

# ECE 451

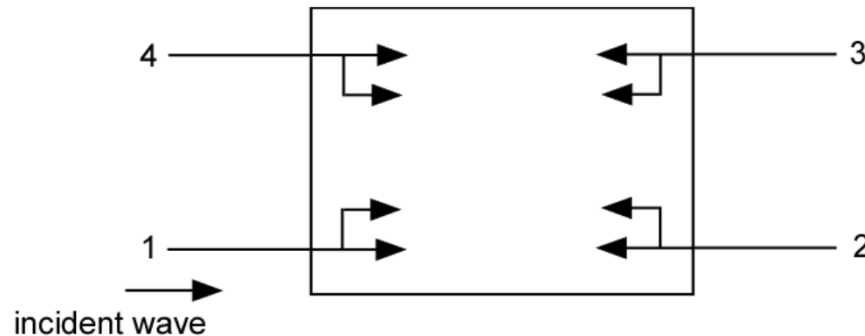
# Advanced Microwave Measurements

## Directional Couplers

Jose E. Schutt-Aine  
Electrical & Computer Engineering  
University of Illinois  
jschutt@emlab.uiuc.edu

# Directional Coupler



- Wave incident in port 1 couples into ports 2 & 3 but NOT in port 4 → ports 1 & 4 are uncoupled
- Wave incident in port 2 couples into ports 1 & 4 but NOT in port 3 → ports 2 & 3 are uncoupled
- In addition, all 4 ports are matched. That is if 3 ports are terminated with  $Z_o$ , the fourth port appears terminated with  $Z_o$ .

# Characteristics of Directional Couplers

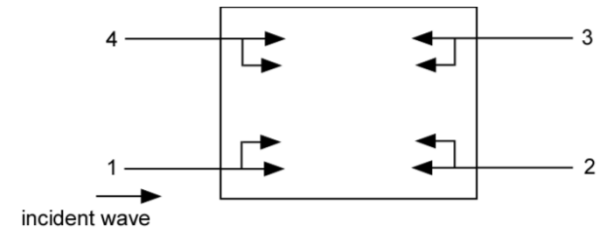
## Coupling

Let  $P_i$  be the incident power in port 1

Let  $P_f$  be the coupled power in port 3

Define coupling in decibels (dB) as:

$$C = 10 \log \frac{P_i}{P_f}$$



## Directivity

Ideally, power coupled into port 4,  $P_b$  should be zero, but in reality it is not which defines the directivity of the coupler

$$D = 10 \log \frac{P_f}{P_b}$$

Ideally, directivity should be infinite

# S-Parameters of Directional Couplers

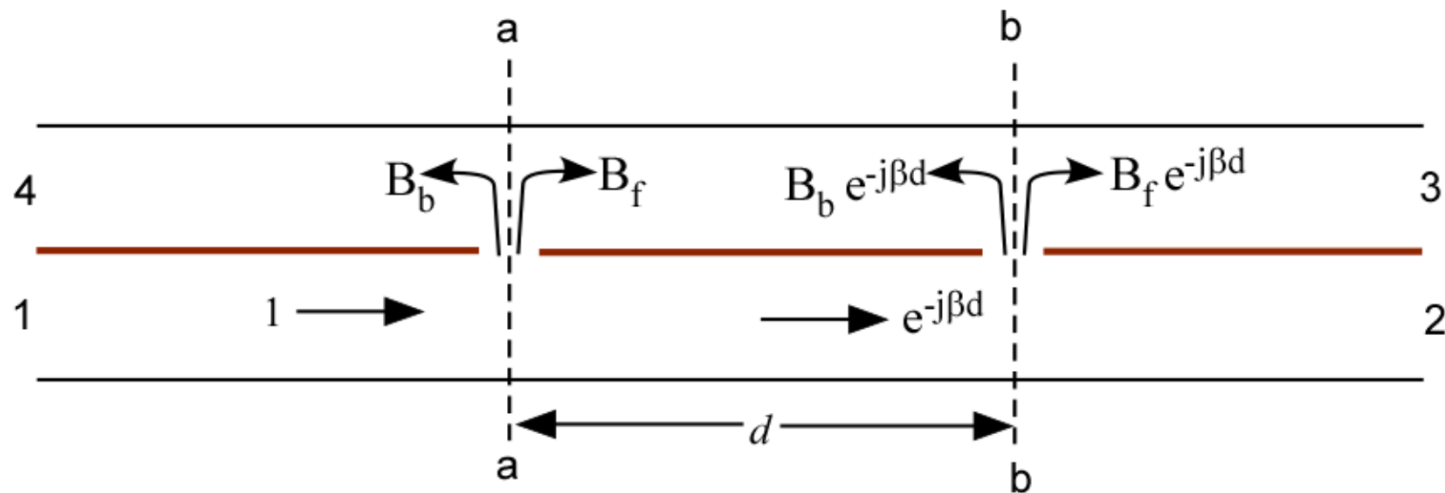
Directional coupler is described by a 4 by 4 S-parameter matrix. It is obvious that  $S_{14}=S_{23}=0 \rightarrow S_{41}=S_{32}=0$ .  
Moreover,  $S_{11}=S_{22}=S_{33}=S_{44}=0$

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{21} & 0 & 0 & S_{24} \\ S_{31} & 0 & 0 & S_{34} \\ 0 & S_{42} & S_{43} & 0 \end{bmatrix}$$

By reciprocity,  $S_{12}=S_{21}$ ,  $S_{13}=S_{31}$ ,  $S_{42}=S_{24}$  and  $S_{43}=S_{34}$

# Design of Directional Coupler

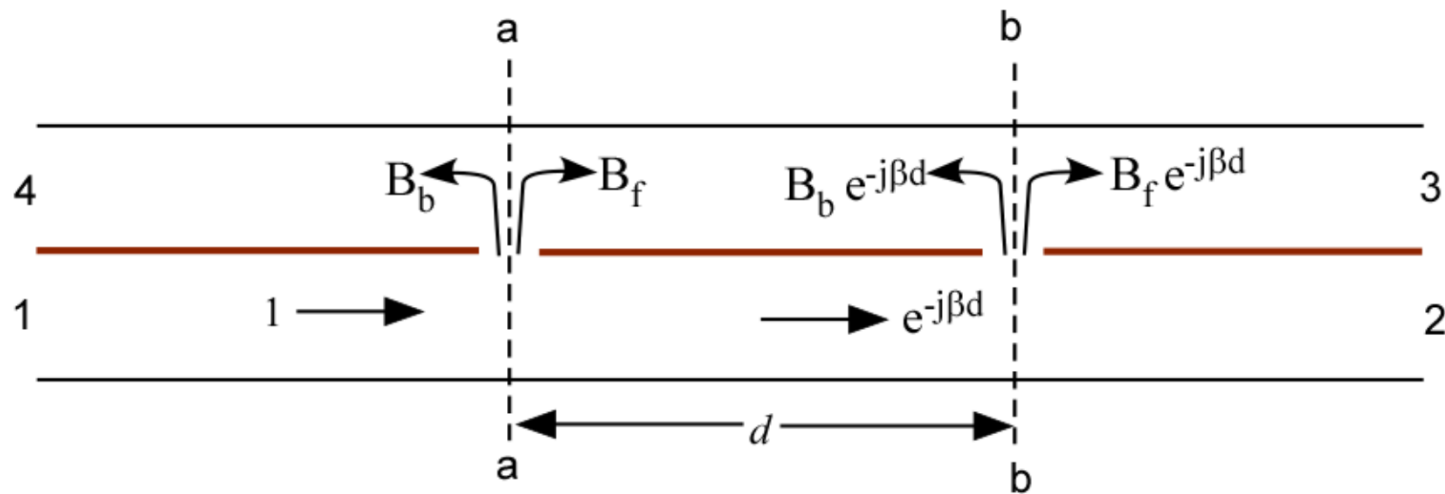
## Aperture Coupling



- Incident wave in port 1 has value 1
- Forward wave coupled in second guide at first hole has value  $B_f$ .
- Backward wave coupled in second guide at first hole has value  $B_b$ .
- $B_b$  and  $B_f$  are the aperture coupling coefficients

# Design of Directional Coupler

## Aperture Coupling



- At second aperture, the field is approximately of the same magnitude but the phase has changed.
- Forward:  $B_f e^{-j\beta d}$  --- Backward:  $B_b e^{-j\beta d}$
- Total forward wave in upper guide at plane bb is  $2B_f e^{-j\beta d}$
- Total backward wave in upper guide at plane aa is given by  $B_b(1+e^{-j2\beta d})$

# Design of Directional Coupler

## Results

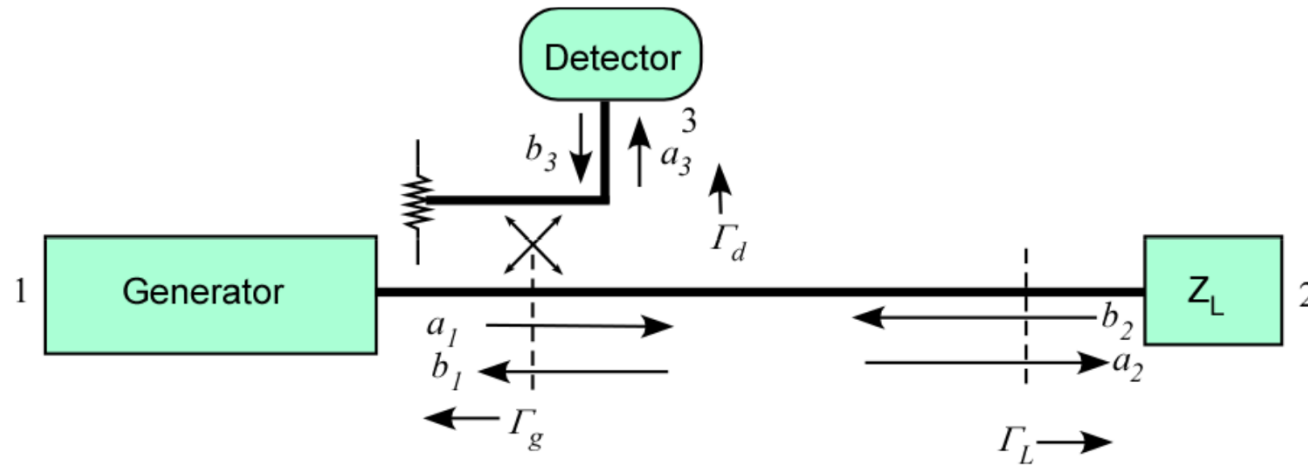
- Forward waves add in phase (same path length)
- Backward waves add out of phase if  $d=\lambda/4$  or if  $d$  is an integer multiple of  $\lambda/4$ .

**Coupling:**  $C = -20 \log 2|B_f|$

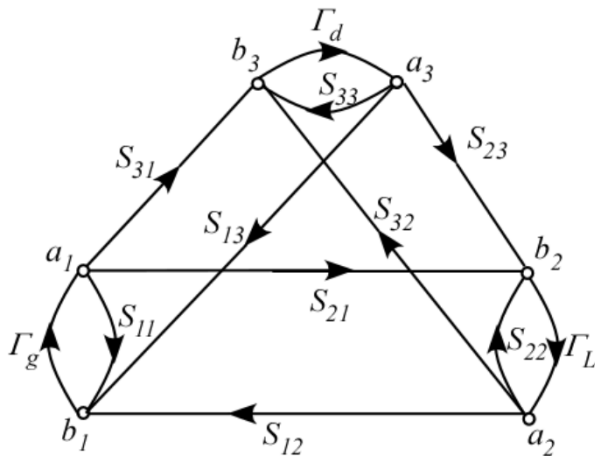
**Directivity:**  $D = 20 \log \frac{2|B_f|}{|B_b| |1 + e^{-2j\beta d}|} = 20 \log \frac{2|B_f|}{|B_b| |\cos \beta d|}$

$$D = 20 \log \frac{|B_f|}{|B_b|} + 20 \log |\sec \beta d|$$

# Flow Graph for Directional Coupler



Assume load is matched  $\Gamma_L=0$



First order loops:

$$\Gamma_g S_{11}, \Gamma_g S_{31} \Gamma_d S_{13}, \Gamma_d S_{33}$$

Second order loop:

$$\Gamma_g S_{11} \Gamma_d S_{33}$$

$$\Delta = 1 - \Gamma_g S_{11} - \Gamma_d S_{33} - \Gamma_g S_{31} \Gamma_d S_{13} + \Gamma_g S_{11} \Gamma_d S_{33}$$