ECE 451
Advanced Microwave Measurements

TL Characterization

Jose E. Schutt-Aine
Electrical & Computer Engineering
University of Illinois
jesa@illinois.edu

Maxwell's Equations

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

Faraday's Law of Induction

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Ampère's Law

$$\nabla \cdot D = \rho$$

Gauss' Law for electric field

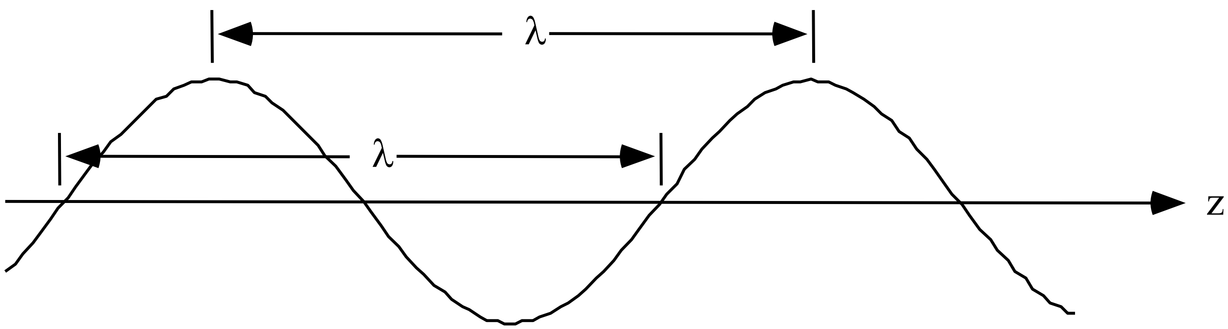
$$\nabla \cdot B = 0$$

Gauss' Law for magnetic field

Constitutive Relations

$$B = \mu H \qquad D = \varepsilon E$$

Wave Propagation



Wavelength : λ

$$\lambda = \frac{\text{propagation velocity}}{\text{frequency}}$$

Why Transmission Lines ?

In Free Space

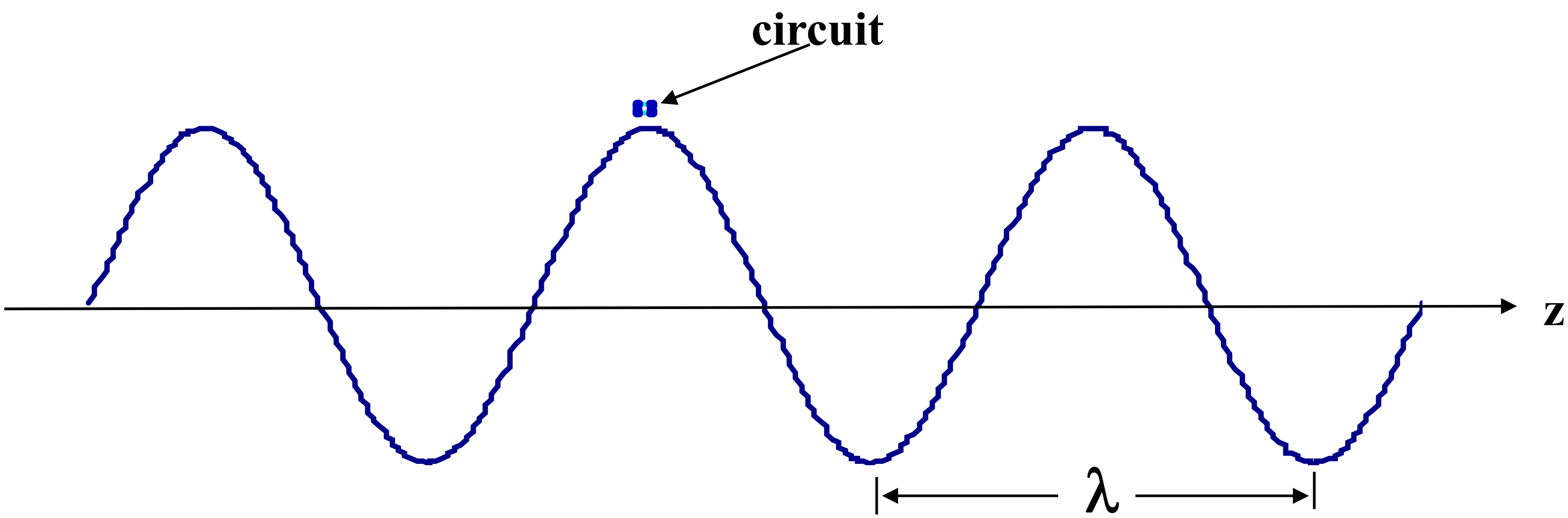
At 10 KHz : $\lambda = 30$ km

At 10 GHz : $\lambda = 3$ cm

Transmission line behavior is prevalent when the structural dimensions of the circuits are comparable to the wavelength.

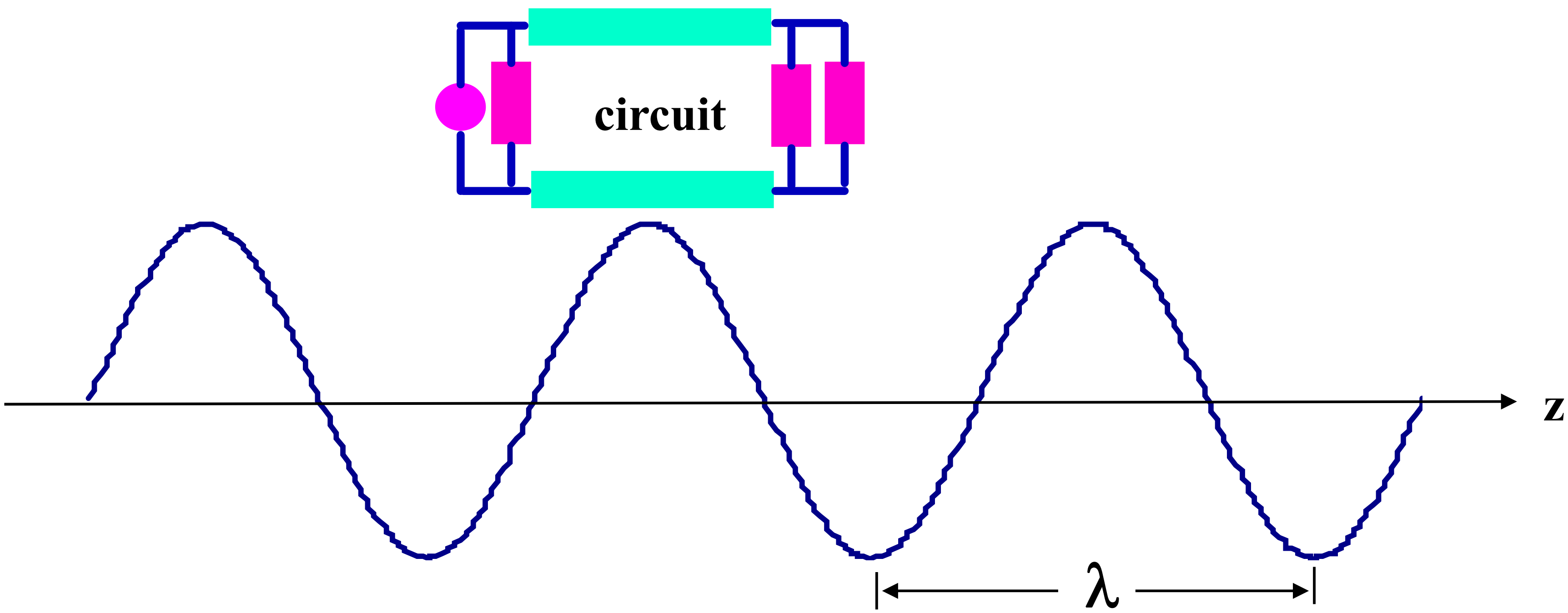
Transmission Line Model

Let d be the largest dimension of a circuit



If d << λ, a lumped model for the circuit can be used

Transmission Line Model



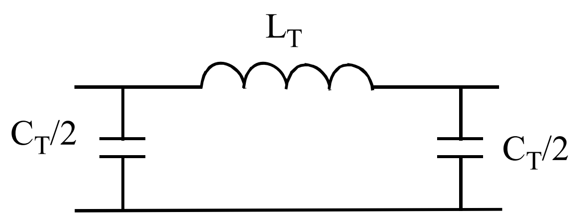
If d ≈ λ, or d > λ then use transmission line model

Modeling Interconnections

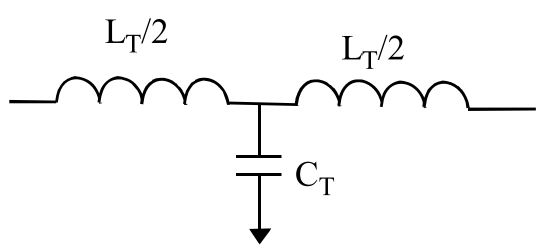
Low Frequency

Short

Mid-range Frequency

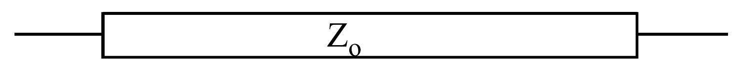


or



Lumped Reactive CKT

High Frequency



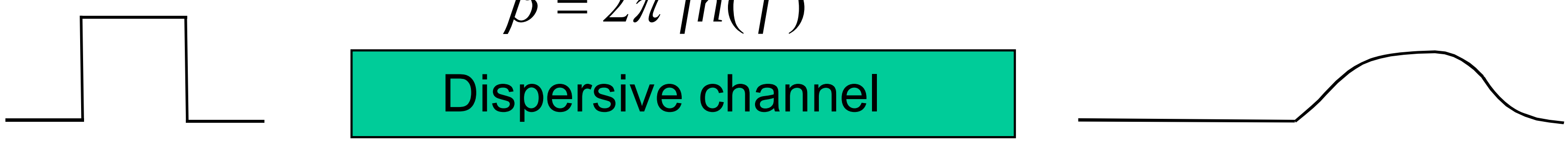
Transmission Line

Dispersion & Velocities

$$\beta = 2\pi f \sqrt{LC}$$

$$velocity = \frac{1}{\sqrt{LC}}$$

$$\beta = 2\pi f h(f)$$

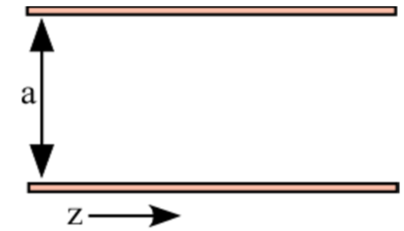


$$velocity = g(f)$$

$$Phase\ velocity = \omega / \beta$$

$$Group\ velocity = \left(\frac{d\beta}{d\omega}\right)^{-1}$$

Parallel-Plate Waveguide



$$\nabla^2 \boldsymbol{E} + \omega^2 \mu\varepsilon \boldsymbol{E} = 0$$

$\mathbf{TE_{mn}}$ modes: $E_z = 0,\ H_z \neq 0,\ H_x \neq 0$

$$E_y = E_o \sin\left(\beta_x x\right) e^{-jkz} \qquad \beta_x = \frac{m\pi}{a} \qquad k = \beta_z = \sqrt{\omega^2 \mu\varepsilon - \left(\frac{m\pi}{a}\right)^2}$$

Wave will propagate if $f > f_c = \dfrac{m}{2a\sqrt{\mu\varepsilon}}$

$\mathbf{TM_{mn}}$ modes: $H_z = 0,\ E_z \neq 0,\ E_x \neq 0$

$$H_y = H_o \cos\left(\beta_x x\right) e^{-jkz}$$

Same dispersion relation as TE_{mn} modes

TEM Mode

Special case *m=0* ➔ TM_0 or TEM mode

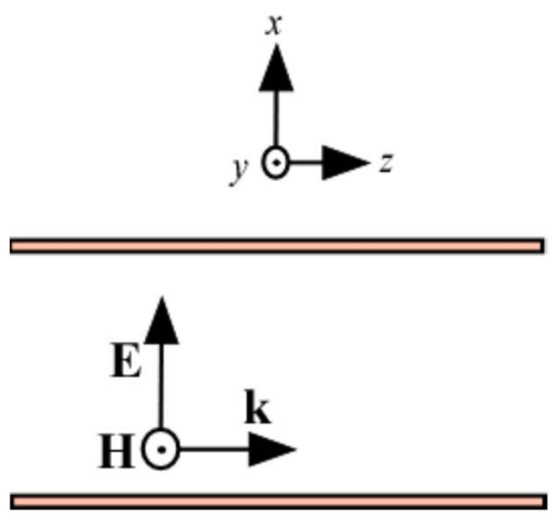
$$E_z = 0,\ H_z = 0,\ E_x \neq 0, H_y \neq 0$$

$$k = \beta_z = \omega\sqrt{\mu\varepsilon}$$

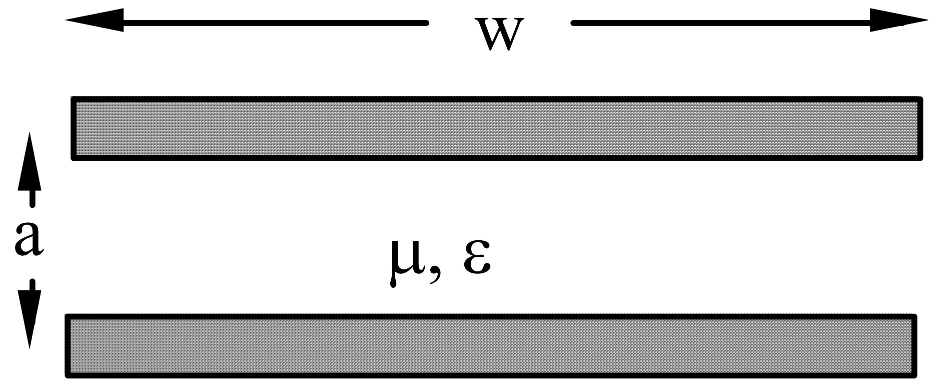
$$E_x = E_o e^{-jkz} = \sqrt{\frac{\mu}{\varepsilon}} H_o e^{-jkz}$$

$$Z_o = \sqrt{\frac{\mu}{\varepsilon}}$$

$$H_y = H_o e^{-jkz}$$



Parallel-Plate in TEM



$$L = \frac{\mu a}{w}$$

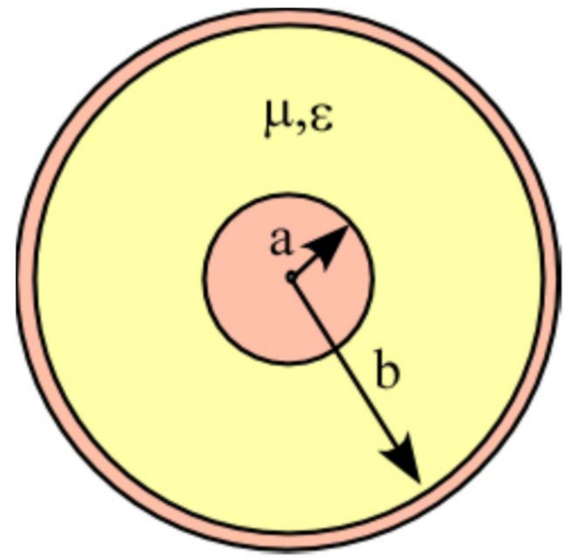
$$C = \frac{\varepsilon w}{a}$$

Coaxial Cables

Laplace's Equation for potential ψ

$$\nabla^2\psi = 0 \Rightarrow \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\psi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\psi}{\partial\phi^2} = 0$$

$$\text{Solution}:\ \psi(\rho,\phi) = \frac{V_o \ln(b/\rho)}{\ln(b/a)}$$



For a TEM mode of propagation

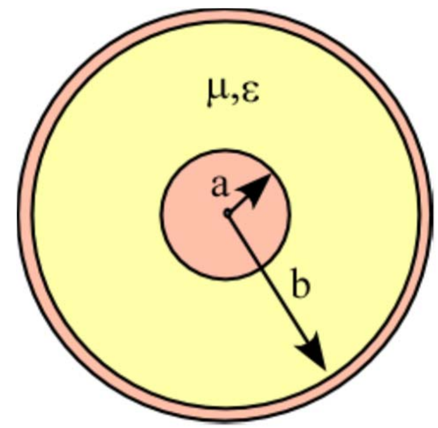
$$L = \frac{\mu}{2\pi}\ln(b/a) \qquad C = \frac{2\pi\varepsilon}{\ln(b/a)}$$

$$\beta = \omega\sqrt{LC} = \omega\sqrt{\mu\varepsilon} \qquad Z_o = \sqrt{L/C} = \frac{\sqrt{\mu/\varepsilon}}{2\pi}\ln(b/a)$$

Coaxial Cables

Higher order modes: TE modes: $H_z = h_z(\rho,\phi)e^{-j\beta z}$

$$\left(\frac{\partial^2}{\partial\rho^2}\frac{1}{\rho}\frac{\partial^2}{\partial\phi^2}+k^2-\beta^2=0\right)h_z=0$$



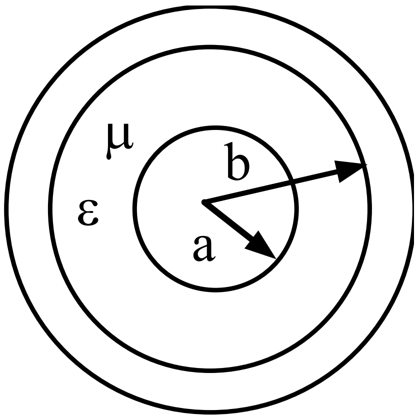
The first higher order mode is the TE_{11} mode

Approximate solution for k_c is: $k_c = \dfrac{2}{a+b}$

From k_c, find cutoff frequency f_c $\quad f_c = \dfrac{ck_c}{2\pi\sqrt{\varepsilon_r}} = \dfrac{vk_c}{2\pi}$

$$f_c = \frac{2c}{(a+b)2\pi\sqrt{\varepsilon_r}} = \frac{c}{\sqrt{\varepsilon_r}\pi(a+b)}$$

Coaxial Line with Losses



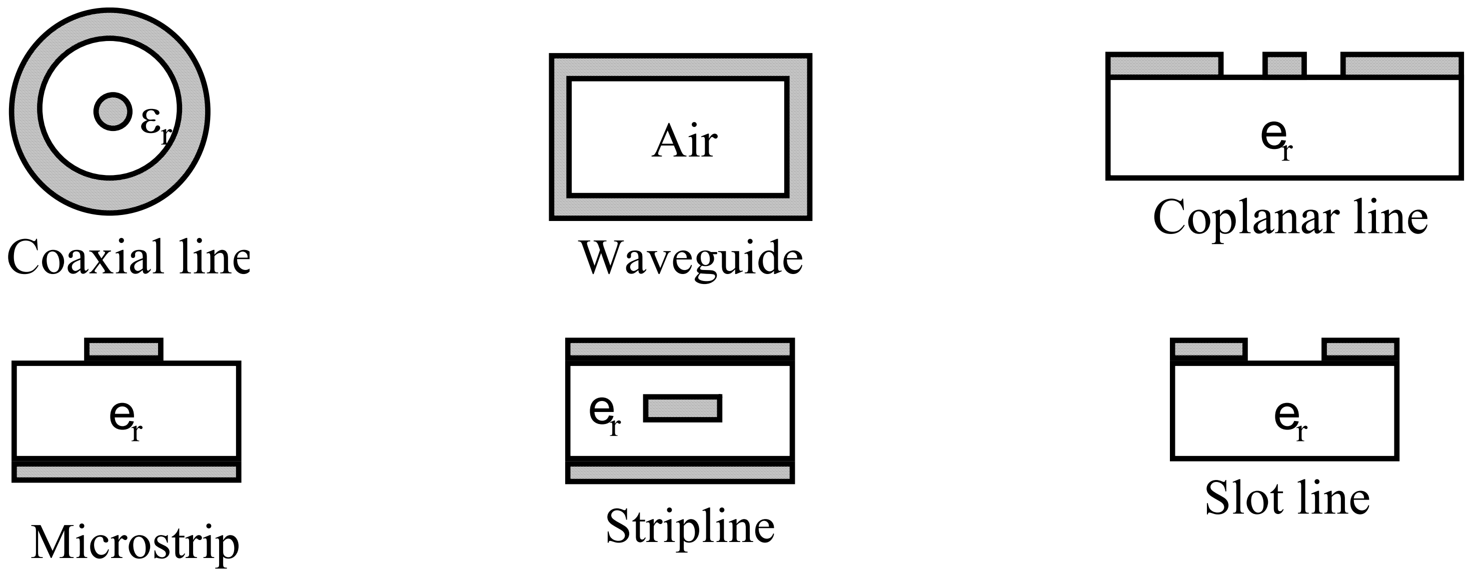
Infinite Conductivity

$$Z_o = \frac{\sqrt{\mu/\varepsilon}}{2\pi}\ln\left(b/a\right)$$

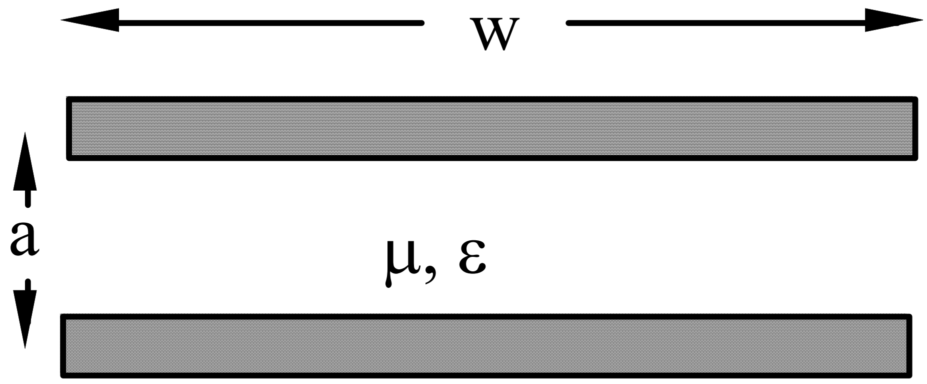
Finite Conductivity

$$Z_o = \frac{\sqrt{\mu/\varepsilon}}{2\pi}\ln\left(b/a\right)\left[1+\frac{\left(1/a+1/b\right)}{4\sqrt{\pi f \mu\sigma}\ln(b/a)}\left(1-j\right)\right]$$

Types of Transmission Lines



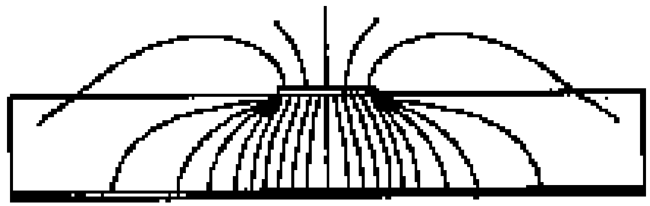
Parallel-plate Transmission Line



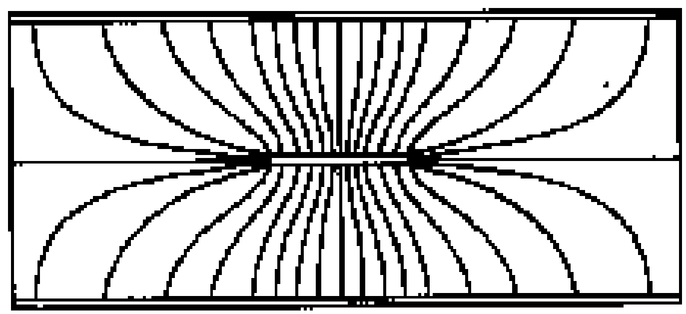
$$L = \frac{\mu a}{w}$$

$$C = \frac{\varepsilon w}{a}$$

Microstrip and Stripline



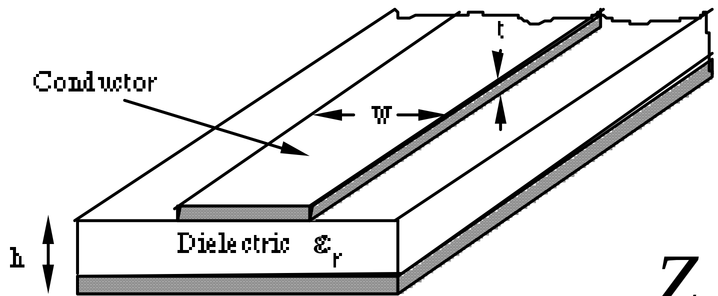
Microstrip



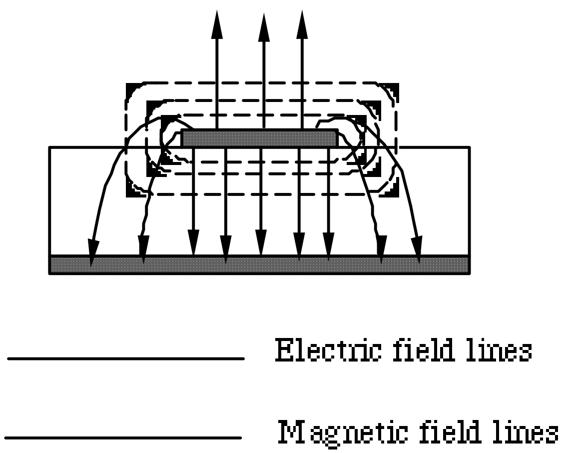
Stripline

Wave propagation in stripline is closer to the TEM mode of propagation and the propagation of velocity is approximately $c/\sqrt{\varepsilon_r}$.

Microstrip – Analysis Equations



(a)



(b)

$w/h < 3.3$

$$Z_o = \frac{119.9}{\sqrt{2(\varepsilon_r + 1)}} \ln\left[4\frac{h}{w} + \sqrt{16\left(\frac{h}{w}\right)^2 + 2} \right]$$

$w/h > 3.3$

$$Z_o = \frac{119.9\pi}{2\sqrt{\varepsilon_r}} \left\{ \frac{w}{2h} + \frac{\ln(4)}{\pi} + \frac{\ln(e\pi^2/16)}{2\pi}\left(\frac{\varepsilon_r - 1}{\varepsilon_r^2}\right) + \frac{\varepsilon_r + 1}{2\pi\varepsilon_r}\left[\ln\frac{\pi e}{2} + \ln\left(\frac{w}{2h} + 0.94\right) \right] \right\}$$

Microstrip Analysis & Synthesis Equations

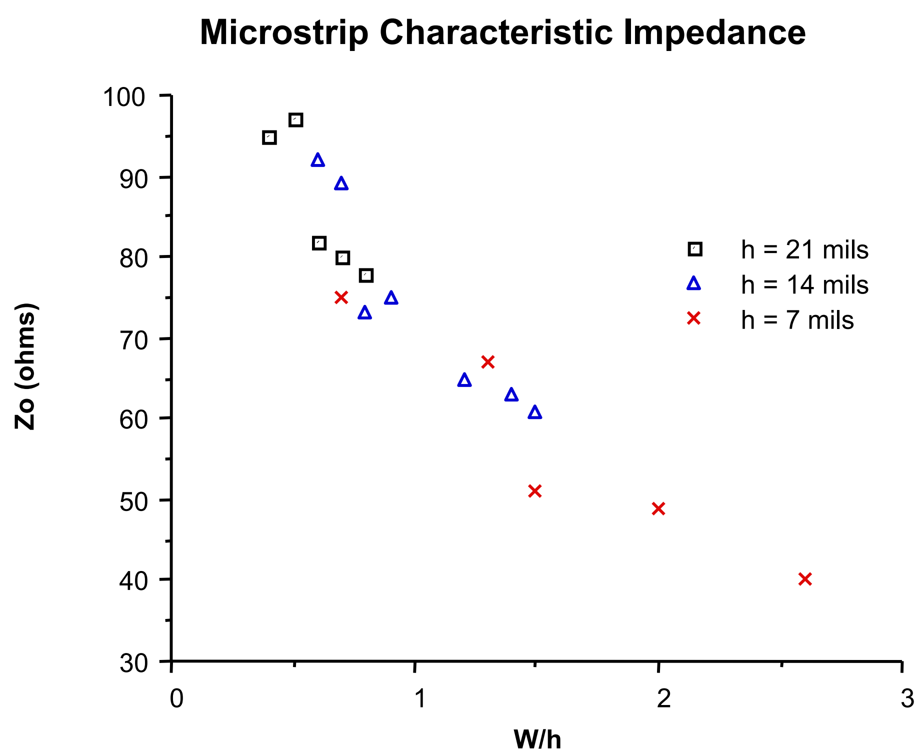
$$\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12d/W}}$$

$$Z_o = \begin{cases} \dfrac{60}{\sqrt{\varepsilon_e}} \ln\left(\dfrac{8d}{W} + \dfrac{W}{4d}\right) & \text{for } W/d \le 1 \\ \dfrac{120\pi}{\sqrt{\varepsilon_e}\left[W/d + 1.393 + 0.667\ln(W/d + 1.444)\right]} & \text{for } W/d \ge 1 \end{cases}$$

$$\frac{W}{d} = \begin{cases} \dfrac{8e^A}{e^{2A} - 2} & \text{for } W/d < 2 \\ \dfrac{2}{\pi}\left[B - 1 - \ln(2B - 1) + \dfrac{\varepsilon_r - 1}{2\varepsilon_r}\left\{\ln(B - 1) + 0.39 - \dfrac{0.61}{\varepsilon_r}\right\}\right] & \text{for } W/d > 2 \end{cases}$$

$$A = \frac{Z_o}{60}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right) \qquad B = \frac{377\pi}{2Z_o\sqrt{\varepsilon_r}}$$

Microstrip



dielectric constant : 4.3.

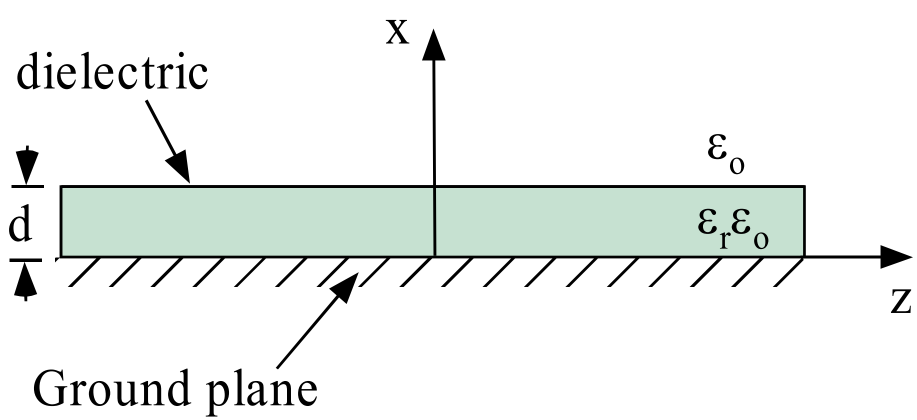
Surface Waves – TM Modes

Assume the form:

$$E_z(x,y,z) = e_z(x,y)e^{-j\beta z}$$

$$\left(\frac{\partial^2}{\partial x^2} + \varepsilon_r k_o^2 - \beta^2\right)e_z(x,y) = 0$$ Inside dielectric

$$\left(\frac{\partial^2}{\partial x^2} + k_o^2 - \beta^2\right)e_z(x,y) = 0$$ Outside dielectric



Dispersion relations

$$k_c^2 = \varepsilon_r k_o^2 - \beta^2$$

$$h^2 = \beta^2 - k_o^2$$

Surface Waves – TM Modes: Solutions

Solutions are:

$$e_z(x,y) = A\sin k_c x + B\cos k_c x$$ Inside dielectric

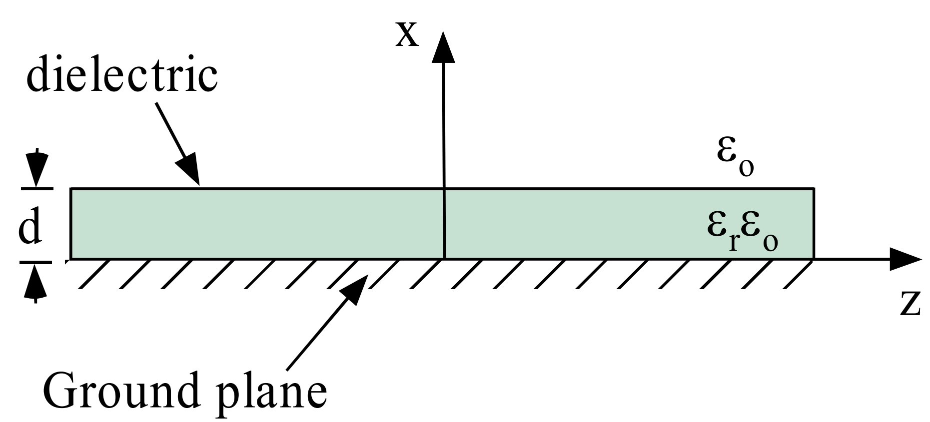
$$e_z(x,y) = Ce^{hx} + De^{-hx}$$ Outside dielectric

$$E_z(x,y,z) = 0 \text{ at x=0}$$

$$E_z(x,y,z) < \infty, \quad for\ x \to \infty$$

$$E_z(x,y,z) \text{ continuous at boundary}$$

$$H_y(x,y,z) \text{ continuous at boundary}$$



$$A\sin k_c d = De^{-hd}$$

$$k_c \tan k_c d = \varepsilon_r h$$

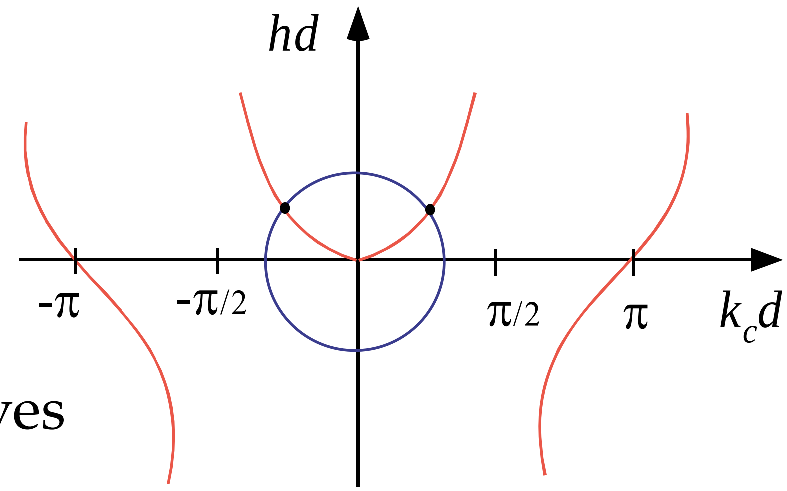
$$\frac{\varepsilon_r A}{k_c}\cos k_c d = \frac{D}{h}e^{-hd}$$

$$k_c^2 + h^2 = (\varepsilon_r - 1)k_o^2$$

Surface Waves – TM Modes: Solutions

$$(k_c d)^2 + (hd)^2 = (\varepsilon_r - 1)(k_o d)^2$$

$$k_c d \tan k_c d = \varepsilon_r hd$$



Solution is found at intersection of curves

First TM mode is TM_o mode

Cutoff frequencies for TM modes are given by:

$$f_c = \frac{nc}{2d\sqrt{\varepsilon_r - 1}}, \qquad n = 0, 1, 2, \ldots$$

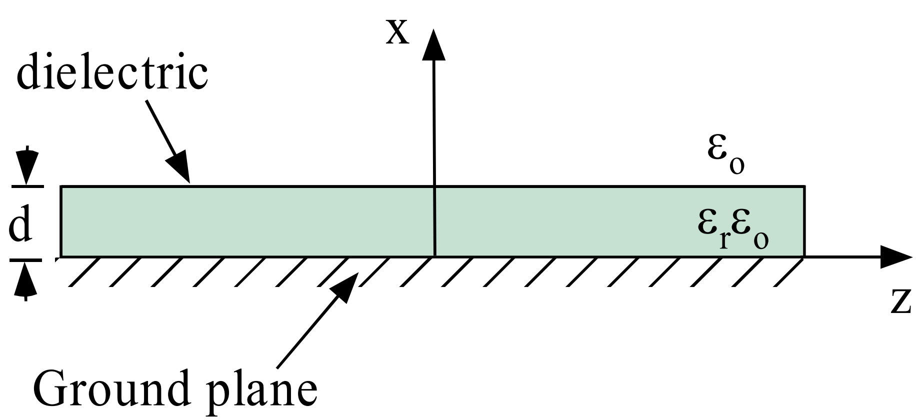
Surface Waves – TE Modes

Assume the form:

$$H_z(x,y,z) = h_z(x,y)e^{-j\beta z}$$

$$\left(\frac{\partial^2}{\partial x^2} + \varepsilon_r k_o^2 - \beta^2\right)h_z(x,y) = 0$$ Inside dielectric

$$\left(\frac{\partial^2}{\partial x^2} + k_o^2 - \beta^2\right)h_z(x,y) = 0$$ Outside dielectric



Dispersion relations

$$k_c^2 = \varepsilon_r k_o^2 - \beta^2$$

$$h^2 = \beta^2 - k_o^2$$

Surface Waves – TE Modes: Solutions

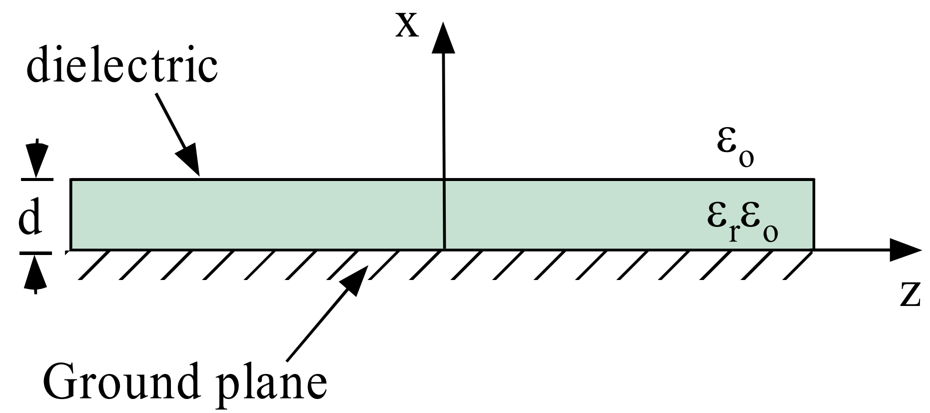
Solutions are:

$$h_z(x,y) = A\sin k_c x + B\cos k_c x$$

Inside dielectric

$$h_z(x,y) = Ce^{hx} + De^{-hx}$$

Outside dielectric



After matching the boundary conditions

$$\frac{-B}{k_c}\sin k_c d = \frac{D}{h}e^{-hd}$$

$$-k_c \cot k_c d = h$$

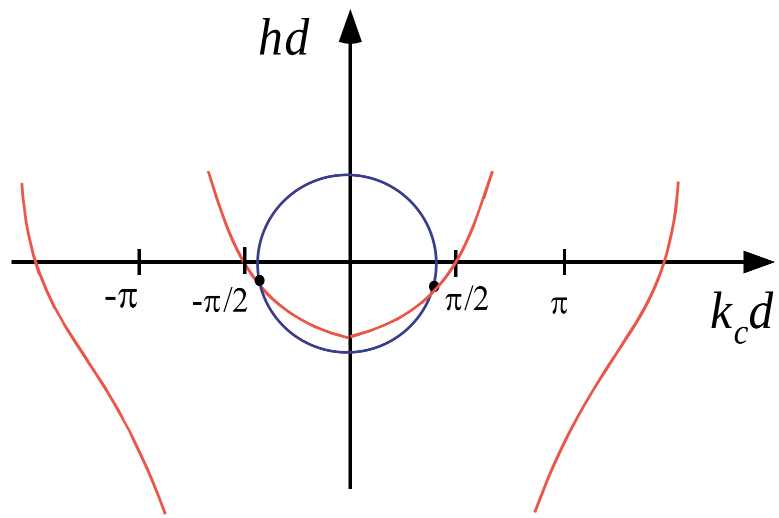
$$B\cos k_c d = De^{-hd}$$

$$k_c^2 + h^2 = \left(\varepsilon_r - 1\right)k_o^2$$

Surface Waves – TE Modes: Solutions

$$\left(k_c d\right)^2 + \left(hd\right)^2 = \left(\varepsilon_r - 1\right)\left(k_o d\right)^2$$

$$-k_c d \cot k_c d = hd$$



Solution is found at intersection of curves

Negative values of *h* must be excluded

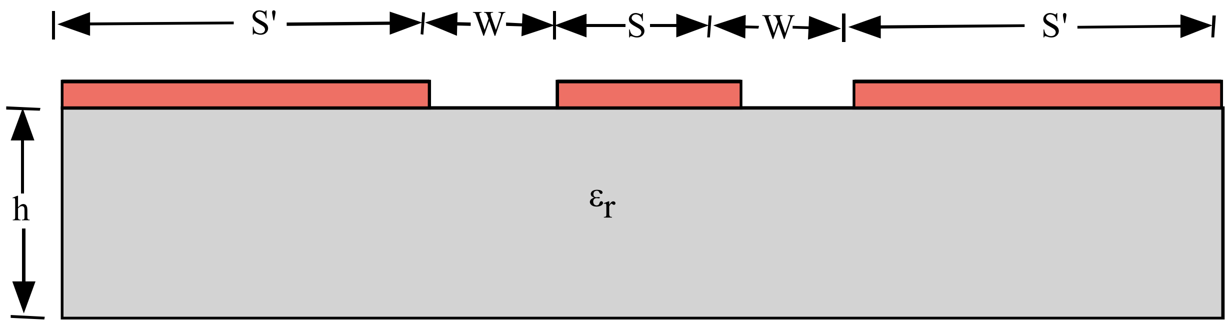
Cutoff frequencies for TE modes are given by:

$$f_c = \frac{(2n-1)c}{4d\sqrt{\varepsilon_r - 1}}, \qquad n = 1, 2, 3, \ldots$$

Modes will occur in the following order:

TM_0, TE_1, TM_1, TE_2, TM_2

Coplanar Waveguide



$$K(k): \text{Complete Elliptic Integral of the first kind}$$

$$k = \frac{S}{S+2W}$$

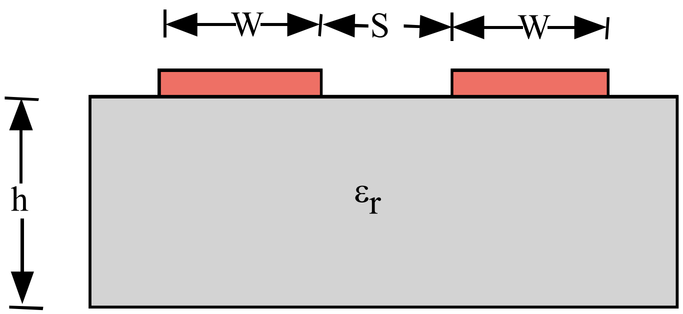
$$Z_{ocp} = \frac{30\pi}{\sqrt{\frac{\varepsilon_r + 1}{2}}} \frac{K'(k)}{K(k)} \text{ (ohm)}$$

$$K'(k) = K(k')$$

$$k' = (1-k^2)^{1/2}$$

$$v_{cp} = \left(\frac{2}{\varepsilon_r + 1} \right)^{1/2} c$$

Coplanar Strips



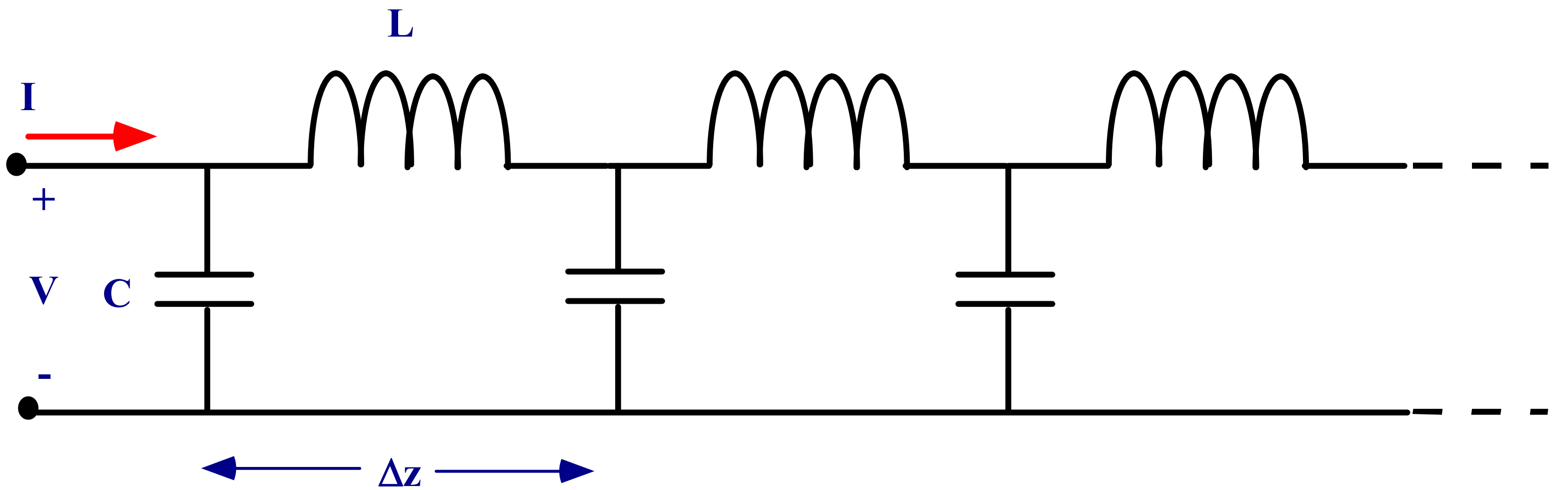
$$Z_{ocs} = \frac{120\pi}{\sqrt{\frac{\varepsilon_r + 1}{2}}} \frac{K'(k)}{K(k)} \text{ (ohm)}$$

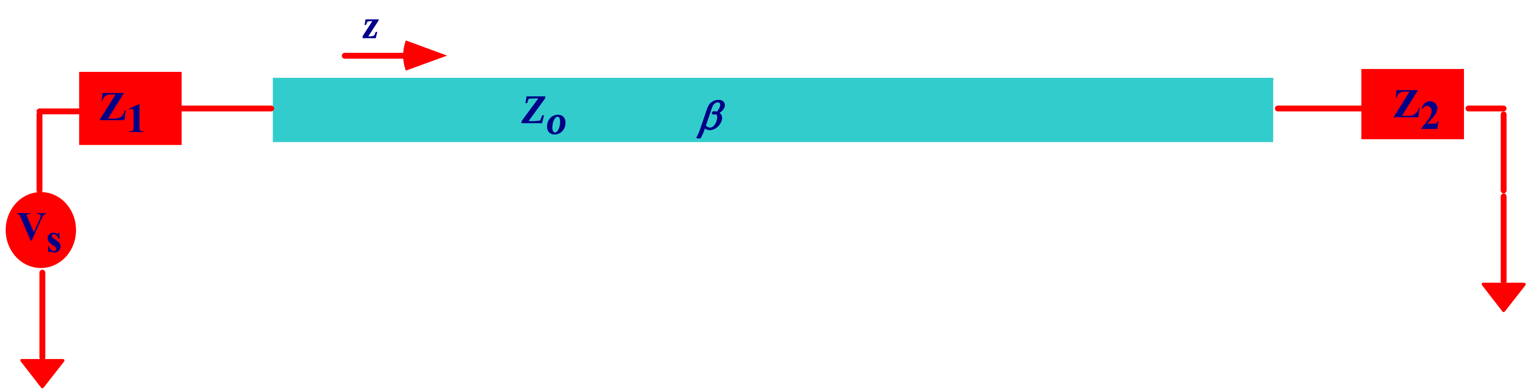
Qualitative Comparison

Characteristic	Microstrip	Coplanar Wguide	Coplanar strips
e_{eff}*	~6.5	~5	~5
Power handling	High	Medium	Medium
Radiation loss	Low	Medium	Medium
Unloaded Q	High	Medium	Low or High
Dispersion	Small	Medium	Medium
Mounting (shunt)	Hard	Easy	Easy
Mounting (series)	Easy	Easy	Easy

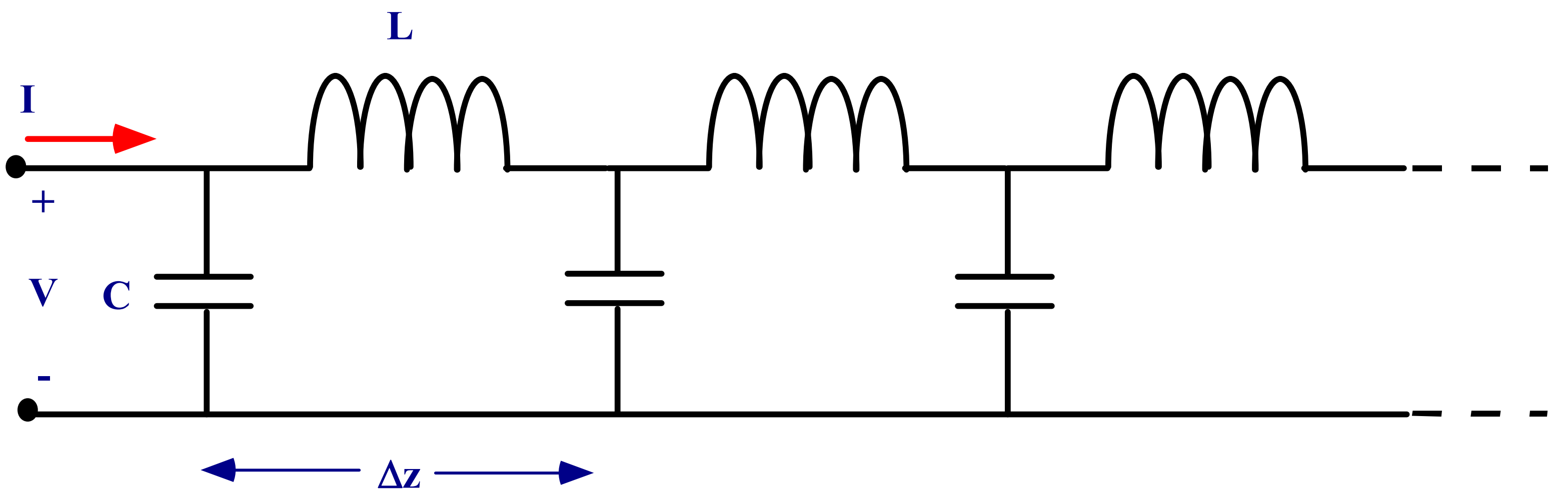
* Assuming ε_r=10 and h=0.025 inch

TEM PROPAGATION





Telegrapher's Equations



$$-\frac{\partial V}{\partial z} = L\frac{\partial I}{\partial t}$$

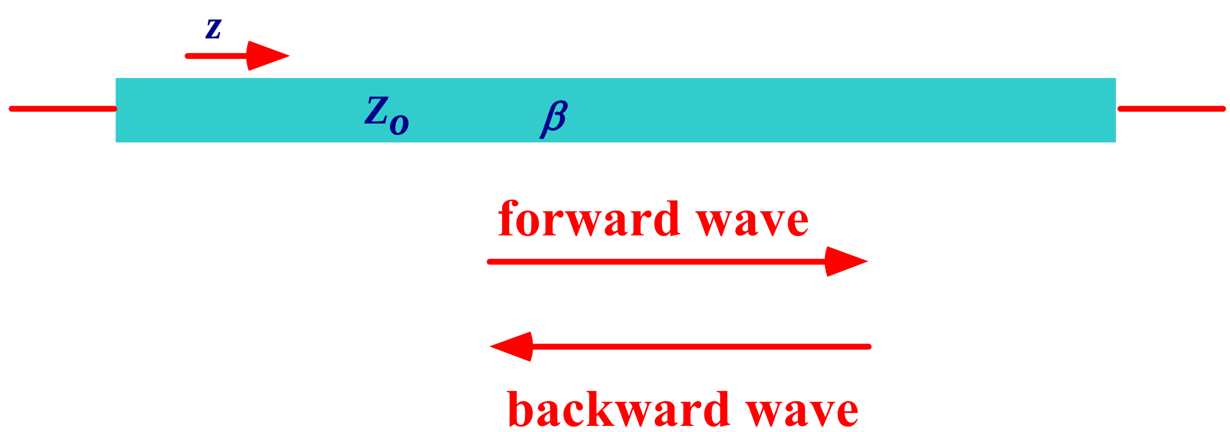
$$-\frac{\partial I}{\partial z} = C\frac{\partial V}{\partial t}$$

***L*: Inductance per unit length.**

***C*: Capacitance per unit length.**

Transmission Line Solutions

(Frequency Domain)



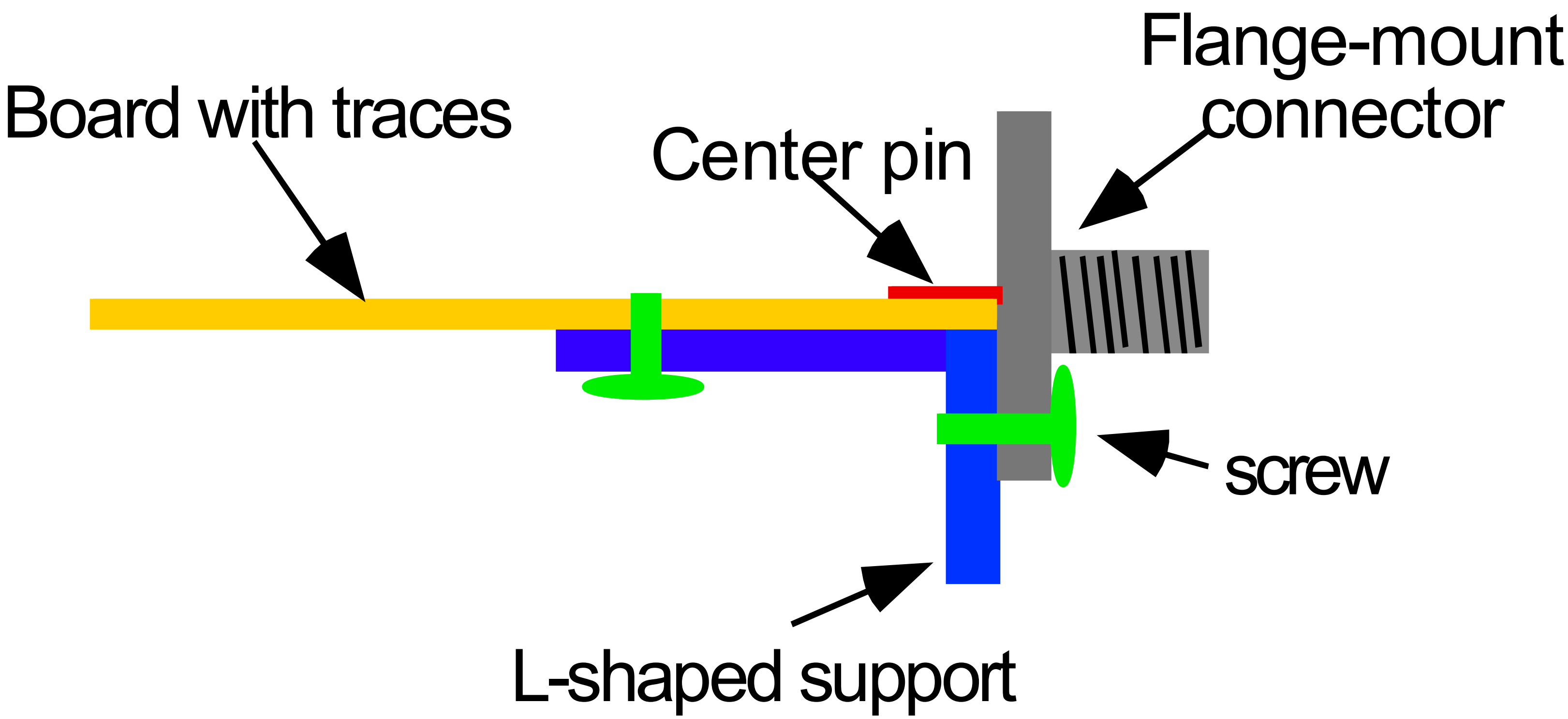
$$\beta = \omega\sqrt{LC}$$

$$V(z) = Ae^{-j\beta z} + Be^{+j\beta z}$$

$$Z_o = \sqrt{\frac{L}{C}}$$

$$I(z) = \frac{1}{Z_o}\left[Ae^{-j\beta z} - Be^{+j\beta z}\right]$$

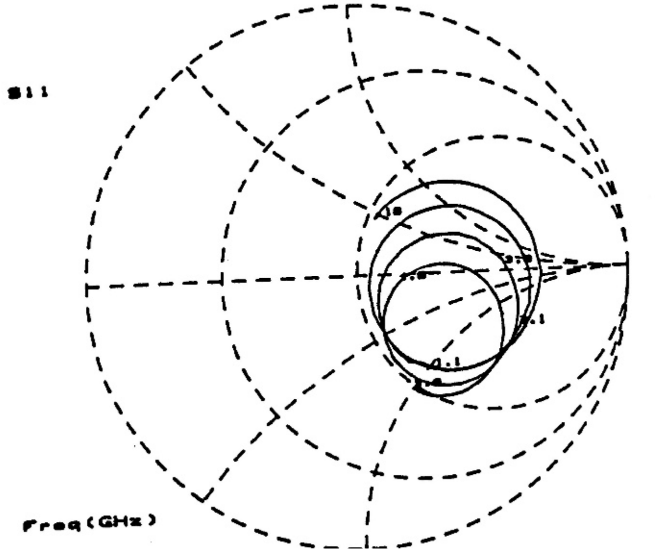
Coaxial-Microstrip Transition

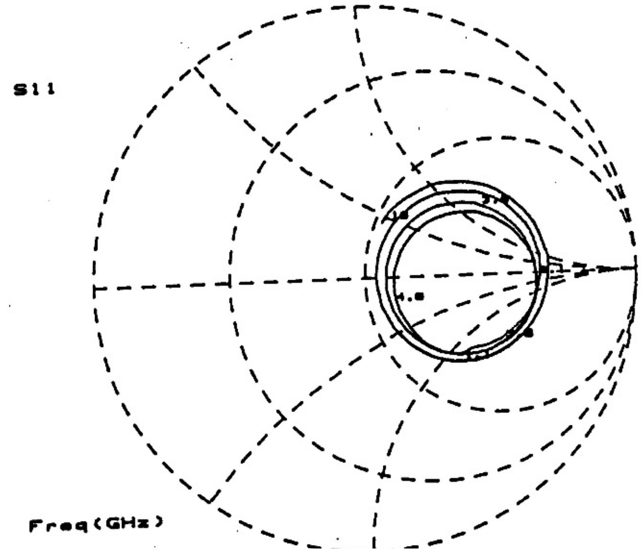


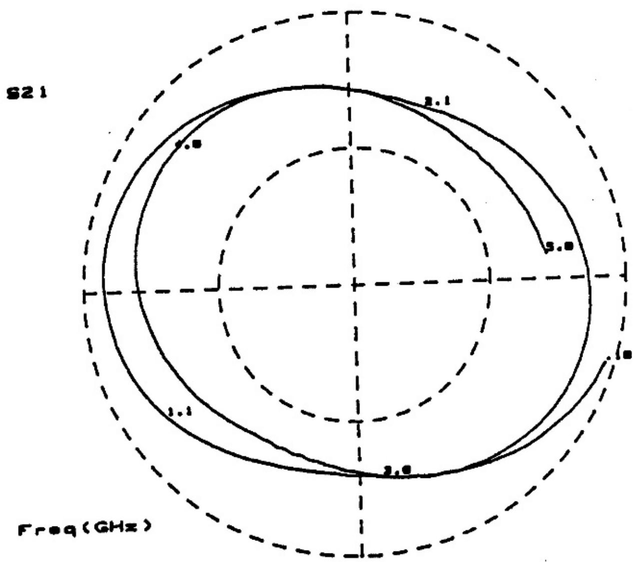
Remove effects of discontinuities before processing data!

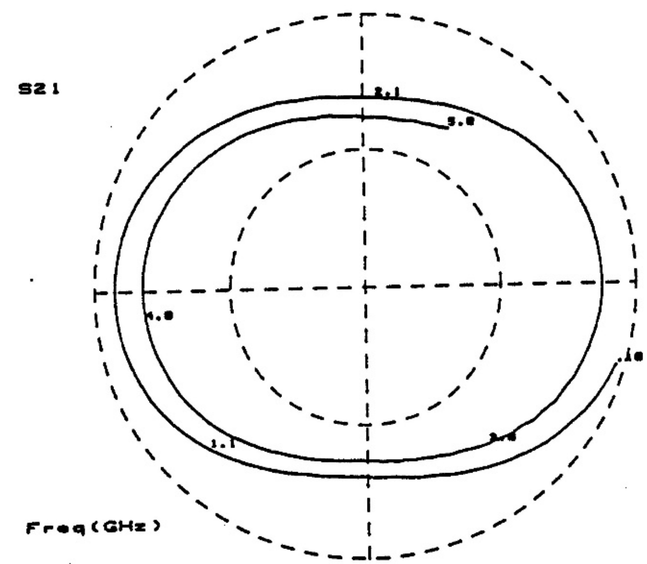
Coaxial-Microstrip Transition

With parasitics | **No parasitics**

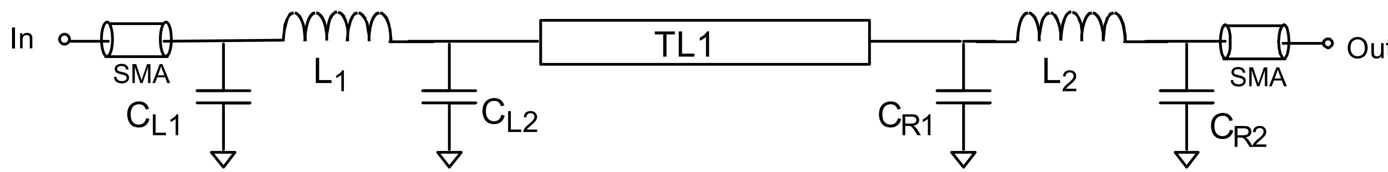




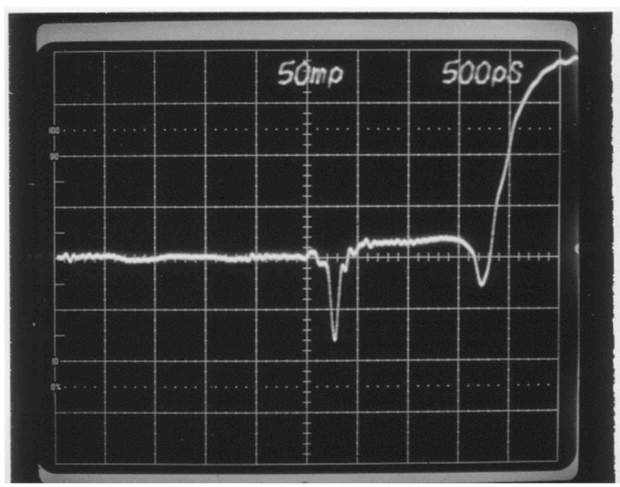




Coaxial-Microstrip Transition

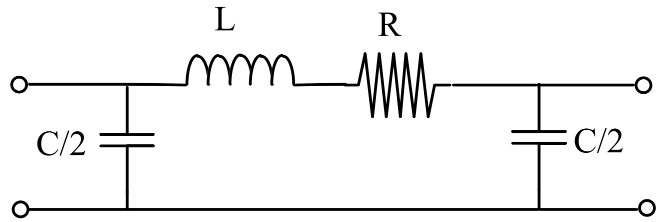


Equivalent Circuit



TDR Plot

Low-Frequency TL Approximation



$$P = (R + j\omega L)(1 + j\omega C Z_o/2)$$

$$Y = j\omega C Z_o/2$$

$$S_{11} = \frac{P - 2YZ_o - YP}{2Z_o + P + 2YZ_o + YP} \qquad S_{21} = \frac{2Z_o}{2Z_o + P + 2YZ_o + YP}$$

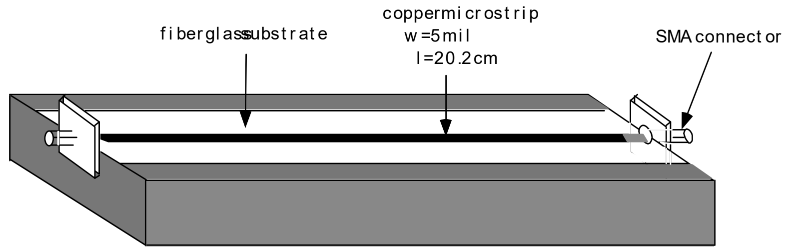
$$A = 2Z_o(1 - S_{21})$$

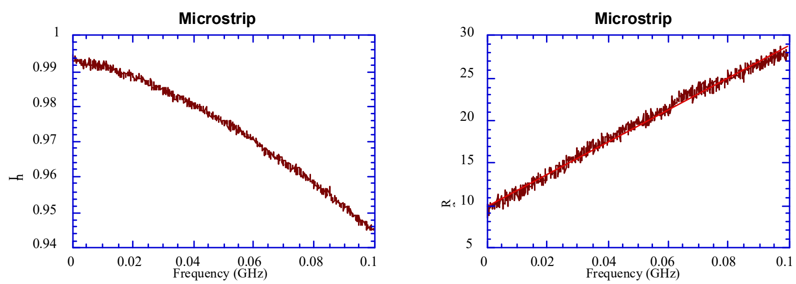
$$Y = \frac{A - 2S_{11}S_{21}Z_o - S_{11}A}{4S_{21}Z_o + 2S_{11}S_{21}Z_o + S_{11}A + A} \qquad P = A - 2YS_{21}Z_oS_{21}(1 + Y)$$

Low-Frequency Model for Microstrip

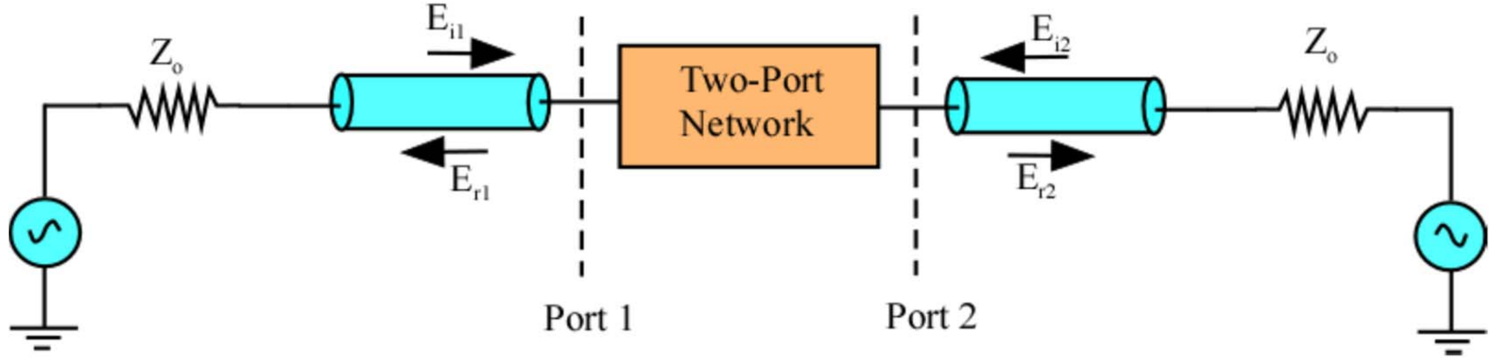
- Lumped Model

- Use extraction algorithm





High-Frequency Characterization



Transmission-Line Scattering Parameters

$$S_{21} = \frac{(1-\Gamma^2)X}{1-\Gamma^2 X^2} \qquad S_{11} = \frac{(1-X^2)\Gamma}{1-\Gamma^2 X^2}$$

$$X = e^{-\gamma d} \qquad Z_c = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\Gamma = \frac{Z_c - Z_o}{Z_c + Z_o} \qquad \gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

TL Extraction Formulas

$$X = e^{-\gamma d} = e^{-j\beta d} e^{-\alpha d} \quad X = e^{-\gamma d} = \frac{\left(S_{11} + S_{21}\right) - \Gamma}{1 - \left(S_{11} + S_{21}\right)\Gamma}$$

$$\Gamma = Q \pm \sqrt{Q^2 - 1} \qquad Q = \frac{\{S_{11}^2 - S_{21}^2\} + 1}{2S_{11}}$$

$$R = \mathrm{Re}\{\gamma Z_c\} \qquad G = \mathrm{Re}\{\frac{\gamma}{Z_c}\}$$

$$L = \frac{1}{\omega}\mathrm{Im}\{Z_c \gamma\} \qquad C = \frac{1}{\omega}\mathrm{Im}\{\frac{\gamma}{Z_c}\}$$

Low-Loss Approximation

If we assume $R << \omega L$

and $G << \omega C$

$$Z_c \cong \sqrt{\frac{L}{C}}$$

$$\gamma \cong \frac{R}{2}\sqrt{\frac{C}{L}} + \mathrm{j}\omega\sqrt{LC} = \frac{\mathrm{R}}{2\mathrm{Z_c}} + j\frac{\omega}{v_p}$$

$$\alpha \cong \frac{\mathrm{R}}{2\mathrm{Z_c}} \qquad \beta \cong \frac{\omega}{\mathrm{v_p}}$$

TL Extraction Formulas

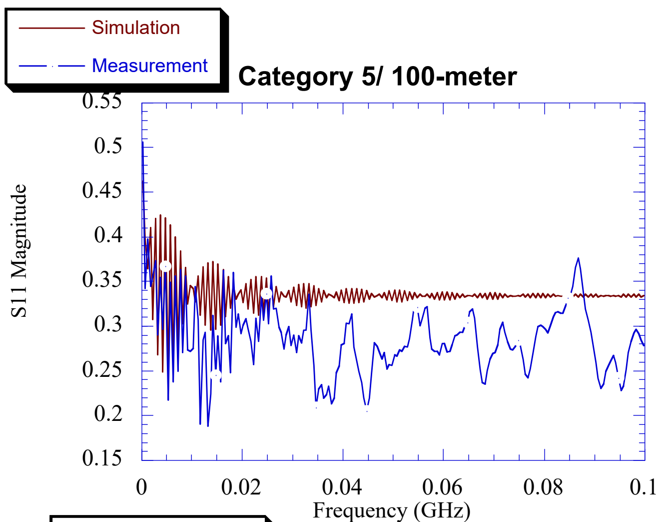
$$\alpha = -\frac{\ln\left(\left|X\right|\right)}{d}$$

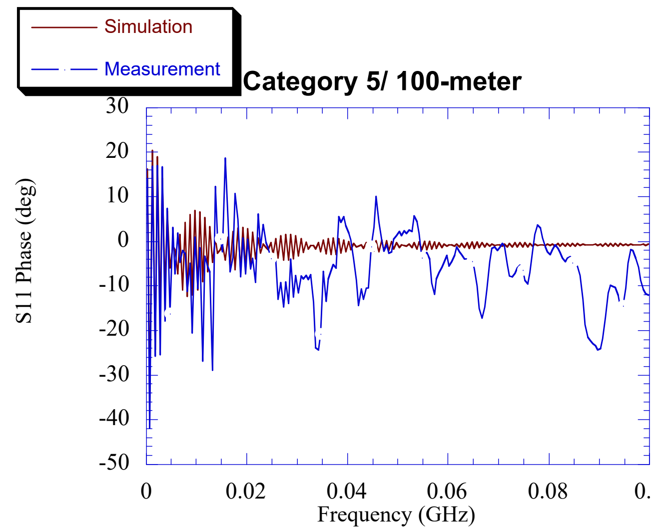
$$R = -\frac{2Z_c \ln\left(\left|X\right|\right)}{d}$$

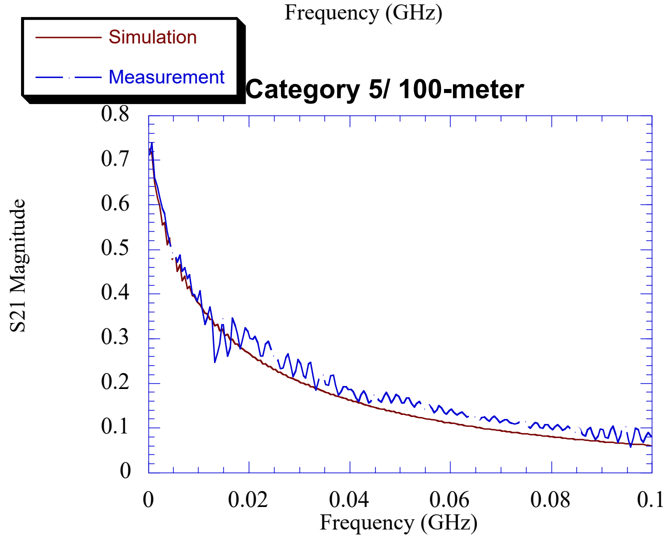
$$\frac{\Delta\phi}{\Delta\omega} = -\frac{d}{v_p}$$

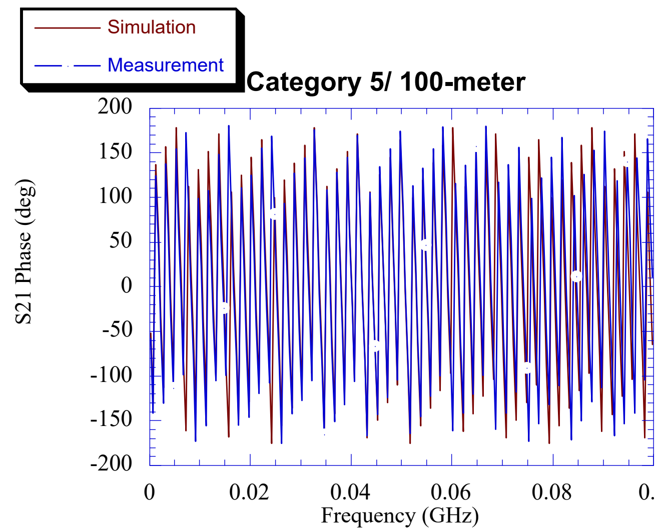
$$v_p = -\frac{d}{\dfrac{\Delta\phi}{\Delta\omega}}$$

Example: Category-5 Cable (long)

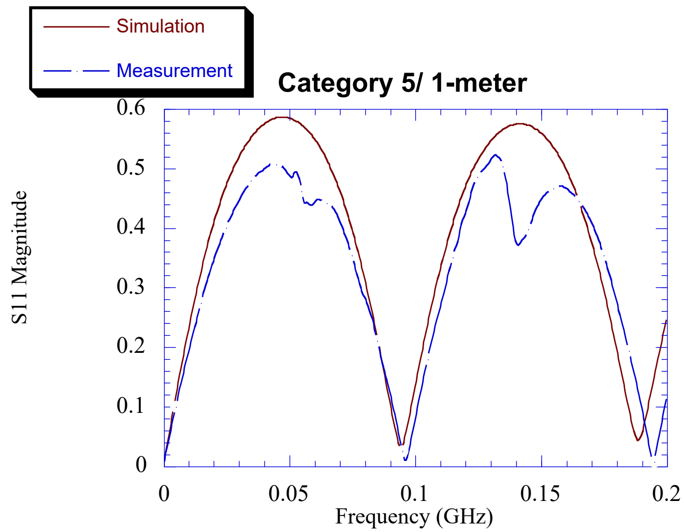


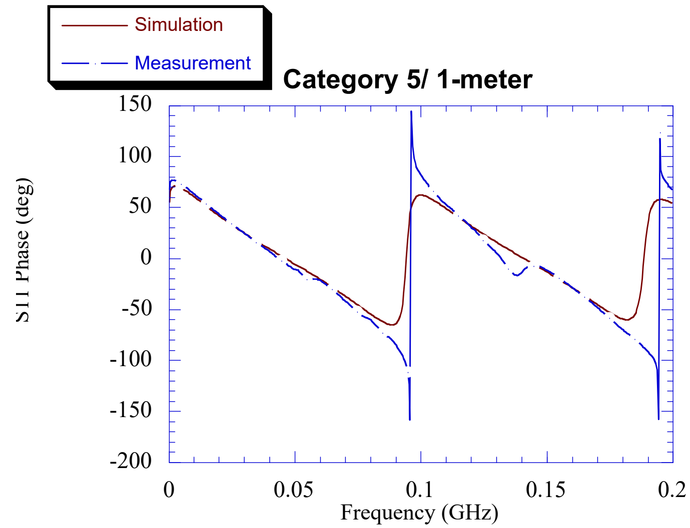


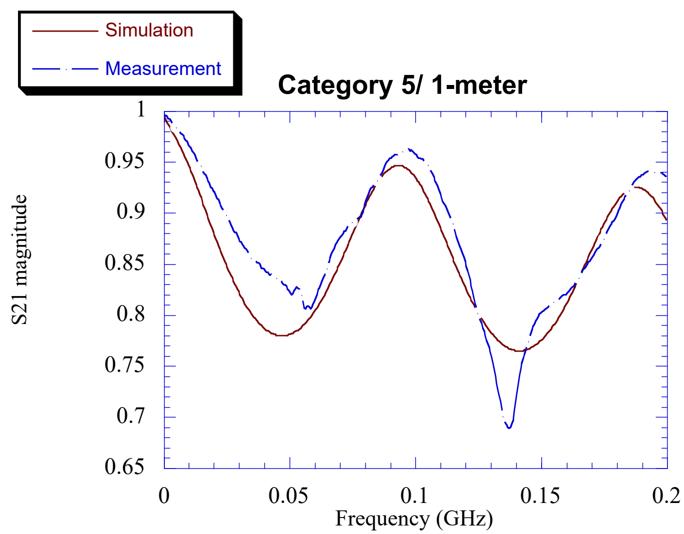


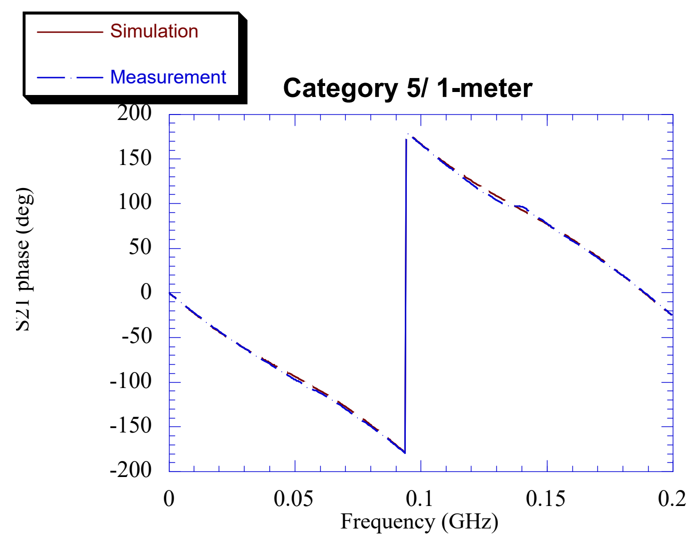


Example: Category-5 Cable (short)

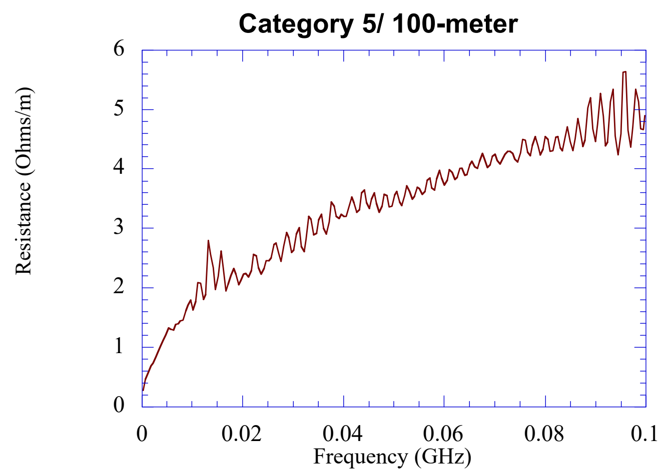


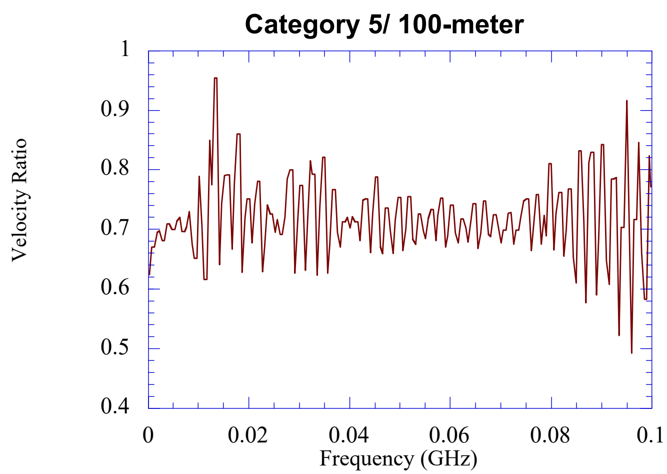






Category-5 Cable – Loss Characteristics





Cable Loss Model

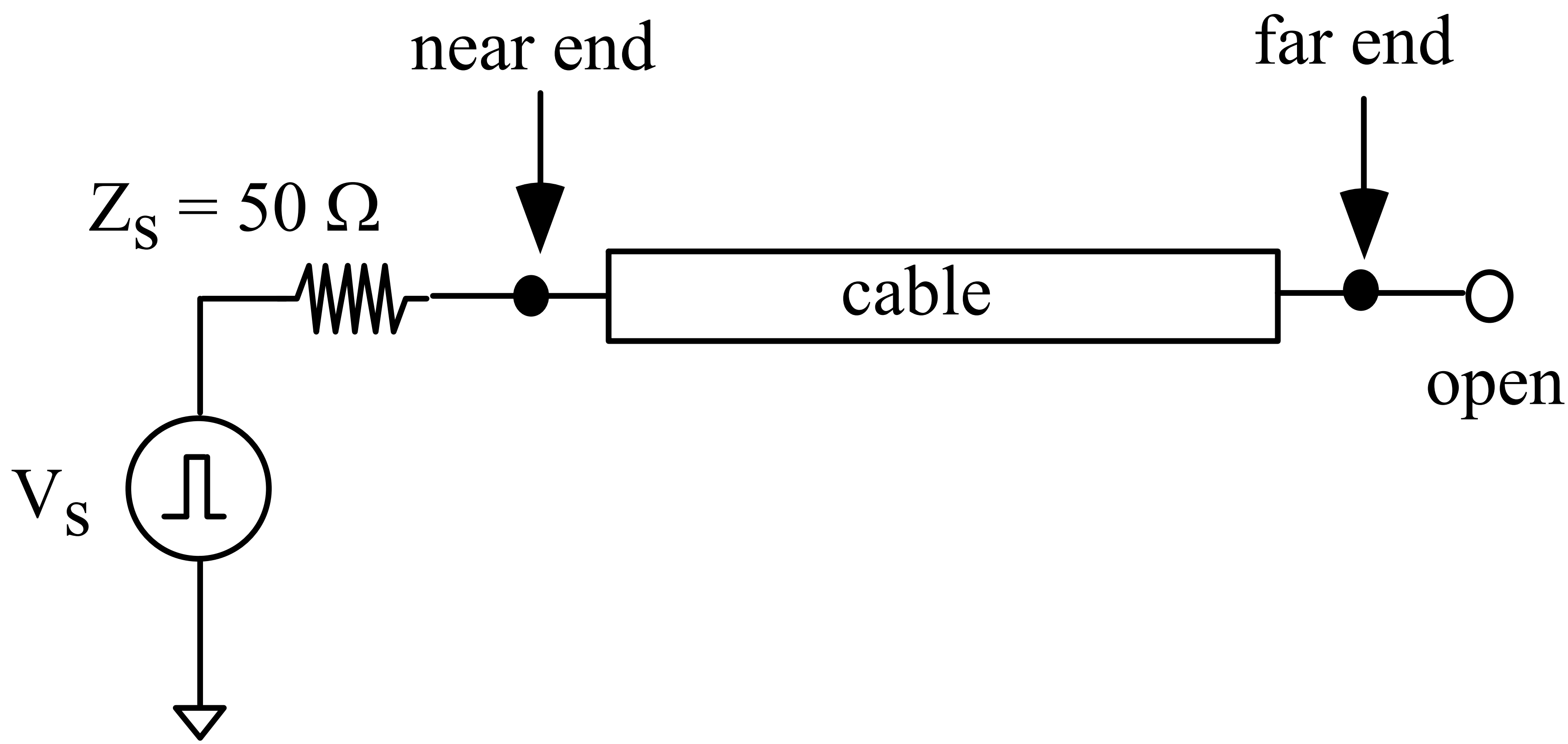
$$R(f) = R_s * f^p$$

$$v_r = v_{ro} + v_{rs} * f$$

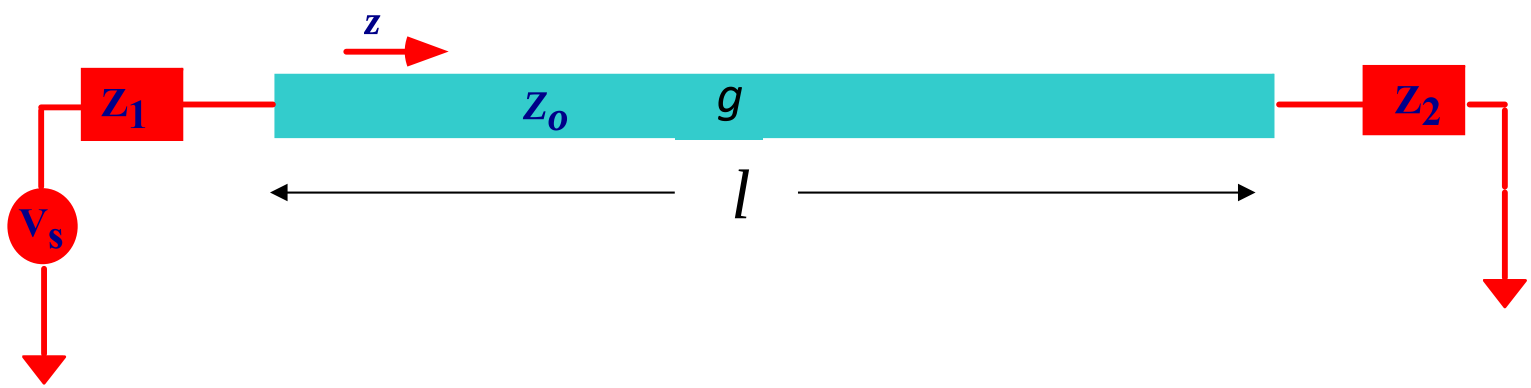
$$Z = R(f) + j\omega L = R_{skin} + j(R_{skin} + \omega L)$$

	Z_o (W)	v_{ro} (m/ns)	v_{rs} (m/ns-GHz)	R_s (Ω/m-GHz^p)	p	f_{max} (GHz)
Category 5	100	0.724	-0.165	15.38	0.482	0.2
24-Ga	100	0.678	1.157	29.03	0.593	0.1
Category 3	100	0.705	11.06	12.31	0.473	0.01
SMA	50	0.700	0.113	7.94	0.415	0.2

Time-Domain Simulations



Lossy Transmission Line

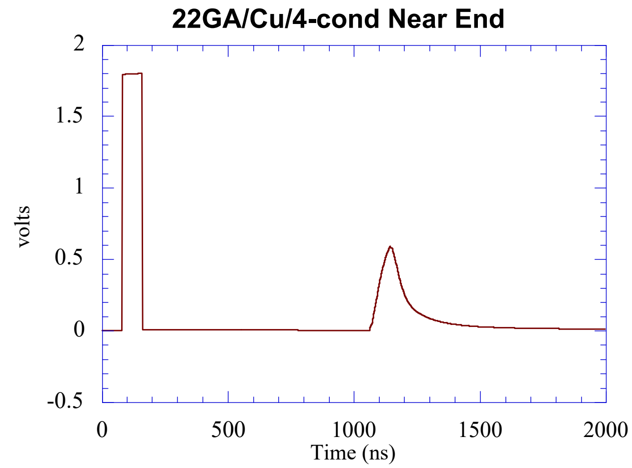


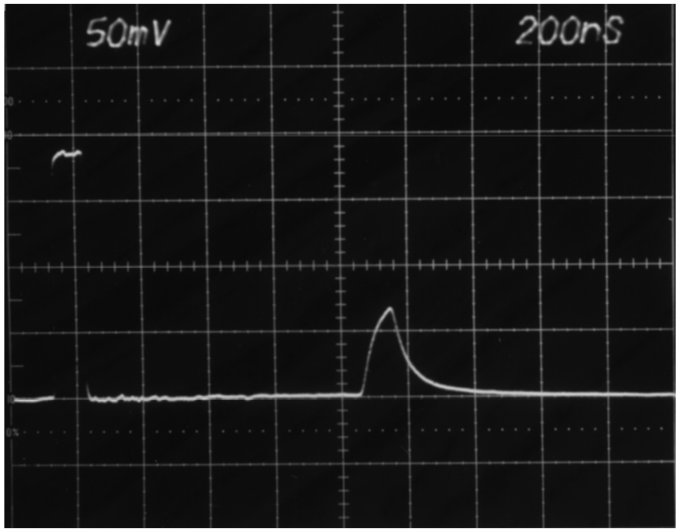
$$V(z) = Ae^{-\alpha z}e^{-j\beta z} + Be^{+\alpha z}e^{+j\beta z}$$

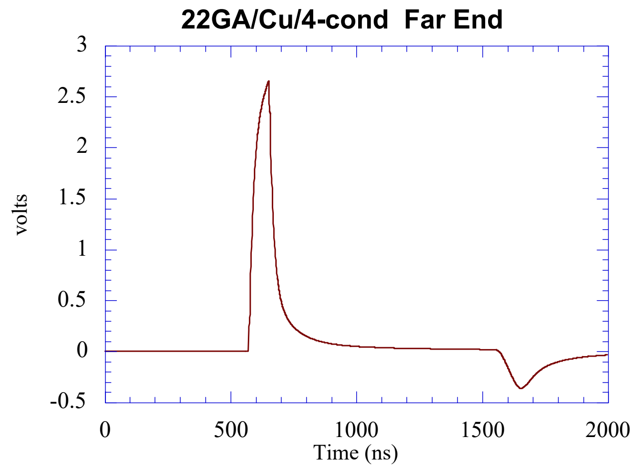
$$I(z) = \frac{1}{Z_o}\left[Ae^{-\alpha z}e^{-j\beta z} - Be^{+\alpha z}e^{+j\beta z}\right]$$

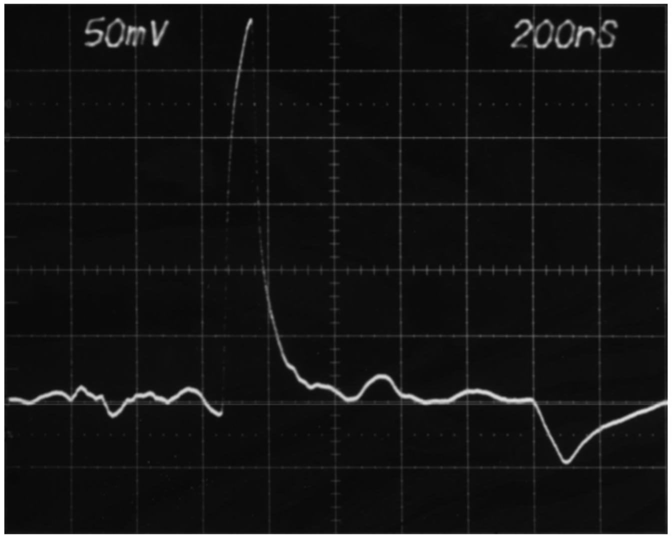
$$Z_o = \sqrt{\frac{\left(R(\omega) + j\omega L\right)}{\left(G + j\omega C\right)}} \qquad \gamma = \alpha + j\beta = \sqrt{\left(R(\omega) + j\omega L\right)\left(G + j\omega C\right)}$$

Pulse Propagation (CAT-5)

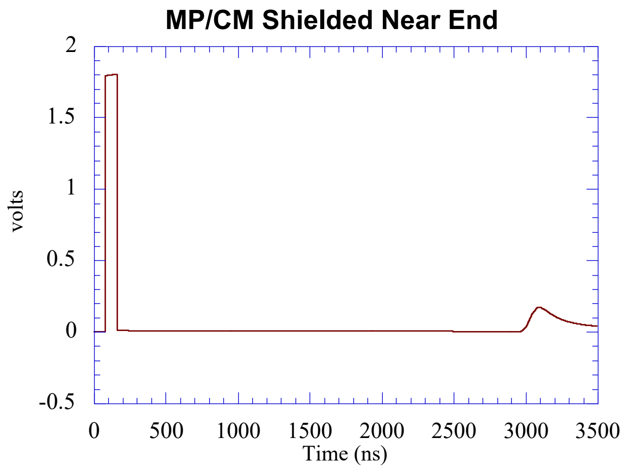


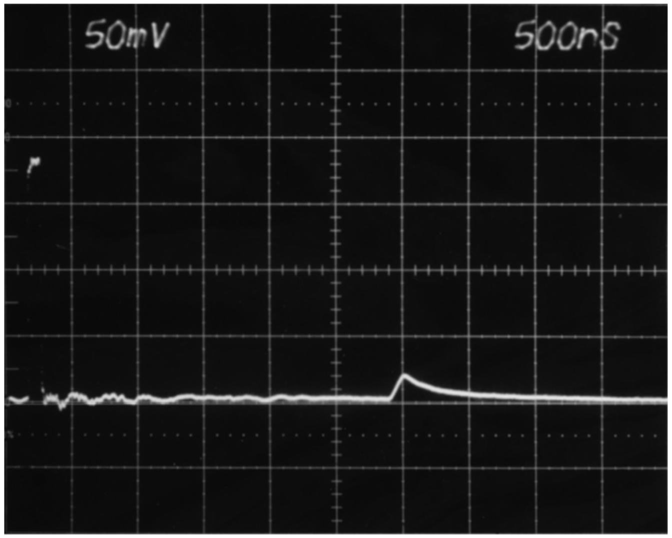


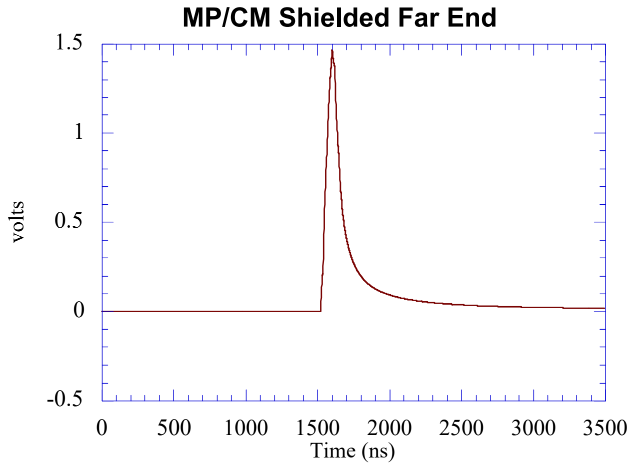


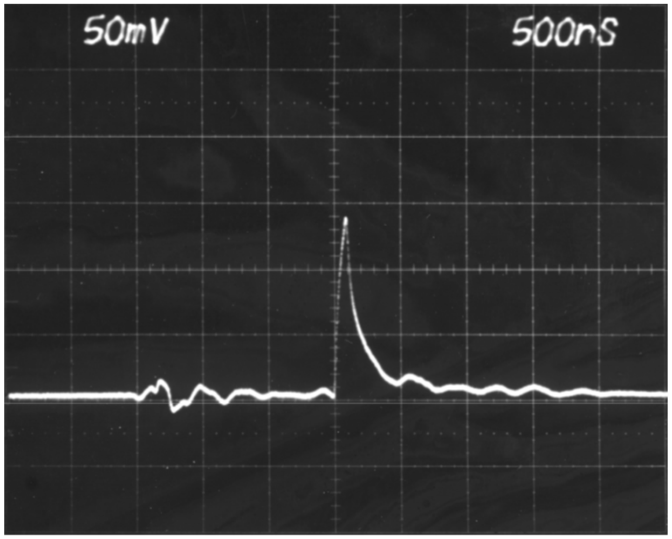


Pulse Propagation (MP/CM)

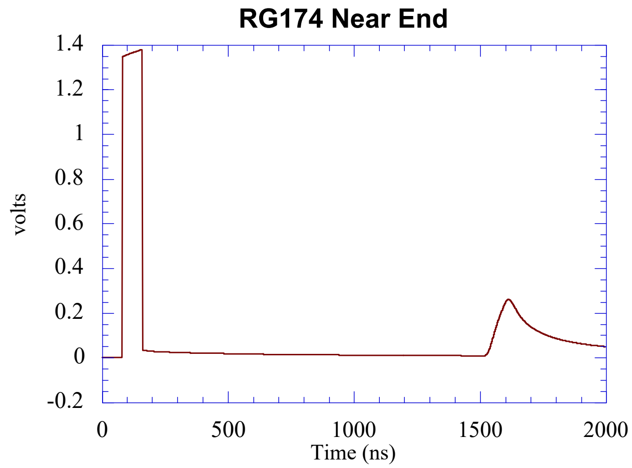


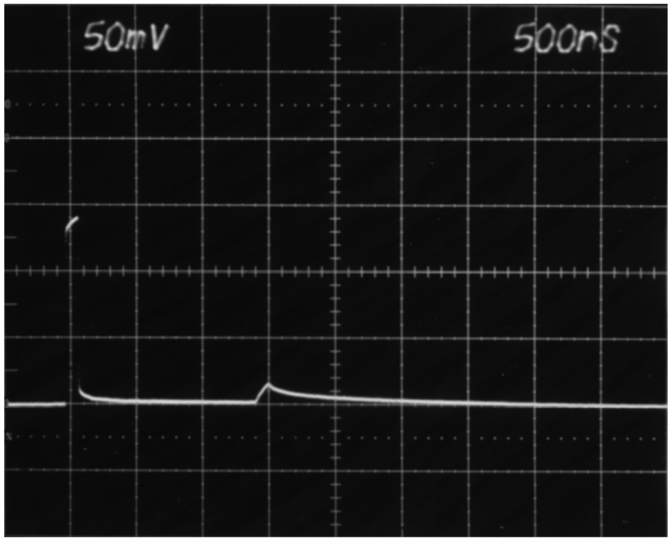


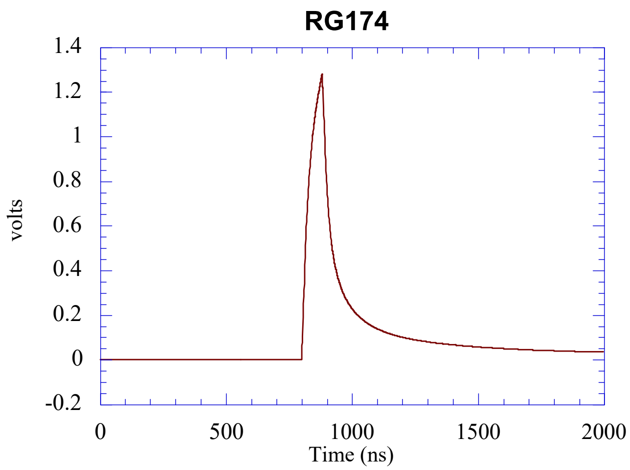


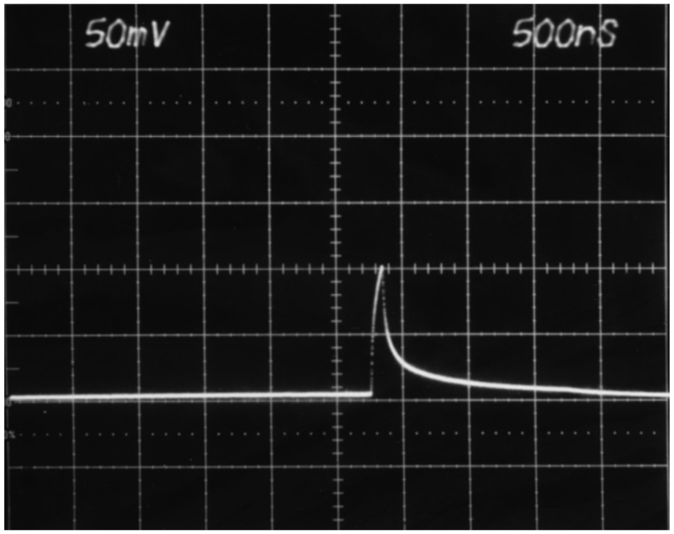


Pulse Propagation (RG174)

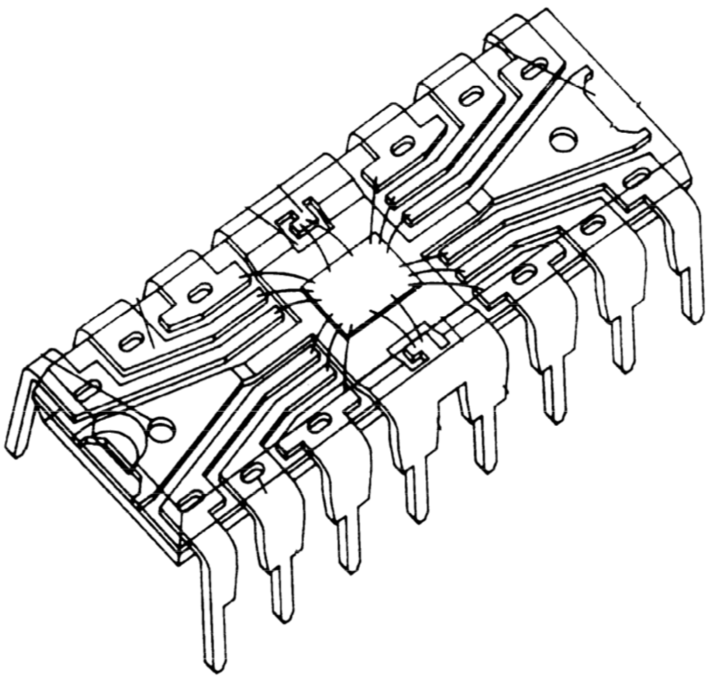






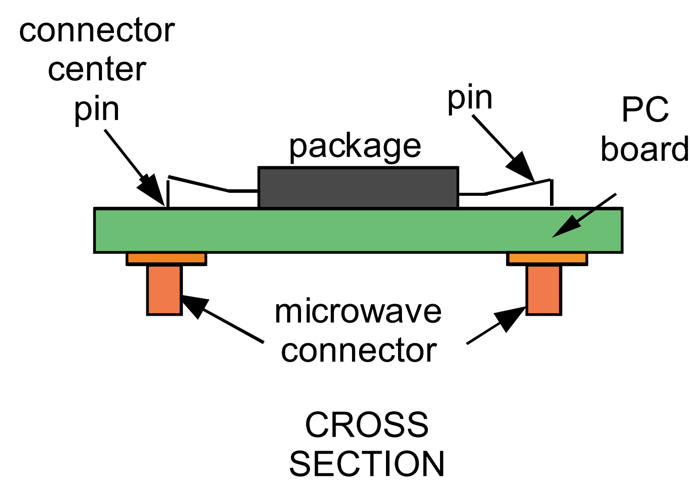


Characterization of DIP Packages



View of DIP Lead Frame

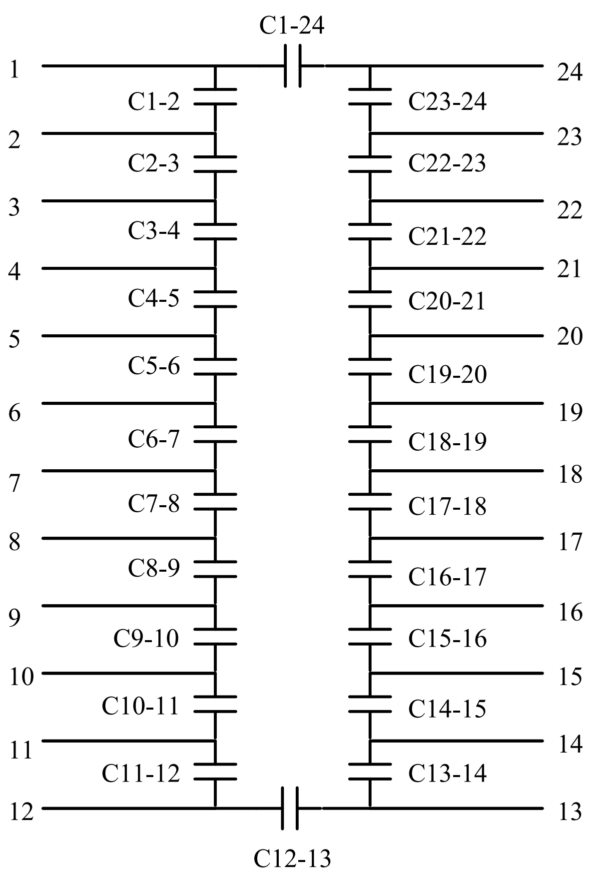
DIP Mounted on PCB with SMA

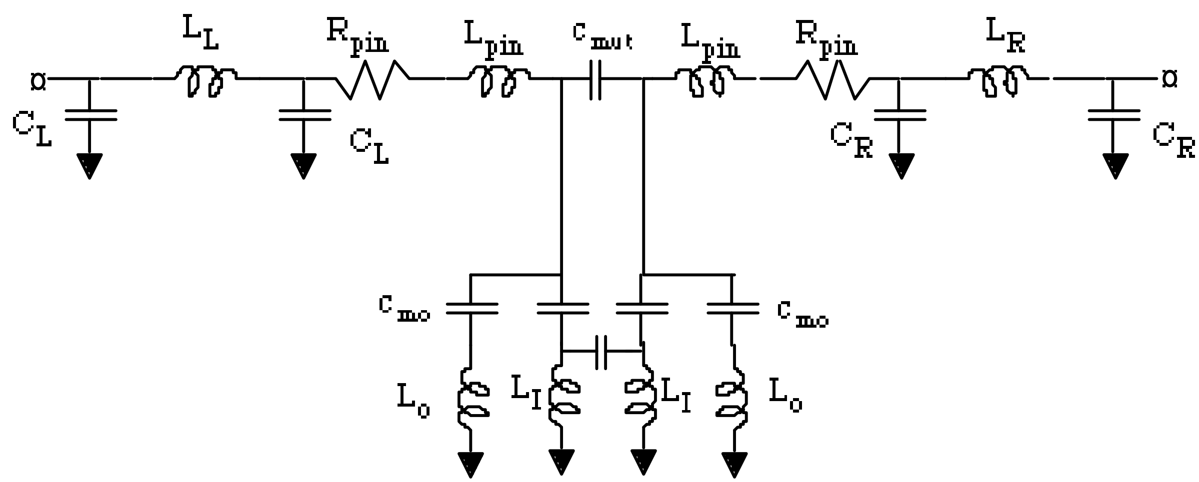


Package Characterization Procedure

- **Devise Model for Package + Environment**
  – Obtain accurate model for topology
  – Determine frequency range for model
- **Calibrate ANA and Perform Measurements**
  – De-embed connector or use TRL
  – S Parameters can be converted
- **Optimize Model using Simulator**
  – ADS or SPICE
  – Select accurate optimization scheme

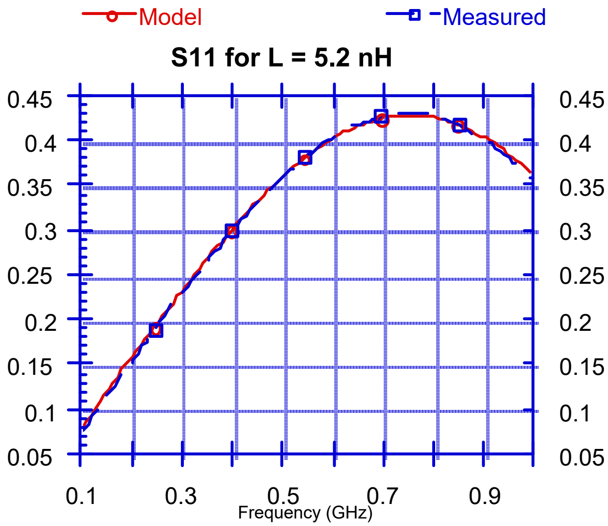
Example – Topology & Model

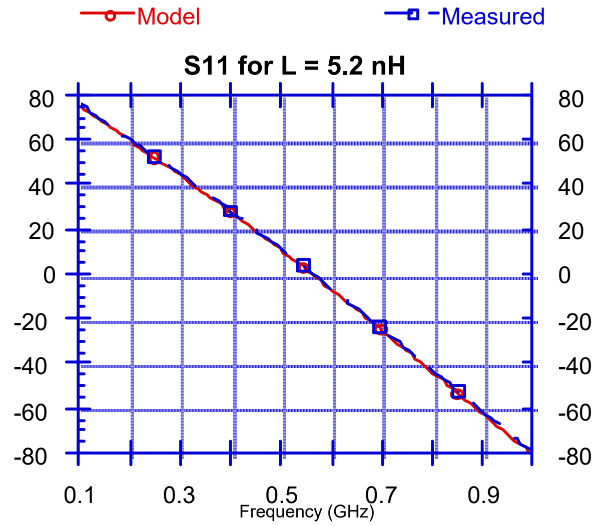


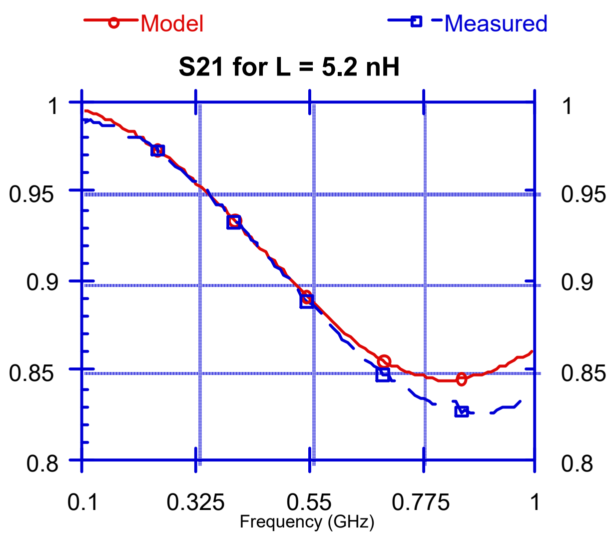


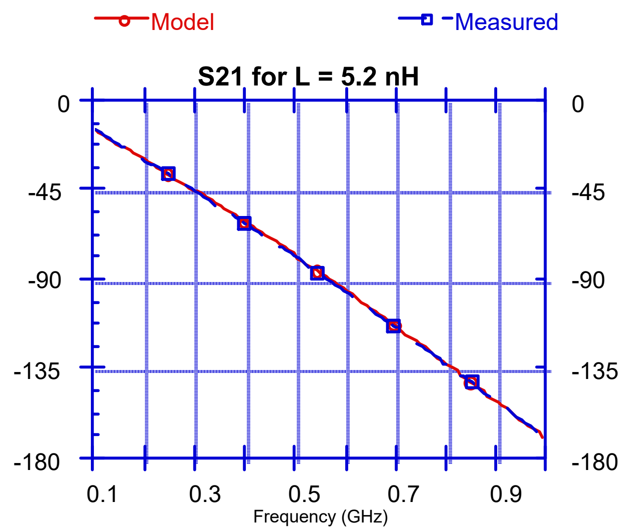
Circuit model for 2-port measurement Between 2 pins

DIP Example – S-Parameters



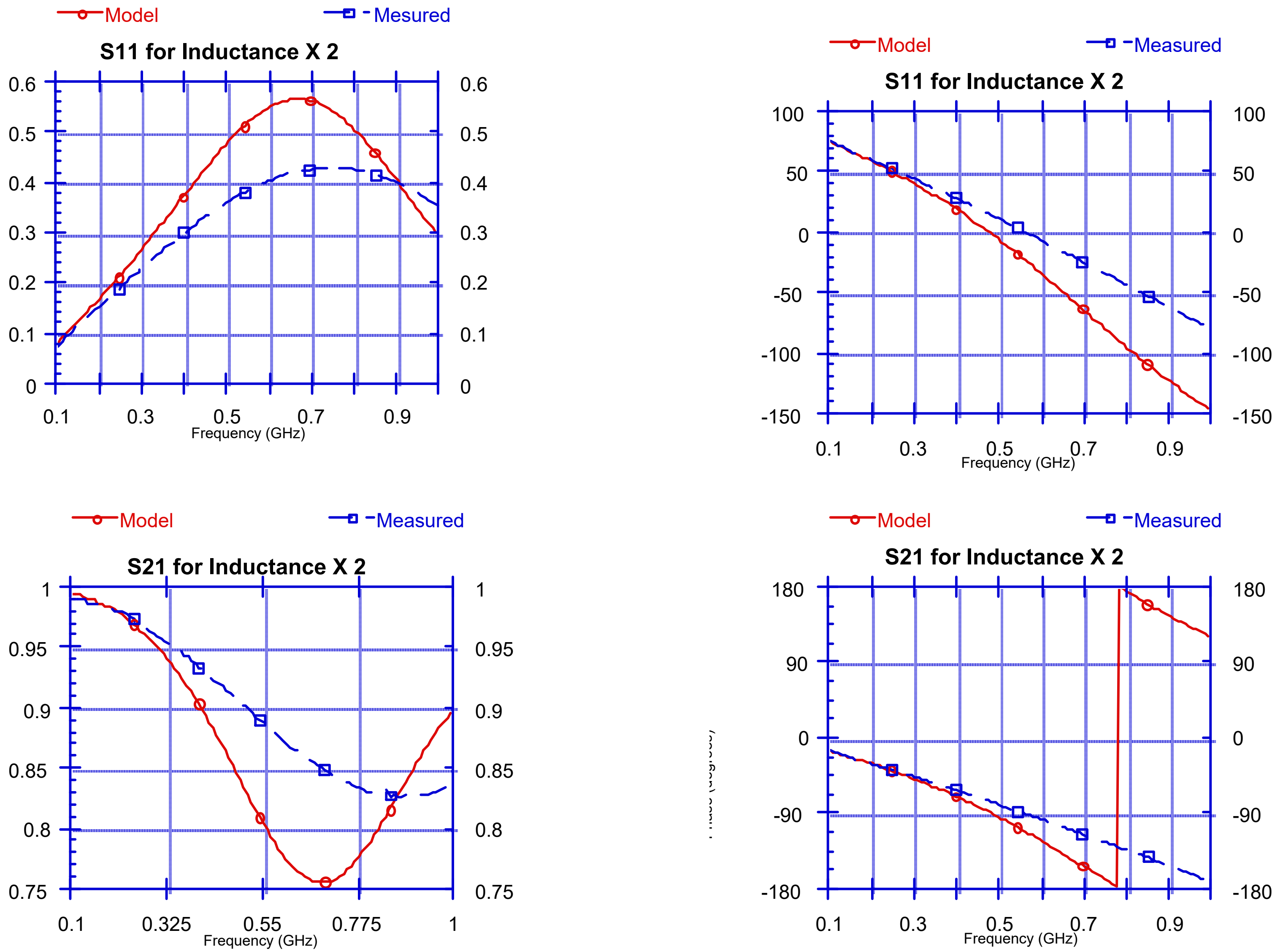






Model and Measured scattering parameters for pins 12-13 for the case were model inductance is assumed to be 5.2 nH.

DIP Example – Inductance



Model and Measured scattering parameters for pins 12-13 for the case were model inductance is assumed to be 10.4 nH.