ECE 451
Advanced Microwave Measurements

Flow Graphs

Jose E. Schutt-Aine
Electrical & Computer Engineering
University of Illinois
jschutt@emlab.uiuc.edu
Flow Graph Definitions

- Voltage waves designated as nodes.
- $S$ parameters designated as branches
- Branches enter dependent nodes and emanate from independent nodes

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$
Two-Port Flow Graph
Flow Graph for Source

\[ b_s = \frac{V_s \sqrt{Z_o}}{Z_s + Z_o} \]

\[ \Gamma_s = \frac{Z_s - Z_o}{Z_s + Z_o} \]

\( b_s \) is power wave associated with power dissipated in a load of value \( Z_o \) connected to the source.
Flow Graph for Load

\[ \Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} \]
Flow Graph for Composite Circuit
Flow Graph of Complete Two-Port
Loop Definitions

- A first order loop is defined as the product of the branches encountered in a journey starting from anode and moving in the direction of the arrows back to that original node.

- A second order loop is defined as the product of any two non-touching first order loops.

- A third order loop is defined as the product of any three non-touching first order loops.
Mason’s Non-Touching Loop Rule

\[ T = \frac{P_1 \left[ 1 - \sum L(1)^{(1)} + \sum L(2)^{(1)} - \ldots \right] + P_2 \left[ 1 - \sum L(1)^{(2)} + \ldots \right] + \ldots}{1 - \sum L(1) + \sum L(2) - \sum L(3) + \ldots} \]

- \( T \): ratio of dependent variable over independent variable
- \( P_k 's \): are the various paths connecting the two variables of interest
- \( L(j)^{(k)} \) is a loop of order \( j \) that does not touch path \( k \)
Example: Find $b_2/b_s$

First Order Loops: $S_{11}\Gamma_s$, $S_{22}\Gamma_L$, $S_{21}S_{12}\Gamma_L\Gamma_s$

Second Order Loops: $S_{11}\Gamma_s S_{22}\Gamma_L$

Paths: $S_{21}$

$$\frac{b_2}{b_s} = \frac{S_{21}}{1 - S_{11}\Gamma_s - S_{22}\Gamma_L - S_{21}S_{12}\Gamma_L\Gamma_s + S_{11}\Gamma_s S_{22}\Gamma_L}$$
Flow Graph for Directional Coupler

Assume load is matched $\Gamma_L = 0$

First order loops:

$$\Gamma_g S_{11}, \quad \Gamma_g S_{31} \Gamma_d S_{13}, \quad \Gamma_d S_{33}$$

Second order loop:

$$\Gamma_g S_{11} \Gamma_d S_{33}$$

$$\Delta = 1 - \Gamma_g S_{11} - \Gamma_d S_{33} - \Gamma_g S_{31} \Gamma_d S_{13} + \Gamma_g S_{11} \Gamma_d S_{33}$$