Causality assessment in S-parameter with Hilbert transform

You are given a measurement data of a passive 2-port network, `allfreq.s2p`. Write a script that calculates the discrete Hilbert transform as in (11). Use the script to:

a) calculate the Hilbert transform of Re(S_{21}). Compare it with Im(S_{21}).

b) calculate the Hilbert transform of Im(S_{21}). Compare it with Re(S_{21}).

**Solution**

Using the discrete version of the Hilbert transform relationship given by [1]:

\[
\hat{f}_k = \begin{cases} 
\frac{2}{\pi} \sum_{n \text{ odd}} f_n \sin(k-n), & k \text{ even} \\
\frac{2}{\pi} \sum_{n \text{ even}} f_n \cos(k-n), & k \text{ odd}
\end{cases}
\]

**Script:**

```python
def dht(H):
    N = len(H)
    Hhilbert = []
    for k in range(N):
        n_start = int(not bool(k%2))
        n = np.arange(n_start,N,2)
        Hhilbert += [2/np.pi*(H[n]/(k-n)).sum()]
    return np.array(Hhilbert)
```

**Plots**

[Plot 1: Graph of Re(S_{21}) vs frequency]

[Plot 2: Graph of Im(S_{21}) vs frequency]

**References**