Operational Amplifiers

- Universal importance (e.g. amplification from microphone to loudspeakers)

General terminal configuration with bias
Operational Amplifiers

Gain = \( A \)

\[ v_{out} = A(v_2 - v_1) \]

Common configuration with bias implied but not shown

- **Signaling**
  1. Differential input stage
  2. Difference between input is amplified
Operational Amplifiers

• Ideal Op Amp

1. Infinite input impedance
2. Zero output impedance
3. Infinite open-loop gain $A \rightarrow \infty$
4. Infinite CMRR or zero common-mode gain
5. Infinite bandwidth

Also, op amps are dc (or direct coupled) amplifiers since they are expected to amplify signals with frequency as low as DC.
Differential & Common-Mode Signals

- Differential input signal $v_{ID} = v_2 - v_1$
- Common-mode input signal $v_{ICm} = 0.5(v_1 + v_2)$

$$v_1 = v_{ICm} - \frac{v_{ID}}{2}$$

$$v_2 = v_{ICm} + \frac{v_{ID}}{2}$$
Ideally, $v_{ICM}$ should be zero to achieve high CMRR.

- Amplifier will amplify the **difference** between the two input signals
Practical Considerations

The output voltage swing of an op amp is limited by the DC power supply. Since op amp can exhibit high gain, power supply voltage fluctuations must be minimized \( \Rightarrow \) use decoupling capacitors from power supply
Inverting Configuration

We introduce $R_F$ (or $R_2$) to reduce gain (from inf)

- When $R_F$ is connected to terminal 1, we talk about negative feedback. If $R_F$ is tied to terminal 2, we have positive feedback

Terminal 2 is tied to ground
Inverting Configuration

Need to evaluate $v_o/v_I$

Assume ideal Op-Amp

Since gain is infinite:

$$\frac{v_o}{A} = (v_2 - v_1) = 0$$

Thus, $v_1 = v_2 = 0$

Note: $A$ is open-loop gain

$v_1$ is virtual ground

$$i_1 = \frac{v_I - v_1}{R_1} = \frac{v_I}{R_1}$$
Inverting Configuration

Since input impedance of OP amp is infinite, current through \( R_F \) is \( i_1 \)

\[
i = i_1 = \frac{v_I}{R_1}
\]

\[
\frac{v_1 - v_o}{R_F} = \frac{0 - v_o}{R_F} = i = \frac{v_I}{R_1}
\]

\[
-\frac{v_o}{R_F} = \frac{v_I}{R_1} \implies v_o = -v_I \frac{R_F}{R_1}
\]
Inverting Configuration

Closed-Loop gain

\[
\frac{v_o}{v_I} = -\frac{R_F}{R_1} = G
\]

Observe that the closed-loop gain is the ratio of external components \(\Rightarrow\) we can make the closed-loop as accurate as we want. Gain is smaller but more accurate.
Inverting Configuration

We assumed that the OP-amp was ideal. If we assume that the gain $A$ is finite $= A$

$$\left( v_2 - v_1 \right) A = v_o \Rightarrow v_1 = -v_o / A$$

$$i_1 = \frac{v_{in} - (-v_o / A)}{R_1} = \frac{v_{in} + v_o / A}{R_1}$$
Inverting Configuration

Still assume infinite input impedance

\[ v_o = -\frac{v_o}{A} - iR_F = -\frac{v_o}{A} - \left(\frac{v_i + v_o}{A}\right)\frac{R_F}{R_1} \]

\[ G = \frac{v_o}{v_i} = \frac{-R_F / R_1}{\left(1 + R_F / R_1\right) / A} = \frac{-AR_F}{R_1 \left(1 + A\right) + R_F} \]

Closed-loop gain for inverting configuration
Inverting Configuration

The reflected impedance of $R_F$ is given by

\[
R_R = \frac{v_1}{i_1} = \frac{-\left(\frac{v_o}{A}\right)R_1}{v_o + \frac{v_o}{A}}
\]

\[
R_R = \frac{-R_1}{A} \frac{A}{1 + \frac{R_1}{G}}
\]

since

\[
\frac{A}{G} = -1 - \frac{-R_1}{R_F}(1 + A)
\]

\[
R_R = \frac{R_F}{1 + A}
\]

⇒ small
Inverting Configuration

Since the reflected impedance is so small, \( v_1 \) is thus very small and the inverting terminal is said to be a virtual ground in this configuration.

We see that

\[
\text{as } A \to \infty, \quad G \to -\frac{R_F}{R_1}
\]

\[
\text{as } A \to \infty, \quad R_R \to 0
\]

Note: To minimize the closed-loop gain (G) on the value of the open-loop gain (A), make

\[
1 + \frac{R_F}{R_1} \ll A
\]
Input and Output Impedances

Inverting Configuration

\[ R_i \equiv \frac{V_i}{i_1} = \frac{V_i}{v_i / R_1} = R_1 \]

- If high gain is required, input impedance will be low
- Output impedance is zero
Example

Find closed-loop gain for $A=10^3$, $A=10^4$ and $A=10^5$ assuming $R_1=1$ kΩ and $R_F=100$ kΩ. Assuming $v_I=0.1$ V, find $v_1$.

Using formulas

| $A$  | $|G|$   | $v_1$       |
|------|---------|-------------|
| $10^3$ | 90.83   | -9.08 mV    |
| $10^4$ | 99.00   | -0.99 mV    |
| $10^5$ | 99.90   | -0.1 mV     |

Note: Since output of inverting configuration is at terminal of VCVS, output impedance of closed-loop amp is zero.
Non-Inverting Configuration

Assume gain is \( \infty \) \( \Rightarrow \) \( v_{ID} = \frac{v_o}{A} \rightarrow 0 \)
Non-Inverting Configuration

\[ v_{ID} = 0 \Rightarrow v_1 \approx v_2 \Rightarrow \text{virtual short} \]

Infinite input impedance \( \Rightarrow i_1 = 0 \)

Therefore

\[
\frac{v_1 - v_o}{R_F} = \frac{0 - v_1}{R_a}
\]

\[
\frac{v_1 \left( R_a + R_F \right)}{R_a} = v_o
\]
Non-Inverting Configuration

Virtual short \( \Rightarrow v_2 = v_I = v_1 \)

\[
 v_I \left( \frac{R_a + R_F}{R_a} \right) = v_o
\]

\[
 G = \frac{v_o}{v_I} = 1 + \frac{R_F}{R_a}
\]

\[
 G = 1 + \frac{R_F}{R_a}
\]
The Buffer Stage

If $R_F \rightarrow 0$, $G = 1 + \frac{R_F}{R_a} \rightarrow 1$

Although voltage gain is low, current gain can be quite high. Buffer stage can be used to interface between processors and switches.
The Voltage Follower

- Unity gain amplifier
- 100% negative feedback

\[ R_{in} = \infty \]
\[ R_{out} = 0 \]