ECE 451
Specialized Transmission Lines

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Radio Spectrum Bands

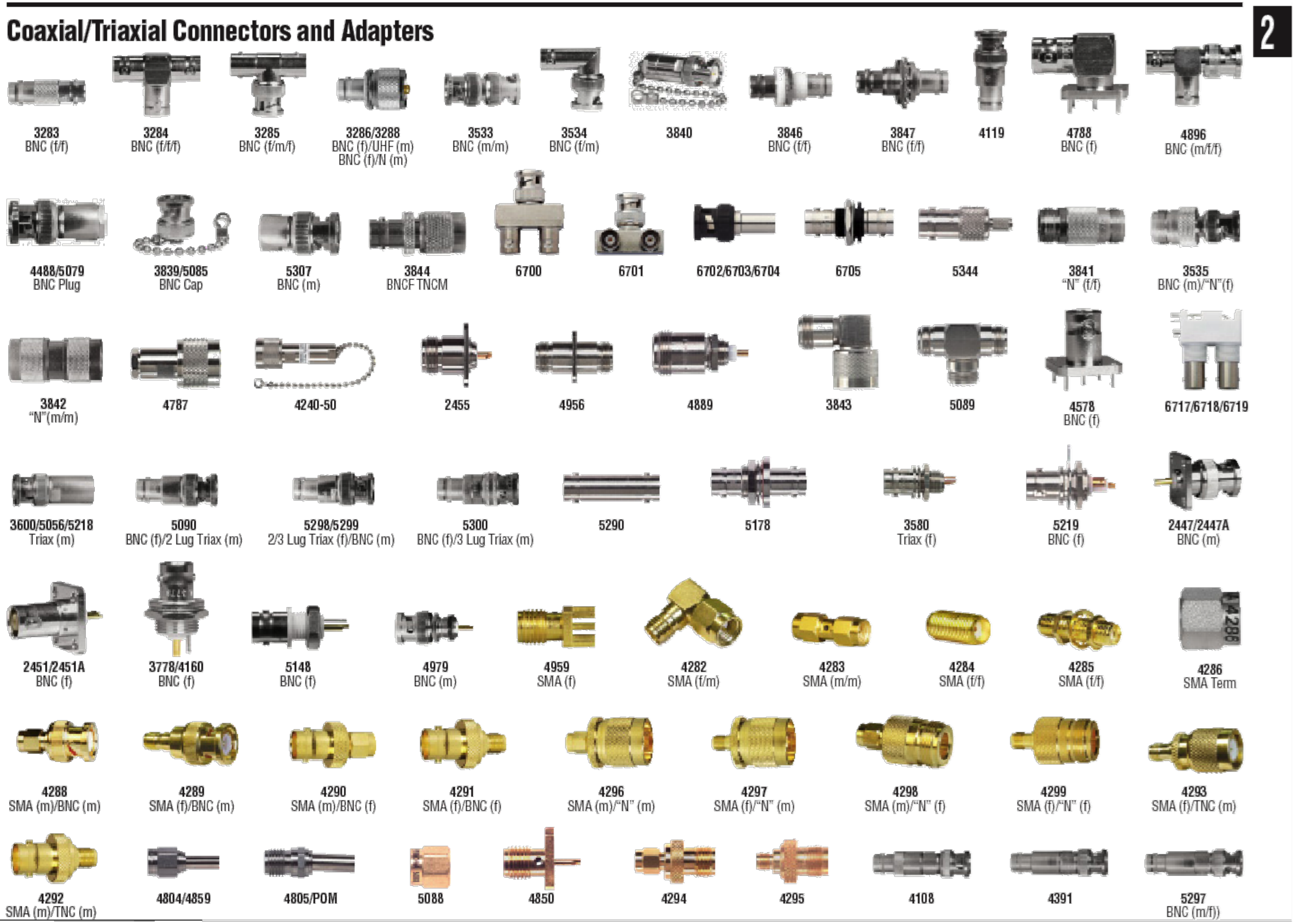
The use of letters to designate bands has long ago been declared obsolete. Nevertheless, people seem to continue to use them and add more letters as users move up the microwave spectrum.

The original definitions had very specific non-overlapping definitions (not counting sub-band definitions). Modern usage seems to associate the bands with waveguide ranges.

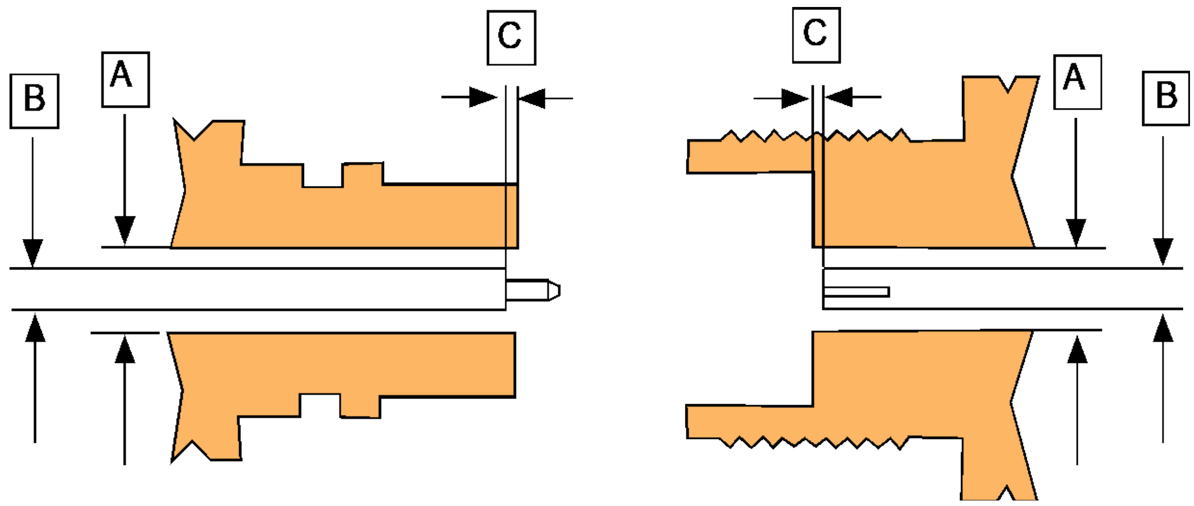
P→ 225-390 MHz
L→ 0.39-1.55 GHz
S→ 1.55-3.9 GHz
C→ 3.9-6.2 GHz
X→ 6.2-10.9 GHz
Ku→ 10.9-18 GHz
K→ 18-26.5 GHz
Ka→ 26.5-40 GHz
Q→ 33-50 GHz

V→ 50-75 GHz
E→ 60-90 GHz
W→ 75-110 GHz
F→ 90-140 GHz
D→ 110-170 GHz
G→ 140-220 GHz
Y→ 170-260 GHz
J→ 220-300 GHz

Coaxial Connectors

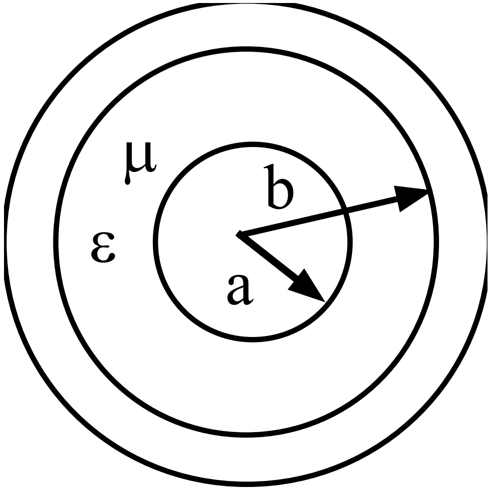


Coaxial Connectors



Type	A	B	C
SMA	0.4	0.05	-
3.5	0.138	-	-
2.9	0.115	0.035	-
2.4	0.095	-	-

Coaxial Transmission Line

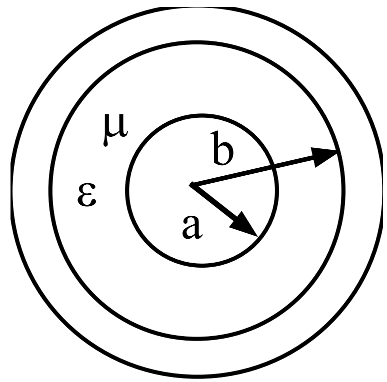


TEM Mode of Propagation

$$L = \mu \ln \frac{b}{a}$$

$$C = \frac{2\pi\varepsilon}{\ln(b/a)}$$

Coaxial Air Lines



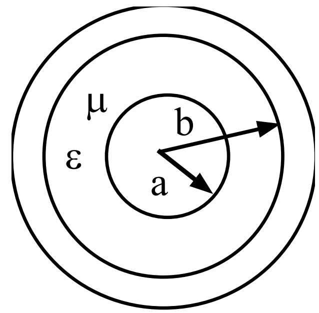
<u>Infinite Conductivity</u>

$$Z_o = \frac{\sqrt{\mu/\varepsilon}}{2\pi}\ln\left(b/a\right)$$

<u>Finite Conductivity</u>

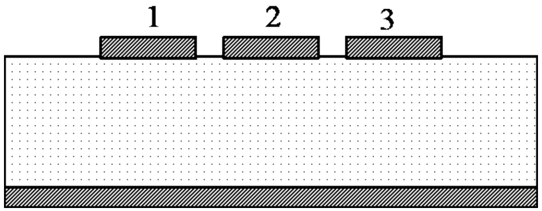
$$Z_o = \frac{\sqrt{\mu/\varepsilon}}{2\pi}\ln\left(b/a\right)\left[1+\frac{\left(1/a+1/b\right)}{4\sqrt{\pi f\mu\sigma}\,\ln(b/a)}\left(1-j\right)\right]$$

Coaxial Connector Standards



Connector	Frequency Range
14 mm	DC - 8.5 GHz
GPC-7	DC - 18 GHz
Type N	DC - 18 GHz
3.5 mm	DC - 33 GHz
2.92 mm	DC - 40 GHz
2.4 mm	DC - 50 GHz
1.85 mm	DC - 65 GHz
1.0 mm	DC - 110 GHz

Three-Line Microstrip



$$-\frac{\partial V_1}{\partial z} = L_{11}\frac{\partial I_1}{\partial t} + L_{12}\frac{\partial I_2}{\partial t} + L_{13}\frac{\partial I_3}{\partial t}$$

$$-\frac{\partial I_1}{\partial z} = C_{11}\frac{\partial V_1}{\partial t} + C_{12}\frac{\partial V_2}{\partial t} + C_{13}\frac{\partial V_3}{\partial t}$$

$$-\frac{\partial V_2}{\partial z} = L_{21}\frac{\partial I_1}{\partial t} + L_{22}\frac{\partial I_2}{\partial t} + L_{23}\frac{\partial I_3}{\partial t}$$

$$-\frac{\partial I_2}{\partial z} = C_{21}\frac{\partial V_1}{\partial t} + C_{22}\frac{\partial V_2}{\partial t} + C_{23}\frac{\partial V_3}{\partial t}$$

$$-\frac{\partial V_3}{\partial z} = L_{31}\frac{\partial I_1}{\partial t} + L_{32}\frac{\partial I_2}{\partial t} + L_{33}\frac{\partial I_3}{\partial t}$$

$$-\frac{\partial I_3}{\partial z} = C_{31}\frac{\partial V_1}{\partial t} + C_{32}\frac{\partial V_2}{\partial t} + C_{33}\frac{\partial V_3}{\partial t}$$

Three-Line – Alpha Mode

Subtract (1c) from (1a) and (2c) from (2a), we get

$$-\frac{\partial V_\alpha}{\partial z} = (L_{11} - L_{13})\frac{\partial I_\alpha}{\partial t}$$
$$-\frac{\partial I_\alpha}{\partial z} = (C_{11} - C_{13})\frac{\partial V_\alpha}{\partial t}$$

This defines the Alpha mode with:

$V_\alpha = V_1 - V_3$ and $I_\alpha = I_1 - I_3$

The wave impedance of that mode is:

$$Z_\alpha = \sqrt{\frac{L_{11} - L_{13}}{C_{11} - C_{13}}}$$

and the velocity is $u_\alpha = \dfrac{1}{\sqrt{(L_{11} - L_{13})(C_{11} - C_{13})}}$

Three-Line – Modal Decomposition

In order to determine the next mode, assume that

$$V_\beta = V_1 + \beta V_2 + V_3$$

$$I_\beta = I_1 + \beta I_2 + I_3$$

$$-\frac{\partial V_\beta}{\partial z} = \left(L_{11} + \beta L_{21} + L_{31}\right)\frac{\partial I_1}{\partial t} + \left(L_{12} + \beta L_{22} + L_{32}\right)\frac{\partial I_2}{\partial t} + \left(L_{13} + \beta L_{23} + L_{33}\right)\frac{\partial I_3}{\partial t}$$

$$-\frac{\partial I_\beta}{\partial z} = \left(C_{11} + \beta C_{21} + C_{31}\right)\frac{\partial V_1}{\partial t} + \left(C_{12} + \beta C_{22} + C_{32}\right)\frac{\partial V_2}{\partial t} + \left(C_{13} + \beta C_{23} + C_{33}\right)\frac{\partial V_3}{\partial t}$$

By reciprocity $L_{32} = L_{23}$, $L_{21} = L_{12}$, $L_{13} = L_{31}$

By symmetry, $L_{12} = L_{23}$

Also by approximation, $L_{22} \approx L_{11}$, $L_{11}+L_{13} \approx L_{11}$

Three-Line – Modal Decomposition

$$-\frac{\partial V_\beta}{\partial z} = \left(L_{11} + \beta L_{12} + L_{13}\right)\left(\frac{\partial I_1}{\partial t} + \frac{\partial I_3}{\partial t}\right) + \left(2L_{12} + \beta L_{11}\right)\frac{\partial I_2}{\partial t}$$

In order to balance the right-hand side into I_β, we need to have

$$\left(2L_{12} + \beta L_{11}\right) I_2 = \beta\left(L_{11} + \beta L_{12} + L_{13}\right) I_2 \approx \beta\left(L_{11} + \beta L_{12}\right) I_2$$

$$2L_{12} = \beta^2 L_{12}$$

or $\beta = \pm\sqrt{2}$

Therefore the other two modes are defined as

The Beta mode with

Three-Line – Beta Mode

The Beta mode with

$$V_\beta = V_1 + \sqrt{2}V_2 + V_3$$

$$I_\beta = I_1 + \sqrt{2}I_2 + I_3$$

The characteristic impedance of the Beta mode is:

$$Z_\beta = \sqrt{\frac{L_{11} + \sqrt{2}L_{12} + L_{13}}{C_{11} + \sqrt{2}C_{12} + C_{13}}}$$

and propagation velocity of the Beta mode is

$$u_\beta = \frac{1}{\sqrt{\left(L_{11} + \sqrt{2}L_{12} + L_{13}\right)\left(C_{11} + \sqrt{2}C_{12} + C_{13}\right)}}$$

Three-Line – Delta Mode

The Delta mode is defined such that

$$V_\delta = V_1 - \sqrt{2}V_2 + V_3$$

$$I_\delta = I_1 - \sqrt{2}I_2 + I_3$$

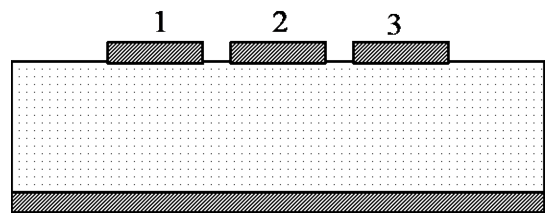
The characteristic impedance of the Delta mode is

$$Z_\delta = \sqrt{\frac{L_{11} - \sqrt{2}L_{12} + L_{13}}{C_{11} - \sqrt{2}C_{12} + C_{13}}}$$

The propagation velocity of the Delta mode is:

$$u_\delta = \frac{1}{\sqrt{\left(L_{11} - \sqrt{2}L_{12} + L_{13}\right)\left(C_{11} - \sqrt{2}C_{12} + C_{13}\right)}}$$

Symmetric 3-Line Microstrip



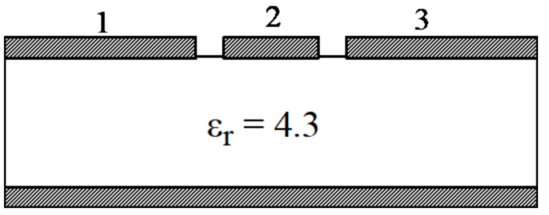
In summary: we have 3 modes for the 3-line system

$$E = \begin{pmatrix} 1 & 0 & -1 \\ 1 & \sqrt{2} & 1 \\ 1 & -\sqrt{2} & 1 \end{pmatrix}$$

- Row 1 ($1 \quad 0 \quad -1$): Alpha mode
- Row 2 ($1 \quad \sqrt{2} \quad 1$): Beta mode*
- Row 3 ($1 \quad -\sqrt{2} \quad 1$): Delta mode*

*neglecting coupling between nonadjacent lines

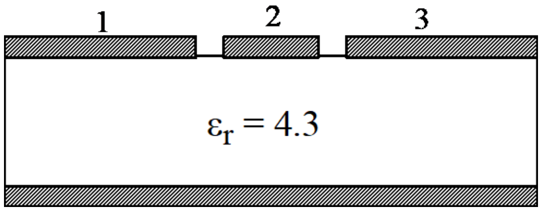
Coplanar Waveguide



$$L(nH/m)=\begin{pmatrix}346 & 162 & 67\\ 152 & 683 & 152\\ 67 & 162 & 346\end{pmatrix} \qquad C(pF/m)=\begin{pmatrix}113 & 17 & 5\\ 16 & 53 & 16\\ 5 & 17 & 113\end{pmatrix}$$

$$E=\begin{pmatrix}0.45 & 0.12 & 0.45\\ 0.5 & 0 & -0.5\\ -0.45 & 0.87 & -0.45\end{pmatrix} \qquad H=\begin{pmatrix}0.44 & 0.49 & 0.44\\ 0.5 & 0 & -0.5\\ -0.10 & 0.88 & -0.10\end{pmatrix}$$

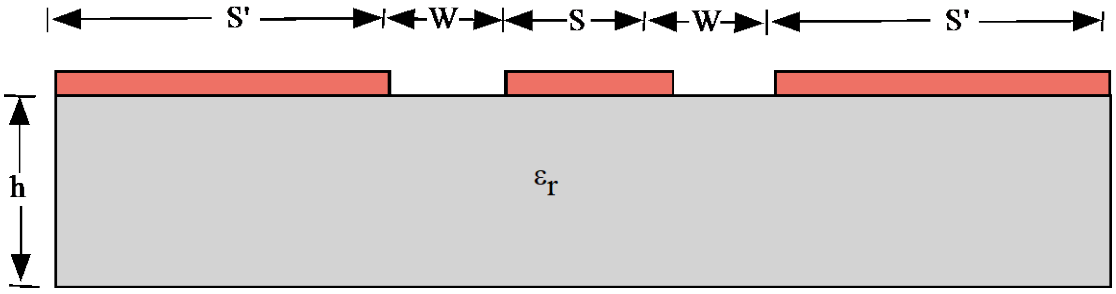
Coplanar Waveguide



$$Z_m(\Omega) = \begin{pmatrix} 73 & 0 & 0 \\ 0 & 48 & 0 \\ 0 & 0 & 94 \end{pmatrix} \qquad Z_c(\Omega) = \begin{pmatrix} 56 & 23 & 8 \\ 22 & 119 & 22 \\ 8 & 23 & 56 \end{pmatrix}$$

$$v_p(m/ns) = \begin{pmatrix} 0.15 & 0 & 0 \\ 0 & 0.17 & 0 \\ 0 & 0 & 0.18 \end{pmatrix}$$

Coplanar Waveguide



$K(k)$: Complete Elliptic Integral of the first kind

$$k = \frac{S}{S+2W}$$

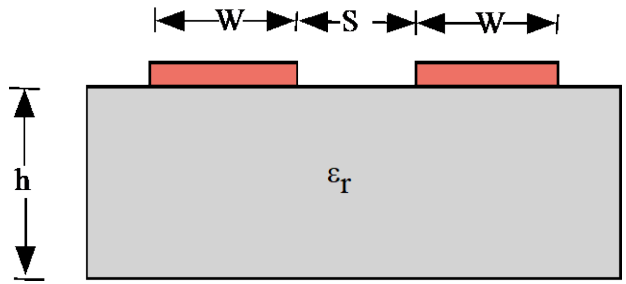
$$Z_{ocp} = \frac{30\pi}{\sqrt{\frac{\varepsilon_r + 1}{2}}} \frac{K'(k)}{K(k)} \text{ (ohm)}$$

$$K'(k) = K(k')$$

$$k' = (1-k^2)^{1/2}$$

$$v_{cp} = \left(\frac{2}{\varepsilon_r + 1}\right)^{1/2} c$$

Coplanar Strips



$$Z_{ocs} = \frac{120\pi}{\sqrt{\frac{\varepsilon_r + 1}{2}}} \frac{K'(k)}{K(k)} \text{ (ohm)}$$

Qualitative Comparison

Characteristic	Microstrip	Coplanar Wguide	Coplanar strips
ε_{eff}*	~6.5	~5	~5
Power handling	High	Medium	Medium
Radiation loss	Low	Medium	Medium
Unloaded Q	High	Medium	Low or High
Dispersion	Small	Medium	Medium
Mounting (shunt)	Hard	Easy	Easy
Mounting (series)	Easy	Easy	Easy

* Assuming ε_r=10 and h=0.025 inch

3-Line Matching Network

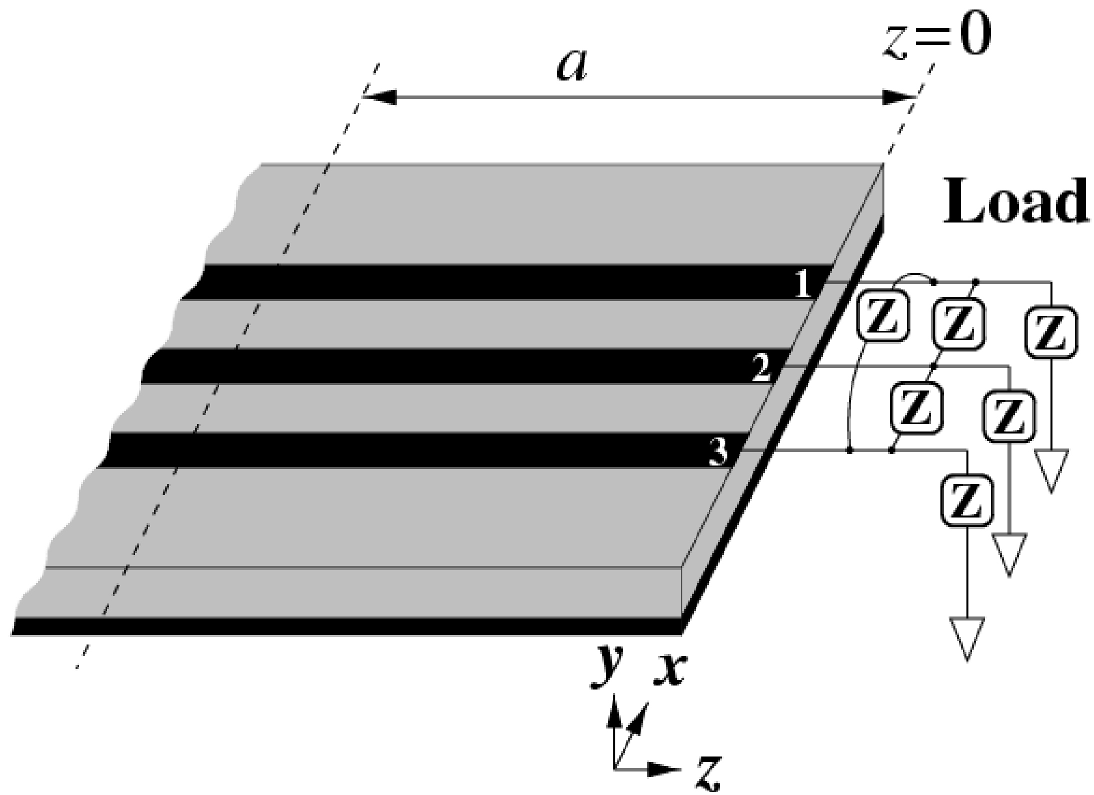
- Extend matching network synthesis to multiconductor transmission line (MTL) systems, specifically coupled microstrip

- Characterize MTL systems by their "normal mode parameters"
  - Mode configurations
  - Propagation constants
  - Characteristic impedance matrices

Longitudinal Immittance Matrix Functions (LIMFs)

- Given terminations, the immittance matrices are expressed as "input" longitudinal functions looking toward the load

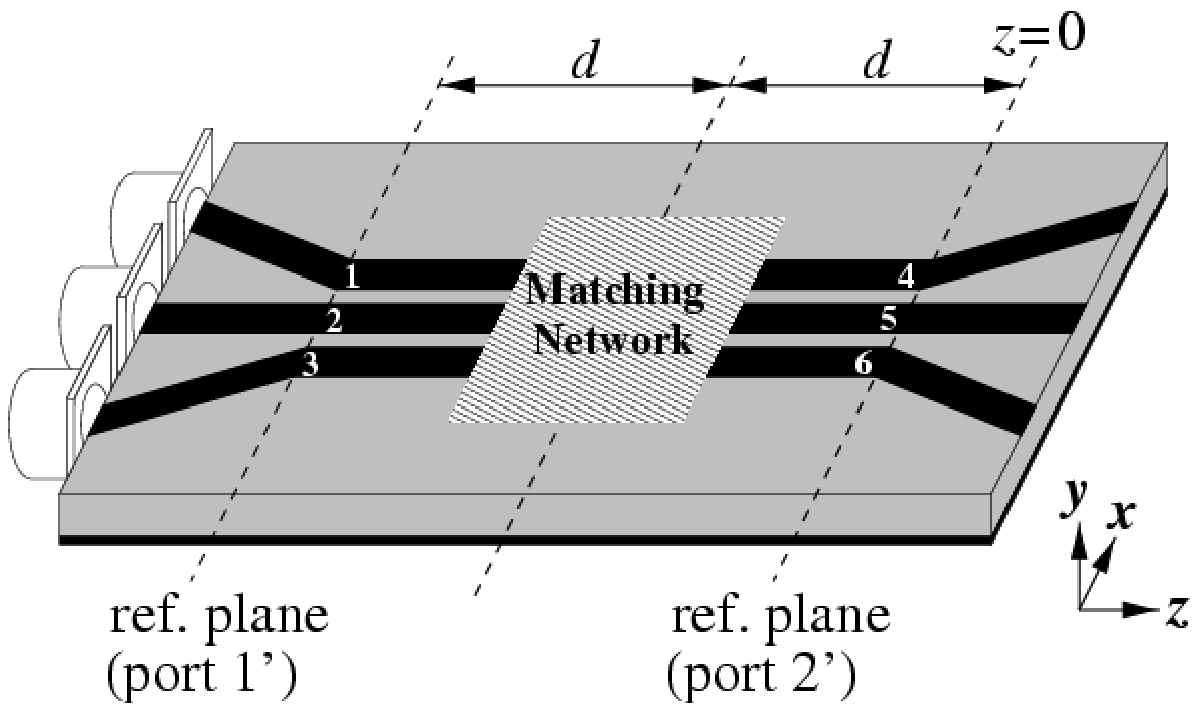
$$\boldsymbol{S}^{\mathrm{m,c}}, \Gamma^{\mathrm{m,c}}_{\mathrm{in,out}},$$
$$\boldsymbol{Y}^{\mathrm{m,c}}_{\mathrm{in,out}}, \boldsymbol{Z}^{\mathrm{m,c}}_{\mathrm{in,out}}$$



Calibration Methods

1st order calibration: 6 distinct TRL calibrations were required to deembed the fan-in and fan-out sections

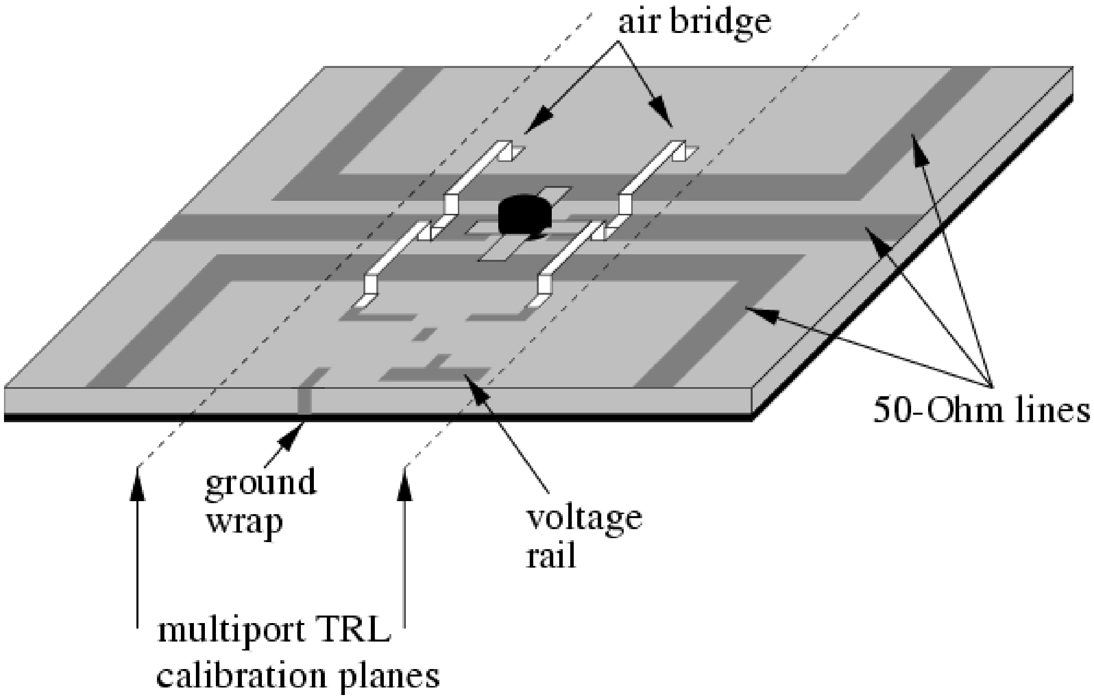
2nd order calibration: 1 multimode TRL calibration required to deembed all fan-ins and fan-outs



Multiconductor transmission line matching network device under test

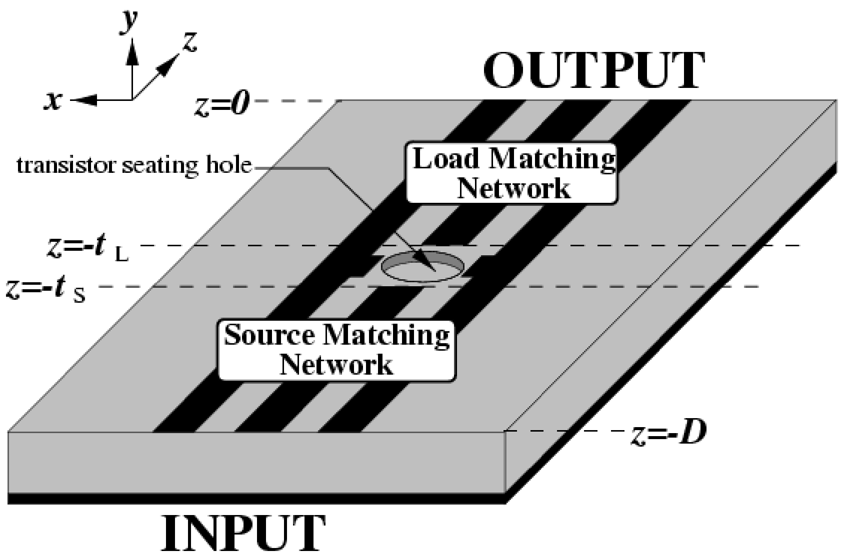
Bias Networks and Multiline/Multimode TRL

- Self-biasing for DC power done using air bridges
- Regulator circuit may be built on board with surface mount/chip components
- Calibration must include all modes and allow for the 6-port discontinuity S-parameter measurement



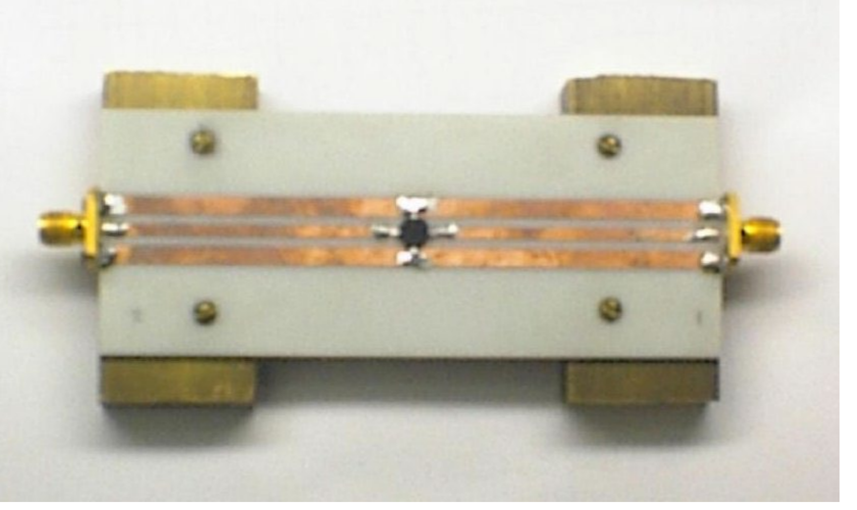
Proposed calibration fixture for transistor amplifier device discontinuity with self-biasing network

3-line Transistor Amplifier



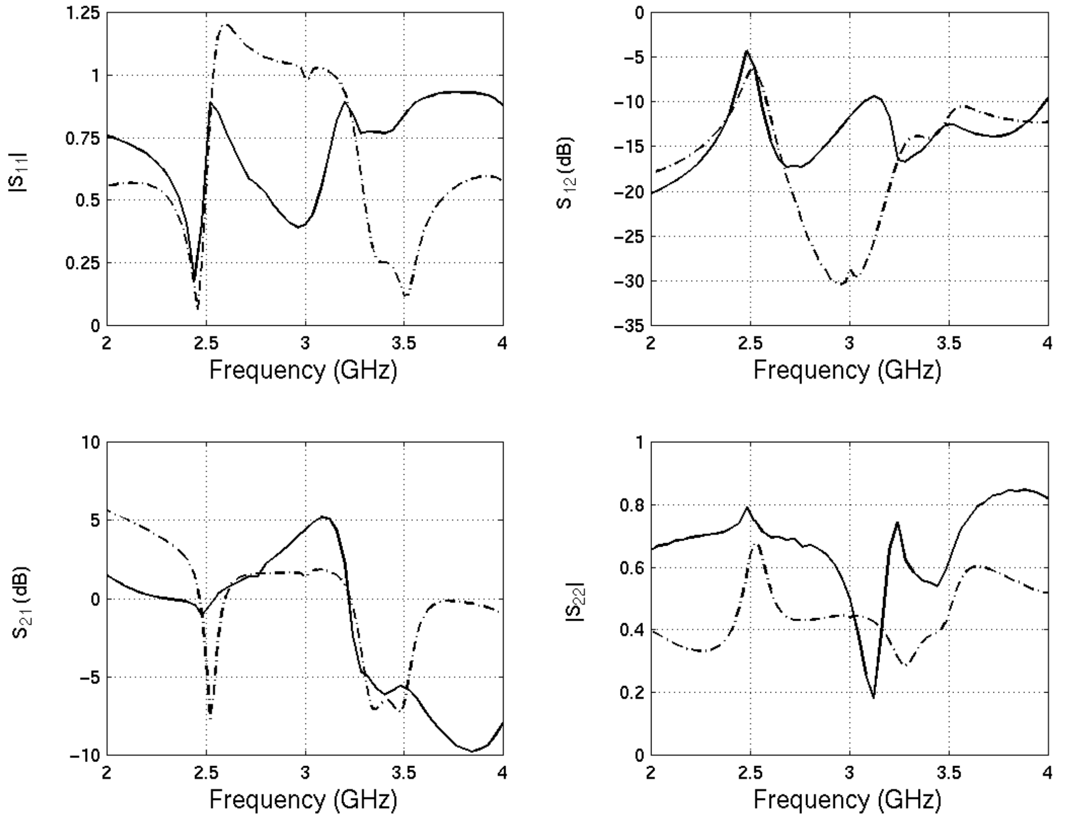
The general amplifier structure with generic matching networks

Note that a transistor seating hole may be present for convenience, but is not necessary



Photograph of a three-line transistor amplifier autobiased w/o matching networks

Transistor Amplifier Matching



S-parameter comparison between unmatched and matched three-line amplifier. Dotted line - unmatched; dashed line - matched. Design frequency: 3.1 GHz.

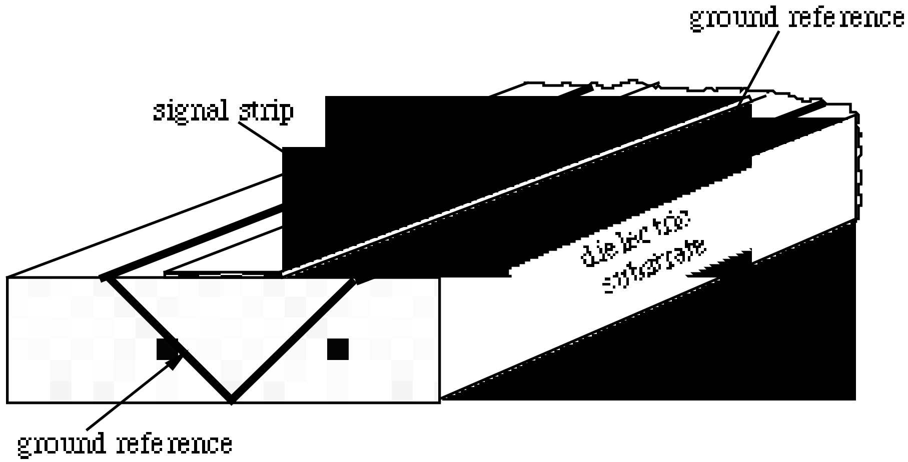
Matching

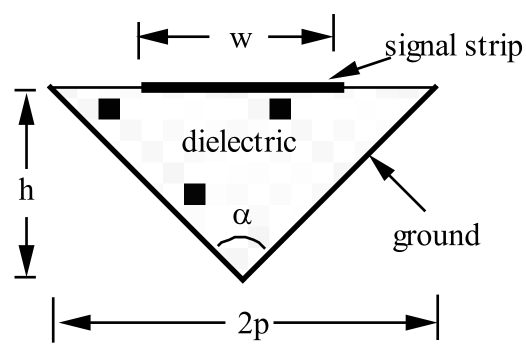
- 2 - 0.8 pF chip cap.
- 2 - 0.4 pF chip cap.

Gains

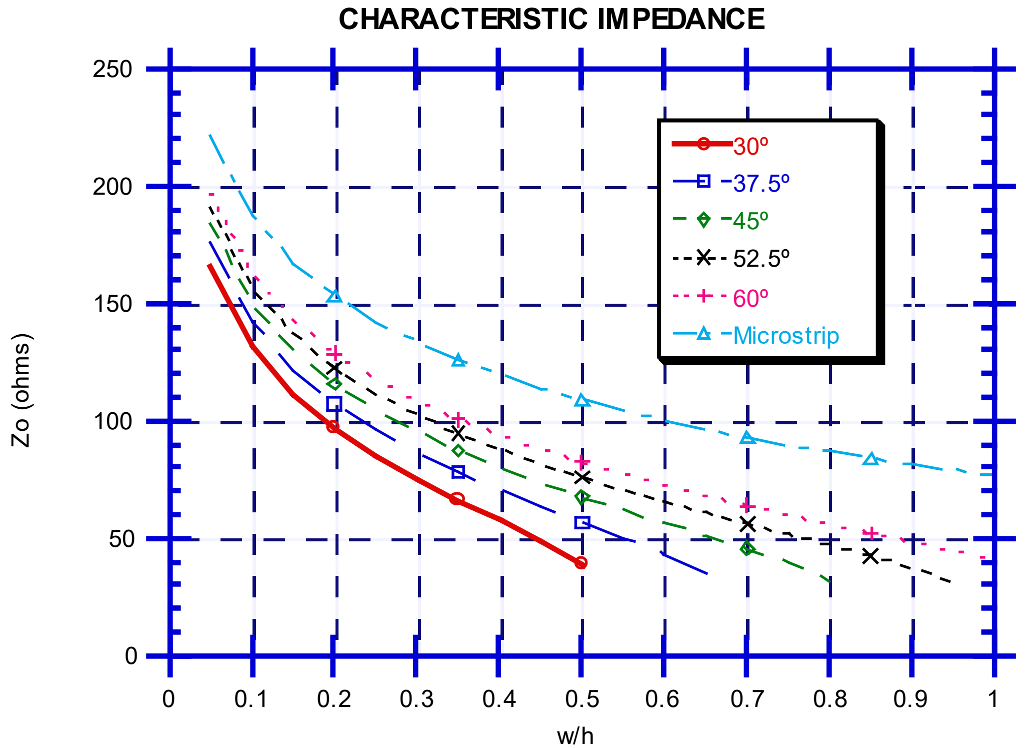
- 3.38 dB @ 3.1 GHz
  m=5.14 / um= 1.76
- 6.22 dB @ 3.22 GHz
  m=6.54 / um= 0.32

V Transmission Line



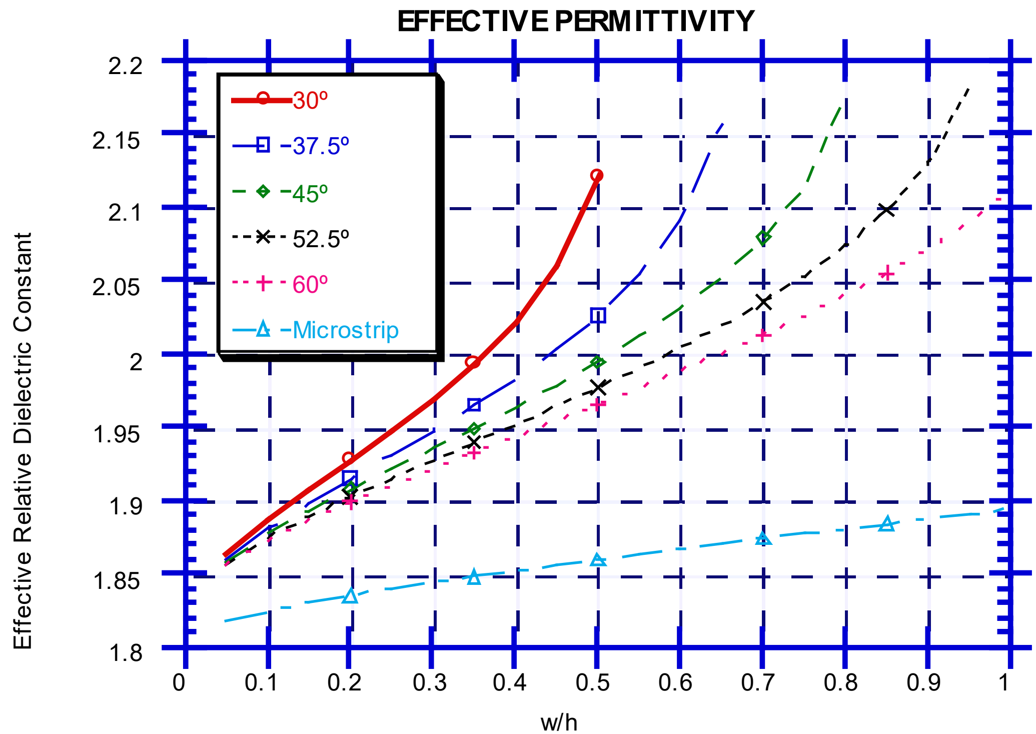


V-Line Characteristic Impedance



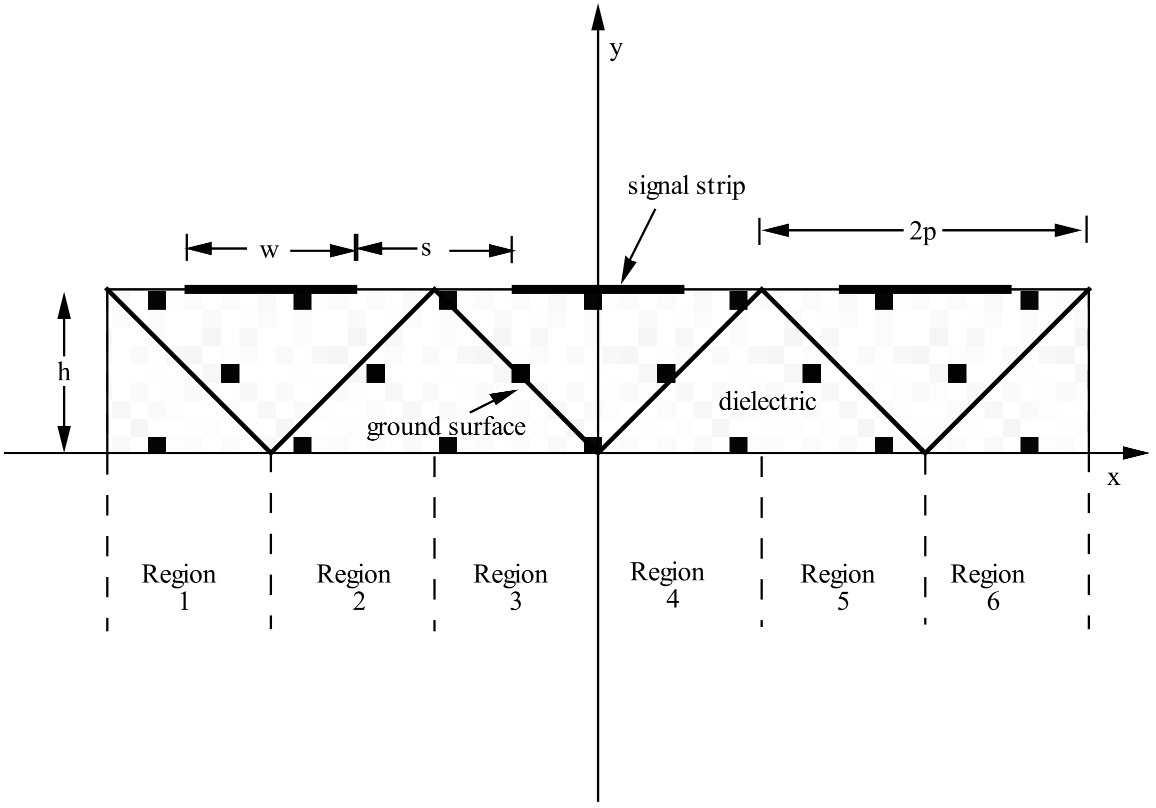
Calculated values of the characteristic impedance for a single-line v-strip structure as a function of width-to-height ratio w/h. The relative dielectric constant is $\varepsilon r = 2.55$.

V-Line – Effective Permittivity



Calculated values of the effective relative dielectric constant for a single-line v-strip structure as a function of width-to-height ratio w/h.
The relative dielectric constant is $\varepsilon r = 2.55$

Three-Line – V Transmission Line



V-Line and Microstrip

Microstrip

C (pF/m) =

52.49	-5.90	-0.57
-5.90	53.27	-5.90
-0.57	-5.90	52.49

L (nH/m) =

609.74	113.46	41.79
113.54	607.67	113.54
41.79	113.46	609.74

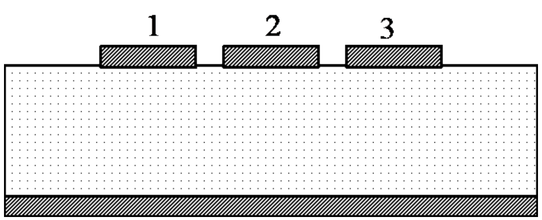
V-line

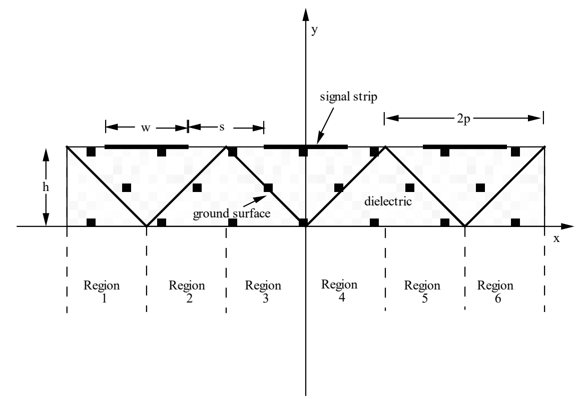
C (pF/m) =

74.00	-0.97	-0.23
-0.97	74.07	-0.97
-0.23	-0.97	74.00

L (nH/m) =

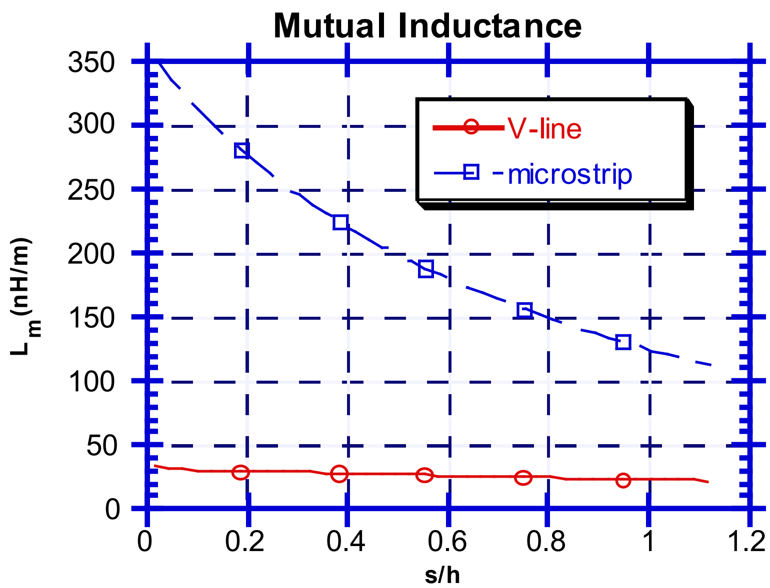
425.84	20.35	7.00
20.36	422.85	20.36
7.00	20.35	425.84

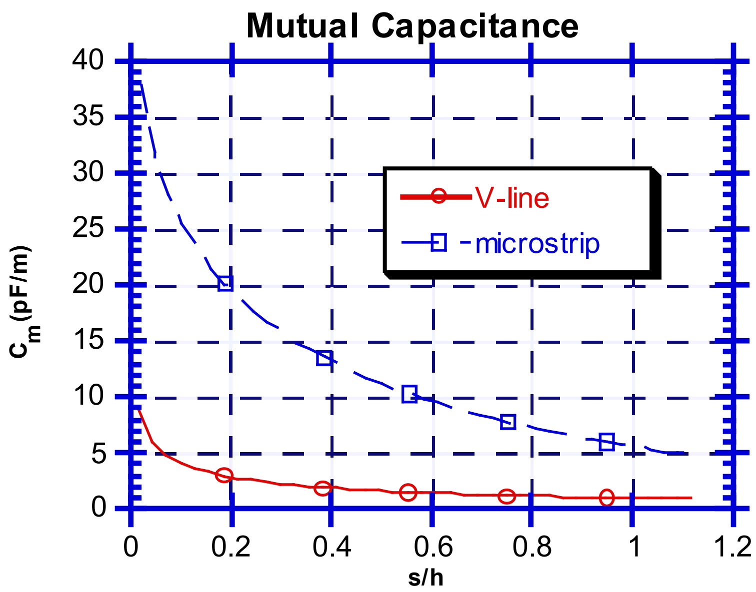




Comparison of the inductance and capacitance matrices between a three-line v-line and microstrip structures. The parameters are p/h = 0.8 mils, w/h = 0.6 and $\varepsilon_r = 4.0$.

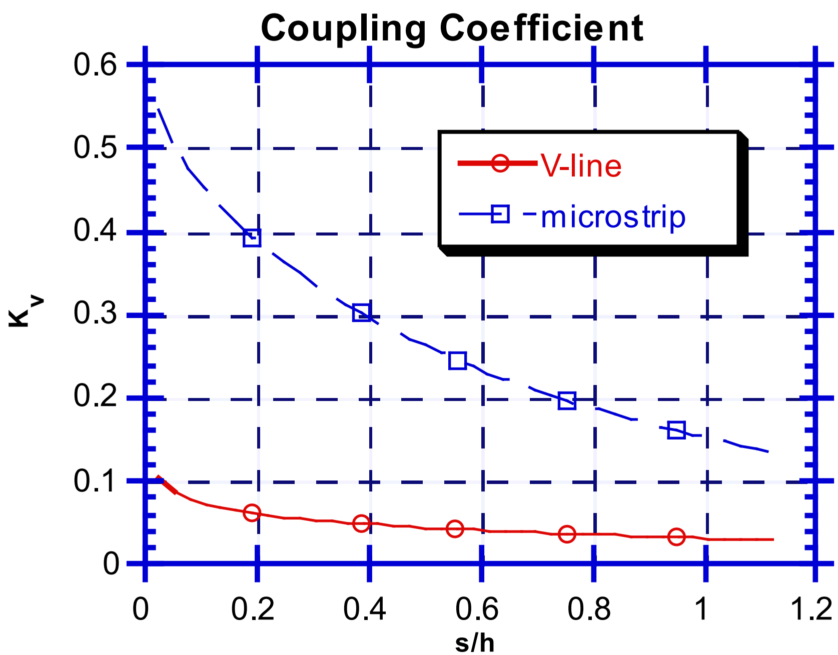
V-Line: Coupling Coefficients





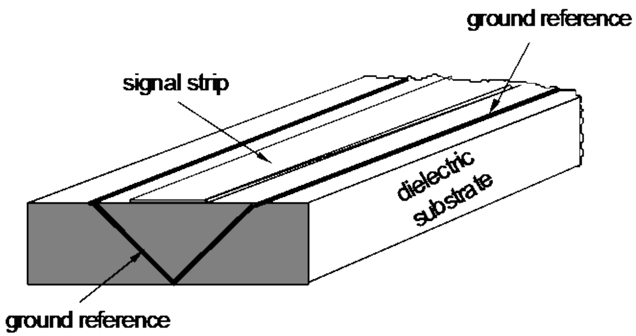
Plot of mutual inductance (top) and mutual capacitance (bottom) versus spacing-to-height ratio for v-line and microstrip configurations. The parameters are w/h = 0.24, εr = 4.0.

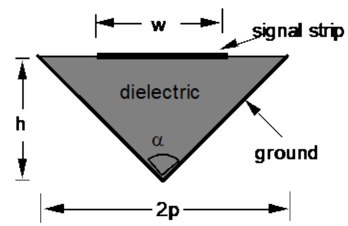
V-Line vs Microstrip: Coupling Coefficients



Plot of the coupling coefficient versus spacing-to-height ratio for v-line and microstrip configurations. The parameters are w/h = 0.24, εr = 4.0.

Advantages of V-Line



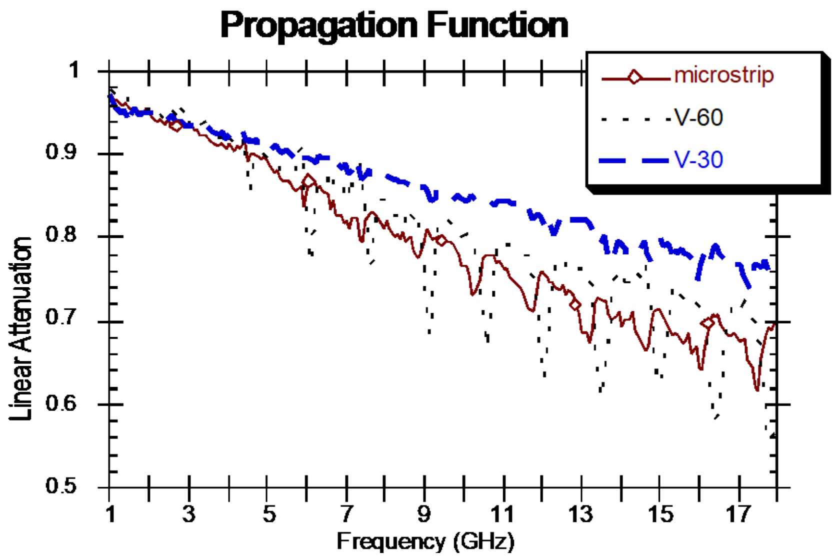


Advantages of V Line

* Higher bandwidth

* Lower crosstalk

* Better transition



V-Line vs Microstrip: Insertion Loss