ECE 451
Coupled Lines

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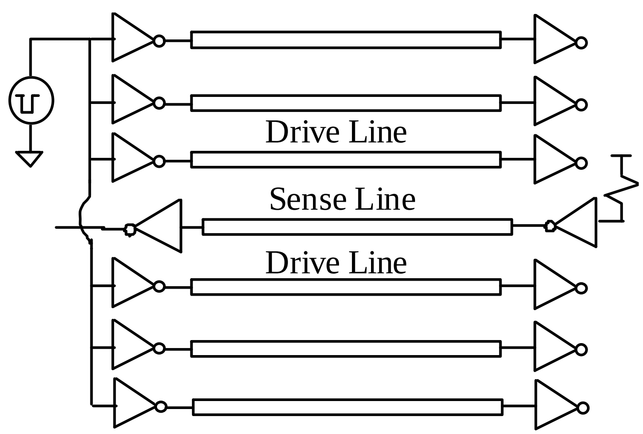
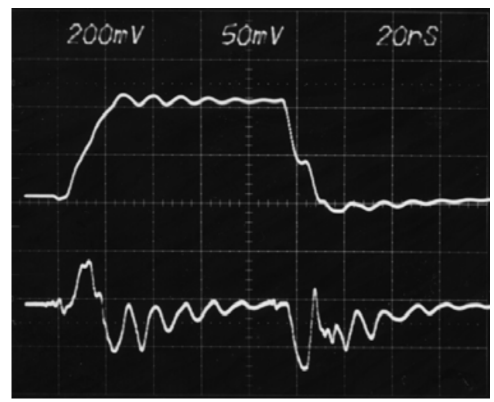
Crosstalk Noise

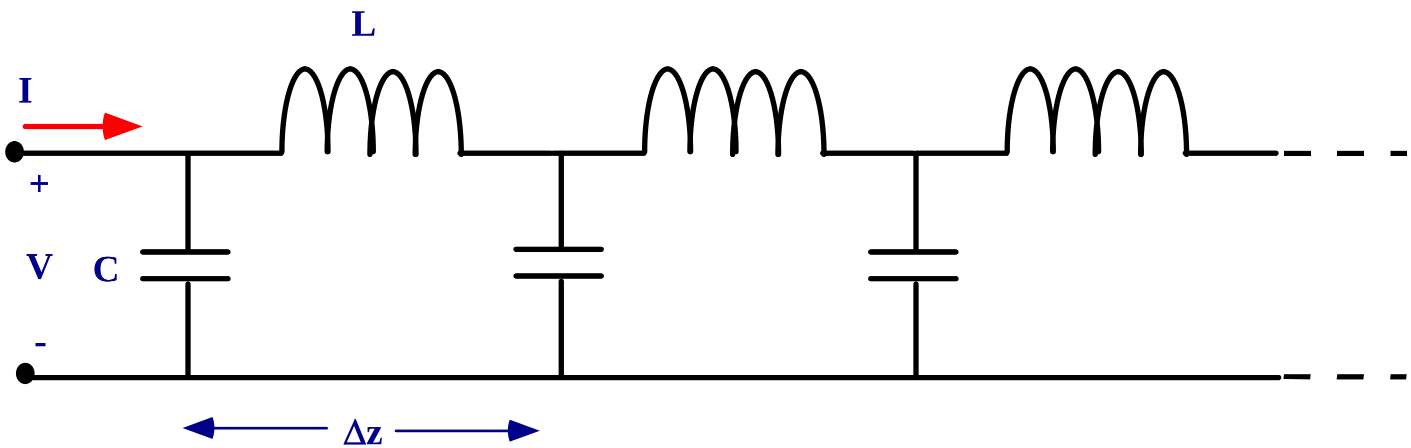
Signal Integrity

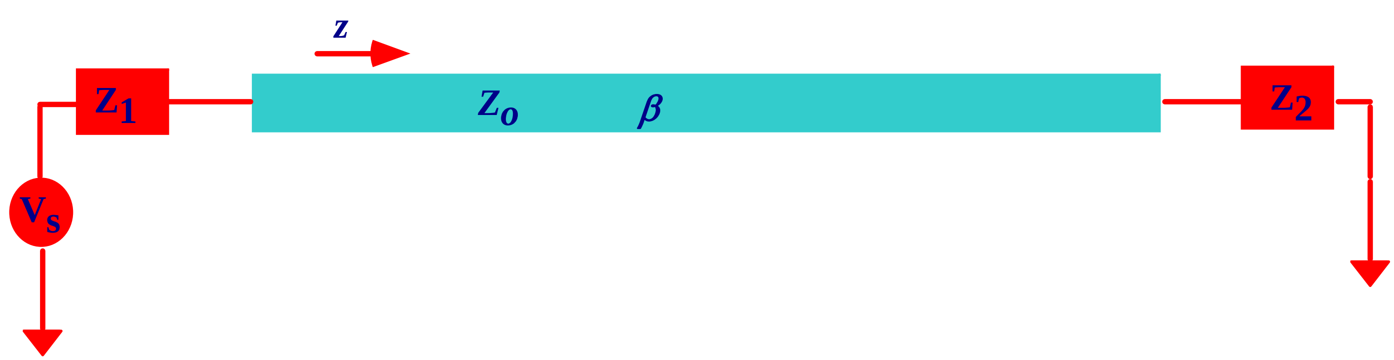
Crosstalk	**Dispersion**	**Attenuation**
Reflection	**Distortion**	**Loss**
Delta I Noise	**Ground Bounce**	**Radiation**



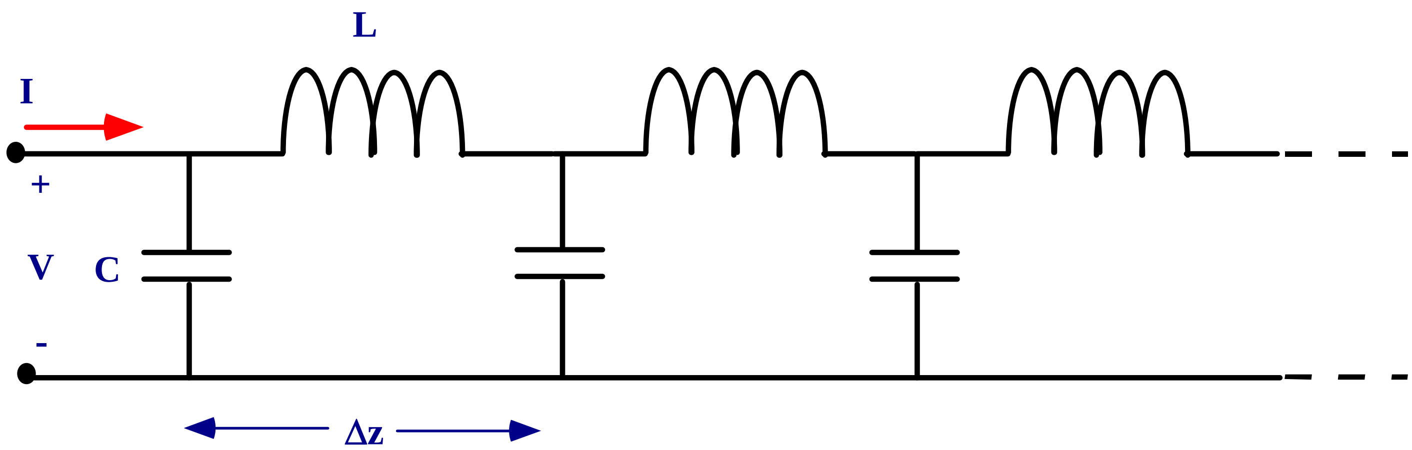


TEM PROPAGATION





Telegrapher's Equations



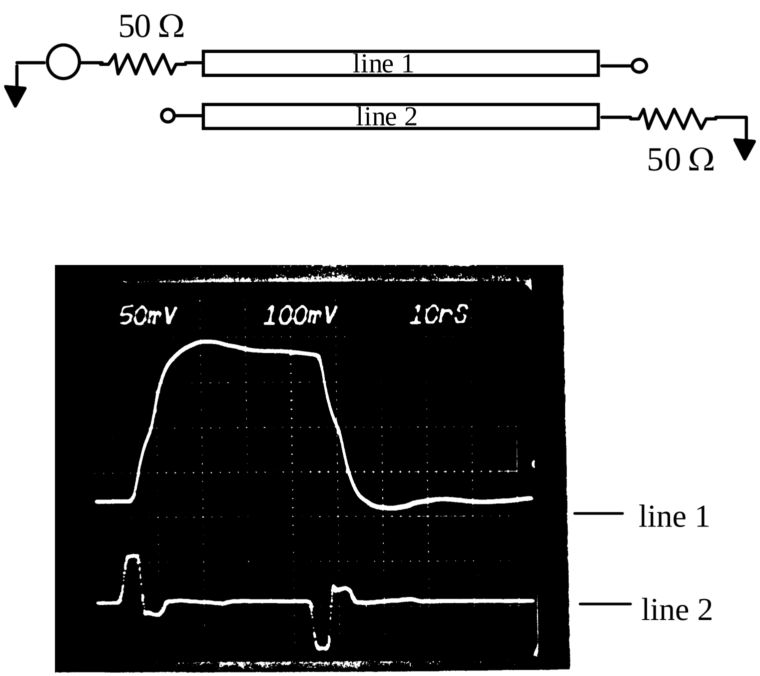
$$-\frac{\partial V}{\partial z} = L\frac{\partial I}{\partial t}$$

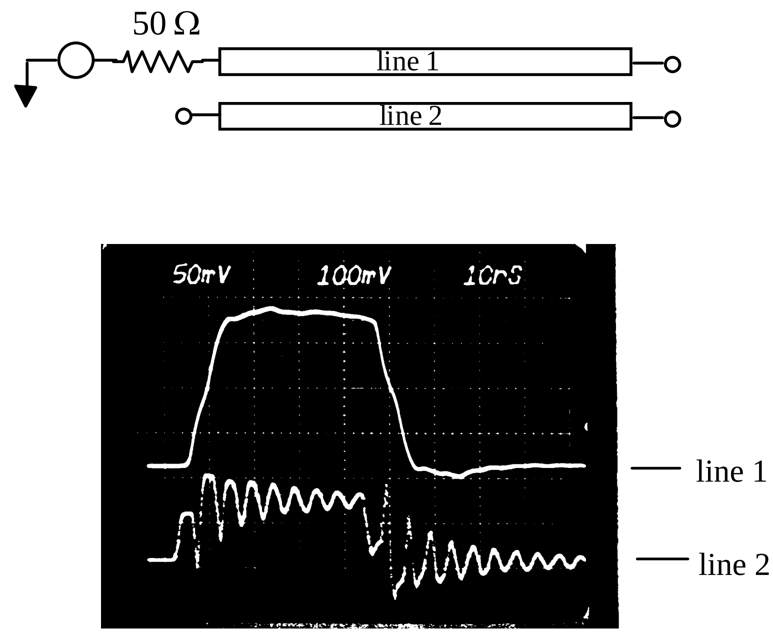
$$-\frac{\partial I}{\partial z} = C\frac{\partial V}{\partial t}$$

***L*: Inductance per unit length.**

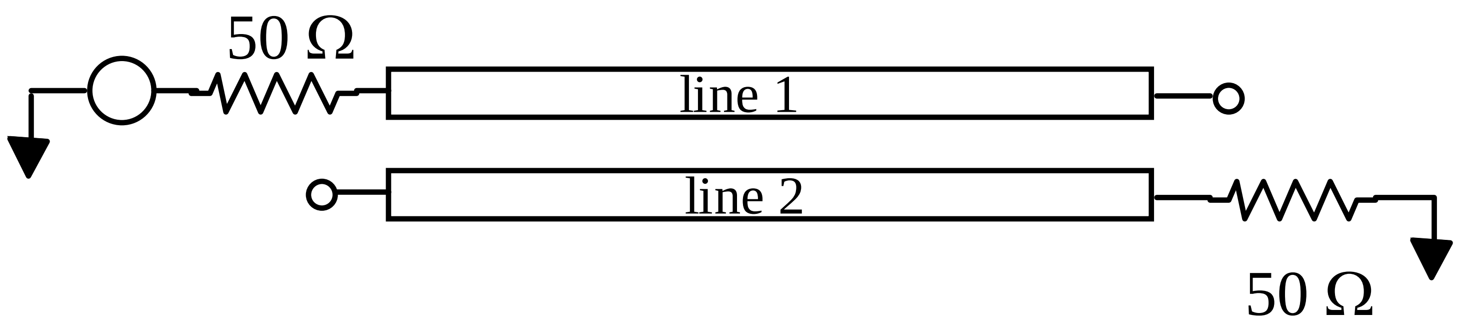
***C*: Capacitance per unit length.**

Crosstalk noise depends on termination

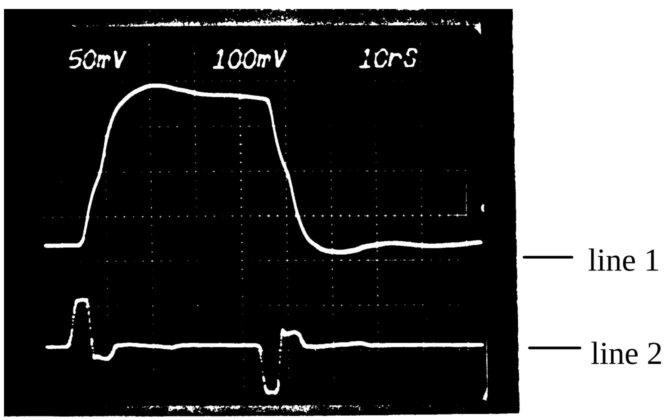




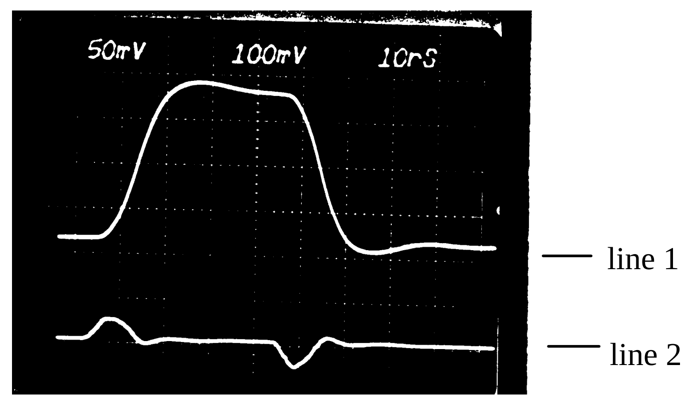
Crosstalk depends on signal rise time



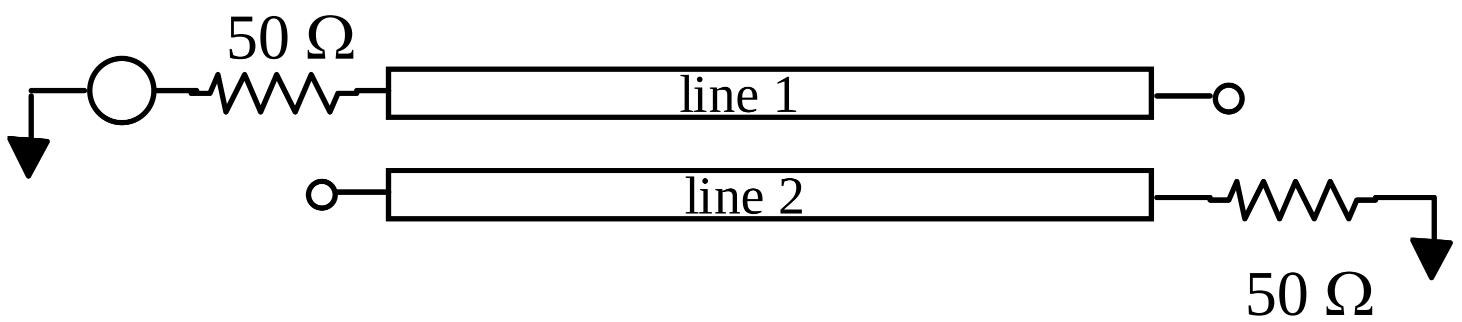
t_r = 1 ns



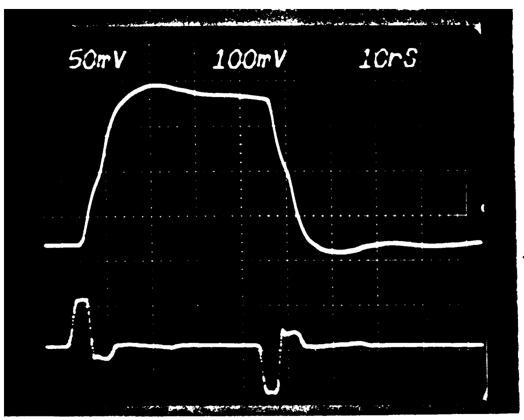
t_r = 7 ns



Crosstalk depends on signal rise time



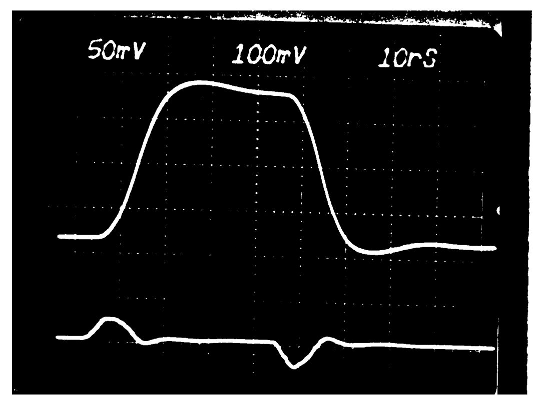
t_r = 1 ns



line 1

line 2

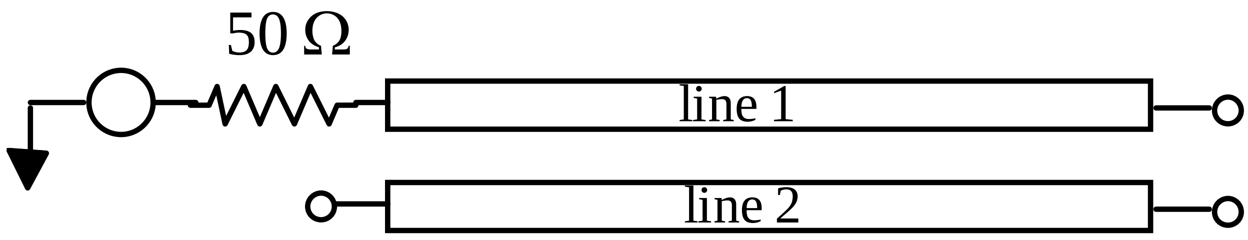
t_r = 7 ns



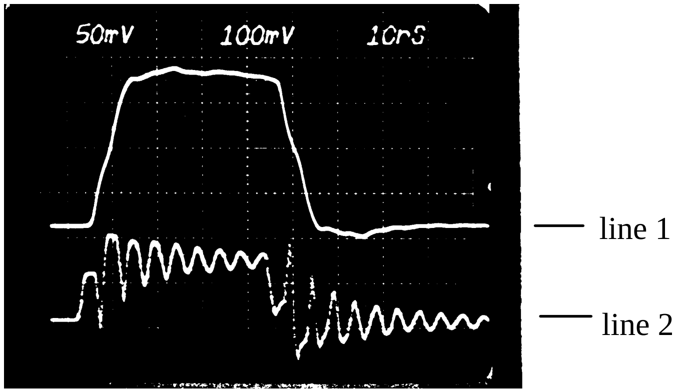
line 1

line 2

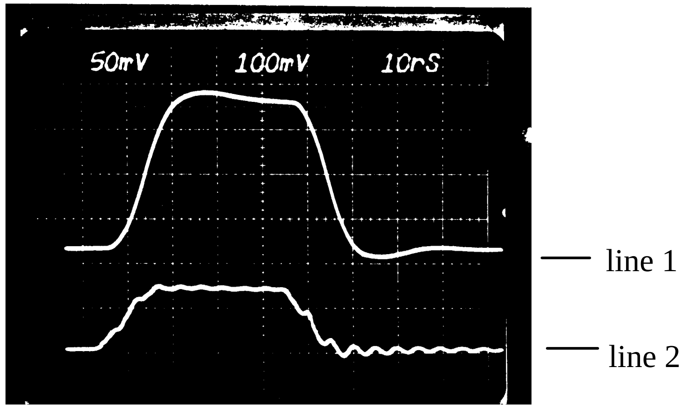
Crosstalk depends on signal rise time



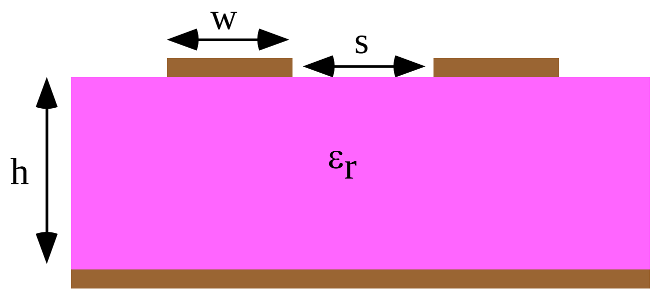
t_r = 1 ns

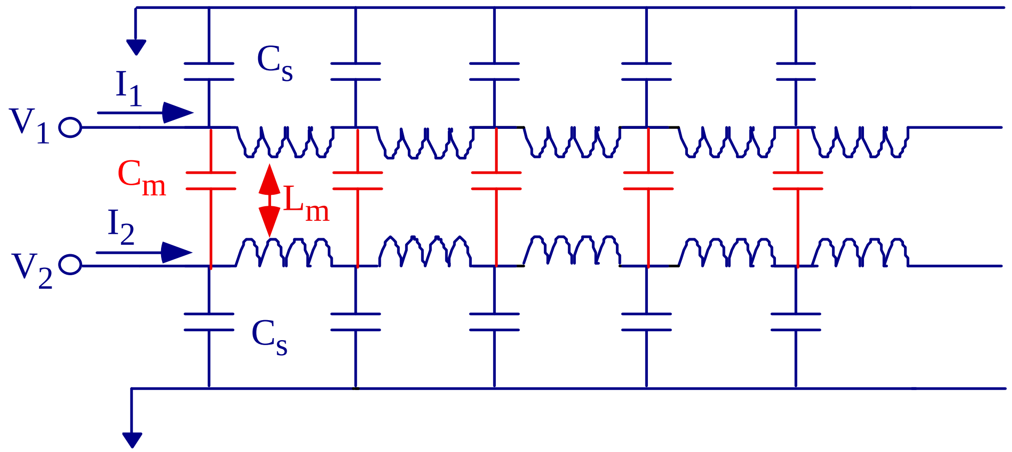


t_r = 7 ns



Coupled Transmission Lines





Telegraphers Equations for Coupled Transmission Lines

Maxwellian Form

$$-\frac{\partial V_1}{\partial z} = L_{11}\frac{\partial I_1}{\partial t} + L_{12}\frac{\partial I_2}{\partial t}$$

$$-\frac{\partial V_2}{\partial z} = L_{21}\frac{\partial I_1}{\partial t} + L_{22}\frac{\partial I_2}{\partial t}$$

$$-\frac{\partial I_1}{\partial z} = C_{11}\frac{\partial V_1}{\partial t} + C_{12}\frac{\partial V_2}{\partial t}$$

$$-\frac{\partial I_2}{\partial z} = C_{21}\frac{\partial V_1}{\partial t} + C_{22}\frac{\partial V_2}{\partial t}$$

Telegraphers Equations for Coupled Transmission Lines

Physical form

$$-\frac{\partial V_1}{\partial z} = L_s \frac{\partial I_1}{\partial t} + L_m \frac{\partial I_2}{\partial t}$$

$$-\frac{\partial V_2}{\partial z} = L_m \frac{\partial I_1}{\partial t} + L_s \frac{\partial I_2}{\partial t}$$

$$-\frac{\partial I_1}{\partial z} = C_s \frac{\partial V_1}{\partial t} + C_m \frac{\partial V_1}{\partial t} - C_m \frac{\partial V_2}{\partial t}$$

$$-\frac{\partial I_2}{\partial z} = -C_m \frac{\partial V_1}{\partial t} + C_m \frac{\partial V_2}{\partial t} + C_s \frac{\partial V_2}{\partial t}$$

Relations Between Physical and Maxwellian Parameters (symmetric lines)

$$L_{11} = L_{22} = L_s$$

$$L_{12} = L_{21} = L_m$$

$$C_{11} = C_{22} = C_s+C_m$$

$$C_{12} = C_{21} = - C_m$$

Even Mode

$$-\frac{\partial V_e}{\partial z} = \left(L_{11} + L_{12}\right)\frac{\partial I_e}{\partial t}$$

$$-\frac{\partial I_e}{\partial z} = \left(C_{11} + C_{12}\right)\frac{\partial I_e}{\partial t}$$

Add voltage and current equations

V_e : Even mode voltage $\quad V_e = \frac{1}{2}\left(V_1 + V_2\right)$

I_e : Even mode current $\quad I_e = \frac{1}{2}\left(I_1 + I_2\right)$

$$Z_e = \sqrt{\frac{L_{11} + L_{12}}{C_{11} + C_{12}}} = \sqrt{\frac{L_s + L_m}{C_s}}$$

Impedance

$$v_e = \frac{1}{\sqrt{(L_{11} + L_{12})(C_{11} + C_{12})}} = \frac{1}{\sqrt{(L_s + L_m)C_s}}$$

velocity

Odd Mode

$$-\frac{\partial V_d}{\partial z} = \left(L_{11} - L_{12}\right)\frac{\partial I_d}{\partial t}$$

$$-\frac{\partial I_d}{\partial z} = \left(C_{11} - C_{12}\right)\frac{\partial I_d}{\partial t}$$

Subtract voltage and current equations

V_d : Odd mode voltage $\quad V_d = \frac{1}{2}\left(V_1 - V_2\right)$

I_d : Odd mode current $\quad I_d = \frac{1}{2}\left(I_1 - I_2\right)$

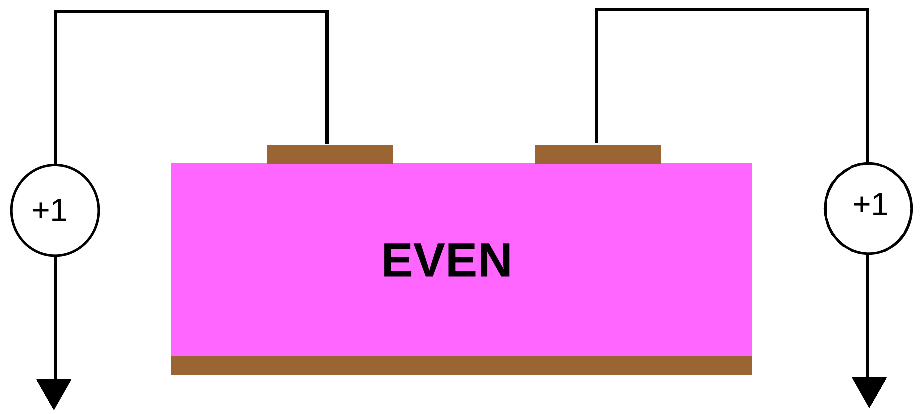
$$Z_d = \sqrt{\frac{L_{11} - L_{12}}{C_{11} - C_{12}}} = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}}$$

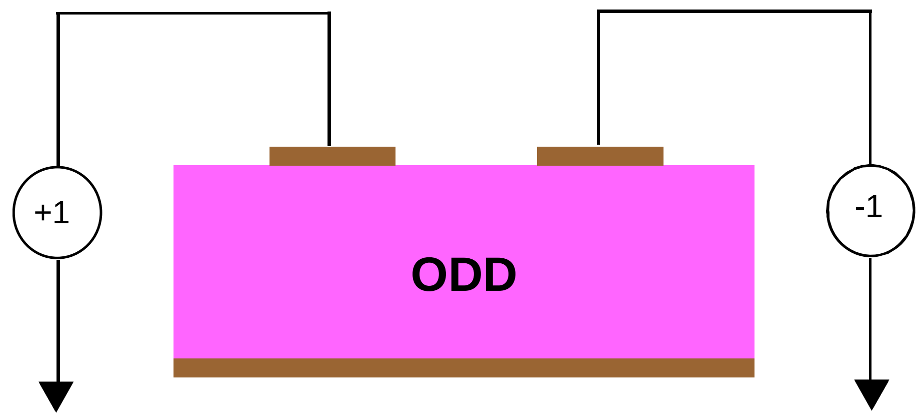
Impedance

$$v_d = \frac{1}{\sqrt{(L_{11} - L_{12})(C_{11} - C_{12})}} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}}$$

velocity

Mode Excitation





PHYSICAL SIGNIFICANCE OF EVEN- AND ODD-MODE IMPEDANCES

*** Z_e and Z_d are the wave resistance seen by the even and odd mode travelling signals respectively.**

*** The impedance of each line is no longer described by a single characteristic impedance; instead, we have**

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Definitions

Even-Mode Impedance: Z_e

Impedance seen by wave propagating through the coupled-line system when excitation is symmetric (1, 1).

Odd-Mode Impedance: Z_d

Impedance seen by wave propagating through the coupled-line system when excitation is anti-symmetric (1, -1).

Common-Mode Impedance: $Z_c = 0.5Z_e$

Impedance seen by a pair of line and a common return by a common signal.

Differential Impedance: $Z_{diff} = 2Z_d$

Impedance seen across a pair of lines by differential mode signal.

EVEN AND ODD-MODE IMPEDANCES

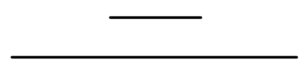
Z_{11}, Z_{22} : Self Impedances

Z_{12}, Z_{21} : Mutual Impedances

For symmetrical lines,

$Z_{11} = Z_{22}$ and $Z_{12} = Z_{21}$

EXAMPLE
(Microstrip)



$\varepsilon_r = 4.3$

$Z_s = 56.4\ \Omega$

Single Line

Dielectric height = 6 mils

Width = 8 mils

Coupled Lines

Height = 6 mils

Width = 8 mils

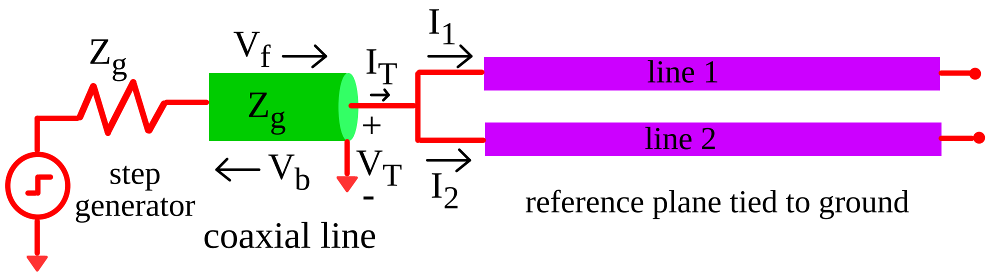
Spacing = 12 mils

$\varepsilon_r = 4.3$

$$\mathbf{Z_e = 68.1\ \Omega \qquad Z_d = 40.8\ \Omega}$$

$$\mathbf{Z_{11} = 54.4\ \Omega \qquad Z_{12} = 13.6\ \Omega}$$

Even Mode

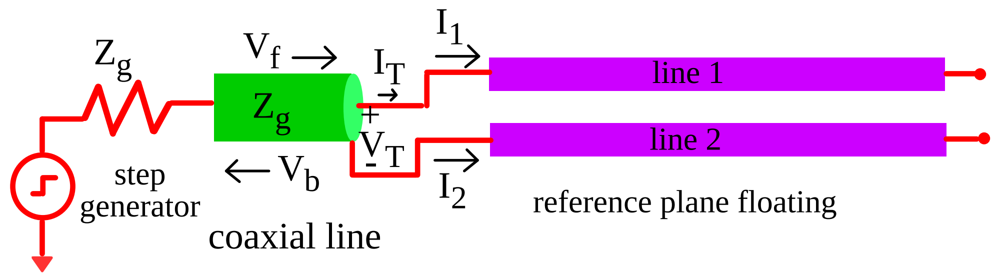


$$I_{tdr} = \left[\frac{a_e(t,0)}{Z_e} + \frac{a_d(t,0)}{Z_d}\right] + \left[\frac{a_e(t,0)}{Z_e} - \frac{a_d(t,0)}{Z_d}\right]$$

$$V_{tdr} = a_e(t,0) - a_d(t,0) \qquad \mathrm{a}_{\mathrm{d}}(t,0) = 0$$

$$\frac{V_{tdr}}{I_{tdr}} = \frac{Z_e}{2} \qquad \mathrm{Z_e} = 2\left(\frac{1+\rho_e}{1-\rho_e}\right)\mathrm{Z_g} \qquad \mathrm{v_e} = \frac{2l}{\tau_e}$$

Odd Mode



$$V_{tdr} = a_e(t,0)+a_d(t,0)-\left[a_e(t,0)-a_d(t,0)\right] = V_f + V_b$$

$$I_{tdr} = \left[\frac{a_e(t,0)}{Z_e} + \frac{a_d(t,0)}{Z_d}\right] \qquad I_{tdr} = -\left[\frac{a_e(t,0)}{Z_e} - \frac{a_d(t,0)}{Z_d}\right]$$

$$a_e(t,0) = 0, \qquad \frac{V_{tdr}}{I_{tdr}} = 2Z_d$$

$$Z_d = \frac{1}{2}\left(\frac{1+\rho_d}{1-\rho_d}\right)Z_g, \qquad v_d = \frac{2l}{\tau_d}$$

EXTRACT INDUCTANCE AND CAPACITANCE COEFFICIENTS

$$L_s = \frac{1}{2}\left[\frac{Z_e}{v_e} + \frac{Z_d}{v_d}\right]$$

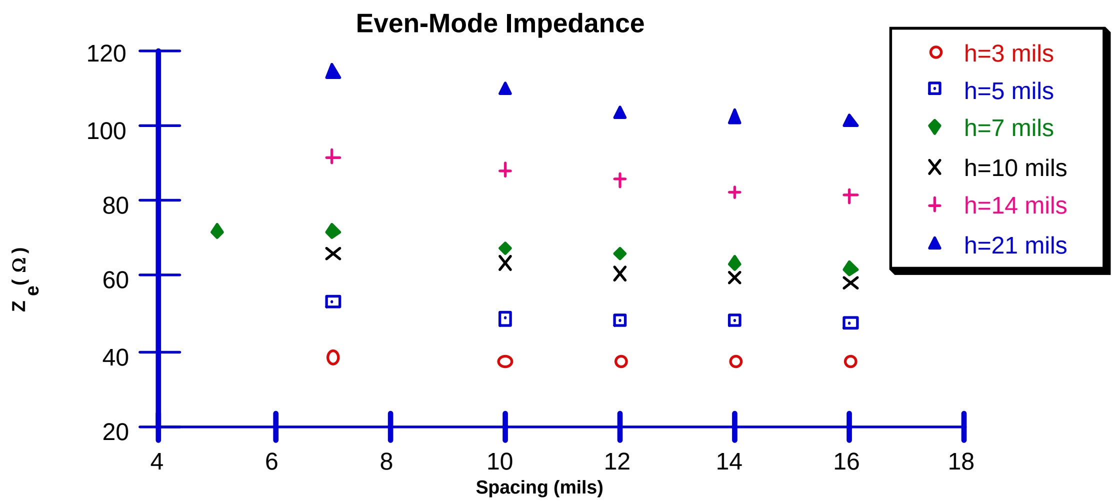
$$C_s = \frac{1}{Z_e v_e}$$

$$L_m = \frac{1}{2}\left[\frac{Z_e}{v_e} - \frac{Z_d}{v_d}\right]$$

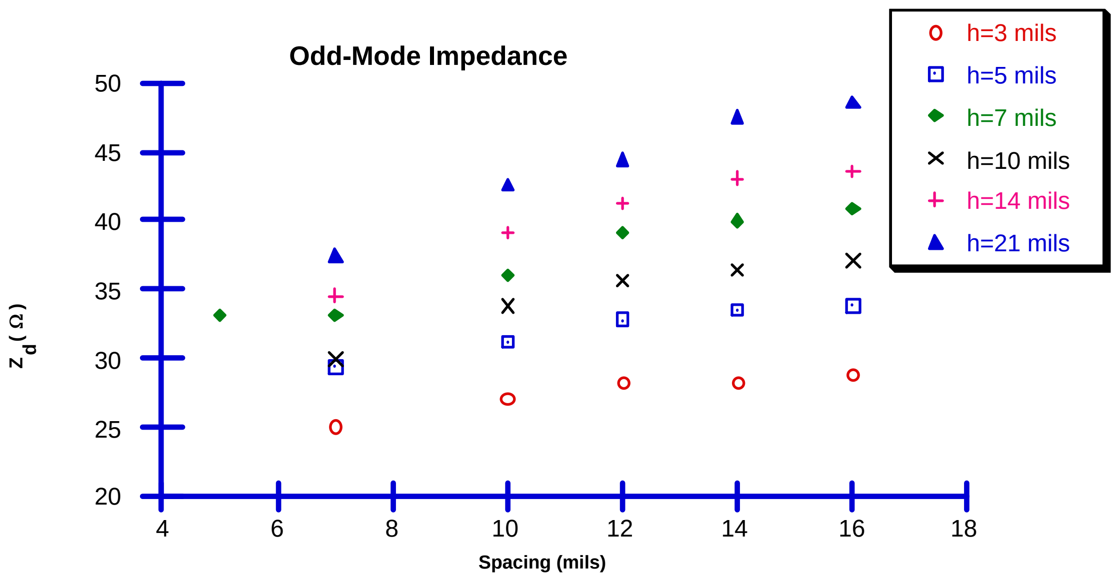
$$C_m = \frac{1}{2}\left[\frac{1}{Z_e v_e} - \frac{1}{Z_d v_d}\right]$$

$$Z_d < Z_s < Z_e$$

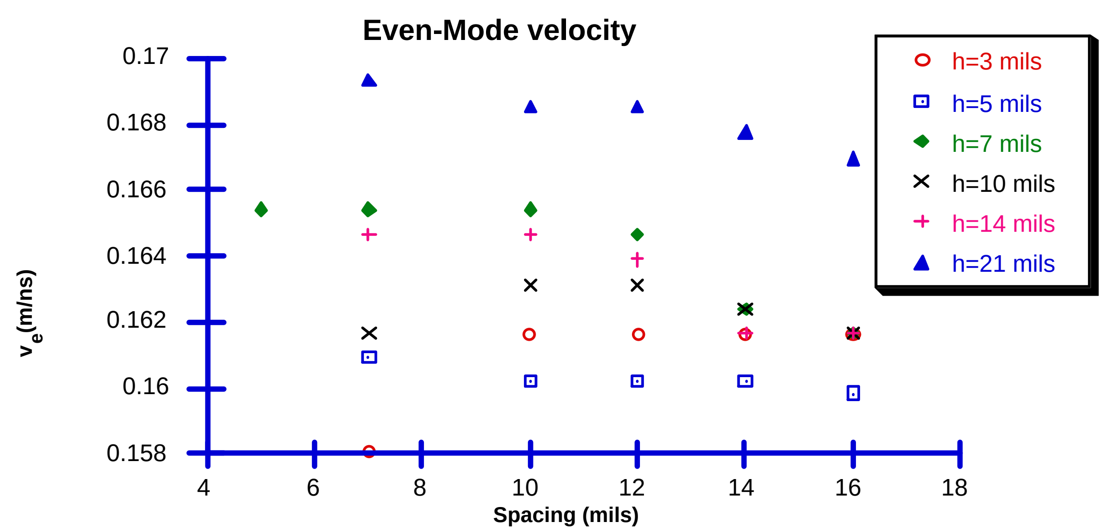
Measured even-mode impedance



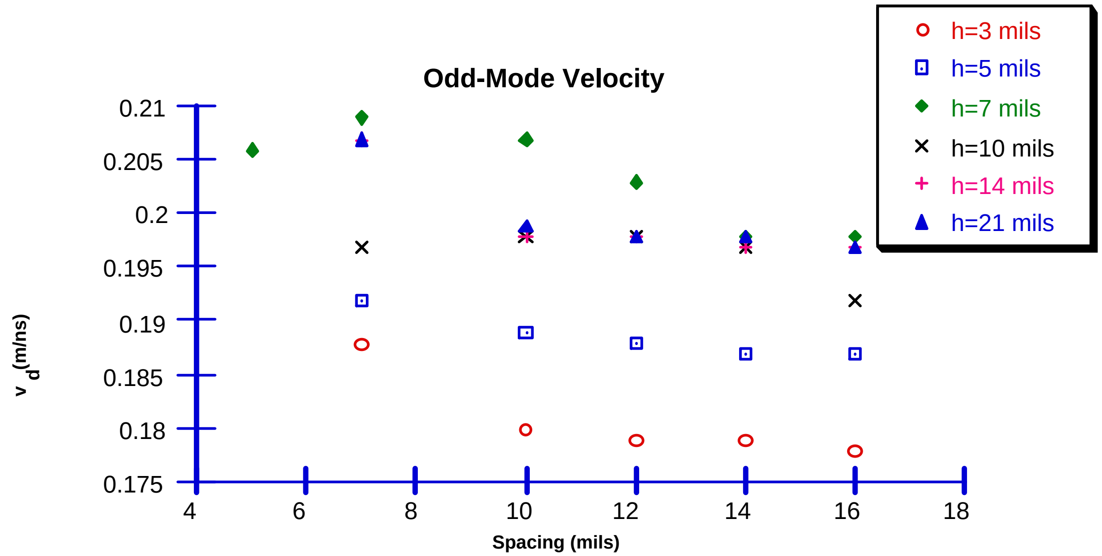
Measured odd-mode impedance



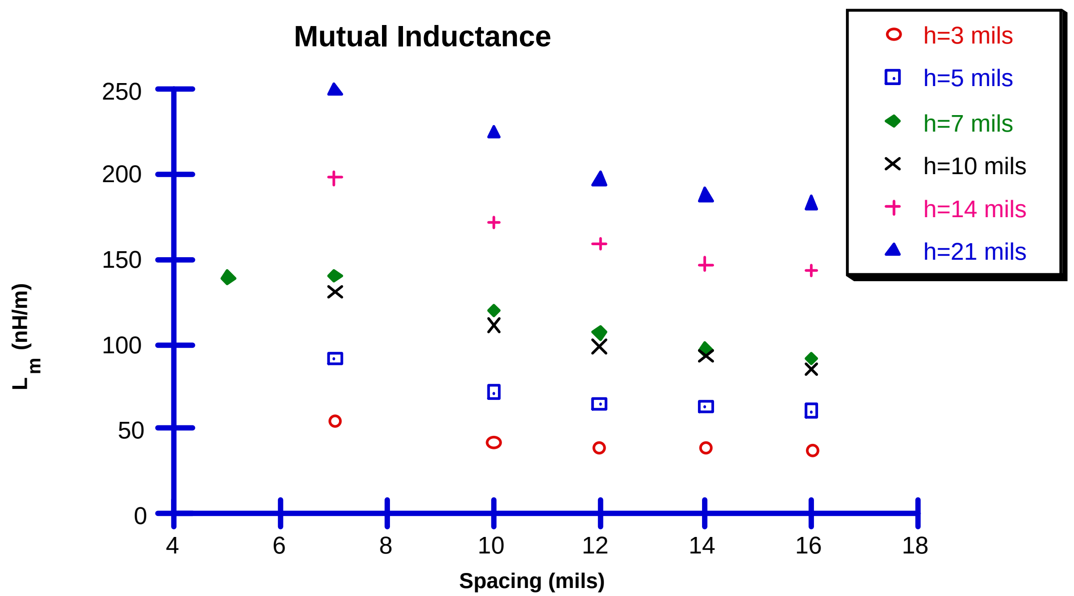
Measured even-mode velocity



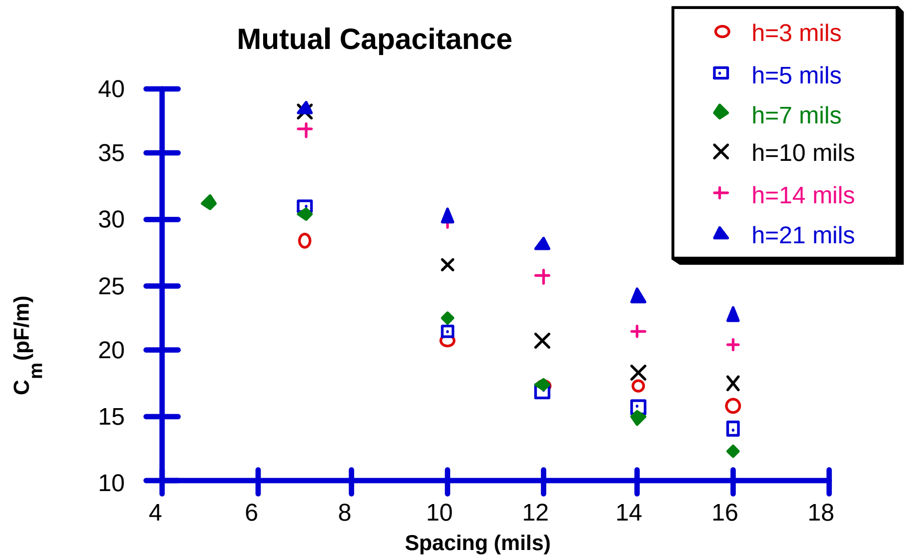
Measured odd-mode velocity



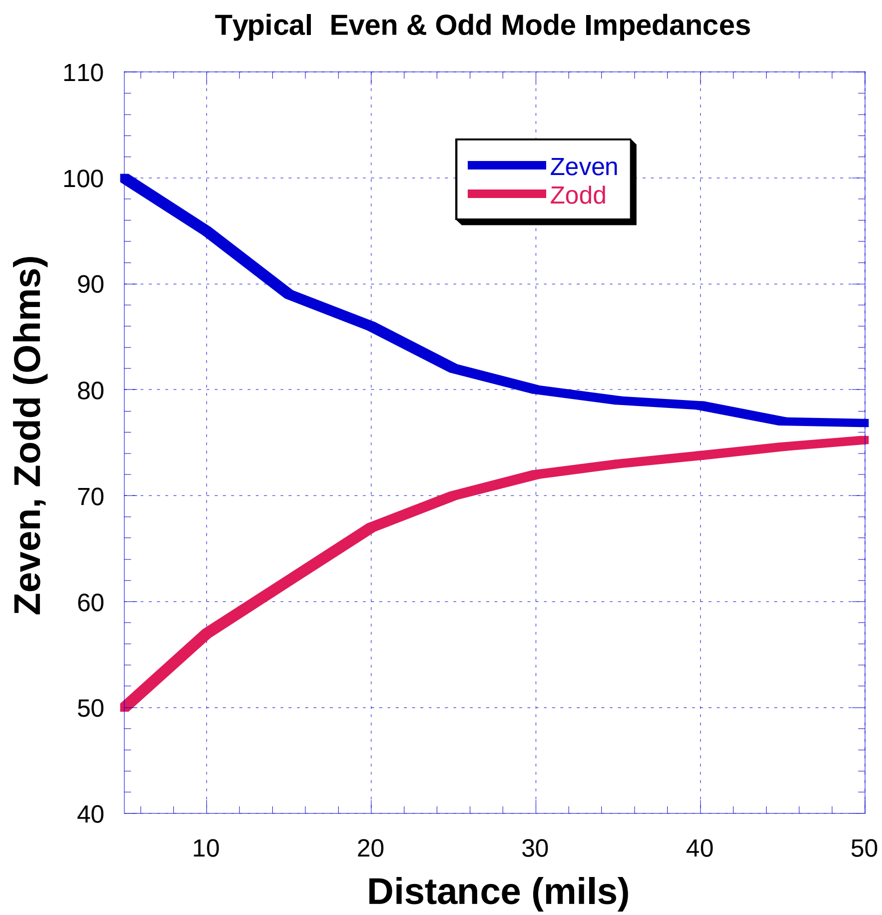
Measured mutual inductance



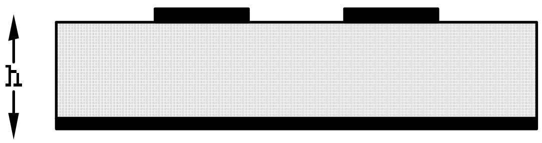
Measured mutual capacitance



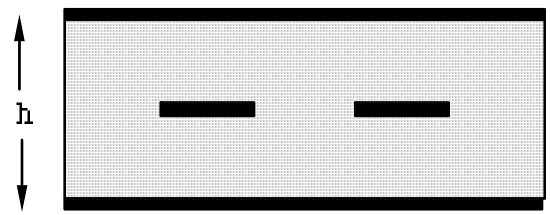
Even & Odd Mode Impedances



Modal Velocities in Stripline and Microstrip

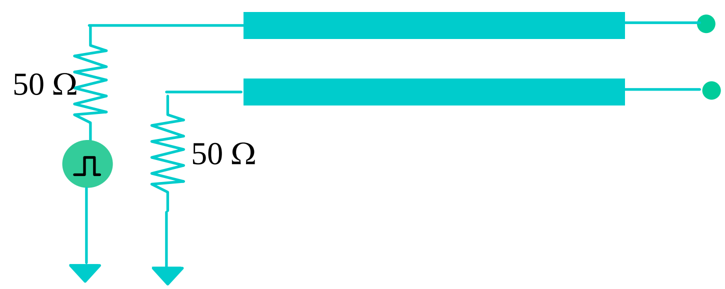


<u>Microstrip</u> : Inhomogeneous structure, odd and even-mode velocities must have different values.



<u>Stripline</u> : Homogeneous configuration, odd and even-mode velocities have approximately the same values.

Microstrip vs Stripline



Microstrip (h =8 mils)

$w = 8$ mils

$\varepsilon_r = 4.32$

$L_s = 377$ nH/m

$C_s = 82$ pF/m

$L_m = 131$ nH/m

$C_m = 23$ pF/m

$v_e = 0.155$ m/ns

$v_d = 0.178$ m/ns

Stripline (h =16 mils)

$w = 8$ mils

$\varepsilon_r = 4.32$

$L_s = 466$ nH/m

$C_s = 86$ pF/m

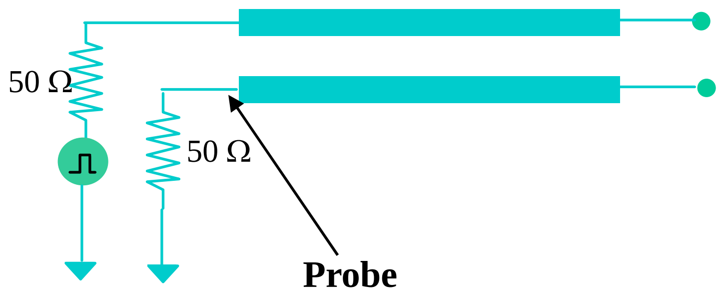
$L_m = 109$ nH/m

$C_m = 26$ pF/m

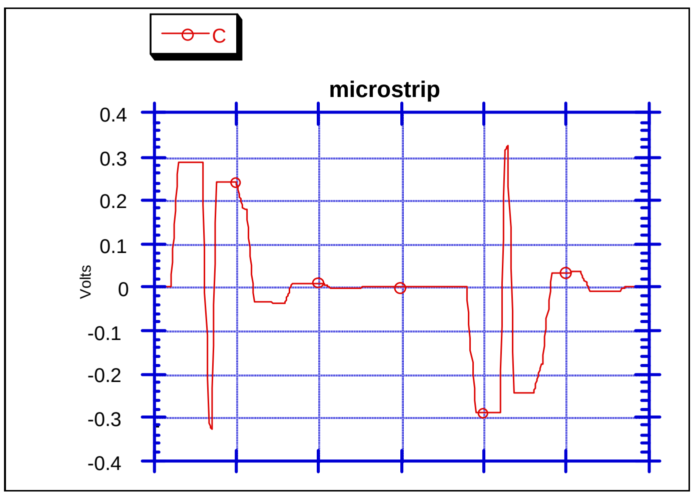
$v_e = 0.142$ m/ns

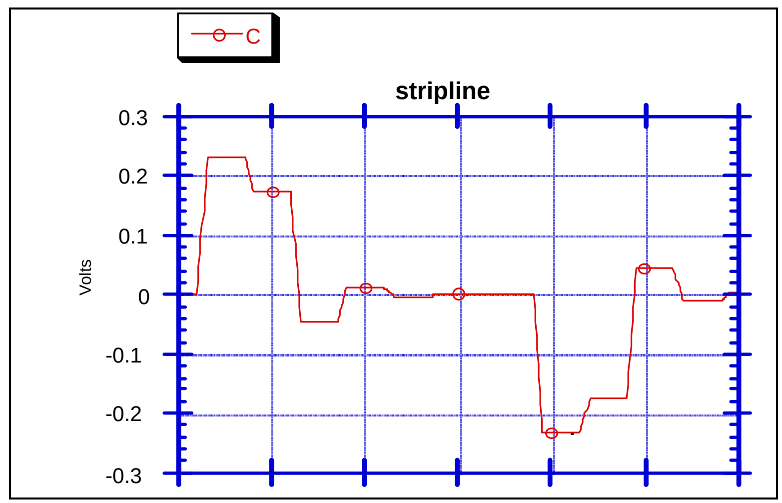
$v_d = 0.142$ m/ns

Microstrip vs Stripline

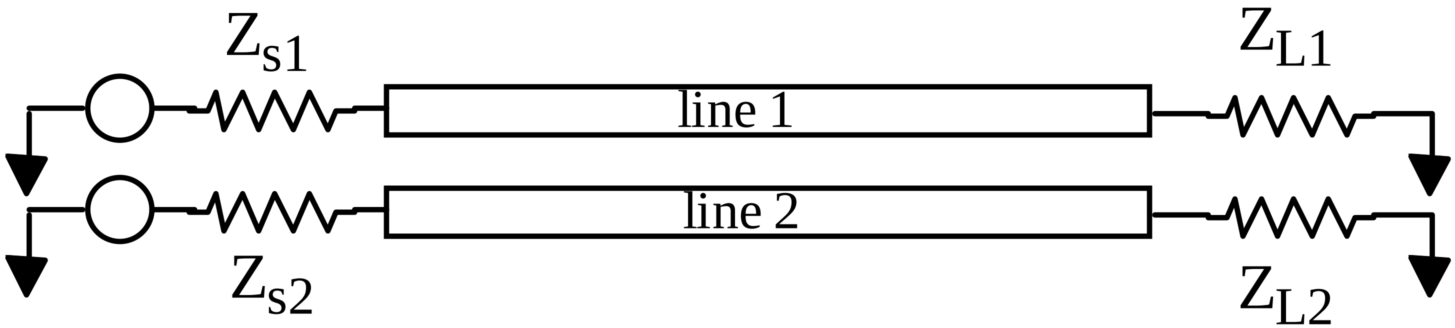


Sense line response at near end





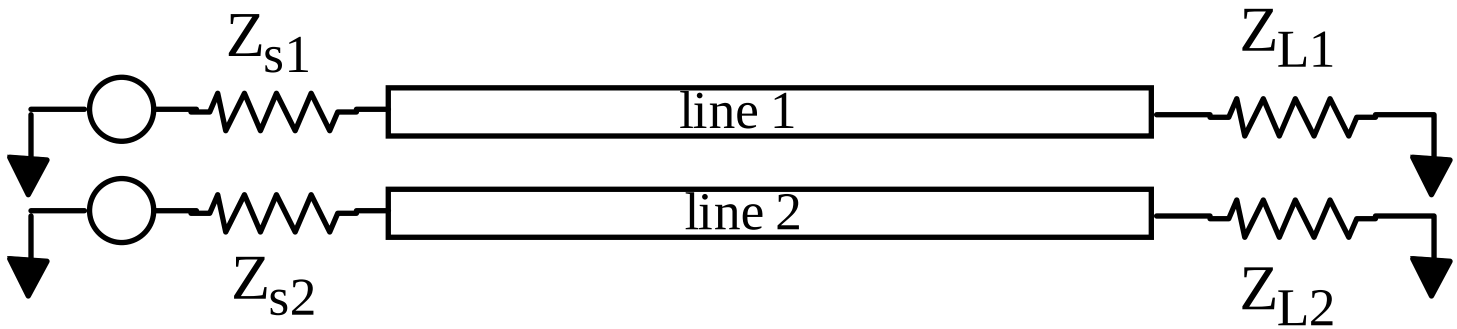
General Solution for Voltages



$$V_1(z) = \underbrace{A_e e^{-\frac{j\omega z}{v_e}} + B_e e^{+\frac{j\omega z}{v_e}}}_{even} + \underbrace{A_d e^{-\frac{j\omega z}{v_d}} + B_d e^{+\frac{j\omega z}{v_d}}}_{odd}$$

$$V_2(z) = \underbrace{A_e e^{-\frac{j\omega z}{v_e}} + B_e e^{+\frac{j\omega z}{v_e}}}_{even} - \underbrace{A_d e^{-\frac{j\omega z}{v_d}} - B_d e^{+\frac{j\omega z}{v_d}}}_{odd}$$

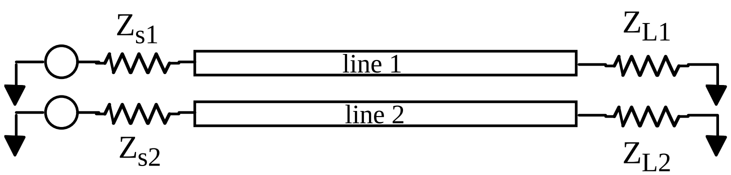
General Solution for Currents

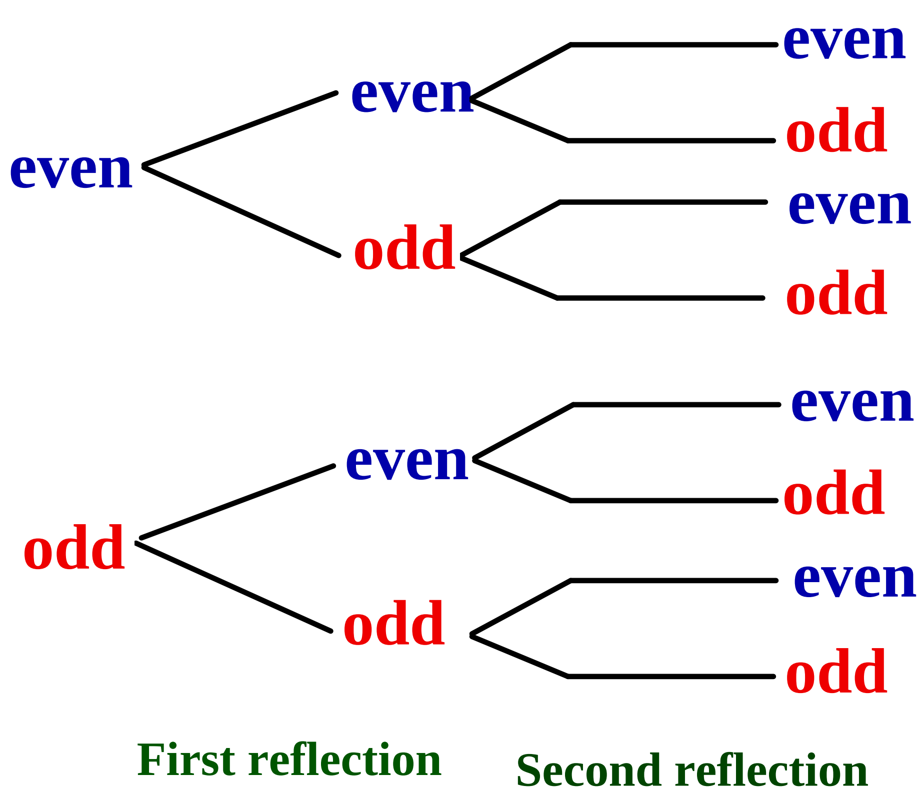


$$I_1(z) = \underbrace{\frac{1}{Z_e}\left[A_e e^{-\frac{j\omega z}{v_e}} - B_e e^{+\frac{j\omega z}{v_e}}\right]}_{even} + \underbrace{\frac{1}{Z_d}\left[A_d e^{-\frac{j\omega z}{v_d}} - B_d e^{+\frac{j\omega z}{v_d}}\right]}_{odd}$$

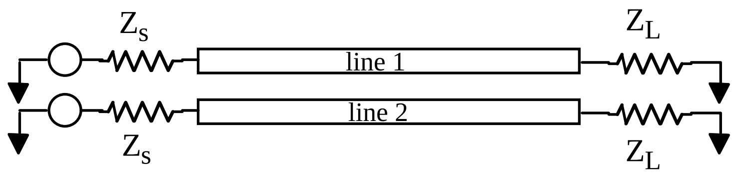
$$I_2(z) = \underbrace{\frac{1}{Z_e}\left[A_e e^{-\frac{j\omega z}{v_e}} - B_e e^{+\frac{j\omega z}{v_e}}\right]}_{even} - \underbrace{\frac{1}{Z_d}\left[A_d e^{-\frac{j\omega z}{v_d}} - B_d e^{+\frac{j\omega z}{v_d}}\right]}_{odd}$$

Coupling of Modes (asymmetric load)





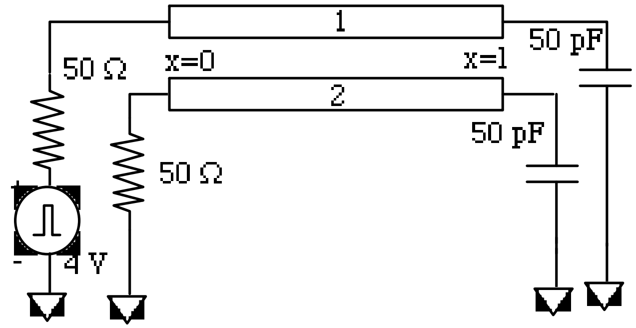
Coupling of Modes (symmetric load)

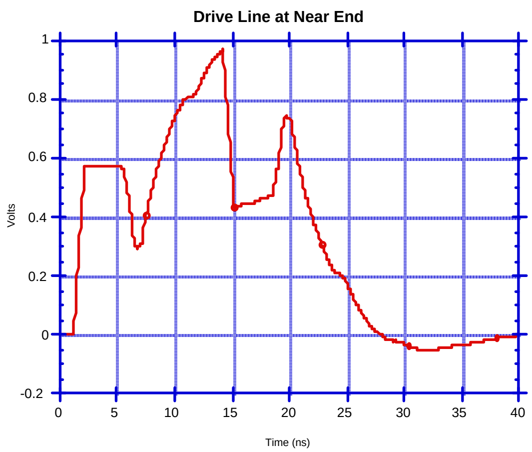


even ——— even ——— even

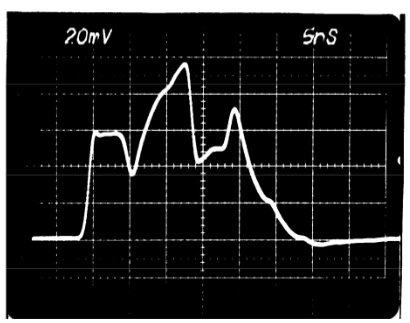
odd ——— odd ——— odd

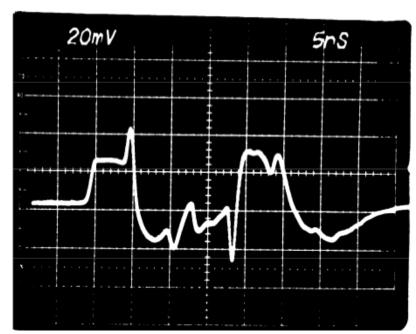
First reflection **Second reflection**


1
2
50 Ω
x=0
x=1
50 pF
50 pF
50 Ω
4 V

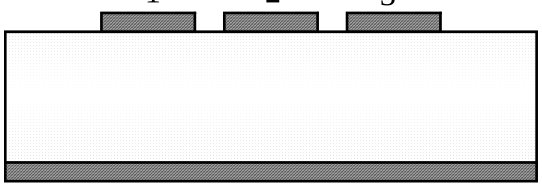

Drive Line at Near End
Volts
1
0.8
0.6
0.4
0.2
0
-0.2
0
5
10
15
20
25
30
35
40
Time (ns)


Sense Line at Near End
Volts
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
0
5
10
15
20
25
30
35
40
Time (ns)


20mV
5nS


20mV
5nS

Three-Line Microstrip



$$-\frac{\partial V_1}{\partial z} = L_{11}\frac{\partial I_1}{\partial t} + L_{12}\frac{\partial I_2}{\partial t} + L_{13}\frac{\partial I_3}{\partial t}$$

$$-\frac{\partial I_1}{\partial z} = C_{11}\frac{\partial V_1}{\partial t} + C_{12}\frac{\partial V_2}{\partial t} + C_{13}\frac{\partial V_3}{\partial t}$$

$$-\frac{\partial V_2}{\partial z} = L_{21}\frac{\partial I_1}{\partial t} + L_{22}\frac{\partial I_2}{\partial t} + L_{23}\frac{\partial I_3}{\partial t}$$

$$-\frac{\partial I_2}{\partial z} = C_{21}\frac{\partial V_1}{\partial t} + C_{22}\frac{\partial V_2}{\partial t} + C_{23}\frac{\partial V_3}{\partial t}$$

$$-\frac{\partial V_3}{\partial z} = L_{31}\frac{\partial I_1}{\partial t} + L_{32}\frac{\partial I_2}{\partial t} + L_{33}\frac{\partial I_3}{\partial t}$$

$$-\frac{\partial I_3}{\partial z} = C_{31}\frac{\partial V_1}{\partial t} + C_{32}\frac{\partial V_2}{\partial t} + C_{33}\frac{\partial V_3}{\partial t}$$

Three-Line – Alpha Mode

Subtract (1c) from (1a) and (2c) from (2a), we get

$$-\frac{\partial V_\alpha}{\partial z} = (L_{11} - L_{13})\frac{\partial I_\alpha}{\partial t}$$
$$-\frac{\partial I_\alpha}{\partial z} = (C_{11} - C_{13})\frac{\partial V_\alpha}{\partial t}$$

This defines the Alpha mode with:

$V_\alpha = V_1 - V_3$ and $I_\alpha = I_1 - I_3$

The wave impedance of that mode is:

$$Z_\alpha = \sqrt{\frac{L_{11} - L_{13}}{C_{11} - C_{13}}}$$

and the velocity is $u_\alpha = \dfrac{1}{\sqrt{(L_{11} - L_{13})(C_{11} - C_{13})}}$

Three-Line – Modal Decomposition

In order to determine the next mode, assume that

$$V_\beta = V_1 + \beta V_2 + V_3$$

$$I_\beta = I_1 + \beta I_2 + I_3$$

$$-\frac{\partial V_\beta}{\partial z} = \left(L_{11} + \beta L_{21} + L_{31}\right)\frac{\partial I_1}{\partial t} + \left(L_{12} + \beta L_{22} + L_{32}\right)\frac{\partial I_2}{\partial t} + \left(L_{13} + \beta L_{23} + L_{33}\right)\frac{\partial I_3}{\partial t}$$

$$-\frac{\partial I_\beta}{\partial z} = \left(C_{11} + \beta C_{21} + C_{31}\right)\frac{\partial V_1}{\partial t} + \left(C_{12} + \beta C_{22} + C_{32}\right)\frac{\partial V_2}{\partial t} + \left(C_{13} + \beta C_{23} + C_{33}\right)\frac{\partial V_3}{\partial t}$$

By reciprocity $L_{32} = L_{23}$, $L_{21} = L_{12}$, $L_{13} = L_{31}$

By symmetry, $L_{12} = L_{23}$

Also by approximation, $L_{22} \approx L_{11}$, $L_{11} + L_{13} \approx L_{11}$

Three-Line – Modal Decomposition

$$-\frac{\partial V_\beta}{\partial z} = \left(L_{11} + \beta L_{12} + L_{13}\right)\left(\frac{\partial I_1}{\partial t} + \frac{\partial I_3}{\partial t}\right) + \left(2L_{12} + \beta L_{11}\right)\frac{\partial I_2}{\partial t}$$

In order to balance the right-hand side into I_β, we need to have

$$\left(2L_{12} + \beta L_{11}\right) I_2 = \beta\left(L_{11} + \beta L_{12} + L_{13}\right) I_2 \approx \beta\left(L_{11} + \beta L_{12}\right) I_2$$

$$2L_{12} = \beta^2 L_{12}$$

or $\quad \beta = \pm\sqrt{2}$

Therefore the other two modes are defined as

The Beta mode with

Three-Line – Beta Mode

The Beta mode with

$$V_\beta = V_1 + \sqrt{2}V_2 + V_3$$

$$I_\beta = I_1 + \sqrt{2}I_2 + I_3$$

The characteristic impedance of the Beta mode is:

$$Z_\beta = \sqrt{\frac{L_{11} + \sqrt{2}L_{12} + L_{13}}{C_{11} + \sqrt{2}C_{12} + C_{13}}}$$

and propagation velocity of the Beta mode is

$$u_\beta = \frac{1}{\sqrt{\left(L_{11} + \sqrt{2}L_{12} + L_{13}\right)\left(C_{11} + \sqrt{2}C_{12} + C_{13}\right)}}$$

Three-Line – Delta Mode

The Delta mode is defined such that

$$V_\delta = V_1 - \sqrt{2}V_2 + V_3$$

$$I_\delta = I_1 - \sqrt{2}I_2 + I_3$$

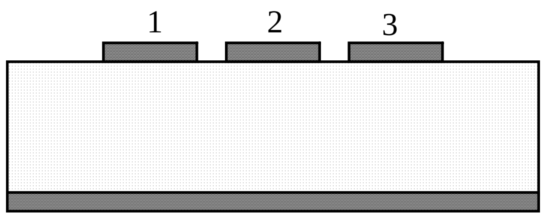
The characteristic impedance of the Delta mode is

$$Z_\delta = \sqrt{\frac{L_{11} - \sqrt{2}L_{12} + L_{13}}{C_{11} - \sqrt{2}C_{12} + C_{13}}}$$

The propagation velocity of the Delta mode is:

$$u_\delta = \frac{1}{\sqrt{\left(L_{11} - \sqrt{2}L_{12} + L_{13}\right)\left(C_{11} - \sqrt{2}C_{12} + C_{13}\right)}}$$

Symmetric 3-Line Microstrip



In summary: we have 3 modes for the 3-line system

$$E = \begin{pmatrix} 1 & 0 & -1 \\ 1 & \sqrt{2} & 1 \\ 1 & -\sqrt{2} & 1 \end{pmatrix}$$

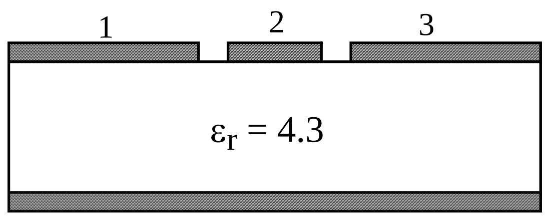
Alpha mode

Beta mode*

Delta mode*

*neglecting coupling between nonadjacent lines

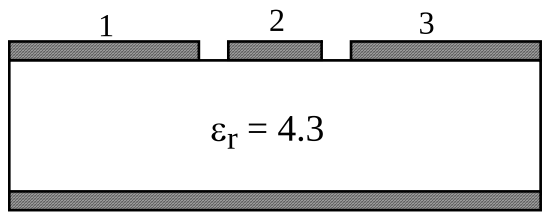
Coplanar Waveguide



$$L(nH/m)=\begin{pmatrix}346 & 162 & 67\\ 152 & 683 & 152\\ 67 & 162 & 346\end{pmatrix}\qquad C(pF/m)=\begin{pmatrix}113 & 17 & 5\\ 16 & 53 & 16\\ 5 & 17 & 113\end{pmatrix}$$

$$E=\begin{pmatrix}0.45 & 0.12 & 0.45\\ 0.5 & 0 & -0.5\\ -0.45 & 0.87 & -0.45\end{pmatrix}\qquad H=\begin{pmatrix}0.44 & 0.49 & 0.44\\ 0.5 & 0 & -0.5\\ -0.10 & 0.88 & -0.10\end{pmatrix}$$

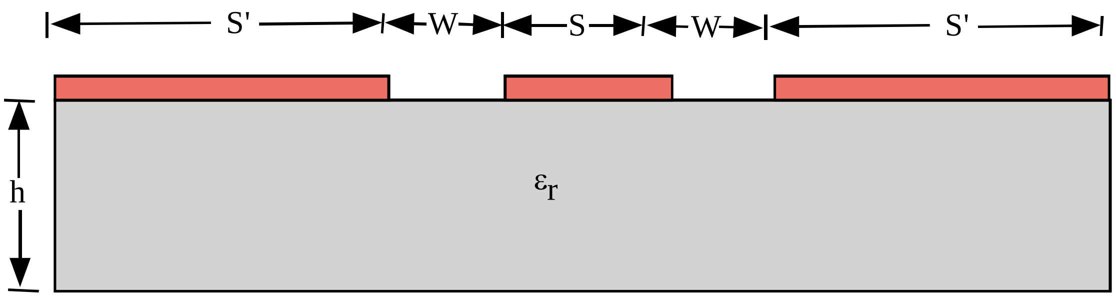
Coplanar Waveguide



$$Z_m(\Omega) = \begin{pmatrix} 73 & 0 & 0 \\ 0 & 48 & 0 \\ 0 & 0 & 94 \end{pmatrix} \qquad Z_c(\Omega) = \begin{pmatrix} 56 & 23 & 8 \\ 22 & 119 & 22 \\ 8 & 23 & 56 \end{pmatrix}$$

$$v_p(m/ns) = \begin{pmatrix} 0.15 & 0 & 0 \\ 0 & 0.17 & 0 \\ 0 & 0 & 0.18 \end{pmatrix}$$

Coplanar Waveguide



$K(k)$: Complete Elliptic Integral of the first kind

$$k = \frac{S}{S+2W}$$

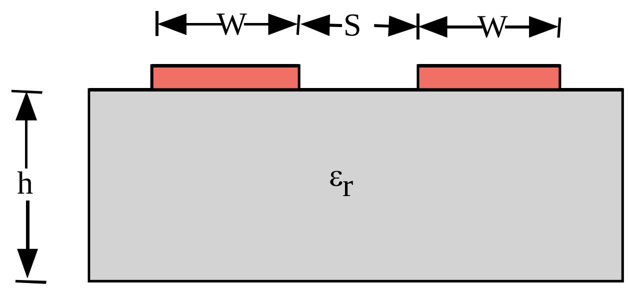
$$K'(k) = K(k')$$

$$k' = (1-k^2)^{1/2}$$

$$Z_{ocp} = \frac{30\pi}{\sqrt{\frac{\varepsilon_r + 1}{2}}} \frac{K'(k)}{K(k)} \text{ (ohm)}$$

$$v_{cp} = \left(\frac{2}{\varepsilon_r + 1}\right)^{1/2} c$$

Coplanar Strips



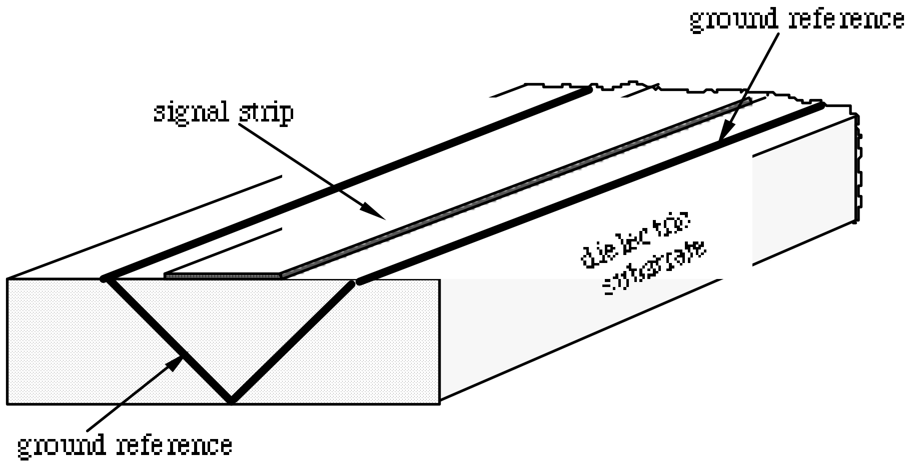
$$Z_{ocs} = \frac{120\pi}{\sqrt{\frac{\varepsilon_r + 1}{2}}} \frac{K'(k)}{K(k)} \text{ (ohm)}$$

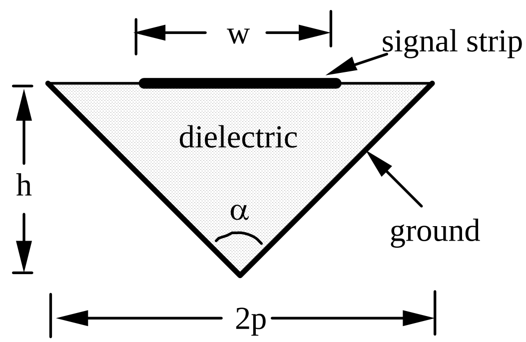
Qualitative Comparison

Characteristic	Microstrip	Coplanar Wguide	Coplanar strips
ε_{eff}*	~6.5	~5	~5
Power handling	High	Medium	Medium
Radiation loss	Low	Medium	Medium
Unloaded Q	High	Medium	Low or High
Dispersion	Small	Medium	Medium
Mounting (shunt)	Hard	Easy	Easy
Mounting (series)	Easy	Easy	Easy

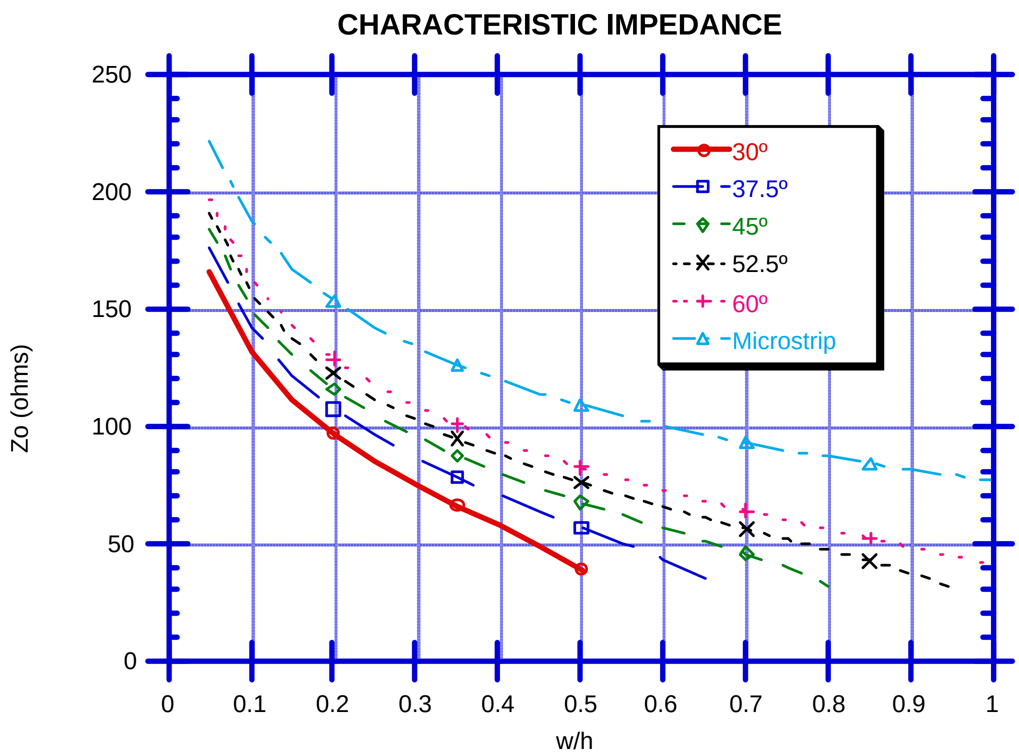
* Assuming ε_r=10 and h=0.025 inch

V Transmission Line



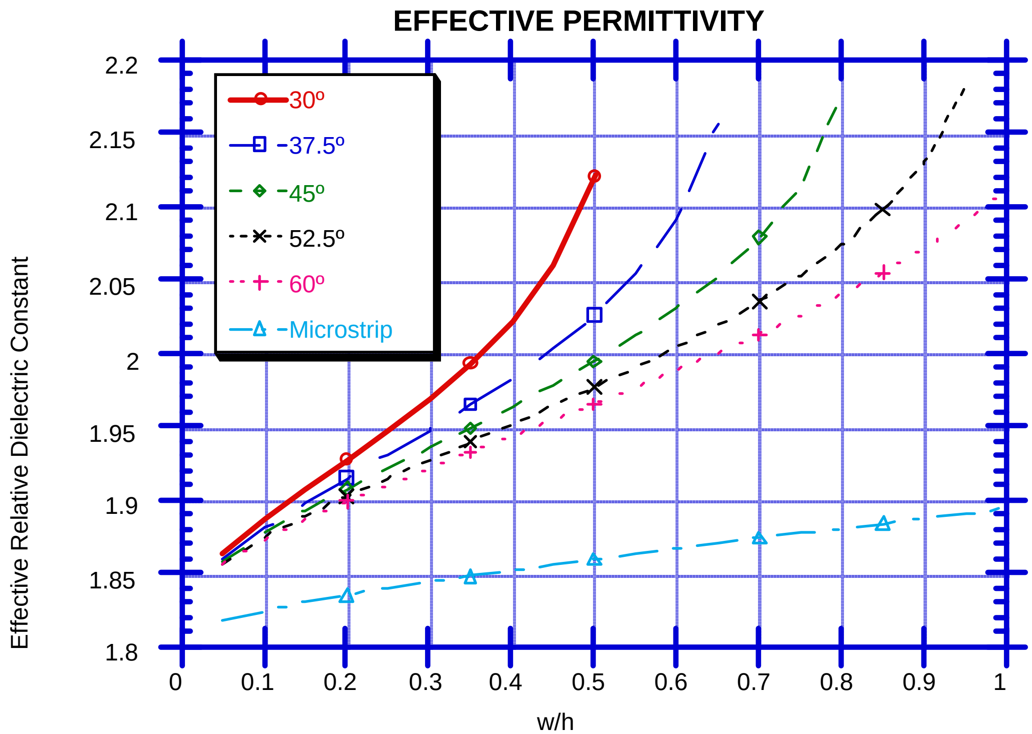


V-Line Characteristic Impedance



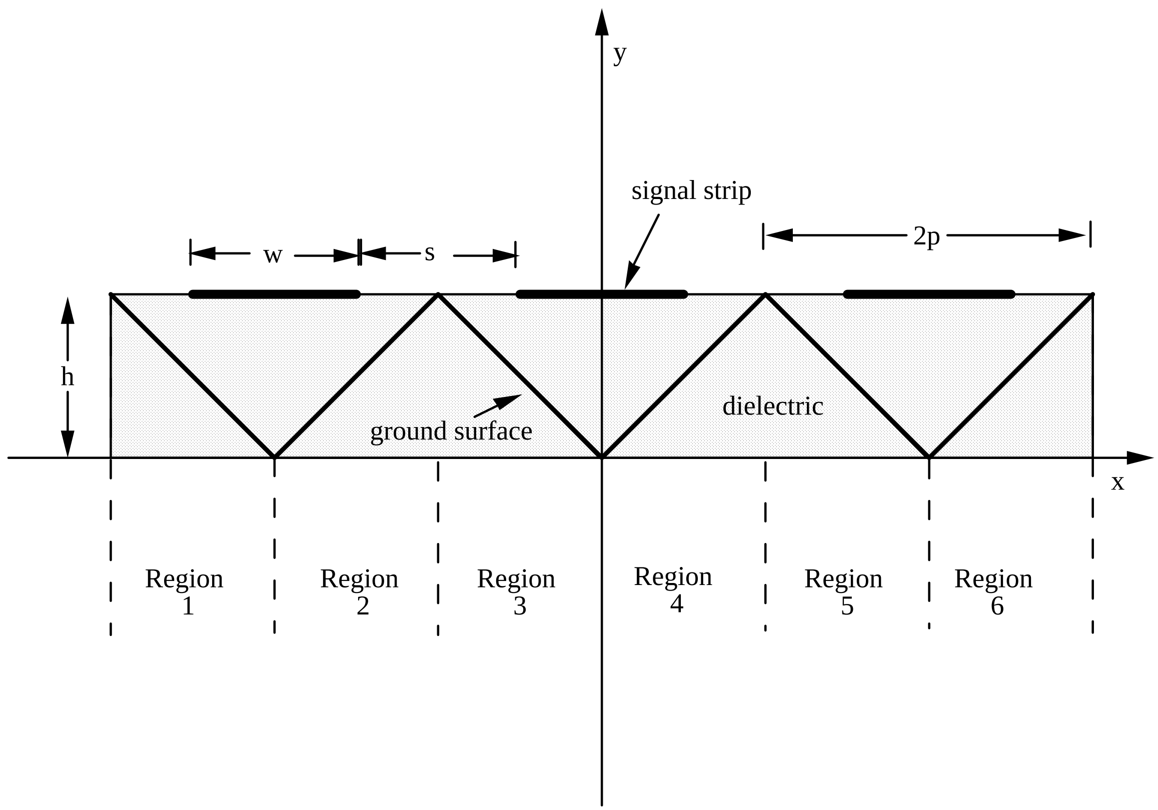
Calculated values of the characteristic impedance for a single-line v-strip structure as a function of width-to-height ratio w/h. The relative dielectric constant is εr = 2.55.

V-Line – Effective Permittivity



Calculated values of the effective relative dielectric constant for a single-line v-strip structure as a function of width-to-height ratio w/h.
The relative dielectric constant is $\varepsilon r = 2.55$

Three-Line – V Transmission Line



V-Line and Microstrip

Microstrip

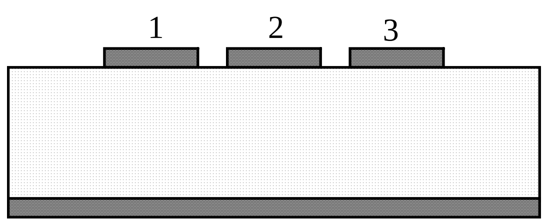
$$C\ (\text{pF/m}) = \begin{matrix} 52.49 & -5.90 & -0.57 \\ -5.90 & 53.27 & -5.90 \\ -0.57 & -5.90 & 52.49 \end{matrix}$$

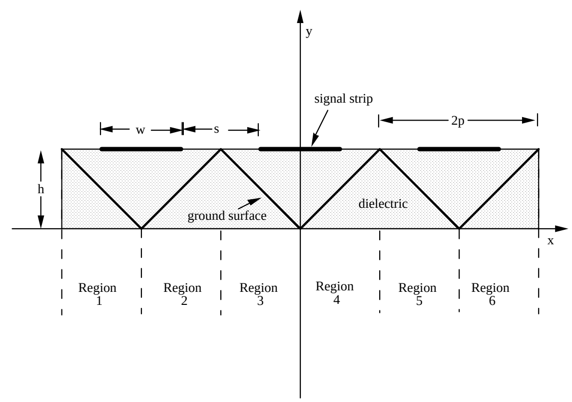
$$L\ (\text{nH/m}) = \begin{matrix} 609.74 & 113.46 & 41.79 \\ 113.54 & 607.67 & 113.54 \\ 41.79 & 113.46 & 609.74 \end{matrix}$$

V-line

$$C\ (\text{pF/m}) = \begin{matrix} 74.00 & -0.97 & -0.23 \\ -0.97 & 74.07 & -0.97 \\ -0.23 & -0.97 & 74.00 \end{matrix}$$

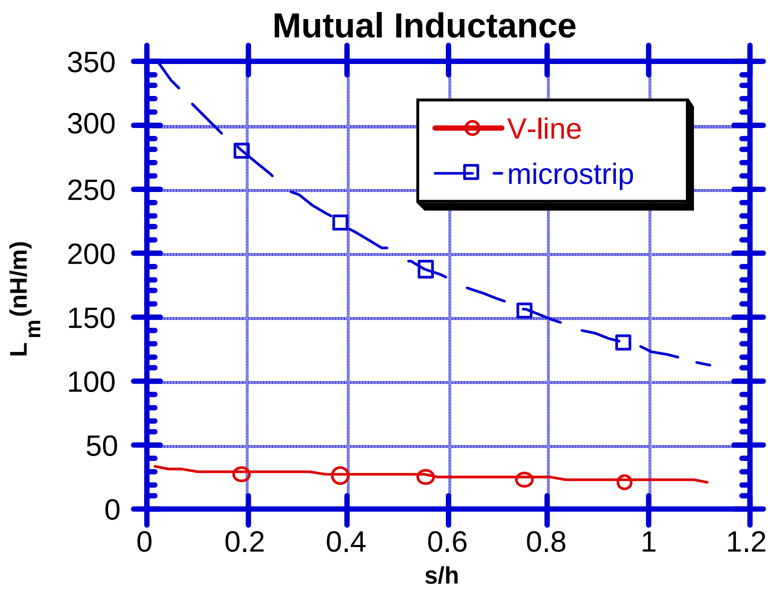
$$L\ (\text{nH/m}) = \begin{matrix} 425.84 & 20.35 & 7.00 \\ 20.36 & 422.85 & 20.36 \\ 7.00 & 20.35 & 425.84 \end{matrix}$$

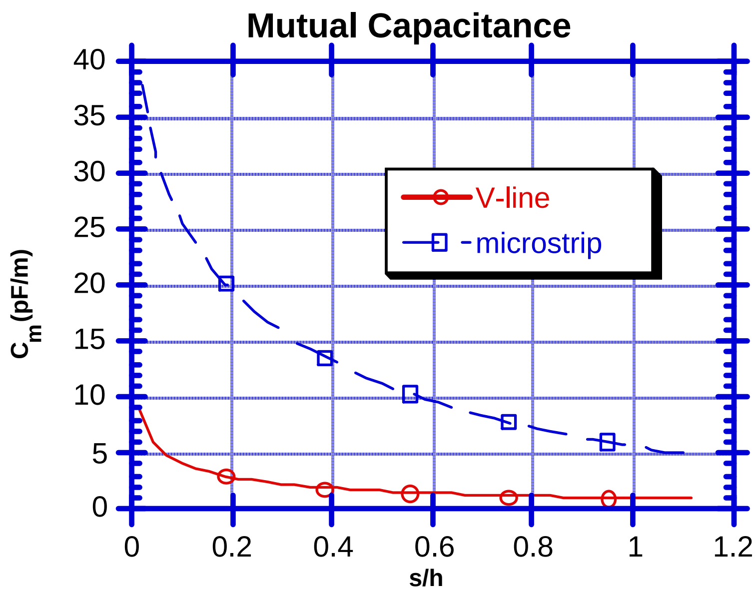




Comparison of the inductance and capacitance matrices between a three-line v-line and microstrip structures. The parameters are p/h = 0.8 mils, w/h = 0.6 and $\varepsilon_r = 4.0$.

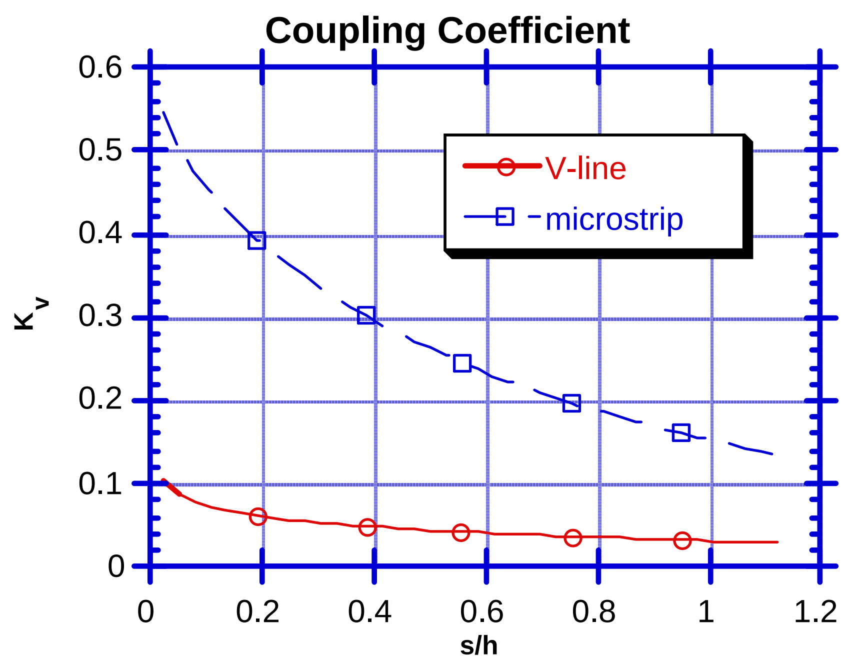
V-Line: Coupling Coefficients





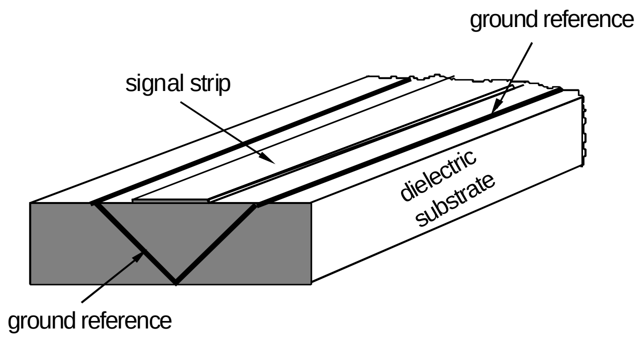
Plot of mutual inductance (top) and mutual capacitance (bottom) versus spacing-to-height ratio for v-line and microstrip configurations. The parameters are w/h = 0.24, εr = 4.0.

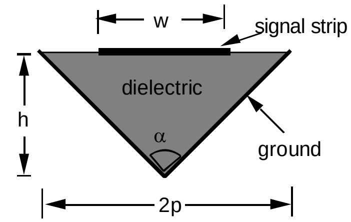
V-Line vs Microstrip: Coupling Coefficients



Plot of the coupling coefficient versus spacing-to-height ratio for v-line and microstrip configurations. The parameters are w/h = 0.24, εr = 4.0.

Advantages of V-Line



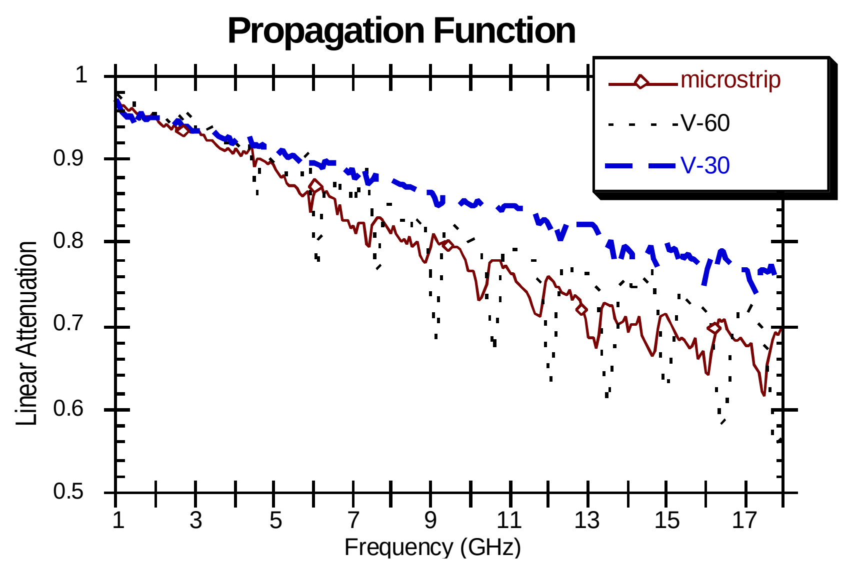


Advantages of V Line

* Higher bandwidth

* Lower crosstalk

* Better transition



V-Line vs Microstrip: Insertion Loss