

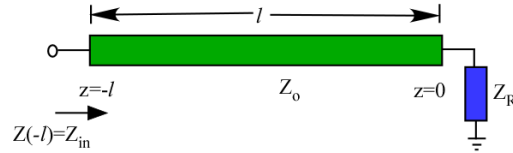
ECE 451 – Formula Sheet - Automated Microwave Measurement

Transmission line equations

$$\Gamma(-l) = \Gamma_R e^{-j2\beta l} = \frac{Z_R - Z_0}{Z_R + Z_0} e^{-j2\beta l}$$

$$Z(-l) = Z_0 \left[\frac{Z_R + jZ_0 \tan \beta l}{Z_0 + jZ_R \tan \beta l} \right]$$

$$Z(z) = Z_0 \frac{1 + \Gamma(z)}{1 - \Gamma(z)} \quad \Gamma(z) = \frac{Z(z) - Z_0}{Z(z) + Z_0}$$

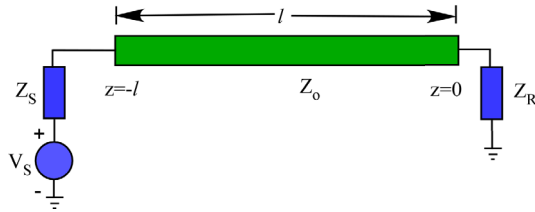


$$V(z) = V_+ e^{-j\beta z} [1 + \Gamma_R e^{+2j\beta z}]$$

$$I(z) = \frac{V_+}{Z_0} e^{-j\beta z} [1 - \Gamma_R e^{+2j\beta z}]$$

$$\beta = \frac{2\pi}{\lambda} = \frac{\omega}{v}$$

Transmission line with voltage source



$$V_+ = \frac{T_s V_s e^{-j\beta l}}{1 - \Gamma_R \Gamma_s e^{-2j\beta l}}$$

$$T_s = \frac{Z_0}{Z_s + Z_0}, \quad \Gamma_s = \frac{Z_s - Z_0}{Z_s + Z_0}, \quad \Gamma_R = \frac{Z_R - Z_0}{Z_R + Z_0}$$

Mason's non-touching loop rule

$$T = \frac{\sum_{i=1}^N P_i [1 + \sum_{j=1}^M (-1)^j L(j)^{(i)}]}{1 + \sum_{j=1}^M (-1)^j L(j)}$$

$$= \frac{P_1 [1 - \sum L(1)^{(1)} + \sum L(2)^{(1)} - \dots] + P_2 [1 - \sum L(1)^{(2)} + \sum L(2)^{(2)} - \dots] + \dots}{1 - \sum L(1) + \sum L(2) - \sum L(3) + \dots}$$

P_i – i^{th} path from source to sink.

$L(j)$ – j^{th} order loop.

$L(j)^{(i)}$ – j^{th} order loop which doesn't touch i^{th} path.

S – Z parameter transformation

$$\mathbf{Z} = \mathbf{Z}_0 (\mathbf{U} + \mathbf{S})(\mathbf{U} - \mathbf{S})^{-1}$$

$$\mathbf{S} = (\mathbf{Z} + \mathbf{Z}_0 \mathbf{U})^{-1} (\mathbf{Z} - \mathbf{Z}_0 \mathbf{U})$$

$\mathbf{U}$: unit matrix

S – ABCD parameters conversion

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{2S_{21}} \begin{bmatrix} (1 + S_{11})(1 - S_{22}) + S_{12}S_{21} & Z_0[(1 + S_{11})(1 + S_{22}) - S_{12}S_{21}] \\ \frac{1}{Z_0}[(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}] & (1 - S_{11})(1 + S_{22}) + S_{12}S_{21} \end{bmatrix}$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} A + \frac{B}{Z_0} - CZ_0 - D & 2(AD - BC) \\ 2 & -A + \frac{B}{Z_0} - CZ_0 + D \end{bmatrix}, \quad \Delta = A + \frac{B}{Z_0} + CZ_0 + D$$

S – T parameters conversion

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} T_{12}T_{22}^{-1} & T_{11} - T_{12}T_{21}T_{22}^{-1} \\ T_{22}^{-1} & -T_{21}T_{22}^{-1} \end{bmatrix}$$

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} S_{12} - S_{11}S_{22}S_{21}^{-1} & S_{11}S_{21}^{-1} \\ -S_{22}S_{21}^{-1} & S_{21}^{-1} \end{bmatrix}$$

S Parameters of Transmission Line of Characteristic Impedance Z_c

Reference system has impedance Z_o

$$S_{11} = S_{22} = \frac{(1 - X^2)\Gamma}{1 - \Gamma^2X^2} \quad S_{12} = S_{21} = \frac{(1 - \Gamma^2)X}{1 - \Gamma^2X^2}$$

$$\Gamma = \frac{Z_c - Z_o}{Z_c + Z_o} = Q \pm \sqrt{Q^2 - 1} \quad Q = \frac{S_{11}^2 - S_{21}^2 + 1}{2S_{11}}$$

$$X = e^{-j\beta l} = \frac{S_{11} + S_{21} - \Gamma}{1 - (S_{11} + S_{21})\Gamma}$$

Reference Plane Extension

$$S' = \Phi S \Phi$$

where S is for the DUT, S' is for the system with the port extensions.

$$\Phi = \begin{bmatrix} e^{-j\phi_1} & 0 \\ 0 & e^{-j\phi_2} \end{bmatrix}$$

ϕ_1 and ϕ_2 are the phase shifts in ports 1 and 2 respectively

Special Case for Lossless System

$$|S_{11}|^2 + |S_{21}|^2 = 1$$

$$|S_{22}|^2 + |S_{12}|^2 = 1$$

If in addition, the network is reciprocal: $S_{12} = S_{21}$, the network is symmetrical: $S_{11} = S_{22}$.