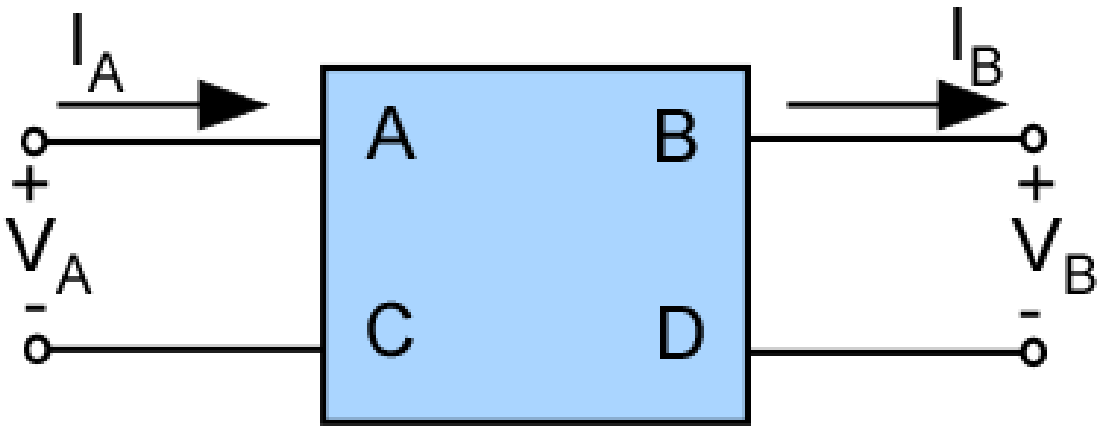
ECE 451
Advanced Microwave Measurements

ABCD Parameters

Jose E. Schutt-Aine
Electrical & Computer Engineering
University of Illinois
jesa@illinois.edu

ABCD -Parameters

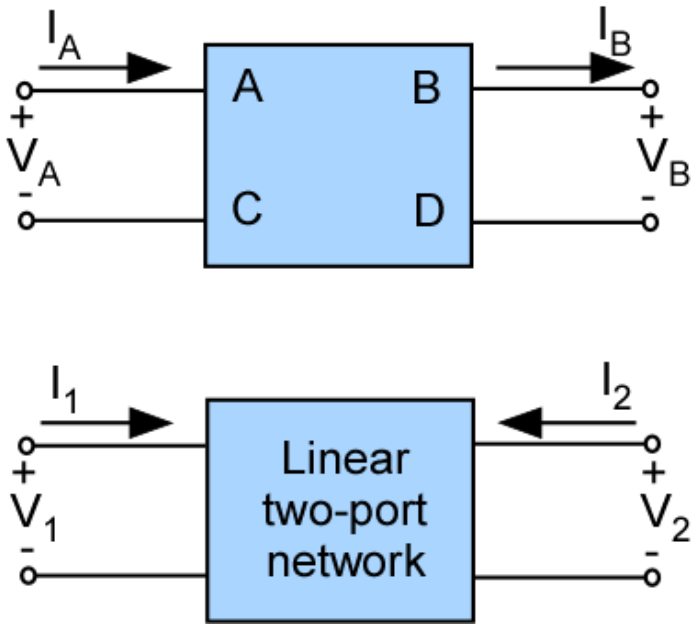


$$V_A = AV_B + BI_B$$

$$I_A = CV_B + DI_B$$

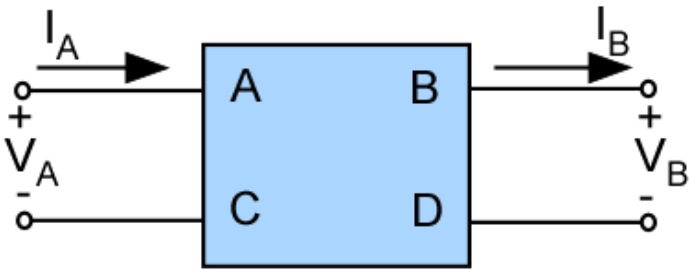
ABCD -Parameters

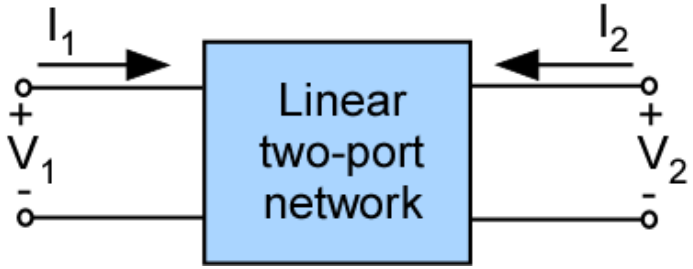
$$V_A = V_1$$
$$V_B = V_2$$
$$I_A = I_1$$
$$I_B = -I_2$$



Relationship with Z parameters is obtained by first expressing ABCD parameters in terms of Z parameters

ABCD -Parameters





From

$$V_A = Z_{11} I_A - Z_{12} I_B$$

$$V_B = Z_{21} I_A - Z_{22} I_B$$

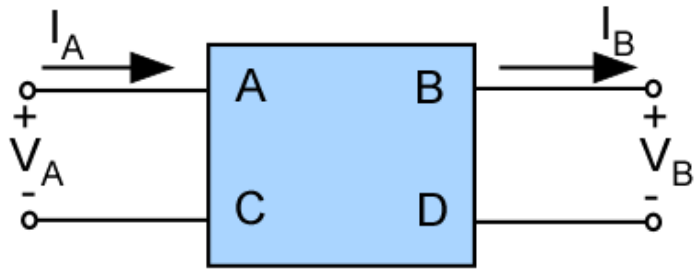
We get

$$A = \frac{Z_{11}}{Z_{21}} \qquad B = \frac{\Delta}{Z_{21}}$$

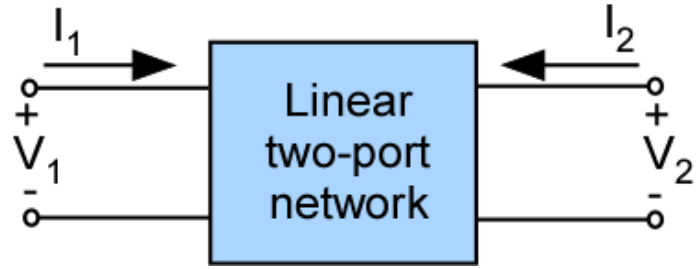
$$C = \frac{1}{Z_{21}} \qquad D = \frac{Z_{22}}{Z_{21}}$$

$$\Delta = Z_{11} Z_{22} - Z_{12} Z_{21}$$

ABCD -Parameters



$$Z_{11} = \frac{A}{C} \qquad Z_{11} = \frac{(AD - BC)}{C}$$



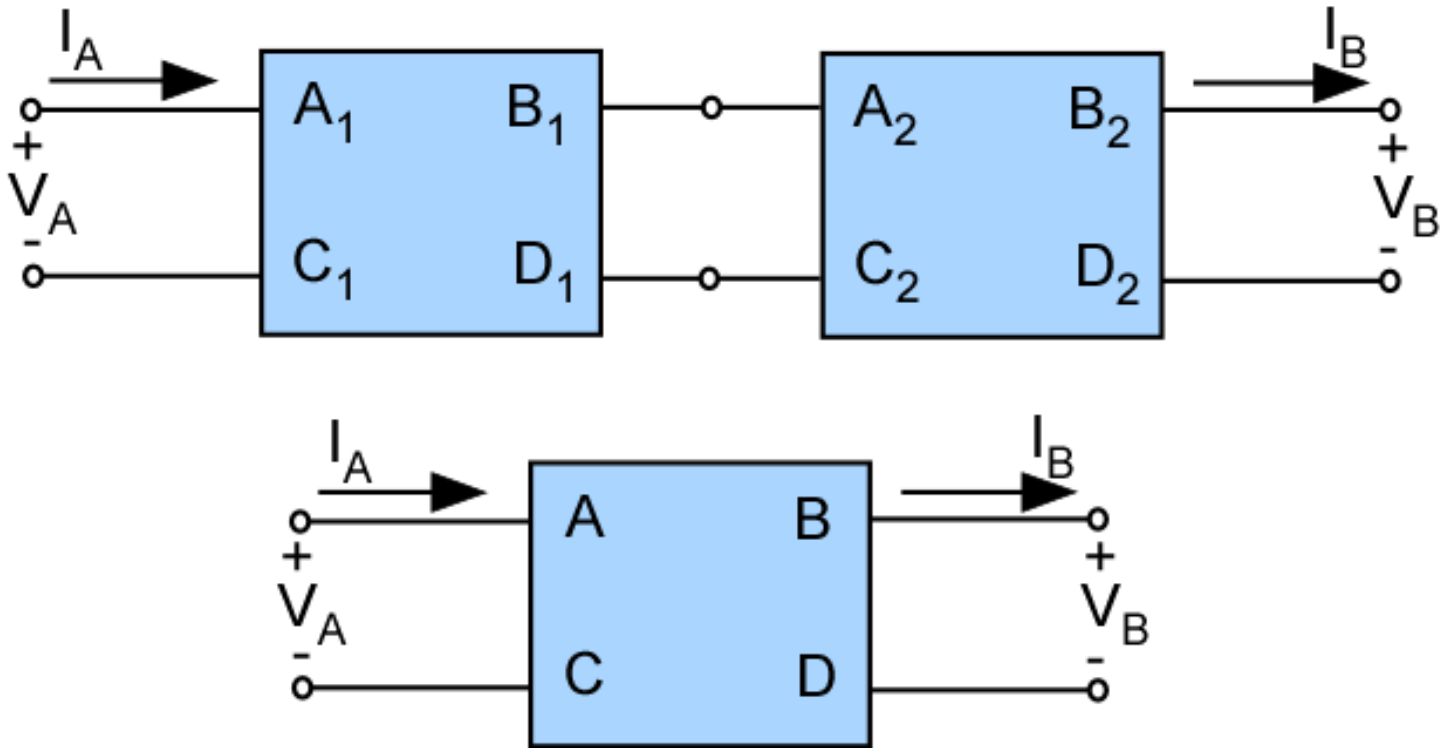
$$Z_{21} = \frac{1}{C} \qquad Z_{22} = \frac{1}{C}$$

For a reciprocal network, $Z_{21} = Z_{12}$, therefore

$$AD - BC = 1$$

Reciprocity condition for ABCD parameters

ABCD -Parameters



When cascading two-ports, it is best to use ABCD parameters. Put voltage and currents in cascadable form with the input variables in terms of the output variables

$$\mathbf{ABCD} = \left(\mathbf{ABCD}\right)_1 \cdot \left(\mathbf{ABCD}\right)_2$$

Scattering Transfer Parameters

In T-Parameters, traveling waves at the input are related to those at the output

$$b_1 = S_{11}a_1 + S_{12}a_2 \qquad b_1 = T_{11}a_2 + T_{12}b_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2 \qquad a_1 = T_{21}a_2 + T_{22}b_2$$

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} T_{12}T_{22}^{-1} & T_{11} - T_{12}T_{21}T_{22}^{-1} \\ T_{22}^{-1} & -T_{21}T_{22}^{-1} \end{pmatrix}$$

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} S_{12} - S_{11}S_{22}S_{21}^{-1} & S_{11}S_{21}^{-1} \\ -S_{22}S_{21}^{-1} & S_{21}^{-1} \end{pmatrix}$$

T parameters can be cascaded $\mathbf{T} = \mathbf{T_A} \cdot \mathbf{T_B}$