ECE 451
Advanced Microwave Measurements

Error Correction

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Network Analyzer

Source provides RF/microwave signal and consists of high-frequency circuitry with internal impedance of 50 Ω.

Test set consists of couplers used to separate signals. There are also power dividers, switches all of which must operate at the RF/microwave frequency of interest.

Signals REF, A, & B are routed to analyzer which down-converts RF signals to intermediate frequency.

Signals are then amplified using low-frequency amplifiers and detected using low-frequency detectors.

Display shows ratios B/REF or A/REF for $S_{11}$, $S_{21}$ in magnitude and/or phase format
Directional Coupler

- Wave incident in port 1 couples into ports 2 & 3 but NOT in port 4 \(\Rightarrow\) ports 1 & 4 are uncoupled
- Wave incident in port 2 couples into ports 1 & 4 but NOT in port 3 \(\Rightarrow\) ports 2 & 3 are uncoupled
- In addition, all 4 ports are matched. That is if 3 ports are terminated with \(Z_o\), the fourth port appears terminated with \(Z_o\).
Characteristics of Directional Couplers

**Coupling**

Let $P_i$ be the incident power in port 1  
Let $P_f$ be the coupled power in port 3  
Define coupling in decibels (dB) as:

$$C = 10 \log \frac{P_i}{P_f}$$

**Directivity**

Ideally, power coupled into port 4, $P_b$ should be zero, but in reality it is not which defines the directivity of the coupler

$$D = 10 \log \frac{P_f}{P_b}$$

Ideally, directivity should be infinite.
S-Parameters of Directional Couplers

Directional coupler is described by a 4 by 4 S-parameter matrix. It is obvious that \( S_{14} = S_{23} = 0 \) \( \Rightarrow \) \( S_{41} = S_{32} = 0 \). Moreover, \( S_{11} = S_{22} = S_{33} = S_{44} = 0 \)

\[
S = \begin{bmatrix}
0 & S_{12} & S_{13} & 0 \\
S_{21} & 0 & 0 & S_{24} \\
S_{31} & 0 & 0 & S_{34} \\
0 & S_{42} & S_{43} & 0
\end{bmatrix}
\]

By reciprocity, \( S_{12} = S_{21}, \ S_{13} = S_{31}, \ S_{42} = S_{24} \) and \( S_{43} = S_{34} \)
Design of Directional Coupler

- Incident wave in port 1 has value 1
- Forward wave coupled in second guide at first hole has value $B_f$.
- Backward wave coupled in second guide at first hole has value $B_b$.
- $B_b$ and $B_f$ are the aperture coupling coefficients
Design of Directional Coupler

Aperture Coupling

- At second aperture, the field is approximately of the same magnitude but the phase has changed.
- Forward: $B_f e^{-j\beta d}$ --- Backward: $B_b e^{-j\beta d}$
- Total forward wave in upper guide at plane bb is $2B_f e^{-j\beta d}$
- Total backward wave in upper guide at plane aa is given by $B_b(1+e^{-j2\beta d})$
Design of Directional Coupler

Results

- Forward waves add in phase (same path length)
- Backward waves add out of phase if \( d = \lambda/4 \) or if \( d \) is an integer multiple of \( \lambda/4 \).

**Coupling:** \[ C = -20 \log 2|B_f| \]

**Directivity:** \[ D = 20 \log \frac{2|B_f|}{|B_b||1 + e^{-2j\beta d}|} = 20 \log \frac{2|B_f|}{|B_b||\cos \beta d|} \]

\[ D = 20 \log \frac{|B_f|}{|B_b|} + 20 \log |\sec \beta d| \]
Flow Graph for Directional Coupler

Assume load is matched $\Gamma_L = 0$

First order loops:

$\Gamma_g S_{11}, \Gamma_g S_{31} \Gamma_d S_{13}, \Gamma_d S_{33}$

Second order loop:

$\Gamma_g S_{11} \Gamma_d S_{33}$

$\Delta = 1 - \Gamma_g S_{11} - \Gamma_d S_{33} - \Gamma_g S_{31} \Gamma_d S_{13} + \Gamma_g S_{11} \Gamma_d S_{33}$
8510C Network Analyzer
Two-Port Measurement

Forward

\[ S_{11} = \frac{b_1}{a_1} \bigg|_{a_2=0} \]

\[ S_{21} = \frac{b_2}{a_1} \bigg|_{a_2=0} \]
Two-Port Measurement

Reverse

\[ S_{12} = \frac{b_1}{a_2} \bigg|_{a_1=0} \quad S_{22} = \frac{b_2}{a_2} \bigg|_{a_1=0} \]
Two-Port Measurement

In general, the measured $S_{11}$, $S_{12}$, $S_{21}$ and $S_{22}$ are not the parameters of the actual DUT.

Need to remove the effects of $X_1$ and $X_2$.

Calibration
One-Port Measurement

The system of cables, couplers, etc… represents a 2-port and must be de-embedded in order to obtain the actual $S_{11}$ of the unknown $S_{11a}$.
One-Port Measurement

Assume that the network analyzer is perfectly matched. Then,

\[ S_{11m} = \frac{b_1}{a_1} = S_{11c} + \frac{S_{12c}S_{21c}S_{11a}}{1 - S_{22c}S_{11a}} \]

All quantities are complex and frequency-dependent!
One-Term Error Model

If we assume that the cables and couplers are a perfect 50-Ω system, then $S_{11c}=S_{22c}=0$. We have a one-term error model.

$$S_{11m} = S_{12c} S_{21c} S_{11a} = T S_{11a}$$

where $T = S_{12c} S_{21c}$

$T$ is not known. To determine $T$, we first measure a short since for a short, $S_{11a}^{(\text{short})} = -1$

We get $S_{11m}^{(\text{short})} = T S_{11a}^{(\text{short})} = -T$

known
One-Term Error Model

Once \( T \) is known, we can then measure the DUT

\[
S_{11m}^{(DUT)} = TS_{11a}^{(DUT)}
\]

From which

\[
S_{11a}^{(DUT)} = \frac{S_{11m}^{(DUT)}}{T} = \frac{-S_{11m}^{(DUT)}}{S_{11m}^{(short)}}
\]

**Practical Observations:**

Since the correction involves a simple complex division, we can do the following

\[
\left| S_{11a}^{(DUT)} \right| = \frac{S_{11m}^{(DUT)}}{S_{11m}^{(short)}} \quad \text{and} \quad \angle S_{11a}^{(DUT)} = \angle S_{11m}^{(DUT)} - \angle S_{11m}^{(short)} \pm 180^\circ
\]
Three-Term Error Model

Network Analyzer + Source

Cables + Couplers + ...

DUT

$E_{DF} [S_{11c}]$

$E_{SF} [S_{22c}]$

$E_{RF} [S_{12c}]$

$b_s \rightarrow a_1 \rightarrow b_1 \rightarrow b_2 \rightarrow a_2 \rightarrow S_{11a}$
Three-Term Error Model

- **E\(_{DF}\):** Directivity of couplers (leak through test ports)
- **E\(_{RF}\):** reflection tracking error (signal path tracking error)
- **E\(_{SF}\):** Source match error

\[
S_{11m} = \Gamma_{in} = S_{11c} + \frac{S_{12c} S_{21c} S_{11a}}{1 - S_{22c} S_{11a}} \quad \text{or} \quad S_{11m} = E_{DF} + \frac{E_{RF} S_{11a}}{1 - E_{SF} S_{11a}}
\]
Three-Term Error Model

**Step 1:**
Use matched load \((Z_o = 50 \, \Omega)\) as DUT \(\Rightarrow S_{11a} = 0\)

\[
S_{11m}^{(\text{load})} = E_{DF} = A
\]  

(1)

**Step 2:**
Use a perfect short as DUT \(\Rightarrow S_{11a} = -1\)

\[
S_{11m}^{(\text{short})} = E_{DF} - \frac{E_{RF}}{1 + E_{SF}} = B
\]  

(2)

**Step 3:**
Use a perfect open as DUT \(\Rightarrow S_{11a} = +1\)

\[
S_{11m}^{(\text{open})} = E_{DF} + \frac{E_{RF}}{1 - E_{SF}} = C
\]  

(3)
Three-Term Error Model

Combining (1), (2), and (3) gives $E_{SF}$, $E_{RF}$ and $E_{DF}$

$$E_{DF} = S_{11m}^{(\text{load})} = A$$

$$E_{SF} = \frac{B + C - 2A}{C - B} = \frac{S_{11m}^{(\text{short})} + S_{11m}^{(\text{open})} - 2S_{11m}^{(\text{load})}}{S_{11m}^{(\text{open})} - S_{11m}^{(\text{short})}}$$

$$E_{RF} = \frac{-2(B - A)(C - A)}{C - B} = \frac{-2(S_{11m}^{(\text{short})} - S_{11m}^{(\text{load})})(S_{11m}^{(\text{open})} - S_{11m}^{(\text{load})})}{S_{11m}^{(\text{open})} - S_{11m}^{(\text{short})}}$$

**Step 4:** Measuring the unknown DUT

This is the actual $S_{11}$ with corrections.

$$S_{11a}^{(\text{DUT})} = \frac{S_{11m}^{(\text{DUT})} - E_{DF}}{E_{RF} + E_{SF}\left[S_{11m}^{(\text{DUT})} - E_{DF}\right]}$$
Alternative Calibration Standards

At very high frequencies, it is difficult to make a good short, open or matched termination. We need to find alternative standards for calibration.

Offset Short

TL of length \( l \) terminated with a short
Offset Short Standard

At $z=0$, $\Gamma = \Gamma_L$

At $z=-l$, $\Gamma (-l) = \Gamma_L e^{-2j\beta l}$

Since $\Gamma_L = -1$, $\Gamma (-l) = \Gamma_{in} = -e^{-2j\beta l} = e^{j\left(\frac{\pi - 4\pi l}{\lambda}\right)}$

Therefore, when calibrating with an offset short, we use: $S^{(offset \ short)}_{11a} = e^{j\theta}$

where $\theta$ is known:
Offset Short Restriction

The offset short will only work if the frequency range is such that

$$0 < l < \lambda / 2$$

This corresponds to a frequency range of

$$f < \frac{\nu}{2l}$$

where $\nu$ is the propagation velocity in the line.
Shielded Open Standard

The shielded open can be modeled as a controlled capacitor.

For a system with reference impedance of $Z_o$, the associated reflection coefficient is:

$$\Gamma_{in} = \frac{1/j\omega C - Z_o}{1/j\omega C + Z_o} = \frac{1 - j\omega CZ_o}{1 + j\omega CZ_o} = \frac{1 - ja}{1 + ja}$$

with $a = \omega CZ_o$

$$\Gamma_{in} = e^{-2j\tan^{-1}a}$$

So, for shielded open, we use $S_{11a}^{(shielded open)} = e^{-2j\tan^{-1}a}$
Motivation: Need to accurately measure the actual directivity error of the system.

Observation: If termination is imperfect, then the measured directivity is the vector sum of the actual directivity and the reflection coefficient of the load.
Sliding Load

With the sliding load, a small $\Gamma$ is willfully introduced and varied in terms of its phase.

By sliding the load at a given frequency point, a circle is defined about the tip of the directivity vector.

We find the best circle that fits the measured $S_{11}$. The center of that circle is the tip of the actual (desired) directivity vector.
Alternate Combinations

- Matched Load
- Offset Short
- Short

- Matched Load
- Short
- Shielded open

- Sliding Load
- Offset Short
- Short

- Sliding Load
- Short
- Shielded open

- Sliding Load
- Offset Short
- Short

- Sliding Load
- Offset Short
- Shielded open
Calibration consists of determining the $i$ and $o$ terms by placing known standards as the $a$ terms.
8-Term Error Model

Calibration Stage: Reflection/Port 1

Placing open, short and load as standards \((S_{11a})\) in port 1 yields

\[
S_{11m}^{(op)} = S_{11i} + \frac{S_{21i}S_{12i}}{e^{j\beta} - S_{21i}}
\]

\[
S_{11m}^{(sh)} = S_{11i} - \frac{S_{21i}S_{12i}}{1 + S_{21i}}
\]

\[
S_{11m}^{(ld)} = S_{11i}
\]
8-Term Error Model

Calibration Stage: Reflection/Port 1

The system is simultaneously solved to give

\[ S_{11i} = S_{11m} \]

\[ S_{22i} = e^{j\beta} \left[ \frac{S_{11m}(\text{op}) - S_{11m}(\text{ld})}{S_{11m}(\text{op}) - S_{11m}(\text{sh})} \right] \left[ \frac{S_{11m}(\text{ld}) - S_{11m}(\text{sh})}{S_{11m}(\text{op}) - S_{11m}(\text{sh})} \right] \]

\[ S_{12i}S_{21i} = \left( 1 + e^{j\beta} \right) \left[ \frac{S_{11m}(\text{op}) - S_{11m}(\text{ld})}{S_{11m}(\text{sh}) - S_{11m}(\text{op})} \right] \left[ \frac{S_{11m}(\text{ld}) - S_{11m}(\text{sh})}{S_{11m}(\text{sh}) - S_{11m}(\text{op})} \right] \]
8-Term Error Model

Calibration Stage: Reflection/Port 2

Same principle can be applied to port 2 to give

\[ S_{11o} = S_{22m}^{(ld)} \]

\[ S_{22o} = \frac{e^{j\beta} \left[ S_{22m}^{(op)} - S_{22m}^{(ld)} \right] - \left[ S_{22m}^{(ld)} - S_{22m}^{(sh)} \right]}{S_{22m}^{(op)} - S_{22m}^{(sh)}} \]

\[ S_{12o} S_{21o} = \frac{(1 + e^{j\beta}) \left[ S_{22m}^{(op)} - S_{22m}^{(ld)} \right] \left[ S_{22m}^{(sh)} - S_{22m}^{(ld)} \right]}{S_{22m}^{(sh)} - S_{22m}^{(op)}} \]
8-Term Error Model

Calibration Stage: Transmission

Next, connect the two ports together for transmission calibration $S_{21a} = S_{12a} = 1$

$$S_{21m}^{(thr)} = \frac{S_{21i} S_{12o}}{1 - S_{22i} S_{22o}}$$

$$S_{12m}^{(thr)} = \frac{S_{12i} S_{21o}}{1 - S_{22i} S_{22o}}$$
8-Term Error Model

Measurement Stage

Insert unknown and provide unit reference signal at input

\[ a_1 = S_{21i} + S_{22i}a_4 \]
\[ a_3 = S_{22o}a_2 \]
\[ S_{11m} = S_{11i} + S_{12i}a_4 \]
\[ S_{21m} = S_{21o}a_2 \]
\[ a_4 = S_{11a}a_1 + S_{12a}a_3 \]
\[ a_2 = S_{21a}a_1 + S_{22a}a_3 \]
8-Term Error Model

Measurement Stage

\[ a_1 = S_{21i} + S_{22i} \left( \frac{S_{11m} - S_{11i}}{S_{21i}} \right) \]

Solve for the \( a \)'s

\[ a_2 = \frac{S_{21m}}{S_{12o}} \]

\[ a_3 = \frac{S_{22o} S_{21m}}{S_{12o}} \]

\[ a_4 = \frac{S_{11m} S_{11i}}{S_{12i}} \]
8-Term Error Model

Measurement Stage

Insert unknown and provide unit reference signal at port 2

\[ b_1 = S_{22i} b_4 \]

\[ b_3 = S_{22o} b_2 + S_{12o} \]

\[ S_{22m} = S_{11o} + S_{12o} b_2 \]

\[ S_{12m} = S_{12i} b_4 \]

\[ b_4 = S_{11a} b_1 + S_{12a} b_3 \]

\[ b_2 = S_{21a} b_1 + S_{22a} b_3 \]
8-Term Error Model

Measurement Stage

Solve for the $b$’s

$$b_1 = \frac{S_{22i}S_{12m}}{S_{12i}}$$

$$b_2 = \frac{S_{22m} - S_{11o}}{S_{12o}}$$

$$b_3 = S_{22o} \left[ \frac{S_{22m} - S_{11o}}{S_{12o}} \right] + S_{21o}$$

$$b_4 = \frac{S_{21m}}{S_{12i}}$$
8-Term Error Model - Solution

\[
S_{11a} = \frac{S_{11m} - S_{11i}}{S_{21i} S_{12i}} \left[ 1 + \frac{S_{22o} \left( S_{22m} - S_{11o} \right)}{S_{12o} S_{21o}} \right] - \frac{S_{22o} S_{21m}}{S_{21i} S_{12o}} \times \frac{S_{12m}}{S_{21o} S_{12i}} \frac{D}{D}
\]

\[
S_{12a} = \frac{S_{12m}}{S_{21o} S_{12i}} \frac{D}{D}
\]

\[
S_{21a} = \frac{S_{21m}}{S_{21i} S_{12o}} \frac{D}{D}
\]

\[
S_{22a} = \frac{S_{22m} - S_{11o}}{S_{21o} S_{12o}} \left[ 1 + \frac{S_{22i} \left( S_{11m} - S_{11i} \right)}{S_{21i} S_{12i}} \right] - \frac{S_{22i} S_{21m}}{S_{21i} S_{12o}} \times \frac{S_{12m}}{S_{21o} S_{12i}} \frac{D}{D}
\]
8-Term Error Model - Solution

\[
D = \left[1 + \frac{S_{22i} \left( S_{11m} - S_{11i} \right)}{S_{21i} S_{12o}} \right] \left[1 + \frac{S_{22o} \left( S_{22m} - S_{11o} \right)}{S_{12o} S_{21o}} \right] - S_{22i} S_{22o} \frac{S_{21m}}{S_{21i} S_{12o}} \frac{S_{12m}}{S_{21o} S_{12i}}
\]

Reference

12-Term Error Model

Forward Mode

Reflection terms
- ESF: Open
- EDF: Short
- ERF: Load

Transmission terms
- ELF: Thru
- ETF: Measure $S_{11m}$ and $S_{21m}$

Isolation term
- EXF: Loads
- Measure $S_{21m}$
12-Term Error Model

Reverse Mode

Reflection terms
- ESR: Open
- EDR: Short
- ERR: Load

Transmission terms
- ELR: Thru
- ETR: Measure $S_{22m}$ and $S_{12m}$

Isolation term
- EXR: Loads
  Measure $S_{12m}$