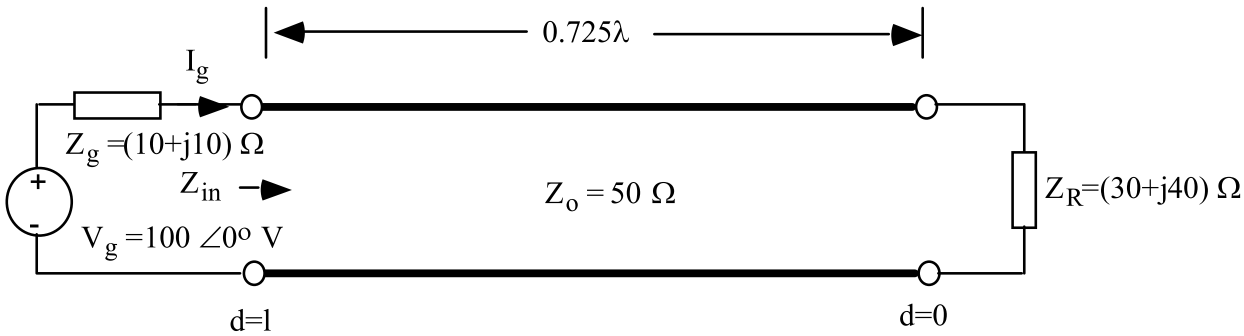
**ECE 451** **Homework 1** **Solutions**




1. Consider the transmission line system shown in the figure above.

   (a) Find the input impedance $Z_{in}$.

$$\Gamma_R = \frac{Z_R - Z_o}{Z_R + Z_o} = \frac{30 + j40 - 50}{30 + j40 + 50} = 0.5\angle 90^\circ$$

$$\Gamma(d = l) = \Gamma_R e^{-2j\beta l} = 0.5\angle 90^\circ e^{-j\frac{4\pi}{\lambda}0.725\lambda} = 0.5\angle -72^\circ$$

$$Z_{in} = Z_o\left[\frac{1+\Gamma(l)}{1-\Gamma(l)}\right] = 50\left[\frac{1+0.5\angle -72^\circ}{1-0.5\angle -72^\circ}\right]$$

$$Z_{in} = 64.36\angle -51.73^\circ$$

$$Z_{in} = (39.86 - j50.54)\Omega$$

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   (b) Find the current drawn from the generator

$$I_g = \frac{V_g}{Z_g + Z_{in}} = \frac{100\angle 0^\circ}{(10 + j10)(39.86 - j50.54)}$$

$$I_g = \frac{100\angle 0^\circ}{(49.86 - j40.54)} = \frac{100\angle 0^\circ}{64.26\angle -39.11^\circ}$$

$$I_g = 1.552\angle 39.11^\circ$$

$$I_g = 1.207 + j0.981$$

(c) Find the time-average power delivered to the load

$$V(l) = Z_{in} I_g = 64.36\angle -51.73^\circ \times 1.552\angle 39.11^\circ = 100.159\angle -12.62^\circ$$

$$\langle P \rangle = \frac{1}{2}\mathrm{Re}\left[V(l) I_g^*\right] = \frac{1}{2}\mathrm{Re}\left[100.159\angle -12.62^\circ \times 1.552\angle -39.11^\circ\right]$$

$$\langle P \rangle = \frac{1}{2}\mathrm{Re}\left[V(l) I_g^*\right] = \frac{1}{2}\mathrm{Re}\left[100.159 \times 1.552 \times \cos 51.73^\circ\right] = 48.26W$$

<P> = 48.26W

2. A lossless transmission line of characteristic impedance 75 Ω is terminated by some complex load, $Z_L$. If the distance from the load to the location of the first impedance minimum is measured to be .304λ and the impedance at that point is 22.5 Ω, using the Smith Chart find:

a) The SWR.

   i. Find $z_{min}=Z_{min}/Z_0=.3$ on the Smith Chart

   ii. Draw constant SWR circle

   iii. Locate real crossing on r>1 side of Smith Chart

   iv. Read off $r_{max}$ = SWR = 3.3

b) The load impedance, $Z_L$.

   i. Rotate from $z_{min}$ towards the load by 0.304λ

   ii. Draw line from point to origin of Smith Chart

   iii. Read off $z_L = 1.6 + j1.5$

   iv. $Z_L = z_L Z_0 = 120 + j112.5$ Ω

c) The reflection coefficient, $\overline{\Gamma}_L$.

   i. Read off θ = 38.5°

   ii. Measure normalized linear distance from origin to $Z_L$: $|\Gamma_L| = .53$

d) Distance to the first voltage maximum.

   i. Rotate from $z_L$ towards generator to real crossing on r>1 side

   ii. $d_{max}$ = .054λ

e) Value of the line impedance at this point.

   i. Read off $z_{max}$ = 3.3

   ii. $Z_{max} = z_{max}Z_0$ = 247.5 Ω

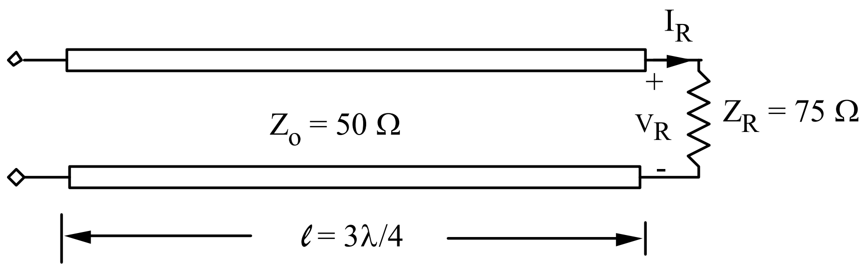
f) The distance to the closest point to the load where the real part of the line impedance is 75 Ω.

   i. Find intersection of SWR circle with r = 1 circle

   ii. Read off distance towards the generator = .134λ

g) Is the impedance at the point located in part f inductive or capacitive?

   i. Point is on bottom portion of Smith Chart so it is capacitive.




3. Given the lossless transmission line shown above,

   (a) Find the reflection coefficient $\Gamma_R$ and the standing wave ratio (SWR) for this loaded transmission line.

   (b) Sketch the standing wave patterns for the magnitude of the voltage along this transmission line in terms of $V_R$.

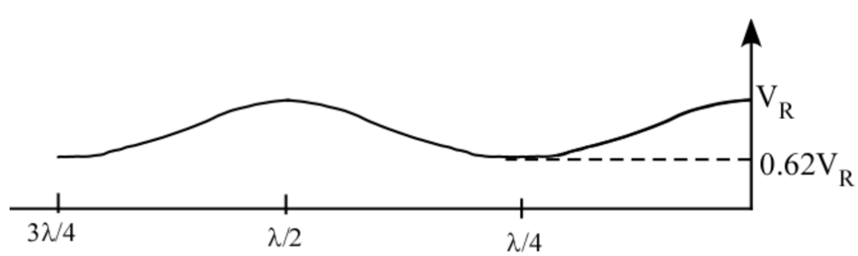
   (c) Determine the impedance at the input of this loaded transmission line.

   (d) If a sinusoidal generator $10 \angle 0^o$ V, which has a source impedance of 100 Ω is connected to this loaded transmission line, what is the time average power delivered to the 75-Ω load.

(a) $\Gamma_R = \dfrac{Z_R - Z_o}{Z_R + Z_o} = \dfrac{75-50}{75+50} = 0.2$

$\text{SWR} = \dfrac{1+0.2}{1-0.2} = 1.5$

(b)




(c) $Z_{in} = \dfrac{Z_o^2}{Z_R} = \dfrac{(50)^2}{75} = \dfrac{2500}{75} = 33.33\ \Omega$

(d) $\langle P \rangle = \dfrac{1}{2}\text{Re}\left[V_{in} I_{in}^*\right] = \dfrac{|V_s|^2 Z_{in}}{2(Z_s + Z_{in})^2} = \dfrac{(100)(33.33)}{2(100+33.33)^2}$, $\langle P \rangle = 0.0937$ watts

4. Answer the following questions using a Smith chart. Clearly identify significant features on the chart. A transmission line with characteristic impedance 50 Ω is terminated by a load of impedance $Z_r = 40 + j50\ \Omega$.

(a) What is the SWR? *Solution: The normalized $z_r$ is 0.8 + j 1.0. Find this point on the Smith Chart, draw the constant SWR circle, read off the SWR=3.0*

(b) What is the phase of $\Gamma_r$? *72 degrees, from the Smith Chart.*

(c) What is the normalized admittance at the load? *0.5-j0.6, from the Smith Chart.*

(d) What is the normalized admittance at $d = 12.2\lambda$ toward the generator from the load? *0.5+j0.6, from the Smith Chart.*

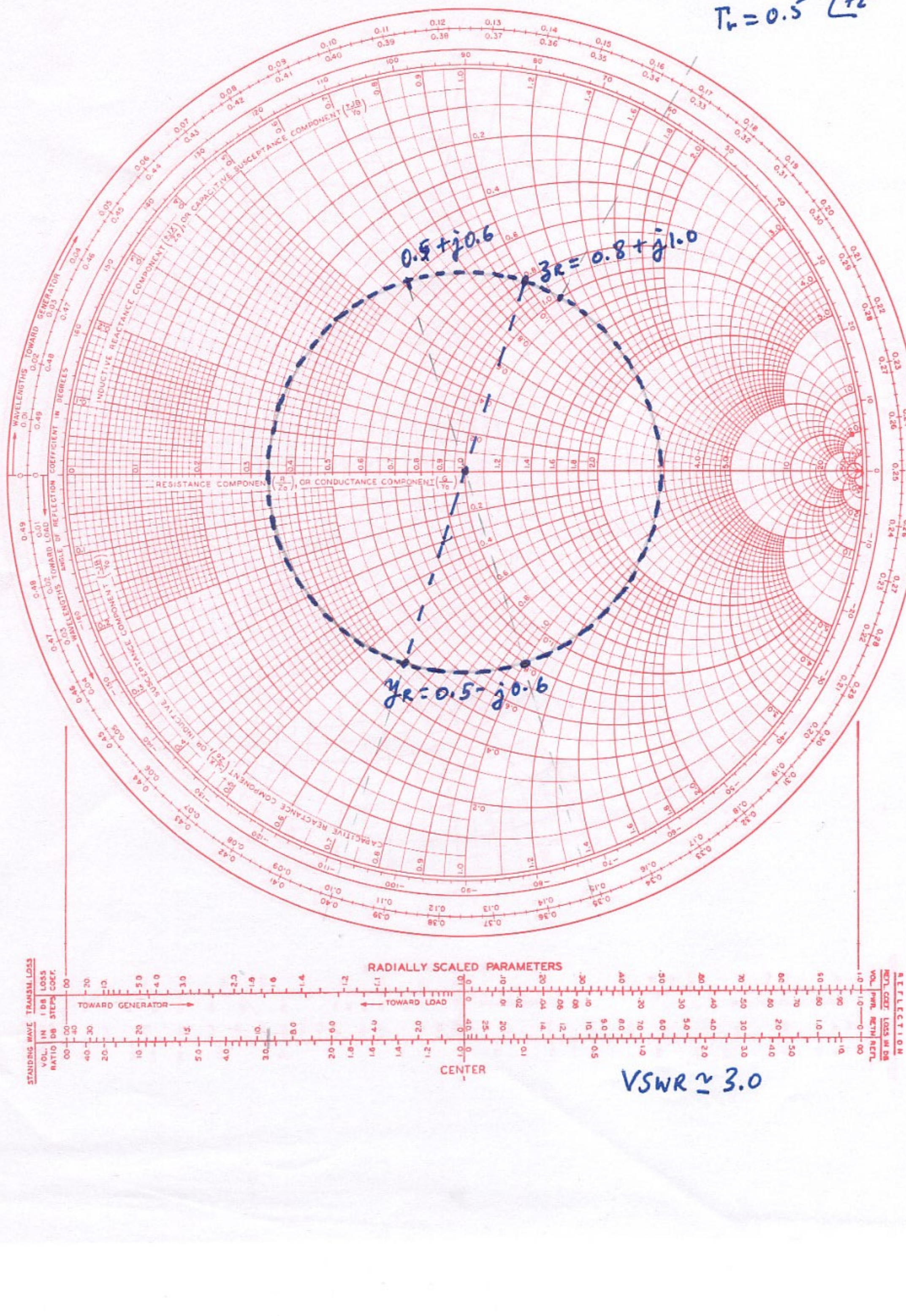
(e) What is the phase of Γ at d = 12.2λ toward the generator from the load? (10 pts) - *72 degrees, from the Smith Chart. Be sure to read the phase of Gamma from the impedance side of the constant SWR circle.*

(f) What is the shortest distance from the load at which a short-circuited stub could be attached to achieve an impedance match? *0.265 wavelengths, from the Smith Chart.*

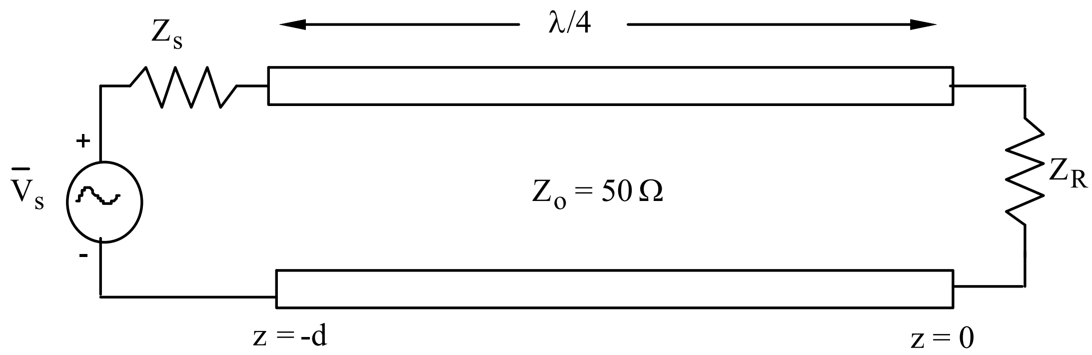
(g) What would the normalized input admittance of the stub be? *-j1.1*

IMPEDANCE OR ADMITTANCE COORDINATES

$\Gamma_L = 0.5 \angle 72°$




VSWR ≃ 3.0




5. For the above figure, let $\overline{V}_s$ = 1 volt, $Z_s$ = 30 Ω, and $Z_R$ = 40 Ω.

   (a) What is the impedance at the input of the line (z = -d)?

   $$Z_{in} = \frac{Z_o^2}{Z_R} = \frac{(50)^2}{40} = 62.5\ \Omega$$

   (b) What is the phasor current through $Z_s$?

   $$I_{in} = \frac{1}{Z_S + Z_{in}} = \frac{1}{30 + 62.5} = 0.01081\ A$$

   (c) What is the phasor current through $Z_R$ ?

   $T_S$=5/8, $\Gamma_S$=-2/8, $\Gamma_R$= -1/9, ,

   V+ = -j0.608 V, $= \frac{-j0.608}{50}\left[1-(-0.111)\right]$

   $I_R$=-j13.51 mA

   (d) What is the time-average power delivered to the load ?

   $$P = \frac{1}{2} Re\left\{V_{in} I_{in}^*\right\} = \frac{1}{2} Re\left\{62.5 \times 0.01081 \times 0.01081\right\}$$

   P=3.65 mW