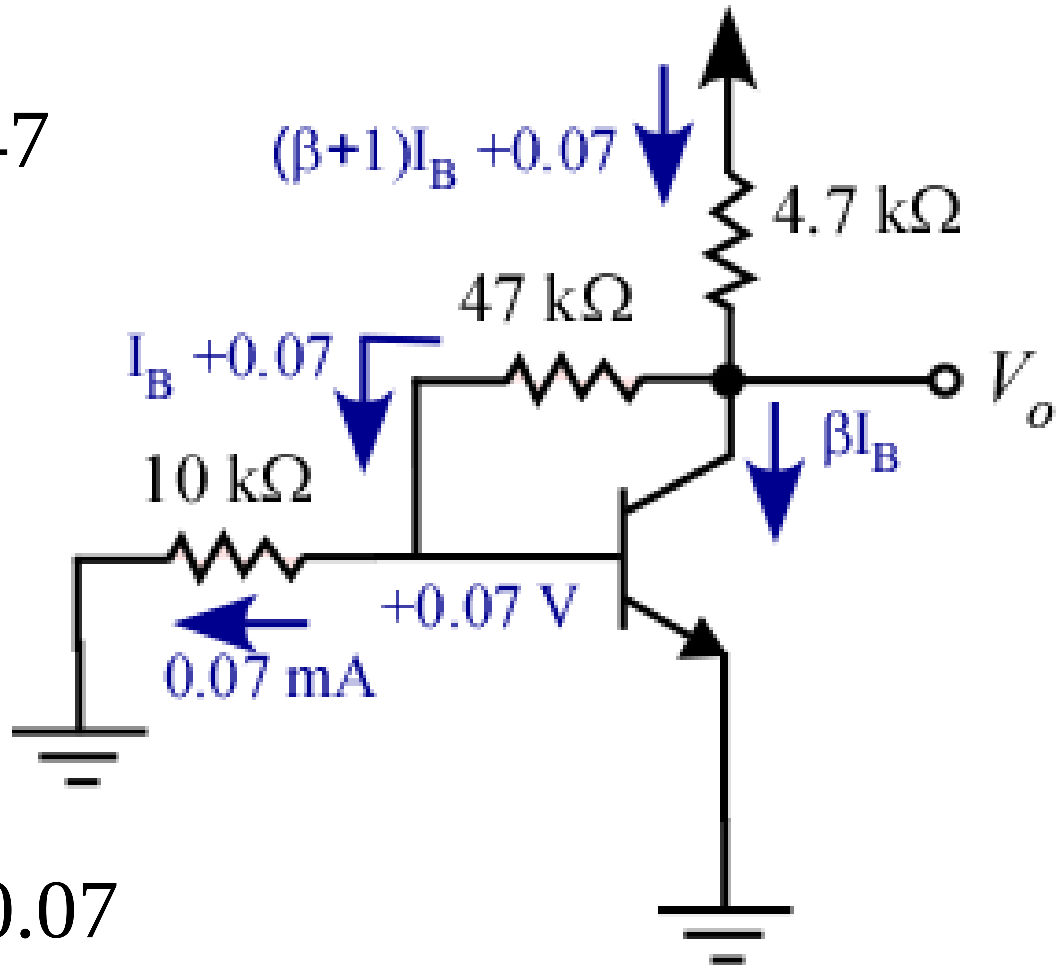
First determine dc operating point

$$V_C = 0.7 + (I_B + 0.07)47$$

Solve for I_B using the following 2 equations

$$V_C = 3.99 + 47 I_B$$

$$\frac{12 - V_C}{4.7} = (\beta + 1) I_B + 0.07$$



Single-Stage – DC Analysis

We get

$$I_B \simeq 0.015\, mA \qquad I_C \simeq 1.5\, mA \qquad V_C \simeq 4.7\, V$$

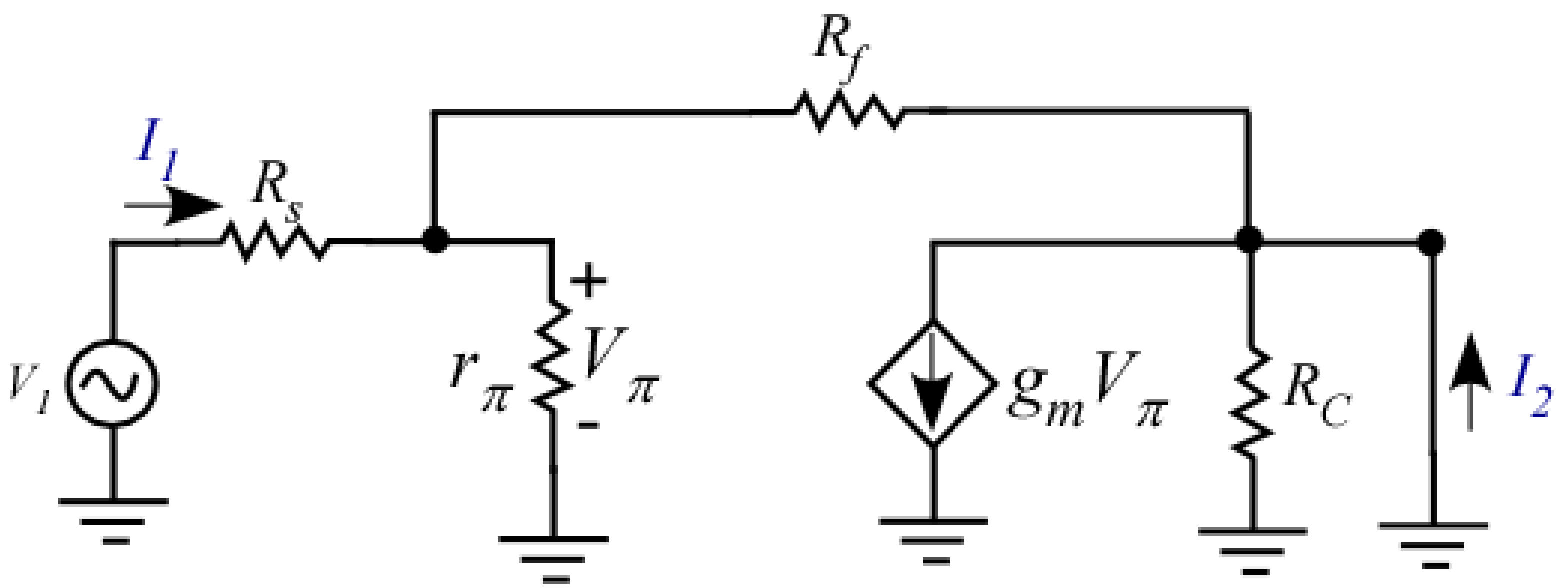
$$g_m = \frac{I_C}{V_T} = \frac{1.5}{25} = 60\, mA/V$$

$$r_\pi = \beta / g_m = 100/60 = 1.666\, k\Omega$$

$$R_S \,||\, r_\pi = \frac{10(1.66)}{11.6} = 1.429\, k\Omega$$

$$R_f \,||\, r_\pi = \frac{47(1.66)}{48.66} = 1.6\, k\Omega$$

Calculating y_{11} for Amplifier



$$y_{11} = \frac{1}{R_S + R_f \,||\, r_\pi} = \frac{1}{10 + 1.6} = 0.086\, mA/V$$

Calculating y_{21} for Amplifier

$$y_{21} = \frac{I_2}{V_1}$$

$$v_\pi = \frac{V_1\left(R_f \parallel r_\pi\right)}{R_S + R_f \parallel r_\pi} = \left(g_m - 1/R_f\right)\frac{V_1\left(R_f \parallel r_\pi\right)}{R_S + R_f \parallel r_\pi}$$

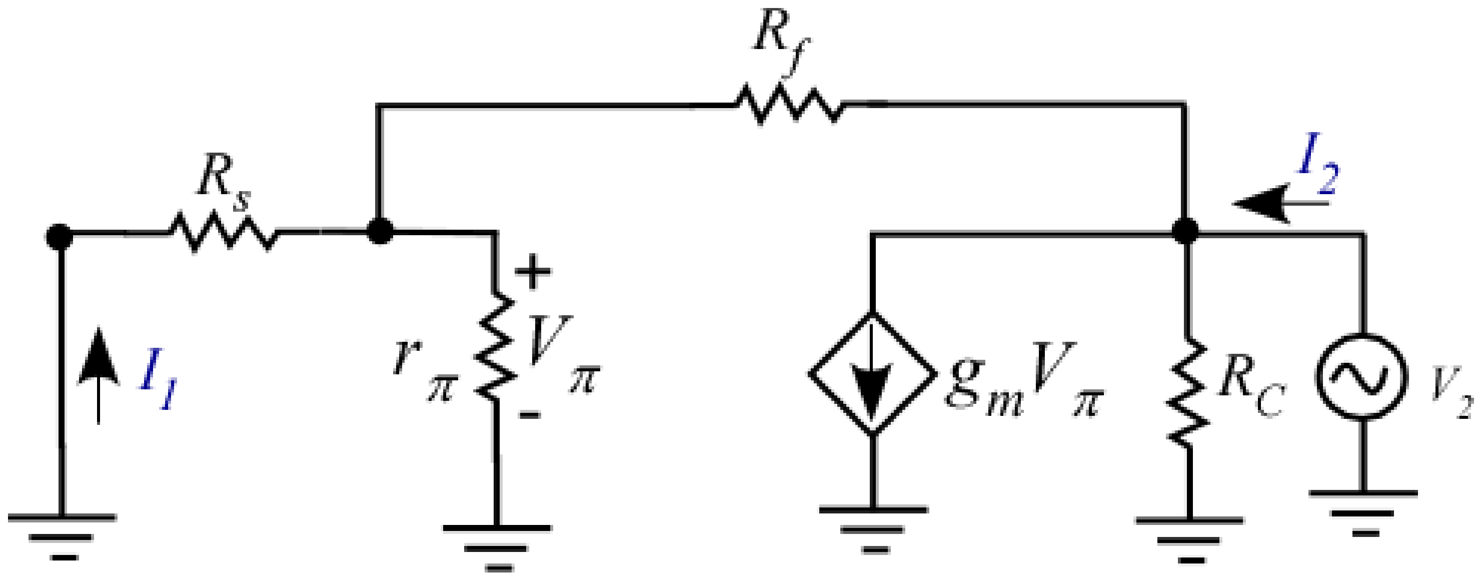
$$I_2 = \left(g_m - 1/R_f\right)\frac{V_1\left(R_f \parallel r_\pi\right)}{R_S + R_f \parallel r_\pi}$$

$$y_{21} = \left(g_m - 1/R_f\right)\frac{\left(R_f \parallel r_\pi\right)}{R_S + R_f \parallel r_\pi}$$

Calculating y_{21} for Amplifier

$$y_{21} = (60 - 0.021)\frac{1.6}{10 + 1.6} = 8.27\ mA/V$$

Calculating y_{12} for Amplifier



$$y_{12} = \frac{I_1}{V_2} = \frac{-v_\pi}{R_S V_2} = \frac{-V_2\left(R_S \,||\, r_\pi\right)}{R_f + \left(R_S \,||\, r_\pi\right)} \frac{1}{R_S} \frac{1}{V_2}$$

$$y_{12} = -\frac{1}{R_S} \frac{\left(R_S \,||\, r_\pi\right)}{R_f + \left(R_S \,||\, r_\pi\right)}$$

Calculating y_{12} for Amplifier

$$y_{12} = -\frac{1.429}{10\left(47+1.429\right)} = -0.00295\ mA/V$$

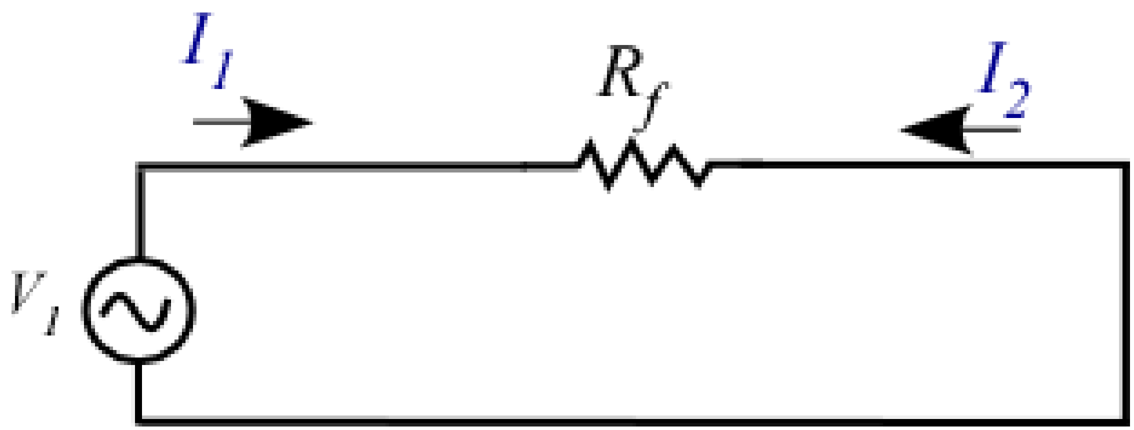
Calculating y_{22} for Amplifier

$$I_2 = \frac{V_2}{R_C} + \underbrace{\frac{g_m V_2 \left(R_S \,\|\, r_\pi\right)}{\left(R_S \,\|\, r_\pi\right) + R_f}}_{g_m v_\pi} + \frac{V_2}{\left(R_S \,\|\, r_\pi\right) + R_f}$$

$$y_{22} = \frac{I_2}{V_2} = \frac{1}{R_C} + \frac{g_m \left(R_S \,\|\, r_\pi\right)}{\left(R_S \,\|\, r_\pi\right) + R_f} + \frac{1}{\left(R_S \,\|\, r_\pi\right) + R_f}$$

$$y_{22} = \frac{I_2}{V_2} = \frac{1}{4.7} + \frac{60(1.429)}{1.429 + 47} + \frac{1}{1.429 + 47} = 2.01\, mA/V$$

y-parameters for Feedback Network



$$y_{11} = \frac{1}{R_f} = 0.021\, mA/V \qquad y_{22} = y_{11}\ by\ \ symmetry$$

$$y_{21} = -\frac{1}{R_f} = -0.021\, mA/V \qquad y_{12} = y_{21}\ by\ \ reciprocity$$

From Feedback Network, we get $\beta = y_{12}$ = -0.021

Basic Amplifier vs Feedback Network

Basic Amplifier

$$Y_A = \begin{bmatrix} 0.086 & -0.003 \\ 8.27 & 2.01 \end{bmatrix}$$

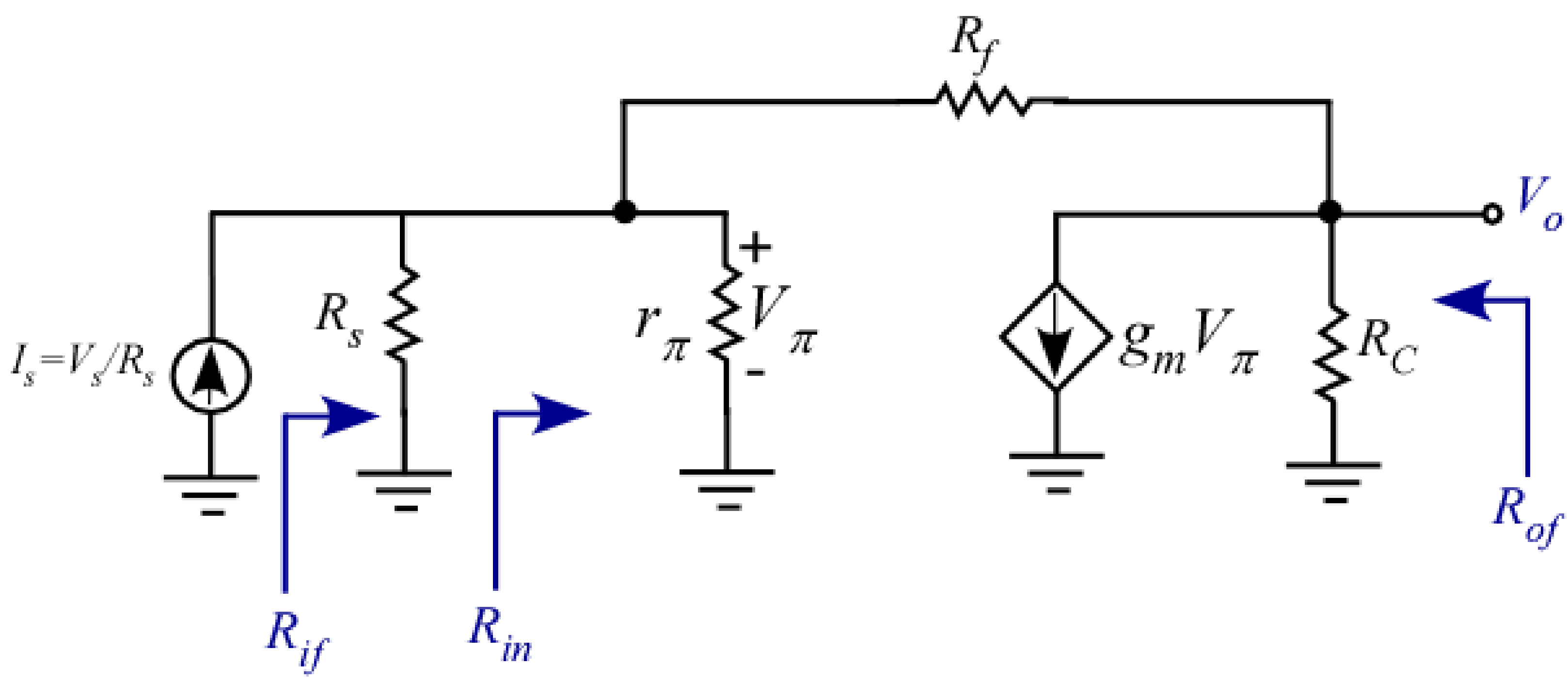
Feedback Network

$$Y_F = \begin{bmatrix} 0.021 & -0.021 \\ -0.021 & 0.021 \end{bmatrix}$$

$$\left|y_{12}\right|_{\substack{basic \\ amplifier}} \ll \left|y_{12}\right|_{\substack{feedback \\ network}}$$

$$\left|y_{21}\right|_{\substack{feedback \\ network}} \ll \left|y_{21}\right|_{\substack{basic \\ amplifier}}$$

Single-Stage – Small-Signal Analysis



Single-Stage – Small-Signal Analysis

The feedback is provided by R_f which samples the output voltage and feeds back a current to be mixed at input

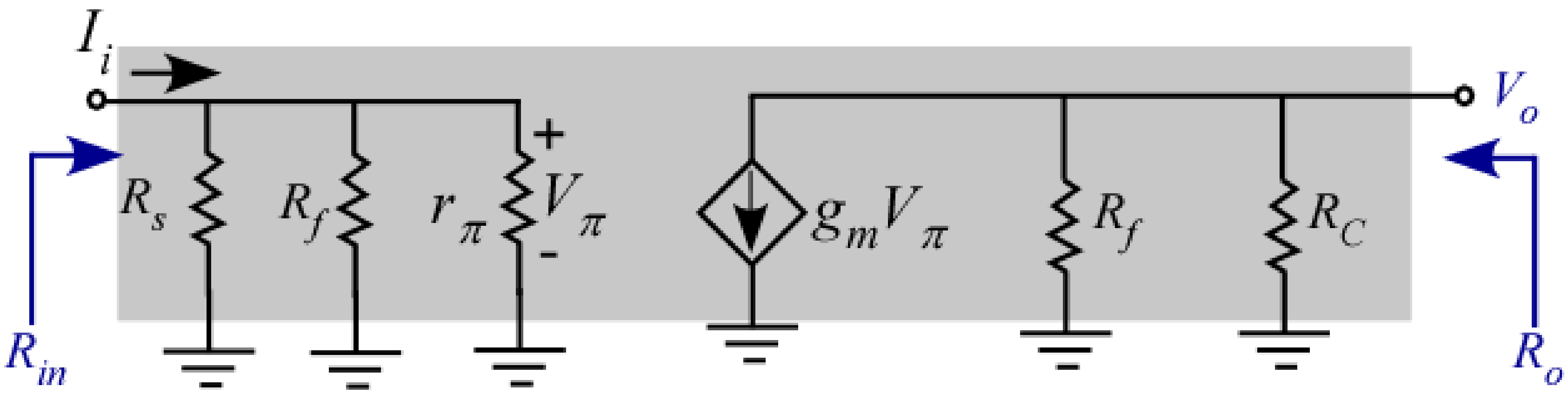
$$V_\pi = I_i\left(R_s \,||\, R_f \,||\, r_\pi\right)$$

$$V_o = -g_m V_\pi \left(R_f \,||\, R_C\right)$$

$$A = \frac{V_o}{I_i} = -g_m\left(R_f \,||\, R_C\right)\left(R_s \,||\, R_f \,||\, r_\pi\right) = -358.7\ k\Omega$$

Transimpedance gain is –358.7 kΩ

Input and Output Resistances

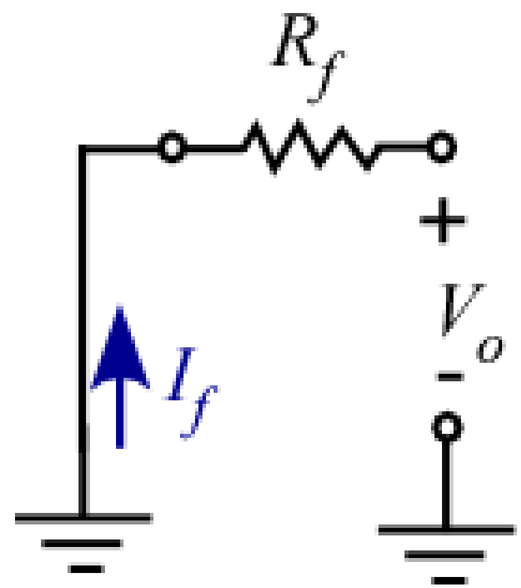


$$R_i = R_s \,||\, R_f \,||\, r_\pi = 1.4\; k\Omega$$

$$R_o = R_C \,||\, R_f = 4.27\; k\Omega$$

Determining β and A_f

$$\beta = \frac{I_f}{V_o} = -\frac{1}{R_f} = -\frac{1}{47\ k\Omega}$$



$$A_f \equiv \frac{V_o}{I_s} = \frac{A}{1+A\beta}$$

$$\frac{V_o}{I_s} = \frac{-358.7}{1+358.7/47} = \frac{-358.7}{8.63} = -41.6\ k\Omega$$

Single-Stage Feedback Amp

Voltage gain is:

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{-41.6}{10} \simeq -4.16\, V/V$$

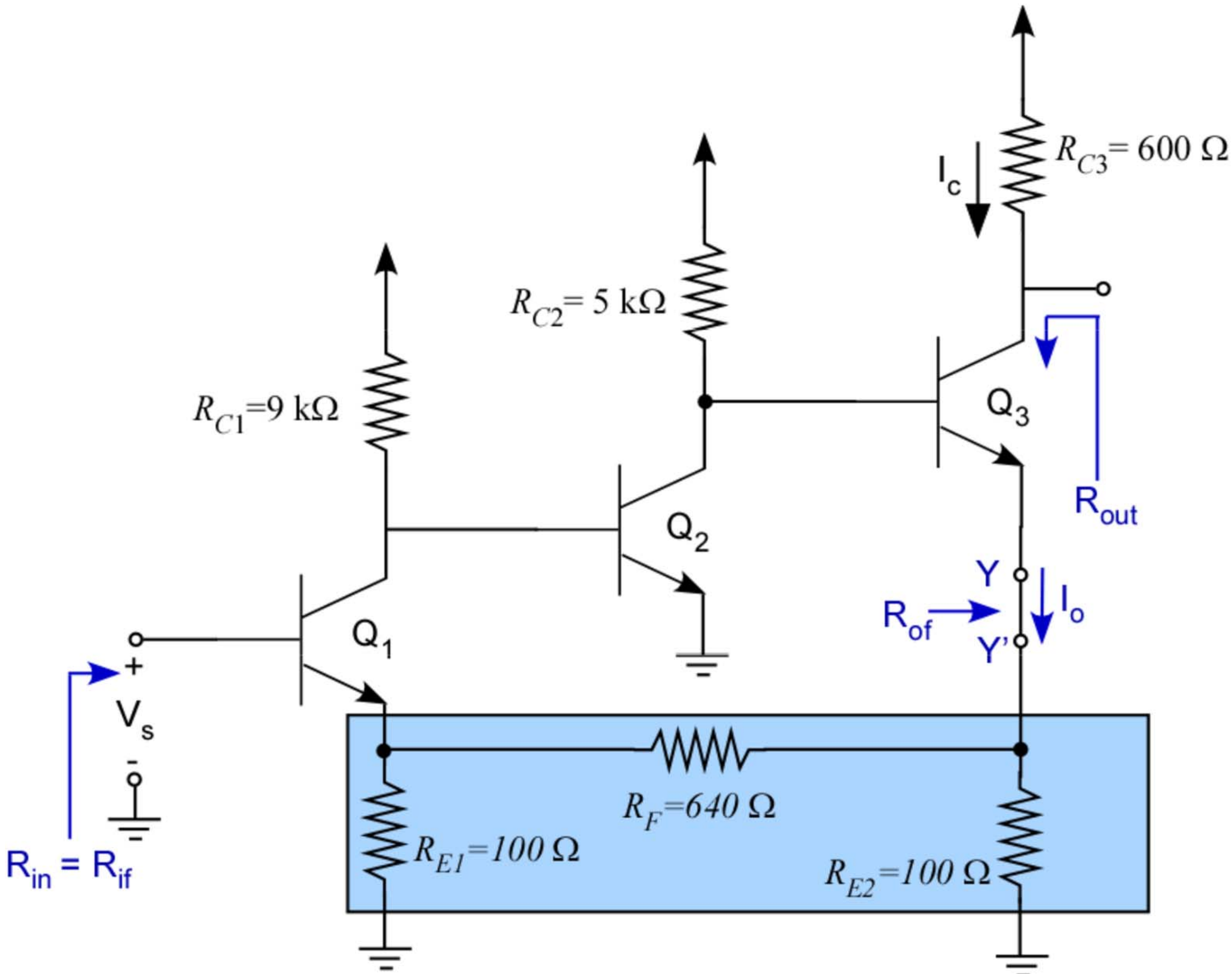
The input resistance with feedback is:

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{1.4}{8.63} = 162.2\,\Omega$$

The output resistance with feedback is:

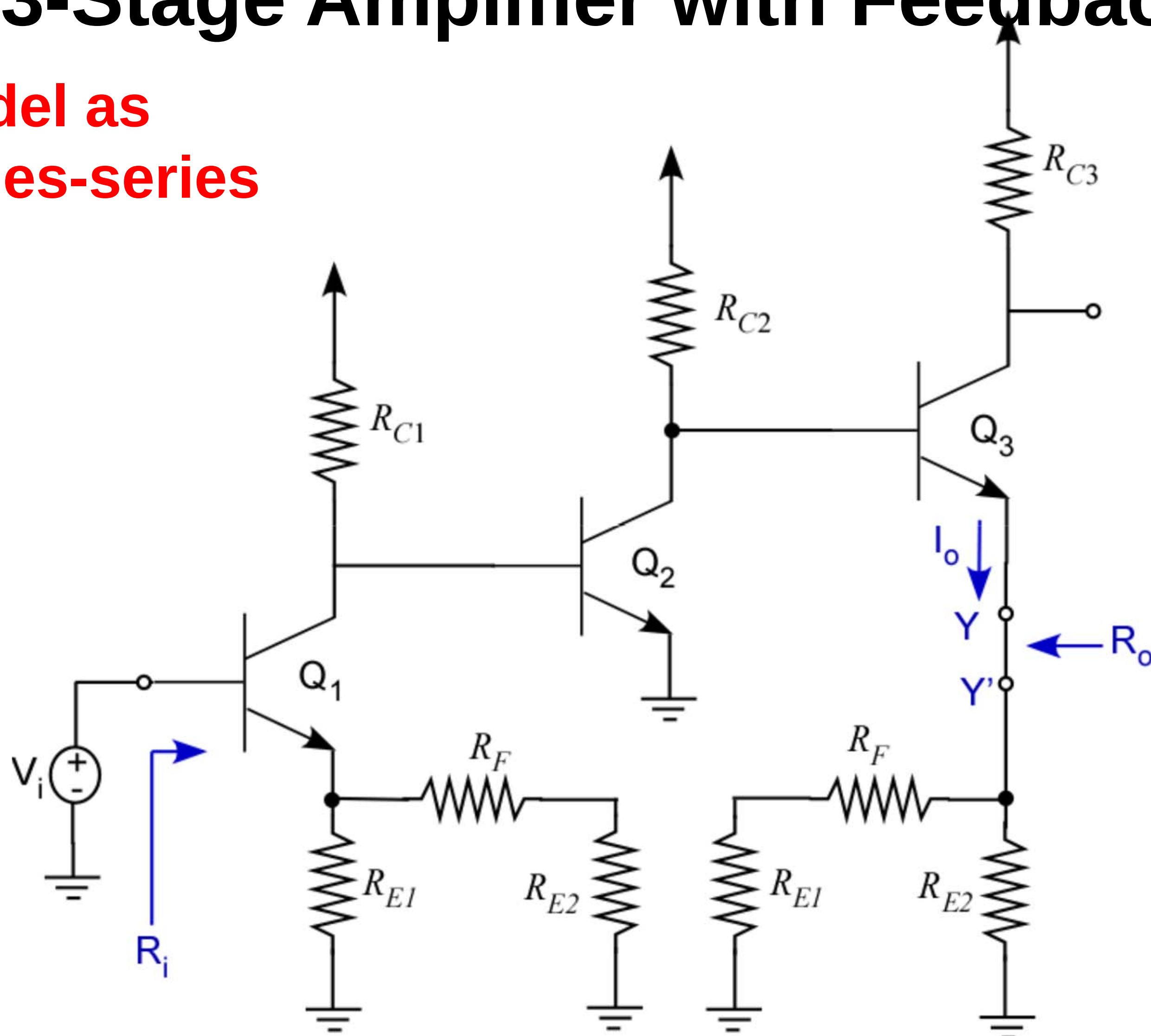
$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{4.27}{8.63} = 495\,\Omega$$

3-Stage Amplifier with Feedback



3-Stage Amplifier with Feedback

Model as series-series



3-Stage Amplifier with Feedback

Gain in first stage is:

$$\frac{V_{c1}}{V_i} = \frac{-\alpha_1\left(R_{C1} \,||\, r_{\pi 2}\right)}{r_{e1} + \left[R_{E1} \,||\, \left(R_F + R_{E2}\right)\right]}$$

First stage parameters are: I_{C1} = 0.6 mA, r_{e1}=41.7 Ω, I_{c2}=1 mA➔ $r_{\pi 2}$= h_{fe}/g_{m2}=100/40 = 2.5 kΩ

Use α_1=0.99, R_{c1}=9 kΩ, R_{E1}=100 Ω, R_F=640 Ω, and R_{E2}=100 Ω

$$\frac{V_{c1}}{V_i} = -14.92\, V/V$$

3-Stage Amplifier with Feedback

Gain in second stage is:

$$\frac{V_{c2}}{V_{c1}} = -g_{m2}\left\{R_{C2} \,||\, \left(h_{fe}+1\right)\left[r_{e3}+\left(R_{E2} \,||\, \left(R_F+R_E\right)\right)\right]\right\}$$

Use g_{m2}=40 mA/V, R_{C2}= 5 kΩ, h_{fe}=100, r_{e3}=25/4,= 6.25 Ω, R_{E2}=100 Ω, R_F = 640 Ω, and R_{E1} = 100 Ω, which gives

$$\frac{V_{c2}}{V_{c1}} = -131.2\,V/V$$

3-Stage Amplifier with Feedback

Gain in third stage is:

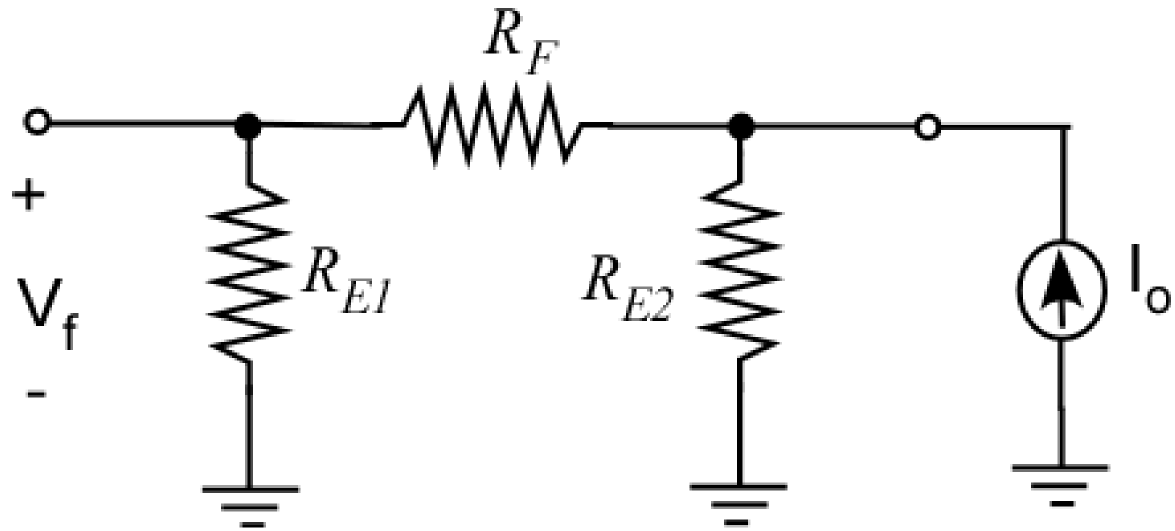
$$\frac{I_o}{V_{c2}} = \frac{I_{e3}}{V_{b3}} = \frac{1}{r_{e3} + \left(R_{E2} \,||\, \left(R_F + R_{E1}\right)\right)}$$

$$\frac{I_o}{V_{c2}} = \frac{1}{6.25 + \left(100 \,||\, 740\right)} = 10.6\, mA/V$$

Combining the 3 stages

$$A = \frac{I_o}{V_i} = -14.92 \times -131.2 \times 10.6 \times 10^{-3} = 20.7\ A/V$$

3-Stage Amplifier with Feedback



determining feedback

$$\beta = \frac{V_f}{I_o} = \frac{R_{E2}}{R_{E2} + R_F + R_{E1}} \times R_{E1}$$

$$\beta = \frac{100}{100 + 640 + 100} \times 100 = 11.9\,\Omega$$

3-Stage Amplifier with Feedback

Closed-loop gain:

$$A_f \equiv \frac{I_o}{V_s} = \frac{A}{1+A\beta} = \frac{20.7}{1+20.7\times 11.9} = 83.7\ mA/V$$

$$\frac{V_o}{V_s} = \frac{-I_C R_{C3}}{V_s} = \frac{-I_o R_{C3}}{V_s} = -A_f R_{C3}$$

$$\frac{V_o}{V_s} = -83.7\times 10^{-3} \times 600 = -50.2\ V/V$$

3-Stage Amplifier with Feedback

Input resistance

$$R_{if} = R_i\left(1 + A\beta\right)$$

$$R_i = \left(h_{fe} + 1\right)\left[r_{e1} + \left(R_{E1} \,||\, R_F + R_{E2}\right)\right] = 13.65\ k\Omega$$

$$R_{if} = 13.65\left(1 + 20.5 \times 11.9\right) = 3.34\ M\Omega$$

Output resistance

$$R_o = \left[R_{E2} \,||\, \left(R_F + R_{E1}\right)\right] + r_{e3} + \frac{R_{C2}}{h_{fe} + 1}$$

R_o=143.9 Ω

3-Stage Amplifier with Feedback

output resistance

$$R_{of} = R_o\left(1 + A\beta\right) = 143.9(1 + 20.7 + 11.9) = 35.6\,k\Omega$$

$$R_{out} = r_o + \left(1 + g_{m3} r_o\right)\left(R_{of} \,||\, r_{\pi 3}\right)$$

$$R_{out} = 25 + \left(1 + 160 \times 25\right)\left(35.6 \,||\, 0.625\right) = 2.5\,M\Omega$$

Important Remarks

1. Most of the forward transmission occurs in the basic amplifier

2. Most of the feedback -or reverse transmission - occurs in the feedback network

3. Care should be taken in the design that these assumptions are valid

Feedback and Frequency Dependence

1. The closed-loop transfer function is a function of frequency

2. The manner in which the loop gain varies with frequency determines the stability or instability of the feedback amplifier

3. The frequency at which the phase of the transfer function is equal to 180^o will be unstable if the magnitude is greater than unity

Feedback and Stability

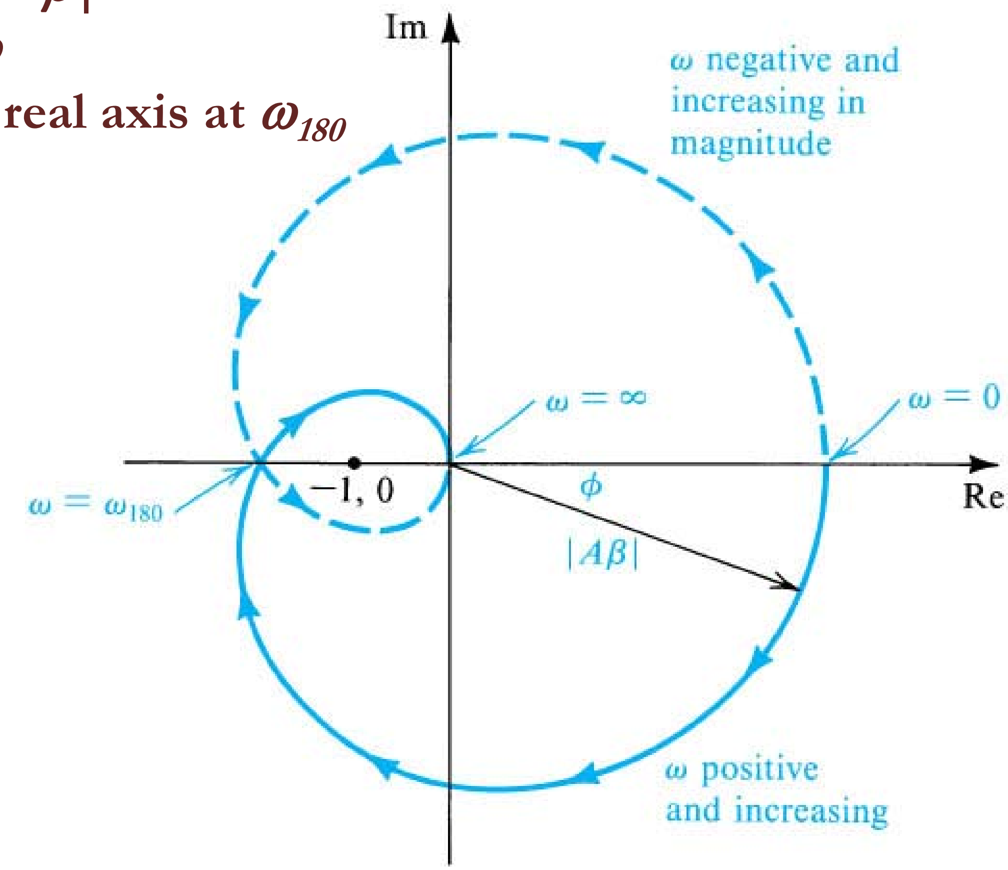
$$A_f(s) = \frac{A(s)}{1 + A(s)\beta(s)}$$

$$A_f(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)\beta(j\omega)}$$

When loop gain $A(j\omega)\beta(j\omega)$ has 180° phase, we have positive feedback

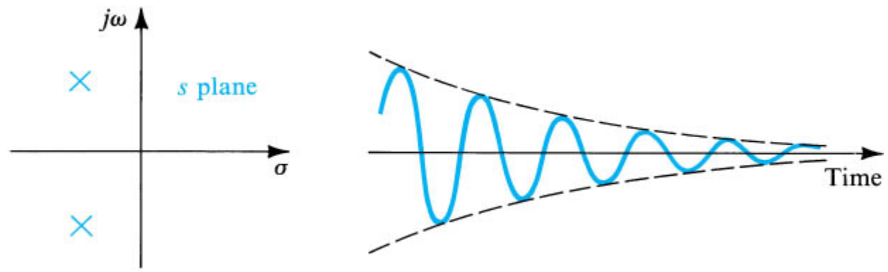
Nyquist Plot

- Radial distance is $|A\beta|$
- Angle is phase of ϕ
- Intersects negative real axis at ω_{180}

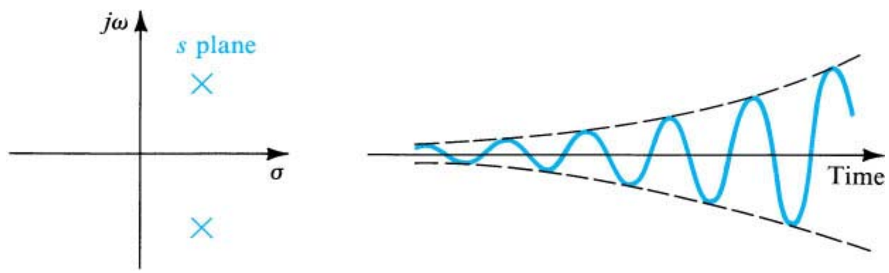


Stability and Pole Location

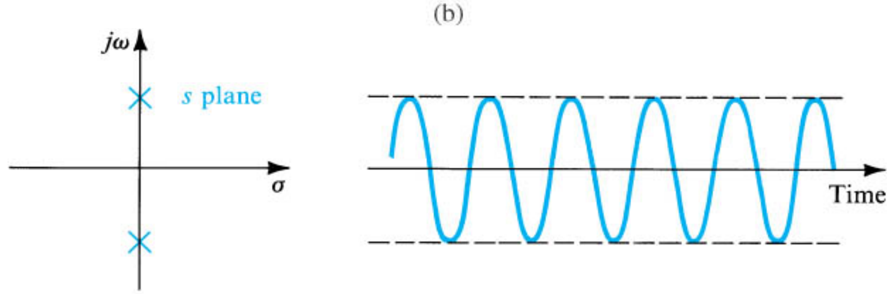
$$v(t) = e^{\sigma_o t}\left[e^{+j\omega t} + e^{-j\omega t}\right] = 2e^{\sigma_o t}\cos\left(\omega_n t\right)$$



(a)

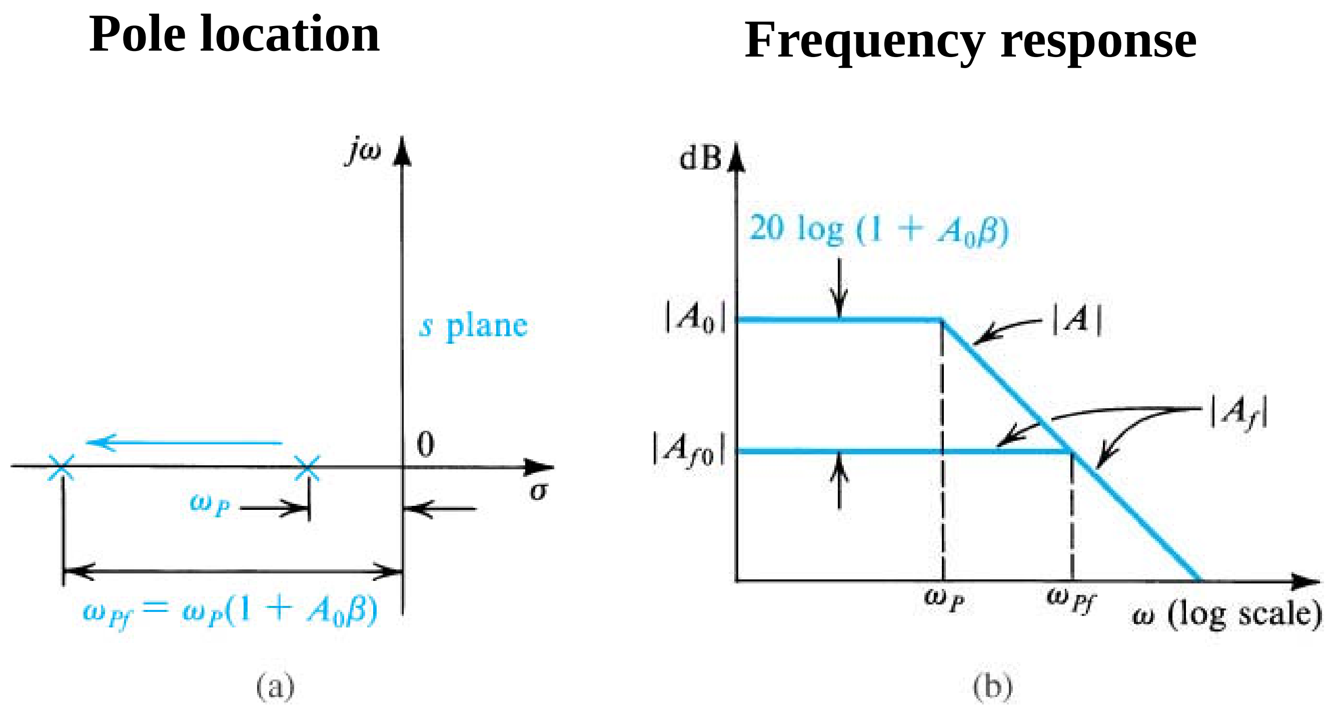


(b)



(c)

Poles of the Feedback Amplifier



Poles are the roots of the *Characteristic Equation*

$$1 + A(s)\beta(s) = 0$$

Single-Pole Amplifier

$$A(s) = \frac{A_o}{1 + s / \omega_P}$$

Closed-loop transfer function is:

$$A_f(s) = \frac{A_o / (1 + A_o \beta)}{1 + s / \omega_P (1 + A_o \beta)}$$

Feedback moves pole along negative real axis to ω_{PF}

$$\omega_{PF} = \omega_P (1 + A_o \beta)$$

Two-Pole Amplifier

$$A(s) = \frac{A_o}{(1 + s/\omega_{P1})(1 + s/\omega_{P2})}$$

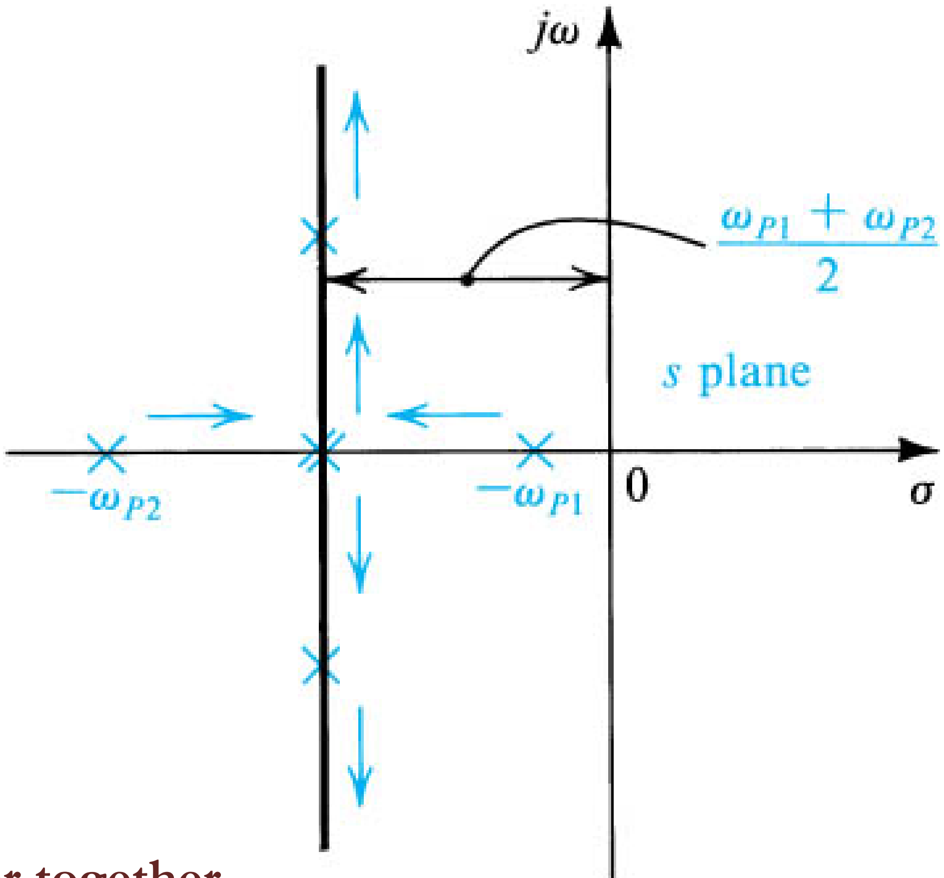
Closed-loop poles are found from *1+A(s)β* = 0

$$s^2 + s(\omega_{P1} + \omega_{P2}) + (1 + A_o\beta)\omega_{P1}\omega_{P2} = 0$$

Closed-loop poles are

$$s = -\frac{1}{2}(\omega_{P1} + \omega_{P2}) \pm \frac{1}{2}\sqrt{(\omega_{P1} + \omega_{P2})^2 - 4(1 + A_o\beta)\omega_{P1}\omega_{P2}}$$

Two-Pole Amplifier Root Locus



- Loop gain increased
- Poles are brought closer together

Gain and Phase Margins

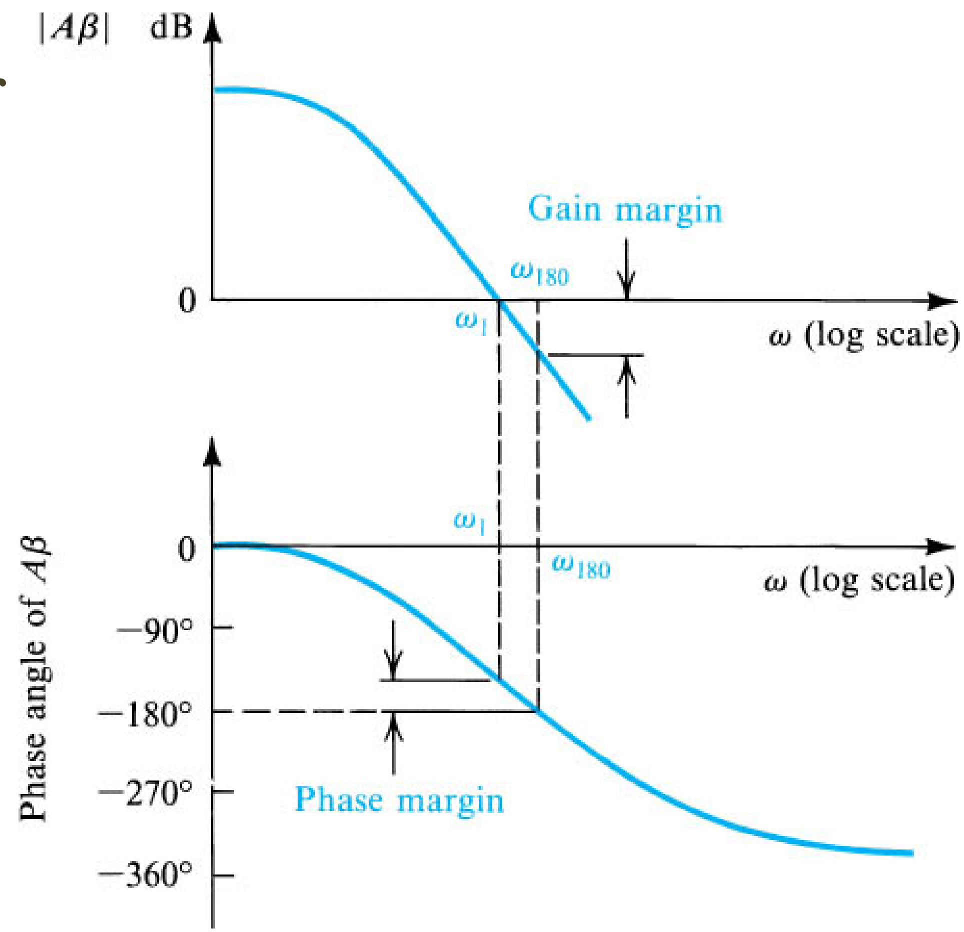
Gain Margin:

Difference between value of $|A\beta|$ at ω_{180} and unity

Phase Margin:

Difference between value of phase when $|A\beta|=1$ and 180^o

If phase angle at frequency when $|A\beta|=1$ is less than 180^o, amplifier is stable, otherwise, amplifier is unstable



Stability Analysis and Bode Plot