ECE 451
Advanced Microwave Measurements

Flow Graphs

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Flow Graph Definitions

- Voltage waves designated as nodes.
- S parameters designated as branches
- Branches enter dependent nodes and emanate from independent nodes

\[ b_1 = S_{11} a_1 + S_{12} a_2 \]
\[ b_2 = S_{21} a_1 + S_{22} a_2 \]
Two-Port Flow Graph
Flow Graph for Source

\[ b_s = \frac{V_S \sqrt{Z_o}}{Z_s + Z_o} \]

\[ \Gamma_s = \frac{Z_s - Z_o}{Z_s + Z_o} \]

\( b_s \) is power wave associated with power dissipated in a load of value \( Z_o \) connected to the source.
Flow Graph for Load

\[ \Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} \]
Flow Graph for Composite Circuit
Flow Graph of Complete Two-Port
Loop Definitions

- A first order loop is defined as the product of the branches encountered in a journey starting from anode and moving in the direction of the arrows back to that original node.

- A second order loop is defined as the product of any two non-touching first order loops.

- A third order loop is defined as the product of any three non-touching first order loops.
Mason’s Non-Touching Loop Rule

\[ T = \frac{P_1 \left[ 1 - \sum L(1)^{(1)} + \sum L(2)^{(1)} - \ldots \right] + P_2 \left[ 1 - \sum L(1)^{(2)} + \ldots \right] + \ldots}{1 - \sum L(1) + \sum L(2) - \sum L(3) + \ldots} \]

*T*: ratio of dependent variable over independent variable

*P_k*: are the various paths connecting the two variables of interest

*L(j)^{(k)}*: is a loop of order *j* that does not touch path *k*
Example: Find $b_2/b_s$

First Order Loops: $S_{11}\Gamma_S$, $S_{22}\Gamma_L$, $S_{21}S_{12}\Gamma_L\Gamma_S$

Second Order Loops: $S_{11}\Gamma_S S_{22}\Gamma_L$

Paths: $S_{21}$

\[
\frac{b_2}{b_s} = \frac{S_{21}}{1 - S_{11}\Gamma_S - S_{22}\Gamma_L - S_{21}S_{12}\Gamma_L\Gamma_S + S_{11}\Gamma_S S_{22}\Gamma_L}
\]