ECE 451
Advanced Microwave Measurements
Lossy Transmission Lines

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Loss in Transmission Lines

Signal amplitude decreases with distance from the source.
Skin Effect in Lines

Low Frequency

High Frequency

Very High Frequency
Skin Effect in Microstrip

Skin Effect in Microstrip

Current density varies as

$$J = J_0 e^{-y/\delta} e^{-jy/\delta}$$

Note that the phase of the current density varies as a function of $y$

$$I = \int_0^{\infty} J_0 w e^{-y/\delta} e^{-jy/\delta} dy = \frac{J_0 w \delta}{1 + j}$$

$$\sigma E_0 = J_0 \Rightarrow E_0 = \frac{J_0}{\sigma}$$

The voltage measured over a section of conductor of length $D$ is:

$$V = E_0 D = \frac{J_0 D}{\sigma}$$
Skin Effect in Microstrip

The skin effect impedance is

\[ Z_{\text{skin}} = \frac{V}{I} = \frac{J_0 D (1 + j)}{\sigma J_0 w \delta} = \frac{D}{w} (1 + j) \sqrt{\pi f \mu \rho} \]

where \( \rho = \frac{1}{\sigma} \) is the bulk resistivity of the conductor

\[ Z_{\text{skin}} = R_{\text{skin}} + jX_{\text{skin}} \]

with

\[ R_{\text{skin}} = X_{\text{skin}} = \frac{D}{w} \sqrt{\pi f \mu \sigma} \]

→ Skin effect has reactive (inductive) component
Lossy Transmission Line

Telegraphers Equation: Time Domain

\[- \frac{\partial V}{\partial z} = RI + L \frac{\partial I}{\partial t}\]

\[- \frac{\partial I}{\partial z} = GV + C \frac{\partial V}{\partial t}\]
Lossy Transmission Line

Telegraphers Equation: Frequency Domain

\[-\frac{\partial V}{\partial z} = (R + j\omega L)I = ZI\]

\[-\frac{\partial I}{\partial z} = (G + j\omega C)V = YV\]
Lossy Transmission Line

Telegraphers Equation: Frequency Domain

\[-\frac{\partial^2 V}{\partial z^2} = (R + j\omega L)(G + j\omega C)V = ZYV = \gamma^2 V\]

\[-\frac{\partial^2 I}{\partial z^2} = (G + j\omega C)(R + j\omega L)I = YZI = \gamma^2 I\]
Lossy Transmission Line

\[ V(z) = A e^{-\alpha z} e^{-j\beta z} + B e^{\alpha z} e^{+j\beta z} \]

\[ I(z) = \frac{1}{Z_0} \left[ A e^{-\alpha z} e^{-j\beta z} - B e^{\alpha z} e^{+j\beta z} \right] \]

\[ Z_0 = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} \]

\[ \gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \]
Lossy Transmission Line

R, L, G, C,
Effects of Losses

- Signal attenuation

- Dispersion  \( \gamma = \alpha(\omega) + j \beta(\omega) = \sqrt{(R + j\omega L)(G + j\omega C)} \)

- Rise time degradation
Special Case – Low Loss

If \( R \ll \omega L \)

and \( G \ll \omega C \)

\[
Z_o = \sqrt{\frac{j\omega L \left(1 + \frac{R}{j\omega L}\right)}{j\omega C \left(1 + \frac{G}{j\omega C}\right)}} \approx \sqrt{\frac{L}{C}}
\]

\[
\gamma = \alpha + j\beta
\]

\[
\alpha \approx \frac{1}{2} \left( R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}} \right)
\]

\[
\beta \approx \omega \sqrt{LC}
\]

\[
\nu_p = \frac{\omega}{\beta} \approx \frac{1}{\sqrt{LC}}
\]
RC Transmission Line

\[ Z_{\text{in}} = \frac{RL \ \text{coth} \left( \frac{RL}{\sqrt{2} \sqrt{\frac{C\omega}{R}}} \right)}{(1+j)} \]

For very high \( \omega \), \( \arg(Z_{\text{in}}) \approx 45^\circ \)

R : series resistance per unit length
C : shunt capacitance per unit length
RC Transmission Line

If \( \omega \ll \frac{2}{RCl^2} \) then

\[
Z_{in} = \frac{Rl}{2} + \frac{1}{jCl\omega} = \frac{R_T}{2} + \frac{1}{jC_T\omega}
\]

\( R_T = Rl \) : total resistance
\( C_T = Cl \) : total capacitance
RC Transmission Line

Pulse Characteristics:
- rise time: 100 ps
- fall time: 100 ps
- pulse width: 4 ns

Line Characteristics
- length: 3 mm
- near end termination: 50 Ω
- far end termination: 65 Ω

Near End Response
- Board
- VLSI
- Submicron
- Deep Submicron

Far End Response
- Board
- VLSI
- Submicron
- Deep Submicron

Logic threshold: 0.175
Long Cable

100m Category-5 Cable

![Graphs showing S11 Magnitude and Phase for Simulation and Measurement for Category 5/100-meter Cable.](image)
Short Cable

1m Category-5 Cable

**Category 5/1-meter Simulation vs. Measurement**

- **S11 Magnitude**
- **S11 Phase (deg)**
- **S21 Magnitude**
- **S21 Phase (deg)**

Frequency (GHz) vs. Magnitude and Phase for Simulation and Measurement.
Category 5 Cable

Resistance and velocity

Category 5/100-meter

Resistance (Ohms/m)

0 0.02 0.04 0.06 0.08 0.1

Frequency (GHz)

Category 5/100-meter

Velocity Ratio

0 0.02 0.04 0.06 0.08 0.1

Frequency (GHz)
Cable Loss Model

\[ R(f) = R_s \cdot f^p \]

\[ v_r = v_{ro} + v_{rs} \cdot f \]

\[ Z = R(f) + j\omega L = R_{\text{skin}} + j(R_{\text{skin}} + \omega L) \]

<table>
<thead>
<tr>
<th></th>
<th>( Z_0 )</th>
<th>( v_{ro} )</th>
<th>( v_{rs} )</th>
<th>( R_s )</th>
<th>( p )</th>
<th>( f_{\text{max}} )</th>
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<td>0.113</td>
<td>7.94</td>
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Lossy TL Simulation

- To simulate lossy TL with resistive loads
  - No closed form solution
  - Simplest method is to use IFFT

\[
\begin{align*}
  v(t, z) &= \text{IFFT}\{A e^{-\alpha z} e^{-j\beta z} + B e^{+\alpha z} e^{+j\beta z}\} \\
  i(t, z) &= \text{IFFT}\left\{\frac{1}{Z_o}\left[A e^{-\alpha z} e^{-j\beta z} + A e^{+\alpha z} e^{+j\beta z}\right]\right\} \\
  Z_o &= \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} \\
  \gamma &= \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \\
  T &= \frac{Z_o}{Z_1 + Z_o}
\end{align*}
\]

\[
\begin{align*}
  A &= \frac{TV_s(\omega)}{1 - \Gamma_1 \Gamma_2 e^{-2\gamma l}} \\
  B &= \Gamma_2 e^{-2\gamma l} A \\
  \Gamma_2 &= \frac{Z_2 - Z_o}{Z_2 + Z_o} \\
  \Gamma_1 &= \frac{Z_1 - Z_o}{Z_1 + Z_o}
\end{align*}
\]
Time-Domain Simulations

\[ Z_S = 50 \, \Omega \]

\[ V_S \]

cable

near end

far end

open
Pulse Propagation (CAT-5)
Pulse Propagation (MP/CM)
Pulse Propagation (RG174)