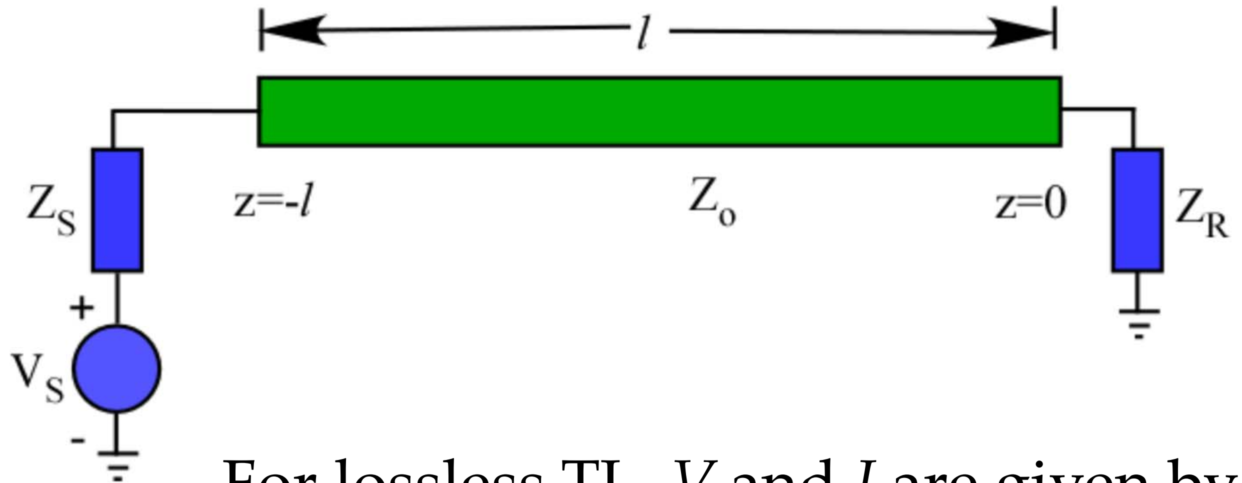
ECE 451
Advanced Microwave Measurements

Transmission Lines - TDR

Jose E. Schutt-Aine
Electrical & Computer Engineering
University of Illinois
jschutt@emlab.uiuc.edu

Determining V_+



For lossless TL, V and I are given by

$$V(z) = V_+ e^{-j\beta z}\left[1 + \Gamma_R e^{+2j\beta z}\right]$$

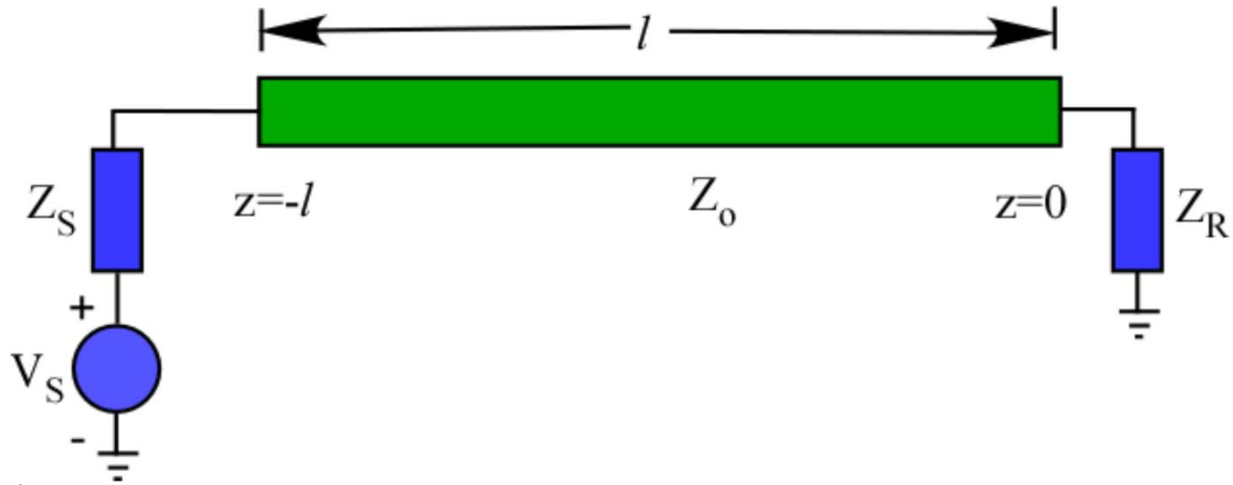
reflection coefficient at the load

$$I(z) = \frac{V_+ e^{-j\beta z}}{Z_o}\left[1 - \Gamma_R e^{+2j\beta z}\right]$$

$$\Gamma_R = \frac{Z_R - Z_o}{Z_R + Z_o}$$

$$At\ z = -l, \quad V_S = Z_S I(-l) + V(-l)$$

Determining V_+



this leads to

$$V_S = V_+ e^{+j\beta l}\left(1+\Gamma_R e^{-2j\beta l}\right) + \frac{Z_S}{Z_o} V_+ e^{+j\beta l}\left(1-\Gamma_R e^{-2j\beta l}\right)$$

or

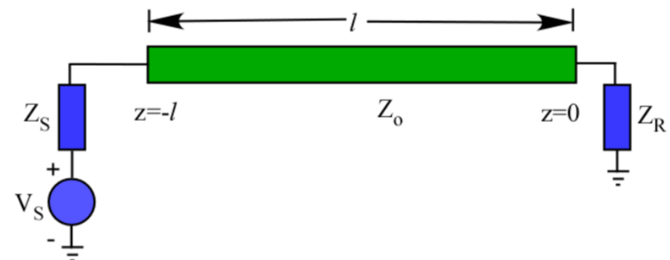
$$V_S = V_+\left(e^{+j\beta l} + \Gamma_R e^{-j\beta l} + \frac{Z_S}{Z_o} e^{+j\beta l} - \Gamma_R \frac{Z_S}{Z_o} e^{-j\beta l}\right)$$

$$V_S = V_+\left(e^{+j\beta l}\left(1+\frac{Z_S}{Z_o}\right) + \Gamma_R e^{-j\beta l}\left(1-\frac{Z_S}{Z_o}\right)\right)$$

Determining V_+

Divide through by $\left(1+\dfrac{Z_S}{Z_o}\right)=\dfrac{1}{T_S}$

$$V_+\left(e^{+j\beta l}-\Gamma_S\Gamma_R e^{-j\beta l}\right)=T_S V_S$$



with $T_S=\left(1+\dfrac{Z_S}{Z_o}\right)^{-1}=\dfrac{Z_o}{Z_S+Z_o}$ and $\Gamma_S=\dfrac{Z_S-Z_o}{Z_S+Z_o}$

From which $$V_+=\frac{T_S V_S e^{-j\beta l}}{1-\Gamma_S\Gamma_R e^{-2j\beta l}}$$

Geometric Series Expansion

$$\text{Since } \left|\Gamma_S \Gamma_R e^{-2j\beta l}\right| \le 1$$

V_+ can be expanded in a geometric series form

$$V_+ = \frac{T_S V_S e^{-j\beta l}}{1 - \Gamma_S \Gamma_R e^{-2j\beta l}}$$

$$V_+ = T_S \sum_{k=0}^{\infty} V_S \Gamma_S^k \Gamma_R^k e^{-2j\beta kl} e^{-j\beta l}$$

$$V(z) = T_S \sum_{k=0}^{\infty} V_S \Gamma_S^k \Gamma_R^k e^{-j\beta l(2k+1)} e^{-j\beta z} + T_S \sum_{k=0}^{\infty} V_S \Gamma_S^k \Gamma_R^{k+1} e^{-j\beta l(2k+1)} e^{+j\beta z}$$

$$\beta = \frac{\omega}{v}$$

$$V(z) = T_S \sum_{k=0}^{\infty} V_S \Gamma_S^k \Gamma_R^k e^{-j\frac{\omega}{v}\left[z+(2k+1)l\right]} + T_S \sum_{k=0}^{\infty} V_S \Gamma_S^k \Gamma_R^{k+1} e^{-j\frac{\omega}{v}\left[-z+(2k+1)l\right]}$$

TL Time-Domain Solution

$$v(z,t) = T_S \sum_{k=0}^{\infty} \Gamma_S^k \Gamma_R^k v_s \left(t - \frac{(2k+1)l + z}{v_o} \right)$$

$$+ T_S \sum_{k=0}^{\infty} \Gamma_S^k \Gamma_R^{k+1} v_s \left(t - \frac{(2k+1)l - z}{v_o} \right)$$

at z=0

$$v(0,t) = T_S \sum_{k=0}^{\infty} \Gamma_S^k \Gamma_R^k v_s \left(t - \frac{(2k+1)l}{v_o} \right)$$

$$+ T_S \sum_{k=0}^{\infty} \Gamma_S^k \Gamma_R^{k+1} v_s \left(t - \frac{(2k+1)l}{v_o} \right)$$

TL Time-Domain Solution

At z=-l

$$v(-l,t) = T_S \sum_{k=0}^{\infty} \Gamma_S^k \Gamma_R^k v_s \left(t - \frac{(2k+1)l + l}{v_o} \right)$$

$$+ T_S \sum_{k=0}^{\infty} \Gamma_S^k \Gamma_R^{k+1} v_s \left(t - \frac{(2k+1)l + l}{v_o} \right)$$

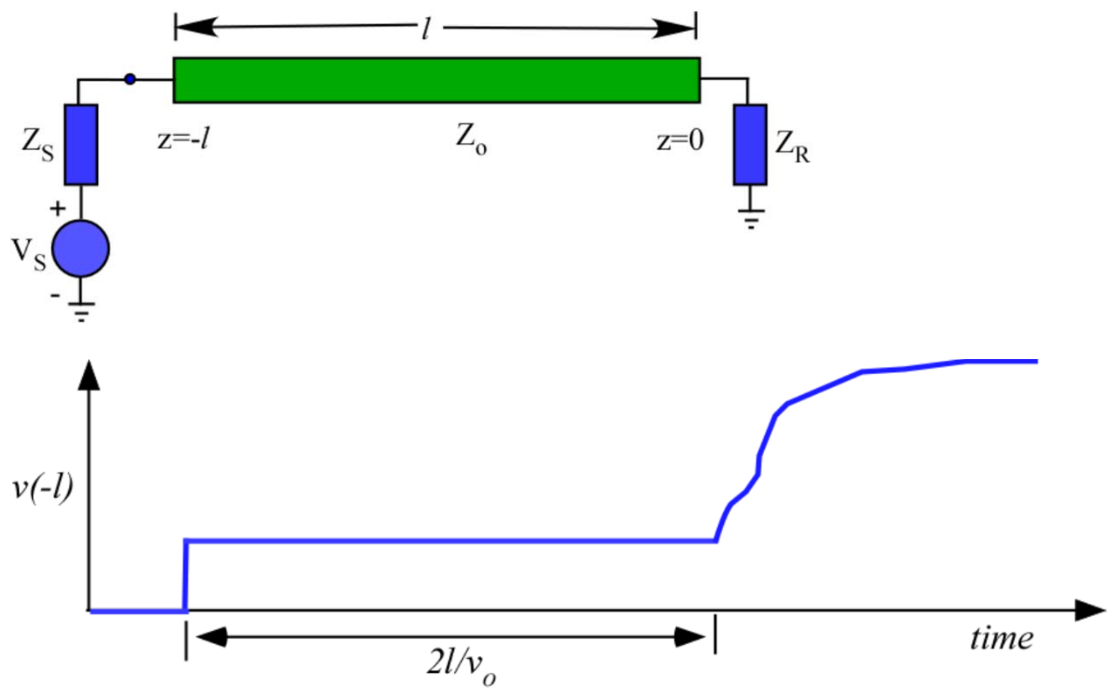
$$v(-l,t) = T_S \sum_{k=0}^{\infty} \Gamma_S^k \Gamma_R^k v_s \left(t - \frac{2kl}{v_o} \right)$$

$$+ T_S \sum_{k=0}^{\infty} \Gamma_S^k \Gamma_R^{k+1} v_s \left(t - \frac{2(k+1)l}{v_o} \right)$$

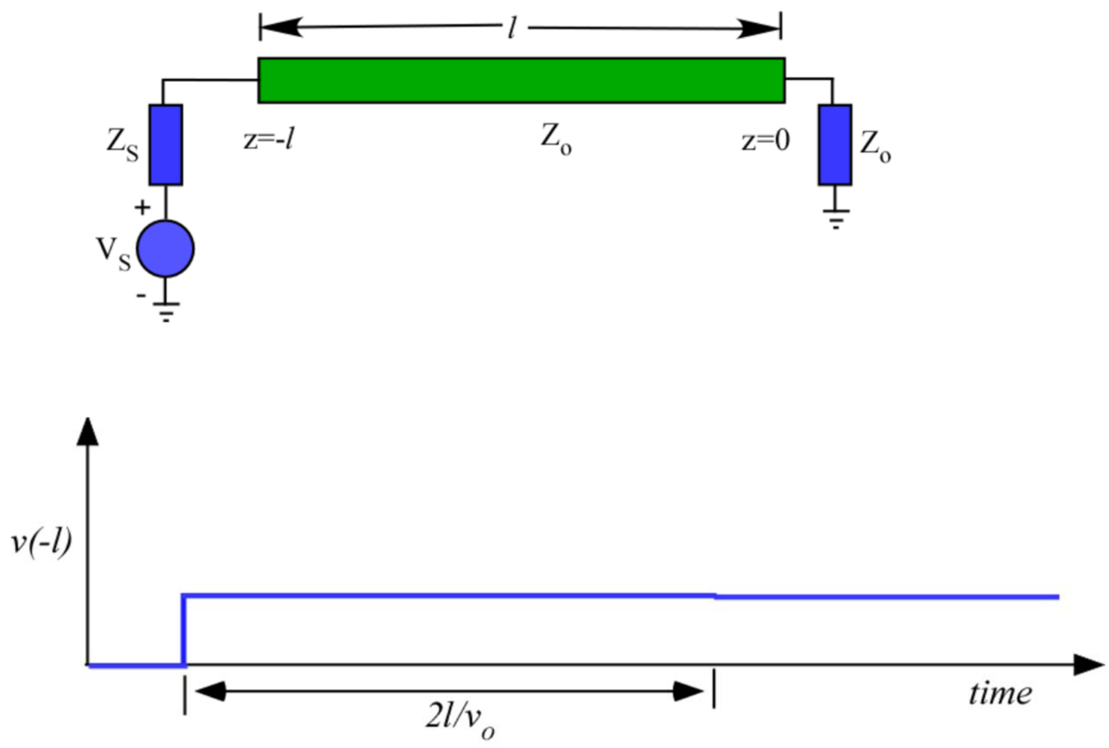
TL - Time-Domain Reflectometer

For TDR, $Z_S = Z_o \rightarrow \Gamma_S = 0$, and retain only $k=1$

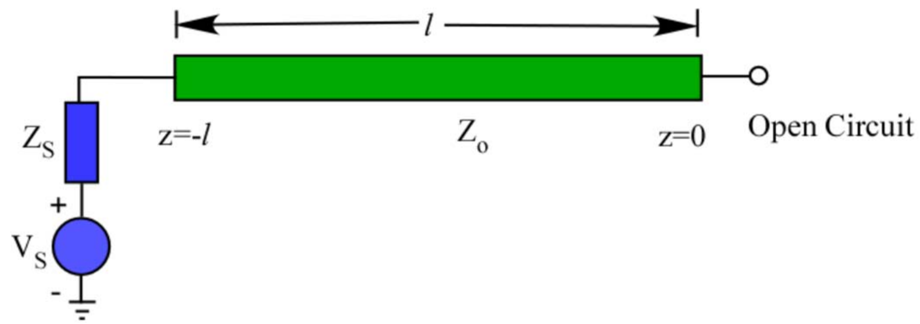
$$v(-l,t) = T_S v_s(t) + T_S \Gamma_R v_s\left(t - \frac{2l}{v_o}\right)$$

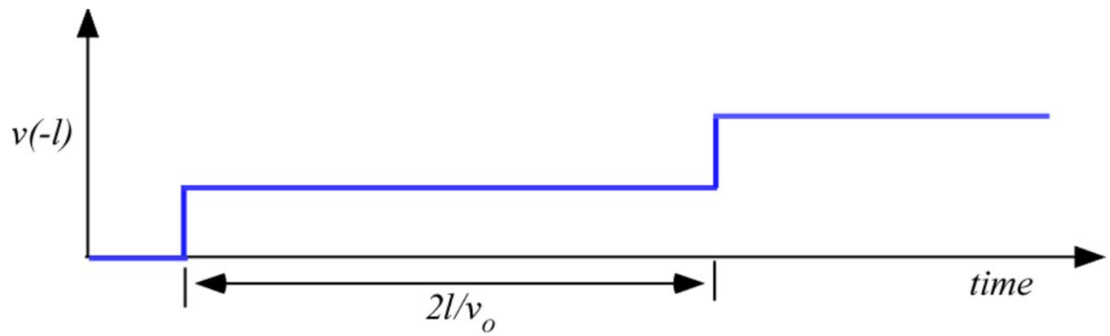


TDR – Matched Load

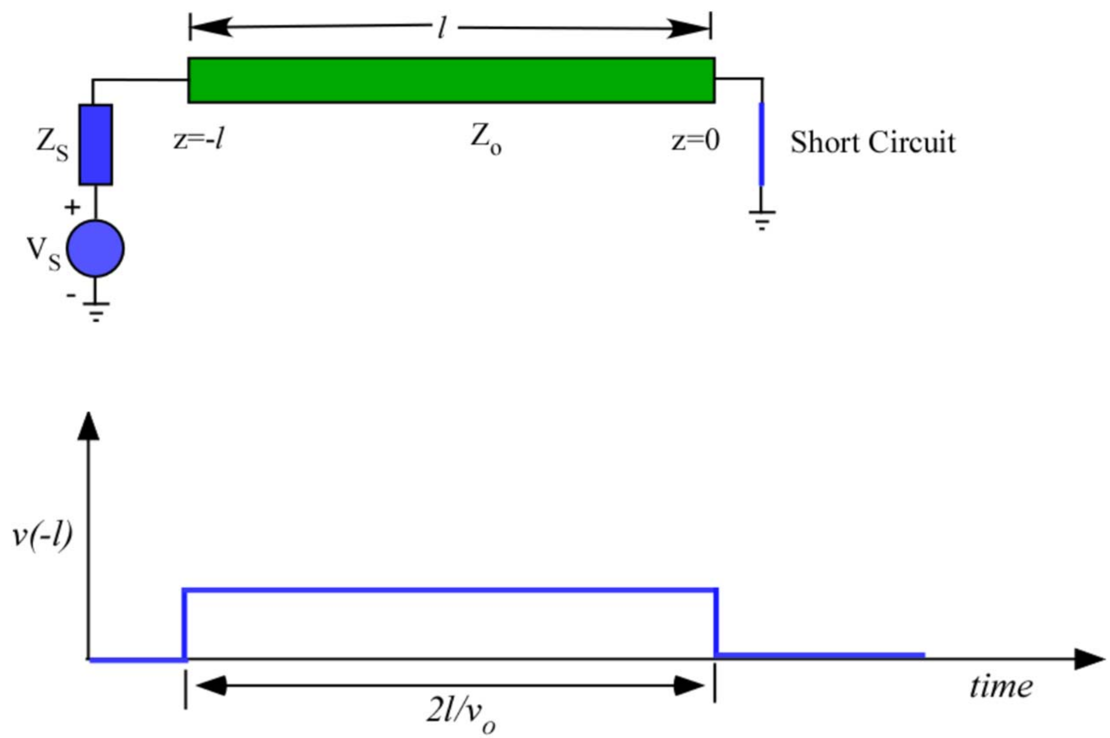


TDR – Open

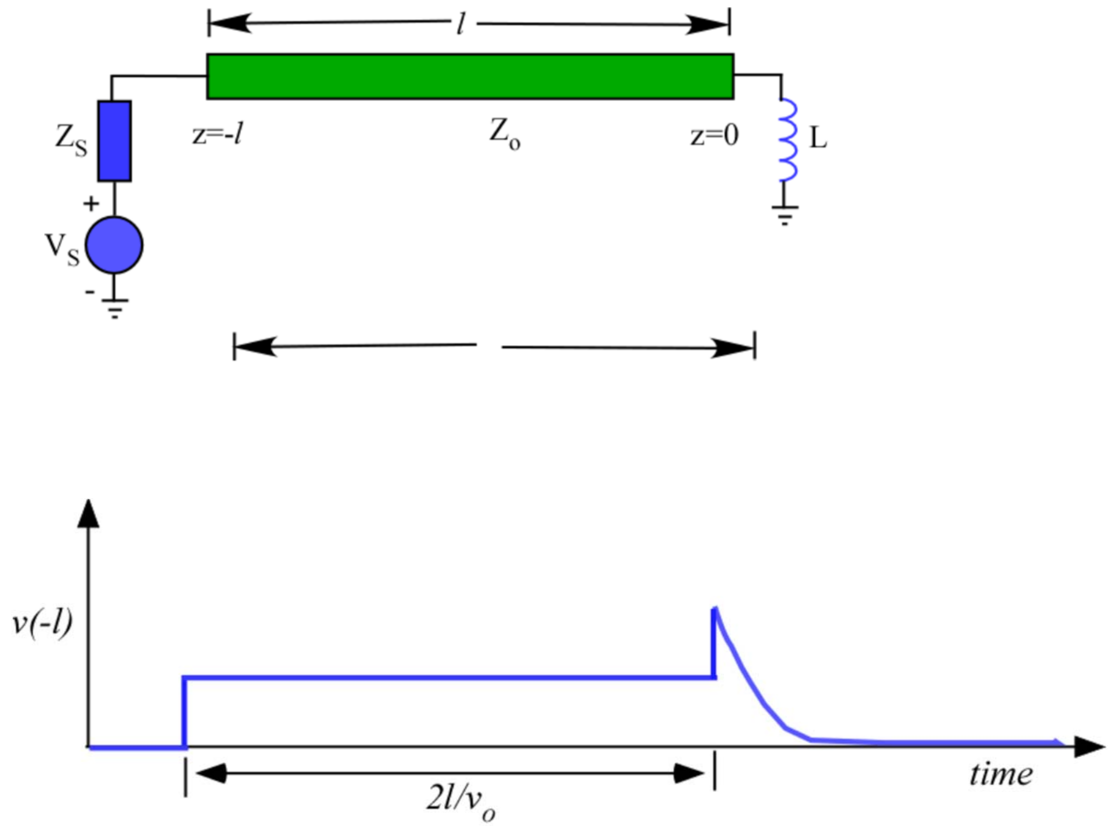




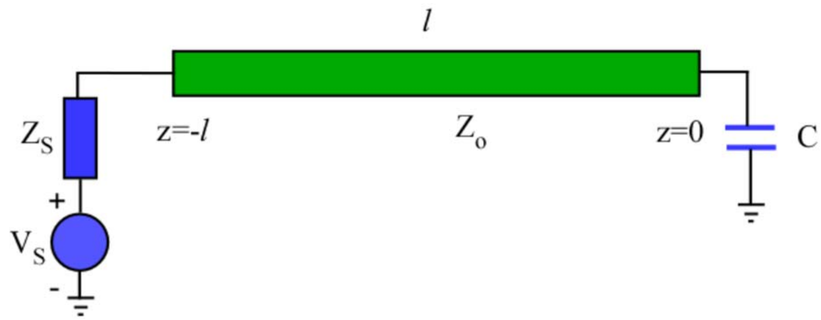
TDR - Short

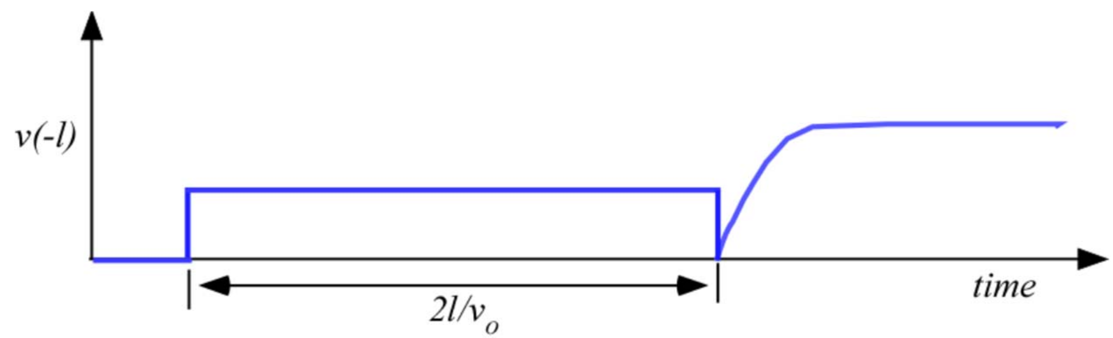


TDR - Inductance

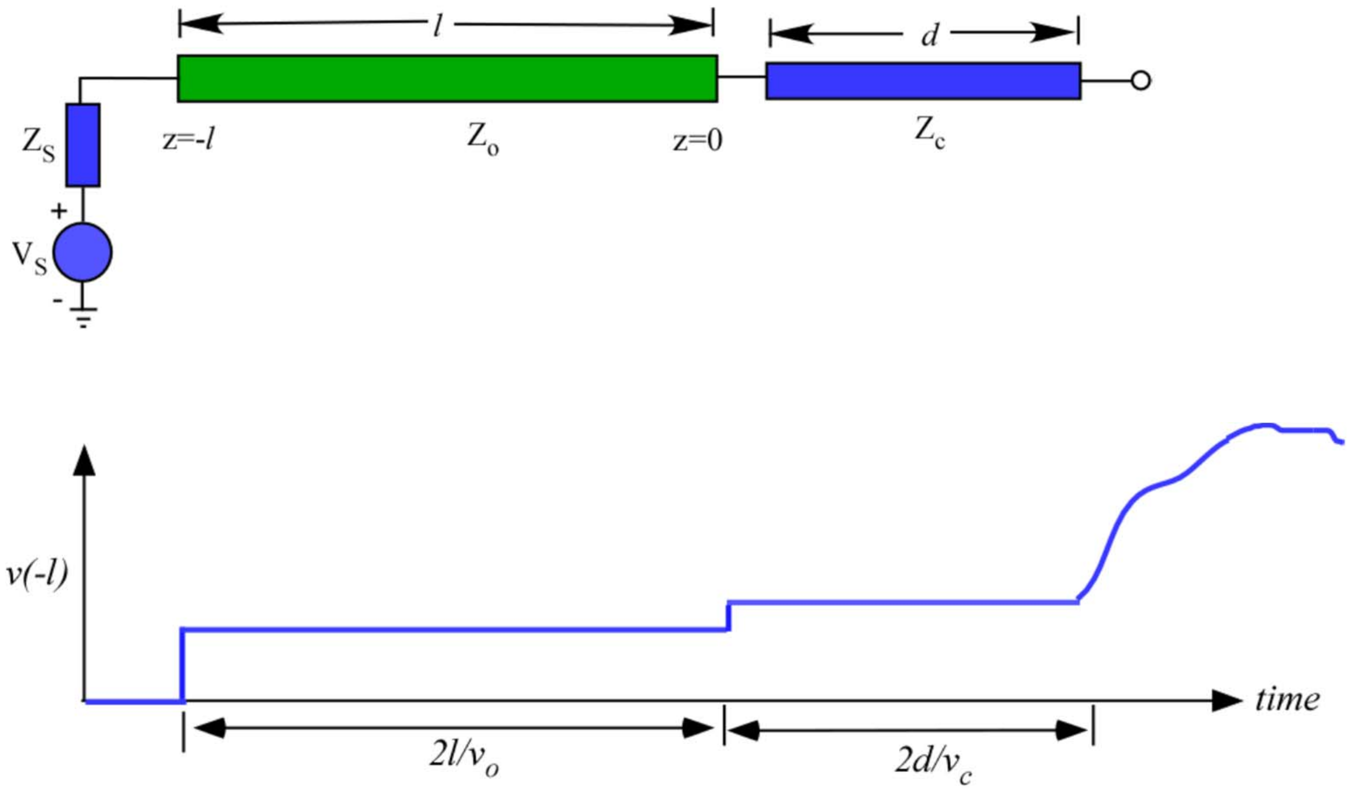


TDR - Capacitance

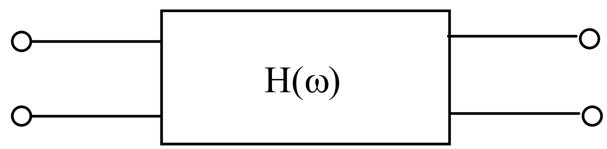




TDR – Transmission Line



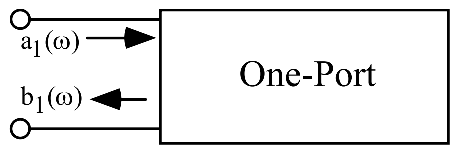
Using Frequency Domain Data for Time-Domain Simulation



Goal: Simulate the time-domain response of a network using frequency-domain measurements.

Motivation: Fast rise time pulse and steps are difficult to design; but high-frequency signals are available

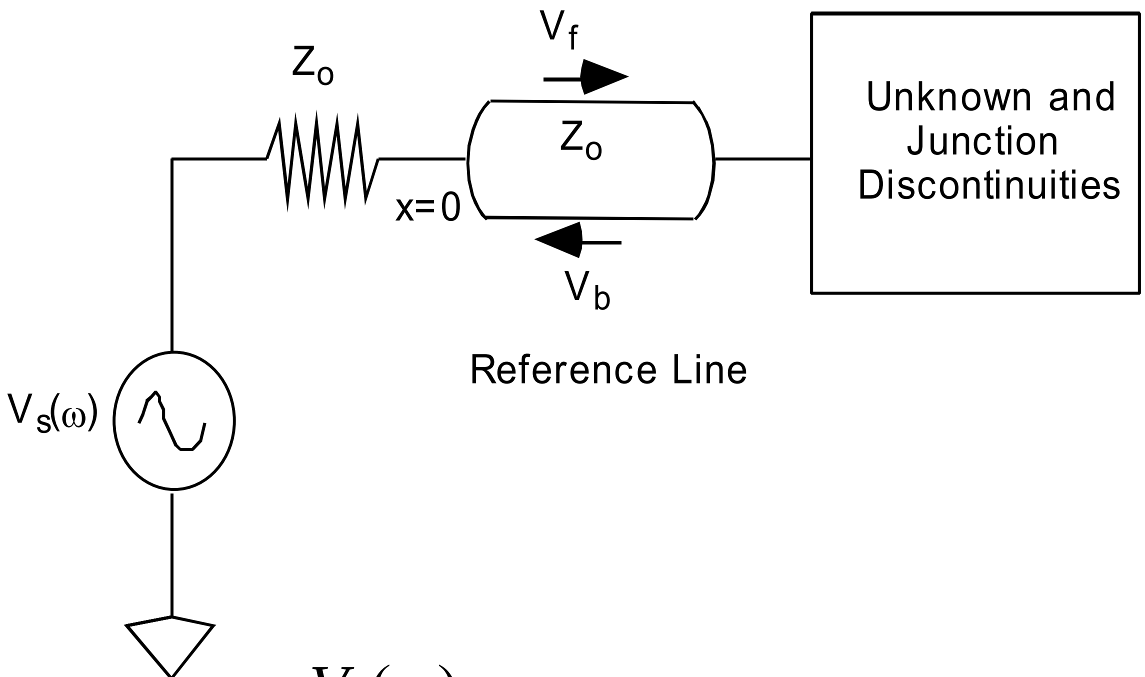
Using Frequency Domain Data for Time-Domain Simulation



Approach

Scattering parameter of one-port network can be measured over a wide frequency range. Since incident and reflected voltage waves are related through the measured scattering parameters, the total voltage can be determined as a function of frequency.

One-Port S-Parameter Measurements



$$V_f(x=0,\ \omega) = \frac{V_s(\omega)}{2}$$

$$V_b(x=0,\ \omega) = \frac{V_s(\omega)}{2} S_{11}(\omega)$$

$$V(x=0,\ \omega) = \frac{V_s(\omega)}{2}\left[1+S_{11}(\omega)\right] = V_o(\omega)$$

Frequency-to-Time Analysis

$S_{11}(\omega)$ is measured experimentally. Assume $v_s(t)$ to be an arbitrary time-domain signal (unit step, pulse, impulse). $V_s(\omega)$ is its transform

$$V_s(\omega) = \int_{-\infty}^{\infty} v_s(t) e^{-j2\pi ft} dt$$

Since the system is linear, its response in the time domain is the superposition of the responses due to all frequencies

$$v_o(t) = \int_{-\infty}^{\infty} \frac{V_s(\omega)}{2} \left[1 + S_{11}(\omega) \right] e^{+j2\pi ft} df$$

Transformation Steps

- Measure $S_{11}(f)$
- Calculate $V_s(\omega)$ analytically
- Evaluate $V_o(\omega)=V_s\,(\omega)\,[1+S_{11}(\omega)]/2$
- Feed $V_o(\omega)$ into inverse Fourier transform to get $v_o(t)$

Addressing Frequency and Time Limitations

1. For negative frequencies use conjugate relation $V(-\omega) = V^*(\omega)$

2. DC value: use lower frequency measurement

3. Rise time is determined by frequency range or bandwidth

4. Time step is determined by frequency range

5. Duration of simulation is determined by frequency step

We wish to evaluate $v(t) = \int_{-\infty}^{+\infty} V(f) e^{j2\pi ft} df$

IFFT calculates: $X(K) = \sum_{I=0}^{N-1} A(I) e^{j\frac{2\pi IK}{N}}$

Fourier Transform

Forward

$$V(f) = \int_{-\infty}^{+\infty} v(t) e^{-j2\pi ft} dt$$

Inverse

$$v(t) = \int_{-\infty}^{+\infty} V(f) e^{j2\pi ft} df$$

In discrete form:

$$v(t) = \Delta f \sum_{q=-N/2}^{N/2-1} V(q\Delta f) e^{j2\pi qt\Delta f}$$

$$v(k\Delta t) = \Delta f \sum_{q=-N/2}^{N/2-1} V(q\Delta f) e^{j2\pi q\Delta t\Delta f}$$

$$\Delta f = \frac{F}{N} \qquad \Delta t = \frac{1}{F} \qquad \Delta f \, \Delta t = \frac{1}{N}$$

Fourier Transform

Moreover, let $v_s(k) = v(k\Delta t)$

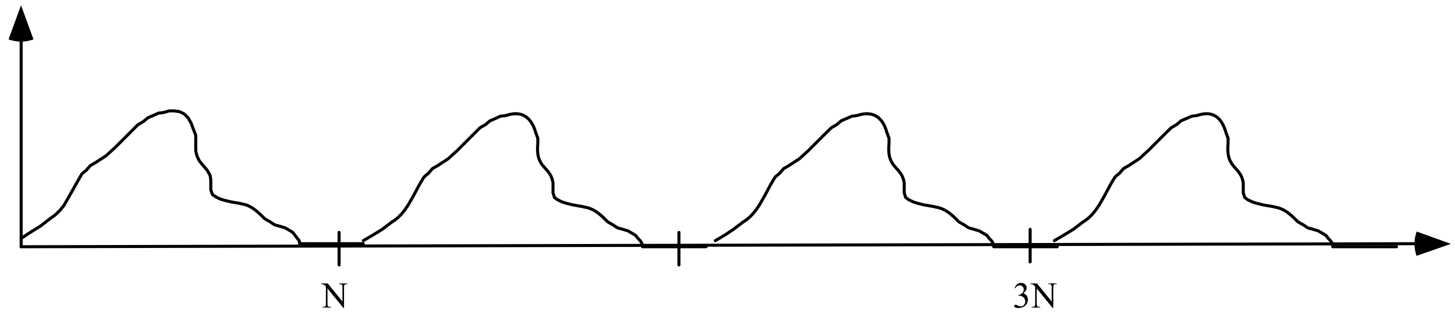
$$V_s(q) = V(q\Delta f)$$

$v_s(k)$ is periodic with period N

$$v_s(k+N) = v_s(k)$$

$$v_s(k) = \frac{F}{N}\sum_{q=-N/2}^{N/2-1} V_s(q) e^{j\frac{2\pi kq}{N}}$$

Discrete Inverse Fourier Transform



Let $p=q+N/2$ which leads to $q=p-N/2$

$$v_s(k) = \frac{F}{N}(-1)^k \sum_{p=0}^{N-1} V_s\left(p - \frac{N}{2}\right) e^{j\frac{2\pi kp}{N}}$$

$$v_s(k) = \frac{F}{N}(-1)^k \sum_{p=0}^{N-1} V_s'\left(p\right) e^{j\frac{2\pi kp}{N}}$$

Discrete Inverse Fourier Transform

$$Let \quad V_s'(p) = V_s\left(p - \frac{N}{2}\right)$$

$$v_s(k) = \frac{F}{N}\sum_{p=0}^{N-1} V_s\left(p - \frac{N}{2}\right) e^{j\frac{2\pi k(p-N/2)}{N}}$$

$$V_s'(p) = V_s\left[\left(p - \frac{N}{2}\right)\Delta f\right] \text{ =value calculated at } \left(p - \frac{N}{2}\right)\Delta f$$

So: from p=0 to p=N/2 negative frequency information is entered.

Discrete Inverse Fourier Transform

p	0	N/2	N-1
Frequency	$-\frac{N}{2}\Delta f$	0	$\frac{N}{2}\Delta f$
$V_s^{'}(p)$	$V\left(-\frac{N}{2}\Delta f\right)$	DC value	$V\left[\left(\frac{N}{2}-1\right)\Delta f\right]$

For *f<0* use relation: $V(-f)=V^*(f) \Leftrightarrow v(t)$ real

Therefore $V_s^{'}\left(\frac{N}{2}-p\right)=V_s^{'*}(p)$

Discrete Inverse Fourier Transform

IFFT routine calculates: $X(K)=\sum_{I=0}^{N-1} A(I)e^{j\frac{2\pi IK}{N}}$

However, we need to evaluate: $v_s(k)=\frac{F}{N}(-1)^k\sum_{p=0}^{N-1} V_s'(p)e^{j\frac{2\pi kp}{N}}$

When output of IFFT is obtained, alternate sign and scale value to obtain actual time-domain values.

Problems & Limitations (in frequency domain)	Consequences (in time domain)	Solution
Discretization	Time-domain response will repeat itself periodically (Fourier series) Aliasing effects	Take small frequency steps. Minimum sampling rate must be the Nyquist rate
Truncation in Frequency	Time-domain response will have finite time resolution (Gibbs effect)	Take maximum frequency as high as possible
No negative frequency values	Time-domain response will be complex	Define negative-frequency values and use V(-f)=V*(f) which forces v(t) to be real
No DC value	Offset in time-domain response, ringing in base line	Use measurement at the lowest frequency as the DC value

Microstrip Line TDR Simulation