ECE 342
Electronic Circuits

Lecture 29
Feedback Examples

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Single-Stage Amplifier with Feedback

We want to determine the small-signal voltage gain $V_o/V_s$, the input resistance and the output resistance $R_{out} = R_{of}$. The transistor has $\beta = 100$.

Model as shunt-shunt
Single-Stage – DC Analysis

First determine dc operating point

\[ V_C = 0.7 + (I_B + 0.07)47 \]

Solve for \( I_B \) using the following 2 equations

\[ V_C = 3.99 + 47I_B \]

\[ \frac{12 - V_C}{4.7} = (\beta + 1)I_B + 0.07 \]
Single-Stage – DC Analysis

We get

\[ I_B \approx 0.015 \text{ mA} \quad I_C \approx 1.5 \text{ mA} \quad V_C \approx 4.7 \text{ V} \]

\[ g_m = \frac{I_C}{V_T} = \frac{1.5}{25} = 60 \text{ mA/V} \]

\[ r_\pi = \beta / g_m = \frac{100}{60} = 1.666 \text{ k}\Omega \]

\[ R_S \parallel r_\pi = \frac{10(1.66)}{11.6} = 1.429 \text{ k}\Omega \]

\[ R_f \parallel r_\pi = \frac{47(1.66)}{48.66} = 1.6 \text{ k}\Omega \]
Calculating $y_{11}$ for Amplifier

\[
y_{11} = \frac{1}{R_s + R_f \parallel r_\pi} = \frac{1}{10 + 1.6} = 0.086 \text{ mA/V}
\]
Calculating $y_{21}$ for Amplifier

\[ y_{21} = \frac{I_2}{V_1} \]

\[ v_\pi = \frac{V_1 \left( R_f \parallel r_\pi \right)}{R_S + R_f \parallel r_\pi} = \left( g_m - \frac{1}{R_f} \right) \frac{V_1 \left( R_f \parallel r_\pi \right)}{R_S + R_f \parallel r_\pi} \]

\[ I_2 = \left( g_m - \frac{1}{R_f} \right) \frac{V_1 \left( R_f \parallel r_\pi \right)}{R_S + R_f \parallel r_\pi} \]

\[ y_{21} = \left( g_m - \frac{1}{R_f} \right) \frac{\left( R_f \parallel r_\pi \right)}{R_S + R_f \parallel r_\pi} \]
Calculating $y_{21}$ for Amplifier

$$y_{21} = \left(60 - 0.021\right) \frac{1.6}{10 + 1.6} = 8.27 \text{ mA/V}$$
Calculating $y_{12}$ for Amplifier

\[
y_{12} = \frac{I_1}{V_2} = -\frac{-V_2}{R_S V_2} = \frac{-V_2 \left( R_S \parallel r_\pi \right)}{R_f + \left( R_S \parallel r_\pi \right) R_S V_2} \frac{1}{1} \frac{1}{V_2}
\]

\[
y_{12} = -\frac{1}{R_S} \frac{\left( R_S \parallel r_\pi \right)}{R_f + \left( R_S \parallel r_\pi \right)}
\]
Calculating $y_{12}$ for Amplifier

\[ y_{12} = -\frac{1.429}{10(47 + 1.429)} = -0.00295 \text{ mA/V} \]
Calculating $y_{22}$ for Amplifier

$$I_2 = \frac{V_2}{R_C} + \frac{g_m V_2 (R_S \parallel r_\pi)}{(R_S \parallel r_\pi) + R_f} + \frac{V_2}{(R_S \parallel r_\pi) + R_f}$$

$$y_{22} = \frac{I_2}{V_2} = \frac{1}{R_C} + \frac{g_m (R_S \parallel r_\pi)}{(R_S \parallel r_\pi) + R_f} + \frac{1}{(R_S \parallel r_\pi) + R_f}$$

$$y_{22} = \frac{I_2}{V_2} = \frac{1}{4.7} + \frac{60(1.429)}{1.429 + 47} + \frac{1}{1.429 + 47} = 2.01 \text{ mA/V}$$
y-parameters for Feedback Network

\[ y_{11} = \frac{1}{R_f} = 0.021 \text{ mA/V} \quad y_{22} = y_{11} \text{ by symmetry} \]

\[ y_{21} = -\frac{1}{R_f} = -0.021 \text{ mA/V} \quad y_{12} = y_{21} \text{ by reciprocity} \]

From Feedback Network, we get \( \beta = y_{12} = -0.021 \)
Basic Amplifier vs Feedback Network

\[
Y_A = \begin{bmatrix}
0.086 & -0.003 \\
8.27 & 2.01
\end{bmatrix}
\]

\[
Y_F = \begin{bmatrix}
0.021 & -0.021 \\
-0.021 & 0.021
\end{bmatrix}
\]

\[
\begin{align*}
|y_{12}|_{\text{basic amplifier}} & \iff |y_{12}|_{\text{feedback network}} \\
|y_{21}|_{\text{feedback network}} & \iff |y_{21}|_{\text{basic amplifier}}
\end{align*}
\]
Single-Stage – Small-Signal Analysis
Single-Stage – Small-Signal Analysis

The feedback is provided by $R_f$ which samples the output voltage and feeds back a current to be mixed at input

$$V_\pi = I_i \left( R_s \parallel R_f \parallel r_\pi \right)$$

$$V_o = -g_m V_\pi \left( R_f \parallel R_C \right)$$

$$A = \frac{V_o}{I_i} = -g_m \left( R_f \parallel R_C \right) \left( R_s \parallel R_f \parallel r_\pi \right) = -358.7 \text{ k}\Omega$$

Transimpedance gain is $-358.7 \text{ k}\Omega$
Input and Output Resistances

\[ R_i = R_s \parallel R_f \parallel r_\pi = 1.4 \, k\Omega \]

\[ R_o = R_C \parallel R_f = 4.27 \, k\Omega \]
Determining $\beta$ and $A_f$

\[ \beta = \frac{I_f}{V_o} = - \frac{1}{R_f} = - \frac{1}{47 \text{ k}\Omega} \]

\[ A_f \equiv \frac{V_o}{I_s} = \frac{A}{1 + A\beta} \]

\[ \frac{V_o}{I_s} = \frac{-358.7}{1 + 358.7/47} = \frac{-358.7}{8.63} = -41.6 \text{ k}\Omega \]
Single-Stage Feedback Amp

Voltage gain is:

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{-41.6}{10} \approx -4.16 \, V / V$$

The input resistance with feedback is:

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{1.4}{8.63} = 162.2 \, \Omega$$

The output resistance with feedback is:

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{4.27}{8.63} = 495 \, \Omega$$
3-Stage Amplifier with Feedback

\[
\begin{align*}
R_{C1} &= 9 \, k\Omega \\
R_{C2} &= 5 \, k\Omega \\
R_{C3} &= 600 \, \Omega \\
R_F &= 640 \, \Omega \\
R_{E1} &= 100 \, \Omega \\
R_{E2} &= 100 \, \Omega \\
R_{in} &= R_{if} \\
I_c &= I_o \\
V_s &= R_{of}
\end{align*}
\]
3-Stage Amplifier with Feedback

Model as series-series
3-Stage Amplifier with Feedback

Gain in first stage is:

\[
\frac{V_{c1}}{V_i} = \frac{-\alpha_1 \left( R_{C1} \parallel r_{\pi2} \right)}{r_{e1} + \left[R_{E1} \parallel (R_F + R_{E2})\right]}
\]

First stage parameters are: \(I_{C1} = 0.6 \text{ mA}\), \(r_{e1} = 41.7 \ \Omega\), \(I_{C2} = 1 \text{ mA}\) \(\Rightarrow r_{\pi2} = h_{fe}/g_{m2} = 100/40 = 2.5 \text{ k}\Omega\)

Use \(\alpha_1 = 0.99\), \(R_{c1} = 9 \text{ k}\Omega\), \(R_{E1} = 100 \ \Omega\), \(R_F = 640 \ \Omega\), and \(R_{E2} = 100 \ \Omega\)

\[
\frac{V_{c1}}{V_i} = -14.92 \ V / V
\]
3-Stage Amplifier with Feedback

Gain in second stage is:

\[
\frac{V_{c2}}{V_{c1}} = -g_{m2} \left\{ R_{C2} \parallel \left( h_{fe} + 1 \right) \left[ r_e + \left( R_{E2} \parallel \left( R_F + R_E \right) \right) \right] \right\}
\]

Use \( g_{m2} = 40 \) mA/V, \( R_{C2} = 5 \) kΩ, \( h_{fe} = 100 \), \( r_e = 25/4 = 6.25 \) Ω, \( R_{E2} = 100 \) Ω, \( R_F = 640 \) Ω, and \( R_{E1} = 100 \) Ω, which gives

\[
\frac{V_{c2}}{V_{c1}} = -131.2 \, V / V
\]
3-Stage Amplifier with Feedback

Gain in third stage is:

\[
\frac{I_o}{V_{c2}} = \frac{I_{e3}}{V_{b3}} = \frac{1}{r_e + \left( R_{E2} \parallel (R_F + R_{E1}) \right)}
\]

\[
\frac{I_o}{V_{c2}} = \frac{1}{6.25 + (100 \parallel 740)} = 10.6 \text{ mA/V}
\]

Combining the 3 stages

\[
A = \frac{I_o}{V_i} = -14.92 \times -131.2 \times 10.6 \times 10^{-3} = 20.7 \text{ A/V}
\]
3-Stage Amplifier with Feedback

determining feedback

\[ \beta = \frac{V_f}{I_o} = \frac{R_{E2}}{R_{E2} + R_F + R_{E1}} \times R_{E1} \]

\[ \beta = \frac{100}{100 + 640 + 100} \times 100 = 11.9 \, \Omega \]
3-Stage Amplifier with Feedback

Closed-loop gain:

$$A_f \equiv \frac{I_o}{V_s} = \frac{A}{1 + A\beta} = \frac{20.7}{1 + 20.7 \times 11.9} = 83.7 \ mA/V$$

$$\frac{V_o}{V_s} = \frac{-I_C R_{C3}}{V_s} = \frac{-I_o R_{C3}}{V_s} = -A_f R_{C3}$$

$$\frac{V_o}{V_s} = -83.7 \times 10^{-3} \times 600 = -50.2 \ V/V$$
3-Stage Amplifier with Feedback

Input resistance

\[ R_{if} = R_i (1 + A\beta) \]

\[ R_i = (h_{fe} + 1) \left[ r_e + \left( R_{E1} \parallel R_F + R_{E2} \right) \right] = 13.65 \, k\Omega \]

\[ R_{if} = 13.65 (1 + 20.5 \times 11.9) = 3.34 \, M\Omega \]

Output resistance

\[ R_o = \left[ R_{E2} \parallel \left( R_F + R_{E1} \right) \right] + r_e + \frac{R_{C2}}{h_{fe} + 1} \]

\[ R_o = 143.9 \, \Omega \]
3-Stage Amplifier with Feedback

output resistance

\[ R_{of} = R_o \left(1 + A\beta \right) = 143.9(1 + 20.7 + 11.9) = 35.6 \, k\Omega \]

\[ R_{out} = r_o + \left(1 + g_{m3}r_o \right)\left(R_{of} \parallel r_{\pi3} \right) \]

\[ R_{out} = 25 + \left(1 + 160 \times 25 \right)(35.6 \parallel 0.625) = 2.5 \, M\Omega \]
Important Remarks

1. Most of the forward transmission occurs in the basic amplifier

2. Most of the feedback -or reverse transmission - occurs in the feedback network

3. Care should be taken in the design that these assumptions are valid
Feedback and Frequency Dependence

1. The closed-loop transfer function is a function of frequency.

2. The manner in which the loop gain varies with frequency determines the stability or instability of the feedback amplifier.

3. The frequency at which the phase of the transfer function is equal to 180° will be unstable if the magnitude is greater than unity.
Feedback and Stability

\[ A_f(s) = \frac{A(s)}{1 + A(s) \beta(s)} \]

\[ A_f(j\omega) = \frac{A(j\omega)}{1 + A(j\omega) \beta(j\omega)} \]

When loop gain \( A(j\omega) \beta(j\omega) \) has 180° phase, we have positive feedback
Nyquist Plot

- Radial distance is \(|A\beta|\)
- Angle is phase of \(\phi\)
- Intersects negative real axis at \(\omega_{180}\)
Stability and Pole Location

\[ \nu(t) = e^{\sigma t} \left[ e^{j\omega t} + e^{-j\omega t} \right] = 2e^{\sigma t} \cos(\omega_n t) \]
Poles of the Feedback Amplifier

Pole location

Frequency response

Poles are the roots of the Characteristic Equation

\[ 1 + A(s) \beta(s) = 0 \]
Single-Pole Amplifier

\[ A(s) = \frac{A_o}{1 + s/\omega_P} \]

Closed-loop transfer function is:

\[ A_f(s) = \frac{A_o / (1 + A_o \beta)}{1 + s/\omega_P (1 + A_o \beta)} \]

Feedback moves pole along negative real axis to \( \omega_{PF} \)

\[ \omega_{PF} = \omega_P (1 + A_o \beta) \]
Two-Pole Amplifier

\[ A(s) = \frac{A_o}{(1 + s / \omega_{P1})(1 + s / \omega_{P2})} \]

Closed-loop poles are found from \( 1 + A(s)\beta = 0 \)

\[ s^2 + s(\omega_{P1} + \omega_{P2}) + (1 + A_o\beta)\omega_{P1}\omega_{P2} = 0 \]

Closed-loop poles are

\[ s = -\frac{1}{2}(\omega_{P1} + \omega_{P2}) \pm \frac{1}{2} \sqrt{\left(\omega_{P1} + \omega_{P2}\right)^2 - 4(1 + A_o\beta)\omega_{P1}\omega_{P2}} \]
Two-Pole Amplifier Root Locus

- Loop gain increased
- Poles are brought closer together
Gain and Phase Margins

Gain Margin:
Difference between value of $|A\beta|$ at $\omega_{180}$ and unity

Phase Margin:
Difference between value of phase when $|A\beta| = 1$ and $180^\circ$

If phase angle at frequency when $|A\beta| = 1$ is less than $180^\circ$, amplifier is stable, otherwise, amplifier is unstable
Stability Analysis and Bode Plot

- $20 \log |A\beta|$ for zero margins
- $20 \log 1/\beta = 50 \text{ dB (unstable)}$
- $20 \log 1/\beta = 85 \text{ dB (stable)}$
- $-20 \text{ dB/decade}$
- $25 \text{ dB gain margin}$
- $X_1$
- $X_2$
- $-40 \text{ dB/decade}$
- $-60 \text{ dB/decade}$
- $f_{180}$
- $-108^\circ$ phase margin
- $72^\circ$ phase margin