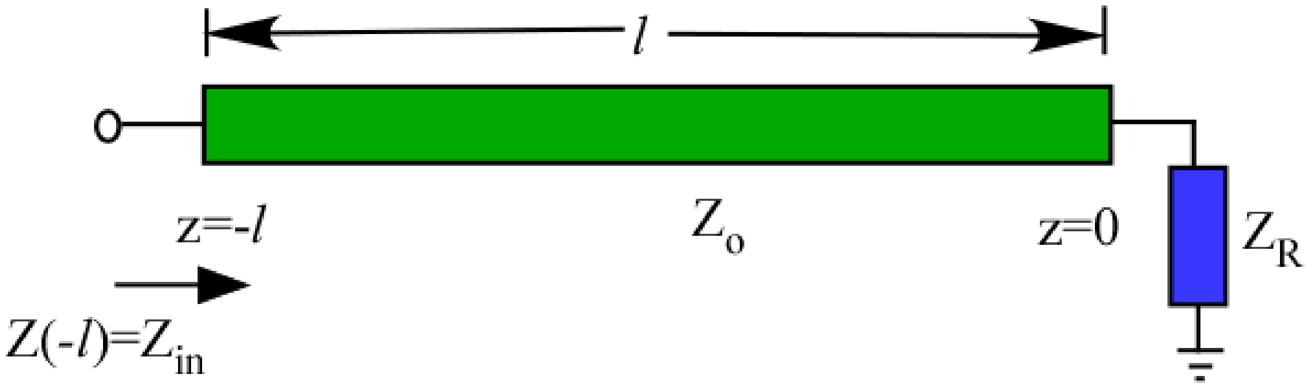
ECE 451
Advanced Microwave Measurements

Review

Jose E. Schutt-Aine
Electrical & Computer Engineering
University of Illinois
jesa@illinois.edu

Generalized Impedance

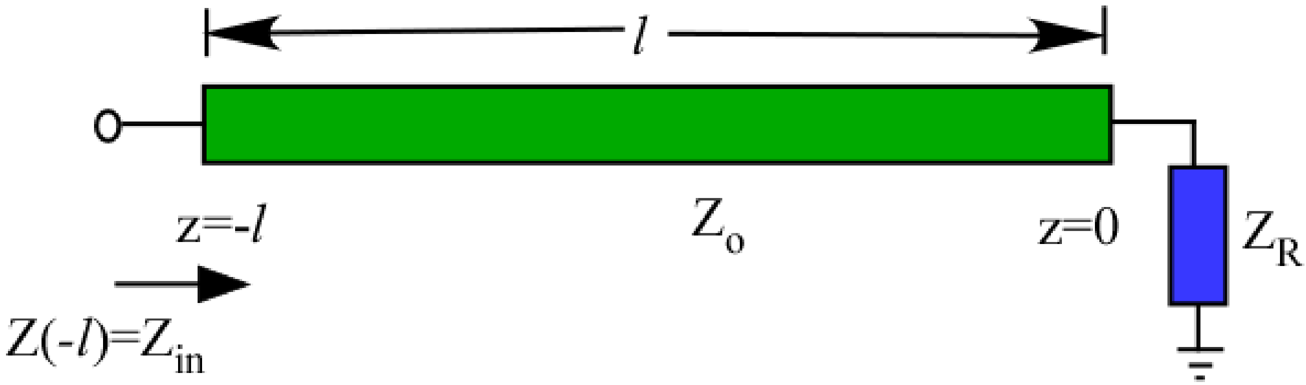


$$Z(z) = \frac{V(z)}{I(z)} = Z_o \left[\frac{e^{-j\beta z} + \Gamma_R e^{+j\beta z}}{e^{-j\beta z} - \Gamma_R e^{+j\beta z}} \right]$$

$$Z(-l) = Z_o \left[\frac{Z_R + jZ_o \tan \beta l}{Z_o + jZ_R \tan \beta l} \right]$$

Impedance transformation equation

Generalized Impedance



- Short circuit Z_R=0, line appears inductive for 0 < l < λ/2

$$Z(-l) = jZ_o \tan \beta l$$

- Open circuit Z_R ➔ inf, line appears capacitive for 0 < l < λ/2

$$Z(-l) = \frac{Z_o}{j \tan \beta l}$$

- If l = λ/4, the line is a quarter-wave transformer

$$Z(-l) = \frac{Z_o^2}{Z_R}$$

Generalized Reflection Coefficient

$$\Gamma(z) = \frac{\text{Backward traveling wave at } z}{\text{Forward traveling wave at } z} = \frac{V_b(z)}{V_f(z)}$$

$$\Gamma(z) = \frac{V_- e^{+j\beta z}}{V_+ e^{-j\beta z}} = \frac{V_-}{V_+} e^{+2j\beta z} = \Gamma_R e^{+2j\beta z}$$

Reflection coefficient transformation equation $\Rightarrow$ $\Gamma(-l) = \Gamma_R e^{-2j\beta l}$

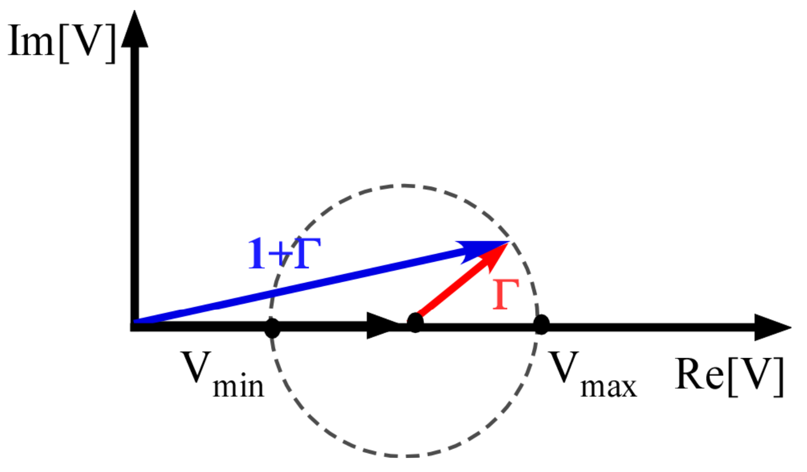
$$Z(z) = Z_o \frac{1+\Gamma(z)}{1-\Gamma(z)}$$

$$\Gamma(z) = \frac{Z(z) - Z_o}{Z(z) + Z_o}$$

Voltage Standing Wave Ratio (VSWR)

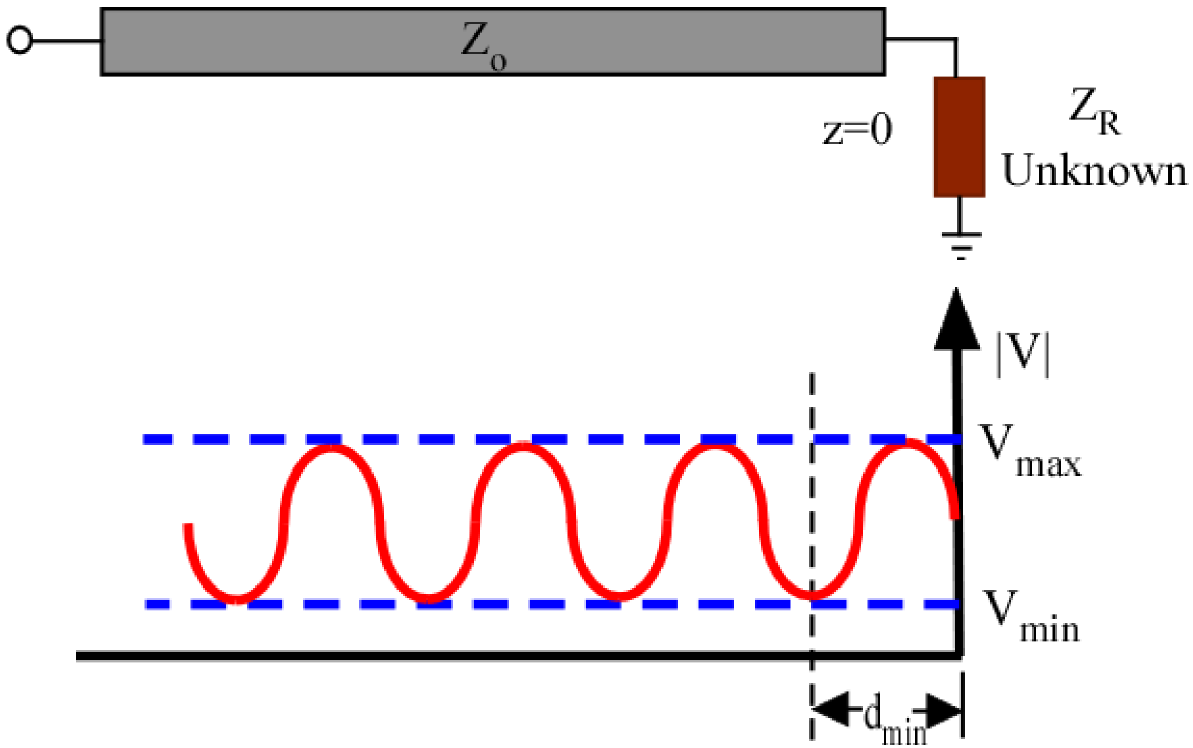
Define Voltage Standing Wave Ratio as:

$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1+|\Gamma_R|}{1-|\Gamma_R|}$$



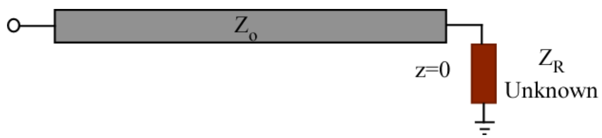
It is a measure of the interaction between forward and backward waves

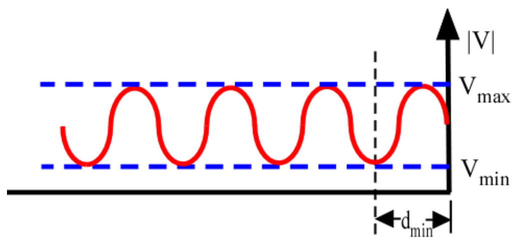
Application: Slotted-Line Measurement



- Measure VSWR = V_{max}/V_{min}
- Measure location of first minimum

Application: Slotted-Line Measurement





At minimum, $\Gamma(z) = \text{pure real} = -\left|\Gamma_R\right|$

Therefore, $\Gamma(-d_{\min}) = \Gamma_R e^{-2j\beta d_{\min}} = -\left|\Gamma_R\right|$

So, $\Gamma_R = -\left|\Gamma_R\right| e^{+2j\beta d_{\min}}$

Since $\left|\Gamma_R\right| = \dfrac{VSWR-1}{VSWR+1}$ **then** $\Gamma_R = -\left(\dfrac{VSWR-1}{VSWR+1}\right) e^{+2j\beta d_{\min}}$

and $Z_R = Z_o\left(\dfrac{1+\Gamma_R}{1-\Gamma_R}\right)$

Summary of TL Equations

Voltage

$$V(z) = V_+ e^{-j\beta z}\left[1+\Gamma_R e^{+2j\beta z}\right]$$

Current

$$I(z) = \frac{V_+}{Z_o} e^{-j\beta z}\left[1-\Gamma_R e^{+2j\beta z}\right]$$

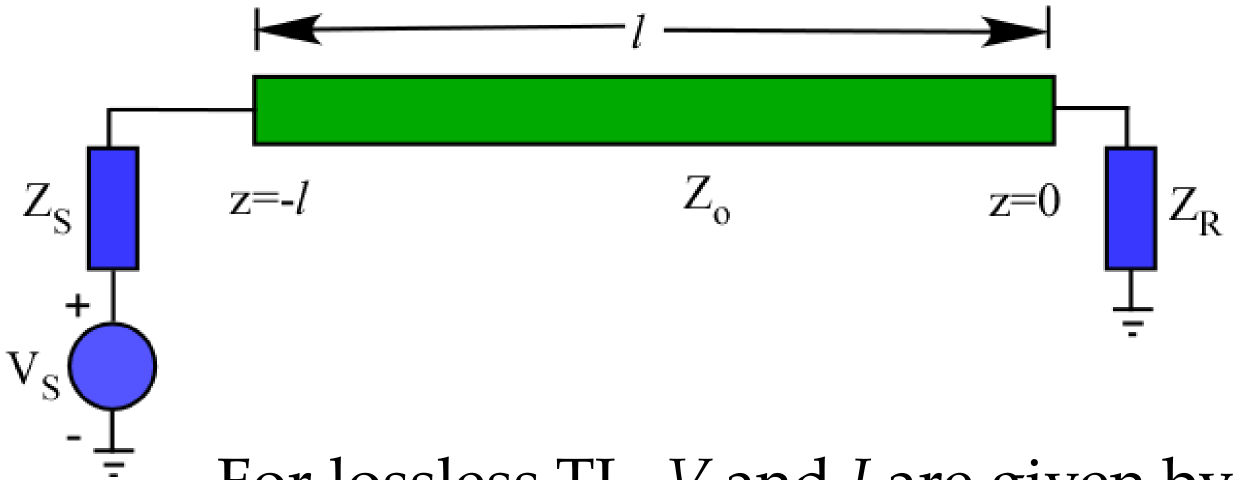
Impedance Transformation ➔ $$Z(-l) = Z_o\left[\frac{Z_R + jZ_o \tan\beta l}{Z_o + jZ_R \tan\beta l}\right]$$

Reflection Coefficient Transformation ➔ $$\Gamma(-l) = \Gamma_R e^{-2j\beta l}$$

Reflection Coefficient – to Impedance ➔ $$Z(z) = Z_o \frac{1+\Gamma(z)}{1-\Gamma(z)}$$

Impedance to Reflection Coefficient ➔ $$\Gamma(z) = \frac{Z(z) - Z_o}{Z(z) + Z_o}$$

Determining V_+



For lossless TL, *V* and *I* are given by

$$V(z) = V_+ e^{-j\beta z}\left[1+\Gamma_R e^{+2j\beta z}\right]$$

reflection coefficient at the load

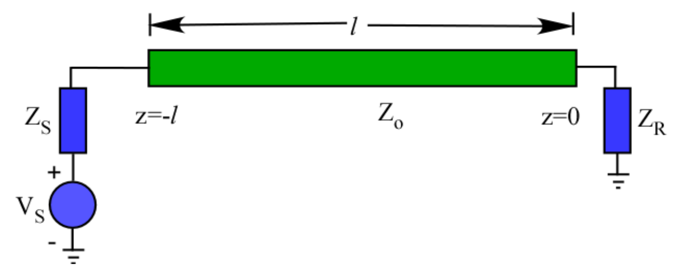
$$\Gamma_R = \frac{Z_R - Z_o}{Z_R + Z_o}$$

$$I(z) = \frac{V_+ e^{-j\beta z}}{Z_o}\left[1-\Gamma_R e^{+2j\beta z}\right]$$

At $z = -l$, $\quad V_S = Z_S I(-l) + V(-l)$

Determining V_+

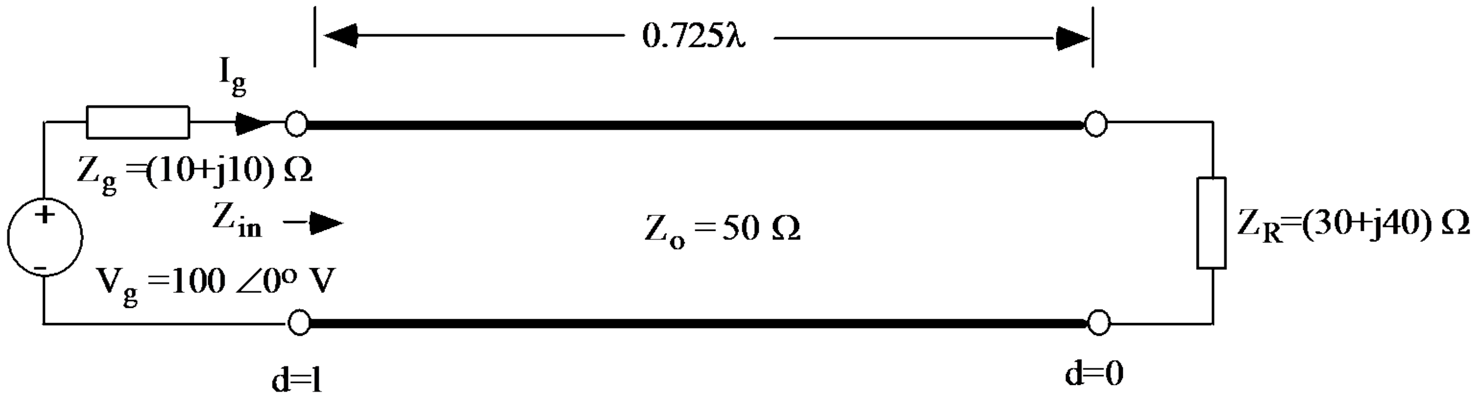
$$V_+\left(e^{+j\beta l}-\Gamma_S\Gamma_R e^{-j\beta l}\right)=T_S V_S$$



with $T_S=\left(1+\frac{Z_S}{Z_o}\right)^{-1}=\frac{Z_o}{Z_S+Z_o}$ and $\Gamma_S=\frac{Z_S-Z_o}{Z_S+Z_o}$

From which $$V_+=\frac{T_S V_S e^{-j\beta l}}{1-\Gamma_S\Gamma_R e^{-2j\beta l}}$$

Example 1



Consider the transmission line system shown in the figure above.

(a) Find the input impedance Z_{in}.

$$\Gamma_R = \frac{Z_R - Z_o}{Z_R + Z_o} = \frac{(30 + j40) - 50}{(30 + j40) + 50} = 0.5\angle 90^\circ$$

$$\Gamma(d = l) = \Gamma_R e^{-2j\beta l} = 0.5\angle 90^\circ e^{-j\frac{4\pi}{\lambda}.0.725\lambda} = 0.5\angle -72^\circ$$

Example 1 – Cont'

$$Z_{in} = Z_o\left[\frac{1+\Gamma(l)}{1-\Gamma(l)}\right] = 50\left[\frac{1+0.5\angle-72^\circ}{1-0.5\angle-72^\circ}\right]\Omega$$

$$Z_{in} = 64.361\angle-51.738^\circ\ \Omega = \left(39.86 - j50.54\right)\Omega$$

(b) Find the current drawn from the generator

$$I_g = \frac{V_g}{Z_g + Z_{in}} = \frac{100\angle 0^\circ}{(10+j10)(39.86-j50.54)} = 1.552\angle 39.11^\circ\ A$$

$$I_g = 1.207 + j0.981\ A$$

Example 1 – Cont’

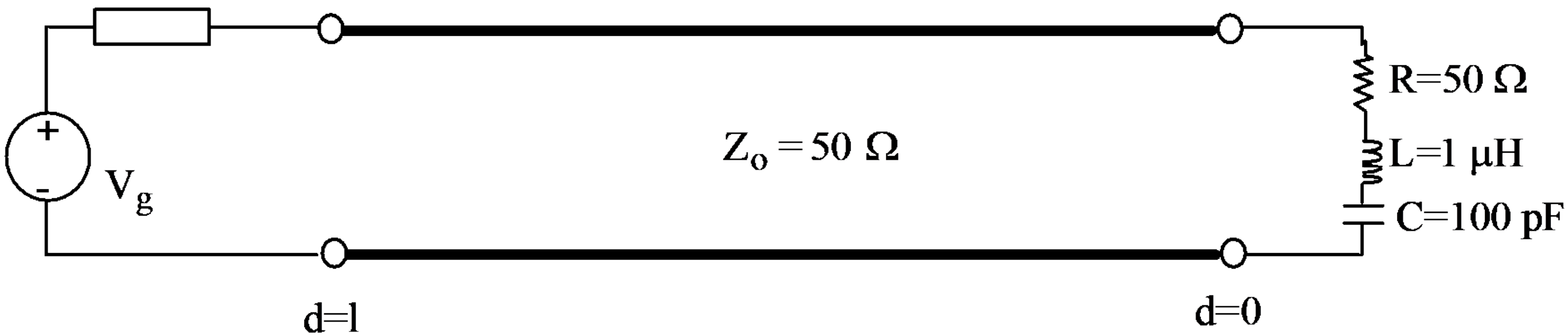
c) Find the time-average power delivered to the load

$$V(l) = Z_{in} I_g = 64.361\angle 51.73^\circ \times 1.5562\angle 39.11^\circ = 100.159\angle -12.62^\circ \frac{1}{2}$$

$$< P >= \frac{1}{2}\text{Real}\left[V(l) I_g^*\right] = \frac{1}{2}\text{Real}\left[100.159\angle -12.62^\circ \times 1.5562\angle -39.11^\circ\right]$$

$$< P >= \frac{1}{2}\text{Real}\left[V(l) I_g^*\right] = 48.26\, W$$

Example 1 – Cont'



In the system shown above, a line of characteristic impedance 50 Ω is terminated by a series R, L, C circuit having the values R = 50 Ω, L = 1 μH, and C = 100 pF.

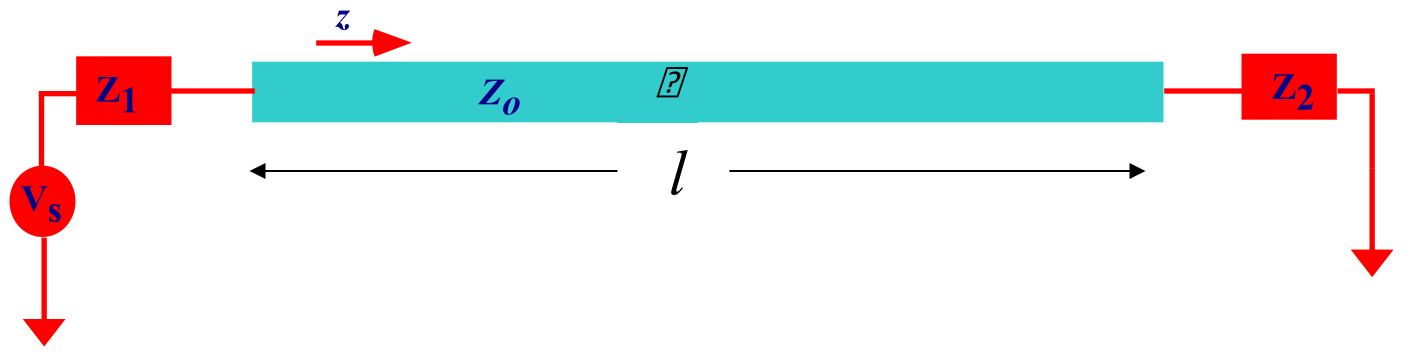
(d) Find the source frequency f_o for which there are no standing waves on the line.

Example 1 – Cont'

No standing wave on the line if $Z_R = R = Z_o$ which occurs at the resonant frequency of the RLC circuit. Thus,

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10^{-6} \times 10^{-10}}} = \frac{10^8}{2\pi}\ Hz = 15.92\ MHz$$

Lossy Transmission Line

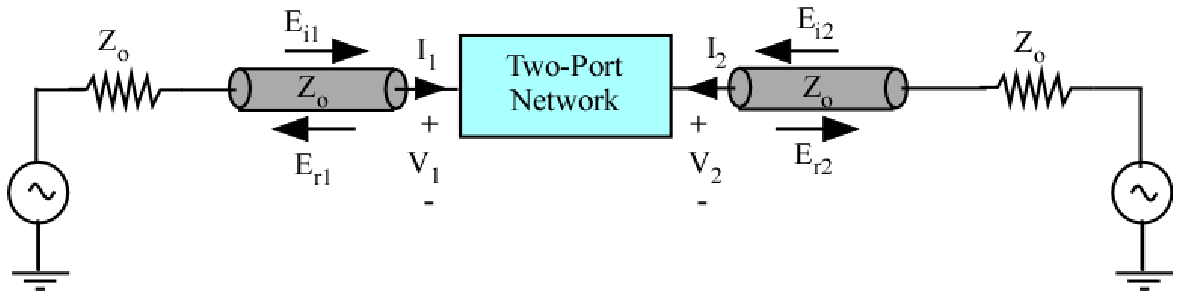


$$V(z) = Ae^{-\alpha z}e^{-j\beta z} + Be^{+\alpha z}e^{+j\beta z}$$

$$I(z) = \frac{1}{Z_o}\left[Ae^{-\alpha z}e^{-j\beta z} - Be^{+\alpha z}e^{+j\beta z}\right]$$

$$Z_o = \sqrt{\frac{\left(R(\omega) + j\omega L\right)}{\left(G + j\omega C\right)}} \qquad \gamma = \alpha + j\beta = \sqrt{\left(R(\omega) + j\omega L\right)\left(G + j\omega C\right)}$$

Wave Approach



$$a_1 = \frac{E_{i1}}{\sqrt{Z_o}} \qquad a_2 = \frac{E_{i2}}{\sqrt{Z_o}}$$

$$b_1 = \frac{E_{r1}}{\sqrt{Z_o}} \qquad b_2 = \frac{E_{r2}}{\sqrt{Z_o}}$$

Z_o is the reference impedance of the system

$$\mathbf{b_1 = S_{11}\, a_1 + S_{12}\, a_2}$$
$$\mathbf{b_2 = S_{21}\, a_1 + S_{22}\, a_2}$$

17

N-Port S Parameters

$$\begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdot & \cdot \\ S_{21} & S_{22} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ a_n \end{bmatrix}$$

$$\mathbf{b} = \mathbf{Sa}$$

If $b_i = 0$, then no reflected wave on port i ➔ port is matched

$$a_i = \frac{V_i^+}{\sqrt{Z_{oi}}}$$

V_i^+ **: incident voltage wave in port *i***

V_i^- **: reflected voltage wave in port *i***

$$b_i = \frac{V_i^-}{\sqrt{Z_{oi}}}$$

Z_{oi} **: impedance in port *i***

Dissipated Power

$$P_d = \frac{1}{2}\mathbf{a}^{\mathrm{T}}\left(\mathbf{U} - \mathbf{S}^{\mathrm{T}}\mathbf{S}^{*}\right)\mathbf{a}^{*}$$

The dissipation matrix D is given by:

$$\mathbf{D} = \mathbf{U} - \mathbf{S}^{\mathrm{T}}\mathbf{S}^{*}$$

Passivity insures that the system will always be stable provided that it is connected to another passive network

For passivity

- **(1) the determinant of D must be** ≥ 0
- **(2) the determinant of the principal minors must be** ≥ 0

Dissipated Power

When the dissipation matrix is 0, we have a lossless network➔

$$\mathbf{S}^{\mathbf{T}}\mathbf{S}^{*} = \mathbf{U}$$

The S matrix is unitary.

For a lossless two-port:

$$|S_{11}|^2 + |S_{21}|^2 = 1$$

$$|S_{22}|^2 + |S_{12}|^2 = 1$$

If in addition the network is reciprocal, then

$$S_{12} = S_{21} \quad \text{and} \quad |S_{11}| = |S_{22}| = \sqrt{1 - |S_{12}|^2}$$

Scattering Transfer Parameters

In T-Parameters, traveling waves at the input are related to those at the output

$$b_1 = S_{11}a_1 + S_{12}a_2 \qquad b_1 = T_{11}a_2 + T_{12}b_2$$

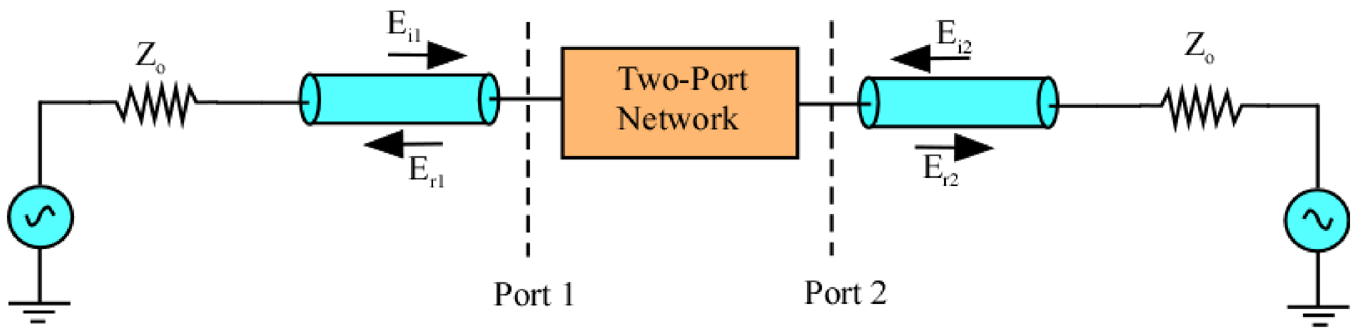
$$b_2 = S_{21}a_1 + S_{22}a_2 \qquad a_1 = T_{21}a_2 + T_{22}b_2$$

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} T_{12}T_{22}^{-1} & T_{11} - T_{12}T_{21}T_{22}^{-1} \\ T_{22}^{-1} & -T_{21}T_{22}^{-1} \end{pmatrix}$$

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} S_{12} - S_{11}S_{22}S_{21}^{-1} & S_{11}S_{21}^{-1} \\ -S_{22}S_{21}^{-1} & S_{21}^{-1} \end{pmatrix}$$

T parameters can be cascaded $\mathbf{T} = \mathbf{T_A} \cdot \mathbf{T_B}$

High-Frequency Characterization



Transmission-Line Scattering Parameters

$$S_{21} = \frac{(1-\Gamma^2)X}{1-\Gamma^2 X^2} \qquad S_{11} = \frac{(1-X^2)\Gamma}{1-\Gamma^2 X^2}$$

$$X = e^{-\gamma d} \qquad Z_c = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\Gamma = \frac{Z_c - Z_o}{Z_c + Z_o} \qquad \gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

TL Extraction Formulas

$$\mathrm{X}=\mathrm{e}^{-\gamma \mathrm{d}} = e^{-j\beta d} e^{-\alpha d}$$

$$X = e^{-\gamma d} = \frac{\left(S_{11}+S_{21}\right)-\Gamma}{1-\left(S_{11}+S_{21}\right)\Gamma}$$

$$\Gamma = Q \pm \sqrt{Q^2-1}$$

$$Q = \frac{\{S_{11}^2 - S_{21}^2\}+1}{2S_{11}}$$

$$R = \mathrm{Re}\{\gamma Z_c\}$$

$$G = \mathrm{Re}\{\frac{\gamma}{Z_c}\}$$

$$L = \frac{1}{\omega}\mathrm{Im}\{\mathrm{Z_c}\gamma\}$$

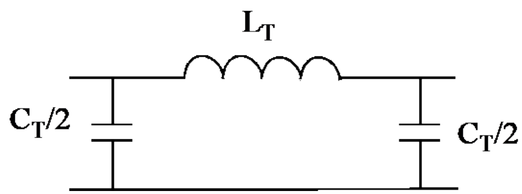
$$C = \frac{1}{\omega}\mathrm{Im}\{\frac{\gamma}{Z_c}\}$$

Modeling Interconnections

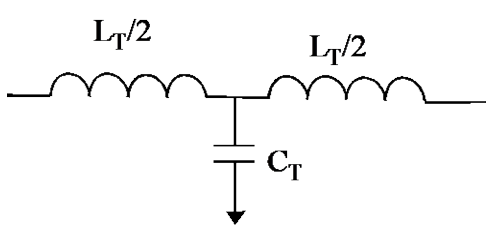
Low Frequency

Short

Mid-range
Frequency

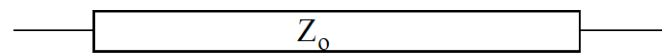


or



Lumped
Reactive CKT

High Frequency



Transmission
Line