ECE 451
Macromodeling

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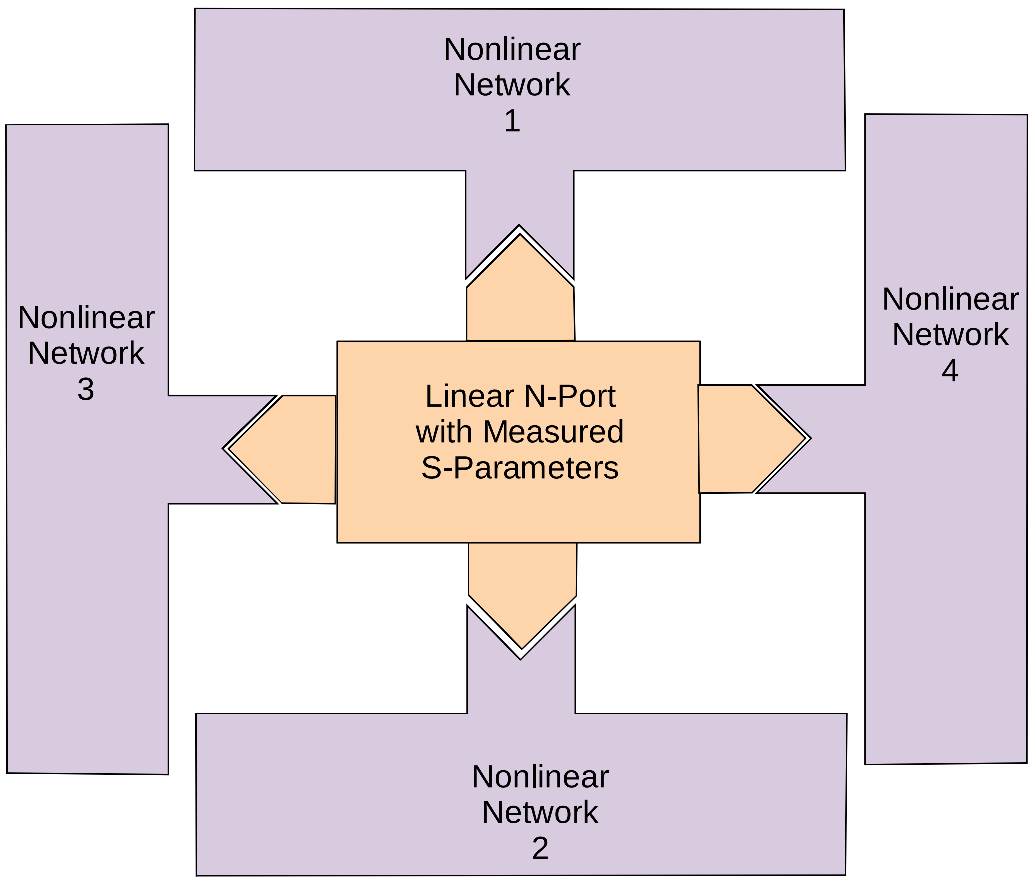
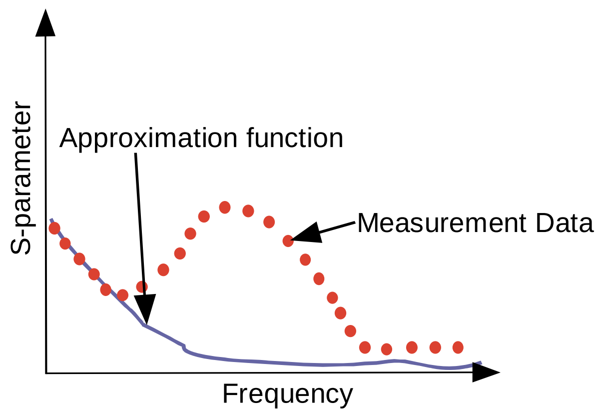
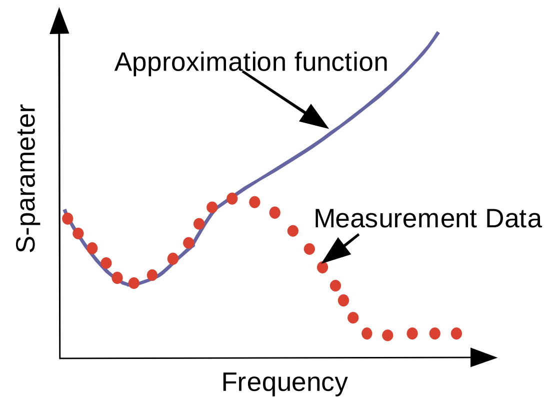
Blackbox Macromodeling

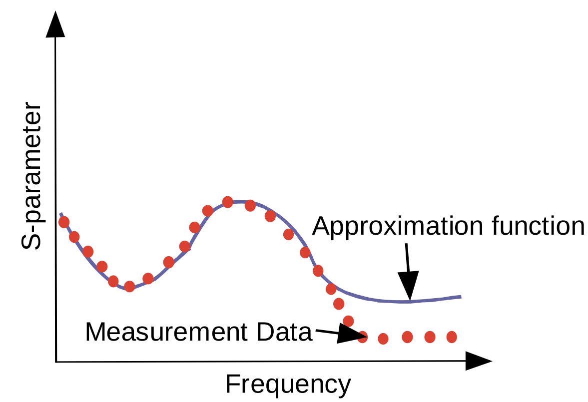


Objective: Perform time-domain simulation of composite network to determine timing waveforms, noise response or eye diagrams

Orders of Approximation



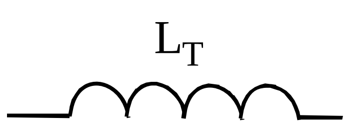


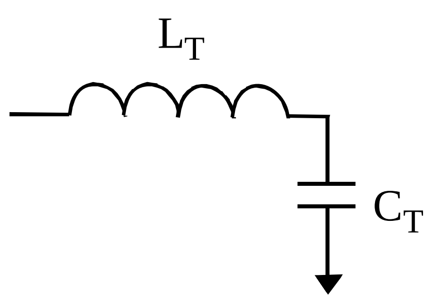


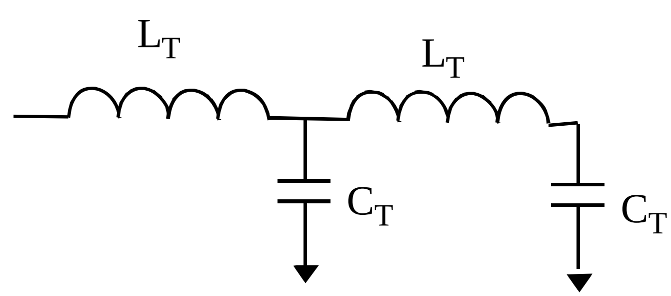
Low order

Medium order

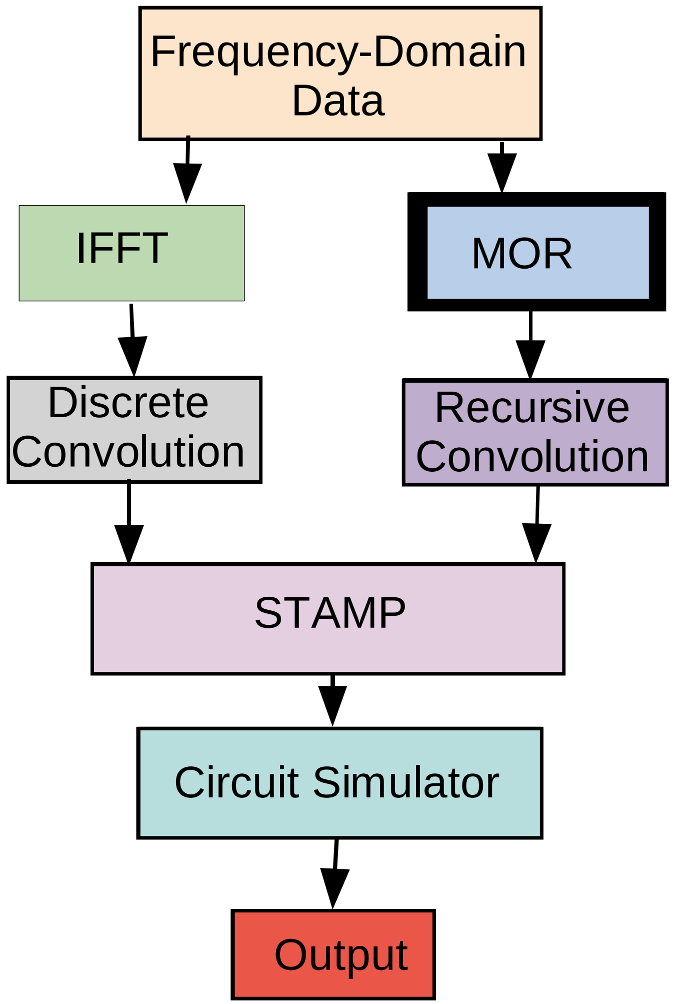
Higher order



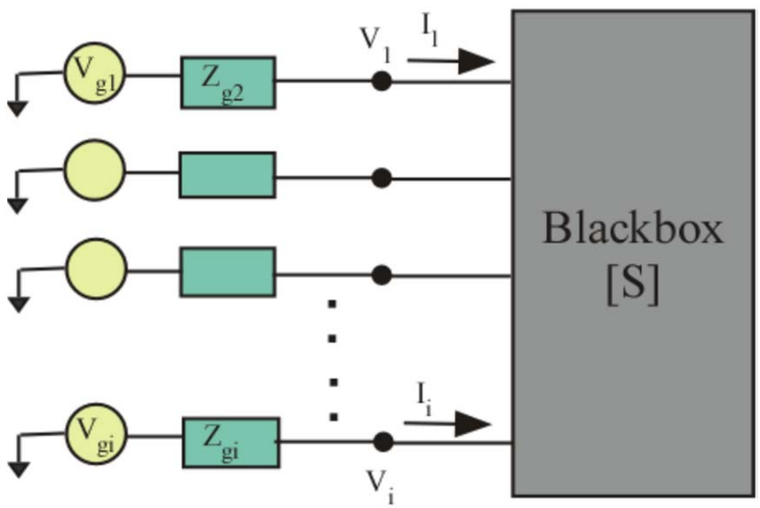




Macromodel Implementation



Blackbox Macromodeling



Terminations are described by a source vector $V_g(\omega)$ and an impedance matrix Z

Blackbox is described by its scattering parameter matrix S

In frequency domain $B=SA$

In time domain $b(t) = s(t)*a(t)$

Convolution: $$s(t)*a(t)=\int_{-\infty}^{\infty} s(t-\tau)a(\tau)d\tau$$

Discrete Convolution

When time is discretized the convolution becomes

$$s(t)*a(t)=\sum_{\tau=1}^{t} s(t-\tau)a(\tau)\Delta\tau$$

Isolating $a(t)$

$$s(t)*a(t)=s(0)a(t)\Delta\tau+\sum_{\tau=1}^{t-1} s(t-\tau)a(\tau)\Delta\tau$$

Since $a(t)$ **is known for** $t < t$**, we have:**

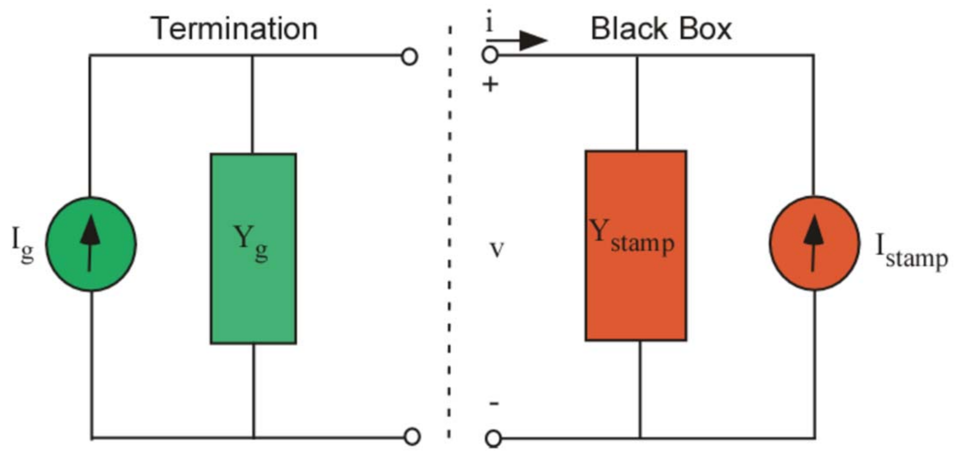
$$H(t)=\sum_{\tau=1}^{t-1} s(t-\tau)a(\tau)\Delta\tau \quad : \quad History$$

Stamp Equations

$$i(t) = Y_{stamp} v(t) - I_{stamp}$$

$$Y_{stamp} = Z_o^{-1}\left[1 + s'(0)\right]^{-1}\left[1 - s'(0)\right]$$

$$I_{stamp} = 2Z_o^{-1}\left[1 + s'(0)\right]^{-1} H(t)$$



$$(Y_g + Y_{stamp})v(t) = I_g + I_{stamp}$$

Convolution Limitations

Frequency-Domain Formulation

$$B(\omega) = S(\omega)A(\omega)$$

Time-Domain Formulation

$$b(t) = s(t) * a(t)$$

Convolution

$$b(t) = s(t) * a(t) = \int_0^t s(t-\tau)a(\tau)d\tau$$

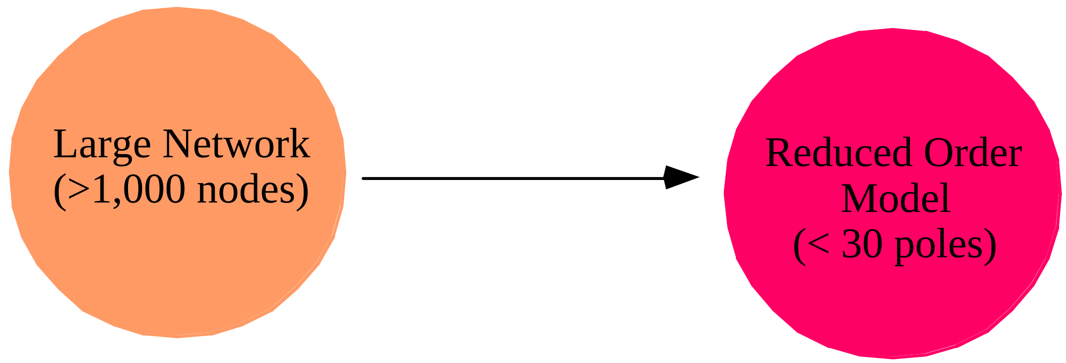
Discrete Convolution

$$s(t) * a(t) = \sum_{\tau=1}^{t} s(t-\tau)a(\tau)\Delta\tau$$

$$H(t) = \sum_{\tau=1}^{t-1} s(t-\tau)a(\tau)\Delta\tau \quad : \quad History$$

Computing History is computationally expensive ➔ Use FD rational approximation and TD recursive convolution

Model-Order Reduction



SPICE

Y(t) v(t) = i(t)

$$\tilde{Y}(\omega) = \left[A_1 + \sum_{i=1}^{L} \frac{a_{1i}}{1 + j\omega / \omega_{c1i}} \right]$$

$Y(\omega)\, V(\omega) = I(\omega)$

↓

Order Reduction

↓

$Y(\omega) \cong \tilde{Y}(\omega)$

↓

Recursive Convolution

↓

$\tilde{Y}(t)\, v(t) = i(t)$

Model-Order Reduction

Objective: Approximate frequency-domain transfer function to take the form:

$$H(\omega) = \left[A_1 + \sum_{i=1}^{L} \frac{a_{1i}}{1 + j\omega / \omega_{c1i}} \right]$$

Methods

- AWE – Pade
- Pade via Lanczos (Krylov methods)
- Rational Function
- Chebyshev-Rational function
- Vector Fitting Method

Model-Order Reduction (MOR)

Question: Why use a rational function approximation?

Answer: because the frequency-domain relation

$$Y(\omega) = H(\omega)X(\omega) = \left[d + \sum_{k=1}^{L} \frac{c_k}{1 + j\omega / \omega_{ck}} \right] X(\omega)$$

will lead to a time-domain *recursive* convolution:

$$y(t) = dx(t-T) + \sum_{k=1}^{L} y_{pk}(t)$$

where

$$y_{pk}(t) = a_k x(t-T)\left(1 - e^{-\omega_{ck}T}\right) + e^{-\omega_{ck}T} y_{pk}(t-T)$$

which is very fast!

Model-Order Reduction

Transfer function is approximated as

$$H(\omega) = d + \sum_{k=1}^{L} \frac{c_k}{1 + j\omega / \omega_{ck}}$$

In order to convert data into rational function form, we need a curve fitting scheme → Use Vector Fitting

History of Vector Fitting (VF)

- **1998 - Original VF formulated by Bjorn Gustavsen and Adam Semlyen***
- **2003 - Time-domain VF (TDVF) by S. Grivet-Talocia.**
- **2005 - Orthonormal VF (OVF) by Dirk Deschrijver, Tom Dhaene, et al.**
- **2006 - Relaxed VF by Bjorn Gustavsen.**
- **2006 - VF re-formulated as Sanathanan-Koerner (SK) iteration by W. Hendrickx, Dirk Deschrijver and Tom Dhaene, et al.**

* B. Gustavsen and A. Semlyen, "Rational approximation of frequency responses by vector fitting," IEEE Trans. Power Del., vol. 14, no. 3, pp 1052–1061, Jul. 1999

Vector Fitting (VF)

Algorithm

$$\begin{bmatrix} \sigma(s) f(s) \\ \sigma(s) \end{bmatrix} \approx \begin{bmatrix} \sum_{n=1}^{N} \frac{c_n}{s - \tilde{a}_n} + d + sh \\ \sum_{n=1}^{N} \frac{\tilde{c}_n}{s - \tilde{a}_n} + 1 \end{bmatrix}$$

$$\left(\sum_{n=1}^{N} \frac{c_n}{s - \tilde{a}_n} + d + sh \right) - \left(\sum_{n=1}^{N} \frac{\tilde{c}_n}{s - \tilde{a}_n} \right) f(s) \approx f(s).$$

Avoid ill-conditioned matrix

Guarantee stability

Converge, accurate

With Good Initial Poles

Solve for $c_n, \tilde{c}_n, d, h$

Can show* that the zeros of *σ(s)* are the poles of *f(s)* for the next iteration

* B. Gustavsen and A. Semlyen, "Rational approximation of frequency responses by vector fitting," IEEE Trans. Power Del., vol. 14, no. 3, pp 1052–1061, Jul. 1999

Issues with MOR

- **Bandwidth**
  - Low-frequency data must be added
- **Passivity**
  - Passivity enforcement
- **High Order of Approximation**
  - Orders > 800 for some serial links
  - Delay need to be extracted

Passivity Assessment

Can be done using S parameter Matrix

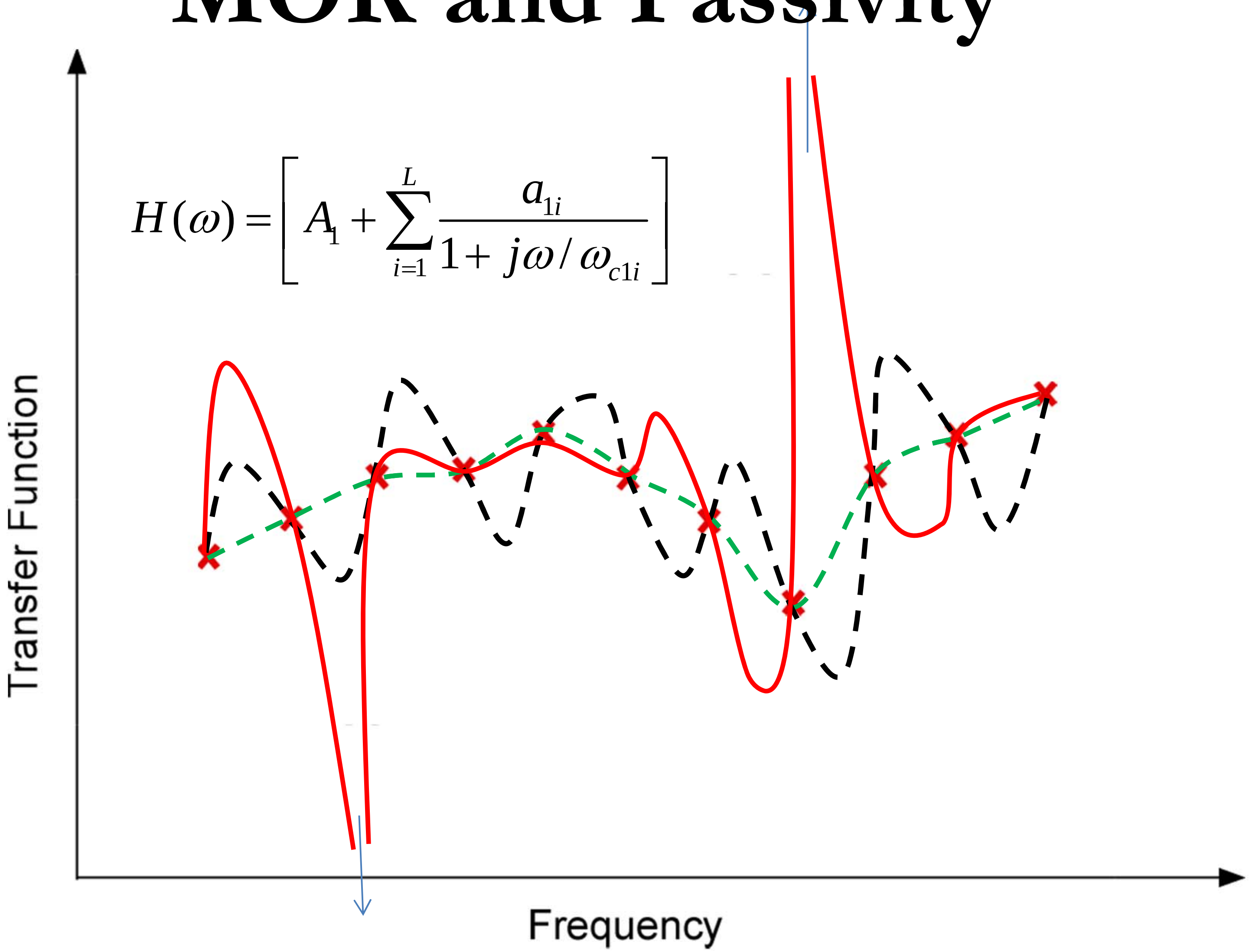
$$\boldsymbol{D} = \left(\boldsymbol{1} - \boldsymbol{S}^{*T}\boldsymbol{S}\right) = Dissipation\ Matrix$$

All the eigenvalues of the dissipation matrix must be greater than 0 at each sampled frequency points.

This assessment method is not very robust since it may miss local nonpassive frequency points between sampled points.

➔ **Use Hamiltonian from State Space Representation**

MOR and Passivity



$$H(\omega) = \left[A_1 + \sum_{i=1}^{L} \frac{a_{1i}}{1 + j\omega/\omega_{c1i}} \right]$$

State-Space Representation

The State space representation of the transfer function is given by

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

The transfer function is given by

$$S(s) = \boldsymbol{C}\left(s\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\boldsymbol{B} + \boldsymbol{D}$$

Passivity Assessment - Procedure

- Approximate all N^2 scattering parameters using Vector Fitting
- Form Matrices A, B, C and D for each approximated scattering parameter
- Form A, B, C and D matrices for complete N-port
- Form Hamiltonian Matrix H

Hamiltonian

Construct Hamiltonian Matrix *M*

$$M = \begin{bmatrix} A - BR^{-1}D^{T}C & -BR^{-1}B^{T} \\ C^{T}S^{-1}C & -A^{T} + C^{T}DR^{-1}B^{T} \end{bmatrix}$$

$$R = \left(D^{T}D - I\right) \ \ and \ \ S = \left(DD^{T} - I\right)$$

The system is passive if *M* has no purely imaginary eigenvalues

If imaginary eigenvalues are found, they define the crossover frequencies ($j\omega$) at which the system switches from passive to non-passive (or vice versa)

➔ **gives frequency bands where passivity is violated**

Size of Hamiltonian

$$M = \begin{bmatrix} A - BR^{-1}D^{T}C & -BR^{-1}B^{T} \\ C^{T}S^{-1}C & -A^{T} + C^{T}DR^{-1}B^{T} \end{bmatrix}$$

M has dimension $2NL$

For a 20-port circuit with VF order of 40, M will be of dimension $2\times 40 \times 20 = 1600$

The matrix M has dimensions 1600×1600

Passivity assessment can be slow…

➔ Eigen-analysis of this matrix is required

Passivity Enforcement Techniques

➔ **Hamiltonian Perturbation Method [1]**

➔ **Residue Perturbation Method [2]**

(1) S. Grivet-Talocia, "Passivity enforcement via perturbation of Hamiltonian matrices," *IEEE Trans. Circuits Syst. I*, vol. 51, no. 9, pp. 1755-1769, Sep. 2004.

(2) D. Saraswat, R. Achar, and M. Nakhla, "A fast algorithm and practical considerations for passive macromodeling of measured/simulated data," *IEEE Trans. Adv. Packag.*, vol. 27, no. 1, pp. 57–70, Feb. 2004.

Passive VF Simulation Code

- ➔ Performs VF with common poles
- ➔ Assessment via Hamiltonian
- ➔ Enforcement: Residue Perturbation Method
- ➔ Simulation: Recursive convolution

Number of Ports	Order	CPU-Time
4-Port	20	1.7 secs
6-port	32	3.69 secs
10-port	34	8.84 secs
20-port	34	33 secs
40	50	142 secs
80	12	255 secs

Passive VF Code - Examples

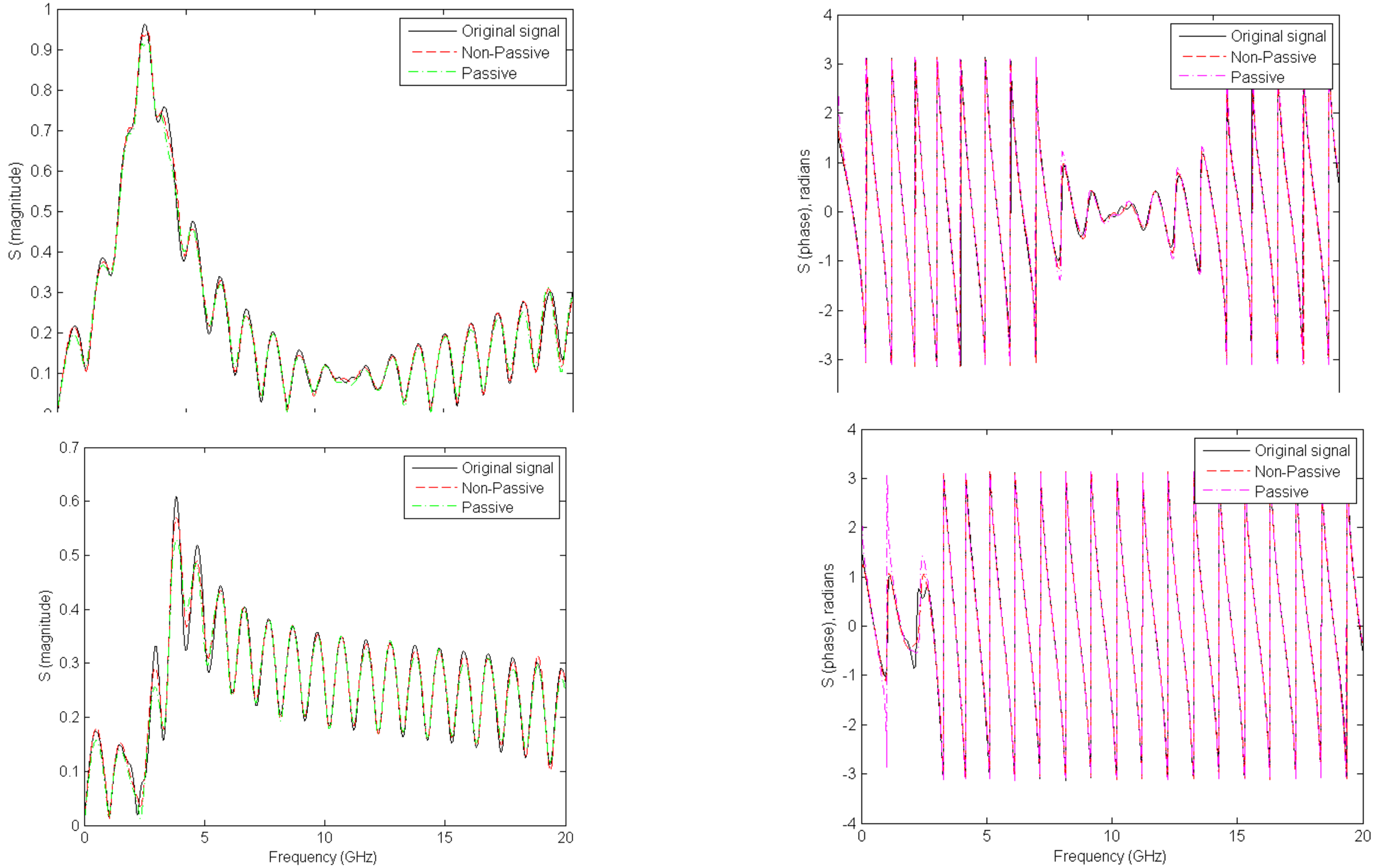
Example 1

4 ports
order = 60

Example 2

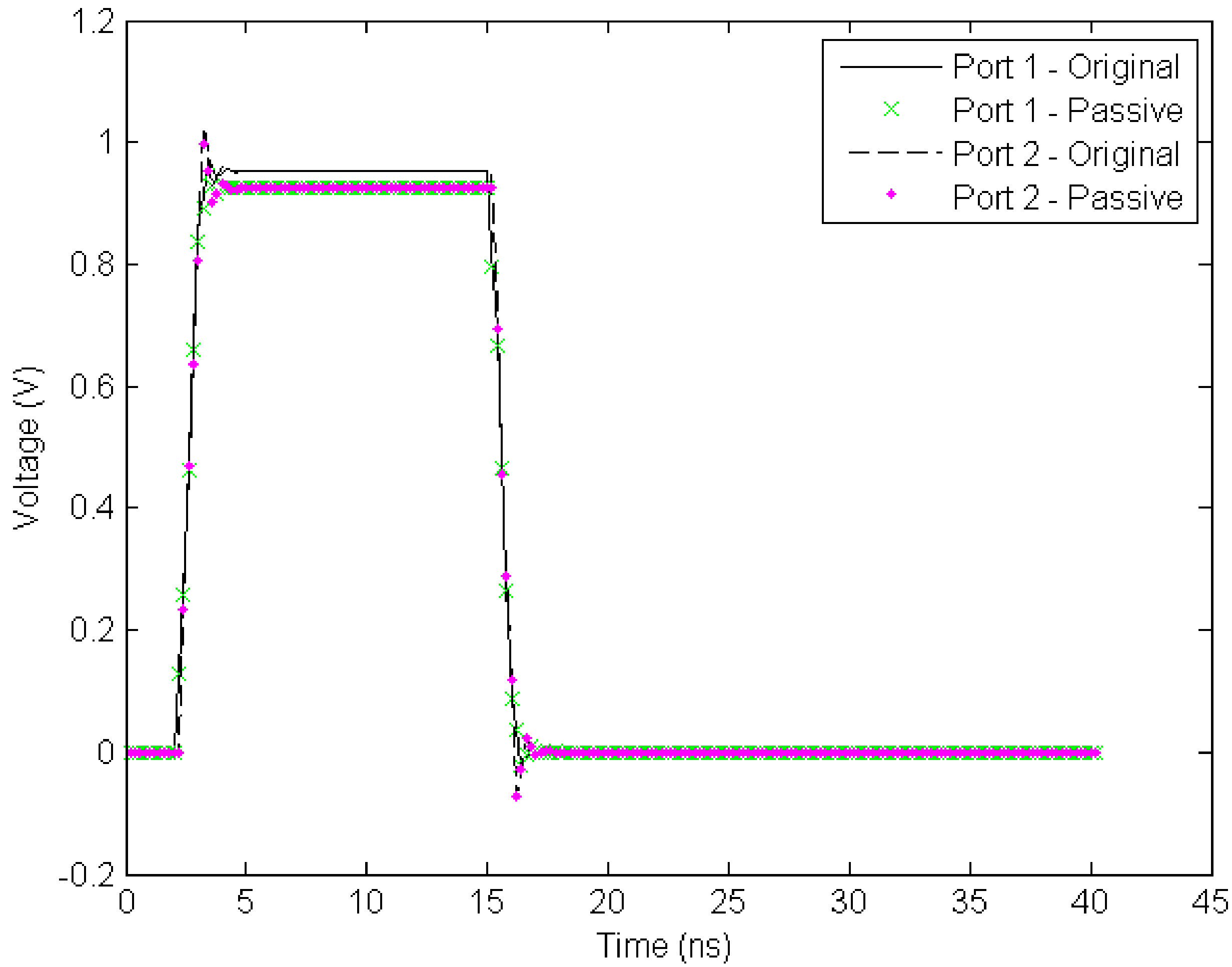
40 ports
order = 50

Two-Port Passivity Enforced VF

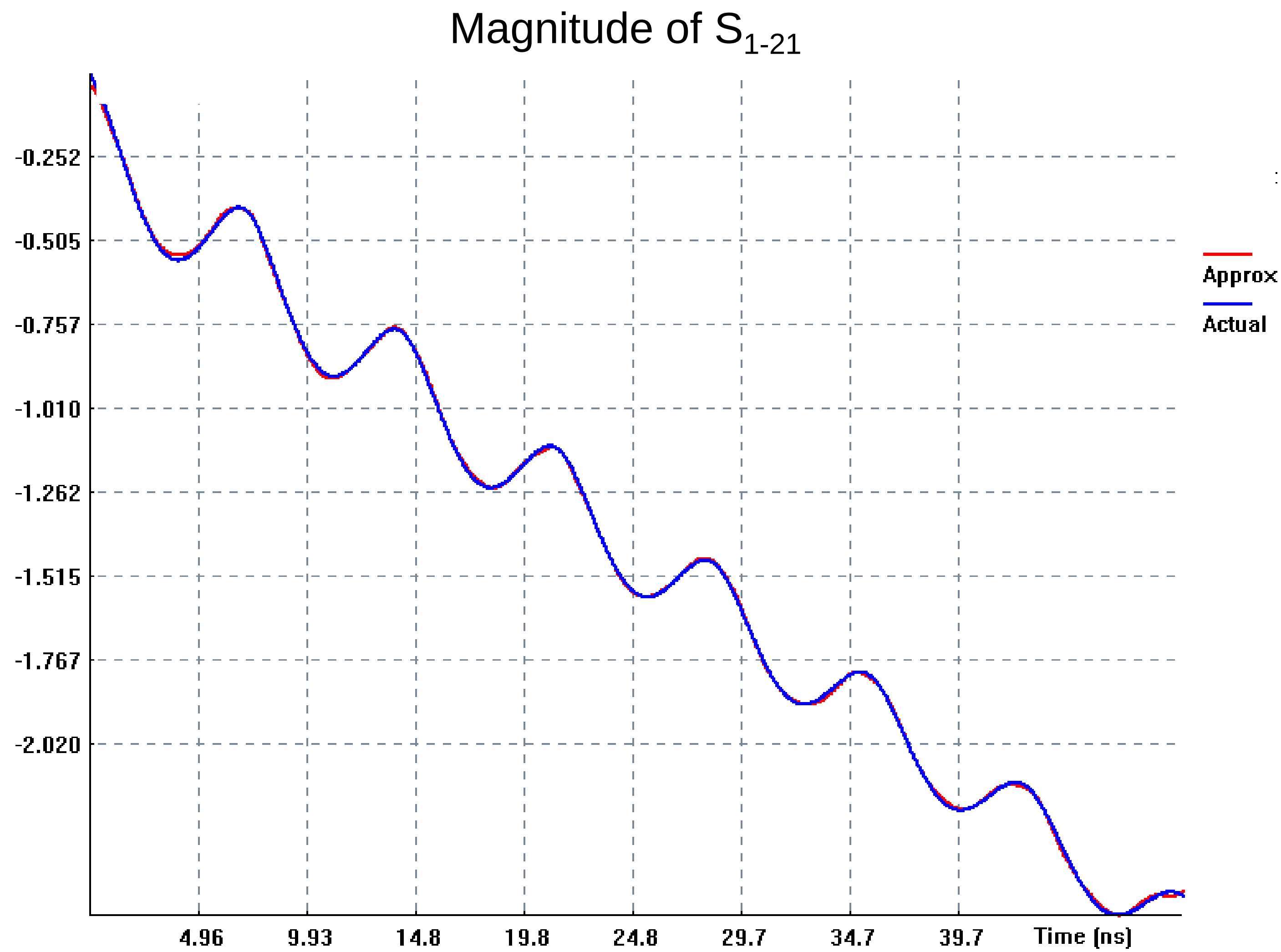


4 ports, 2039 data points - VFIT order = 60 (4 iterations ~6-7mins), Passivity enforcement: 58 Iterations (~1hour)

Passive Time-Domain Simulation

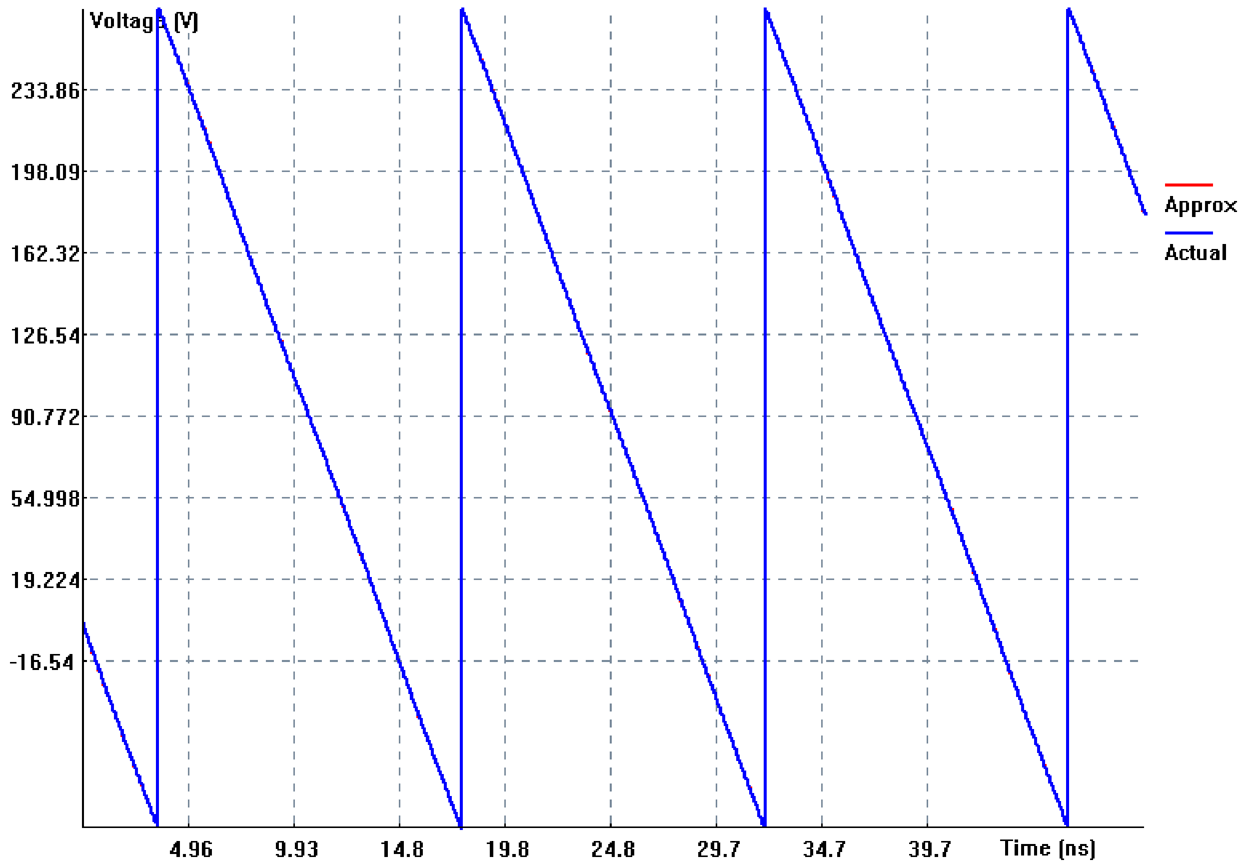


40-Port Passivity Enforced VF



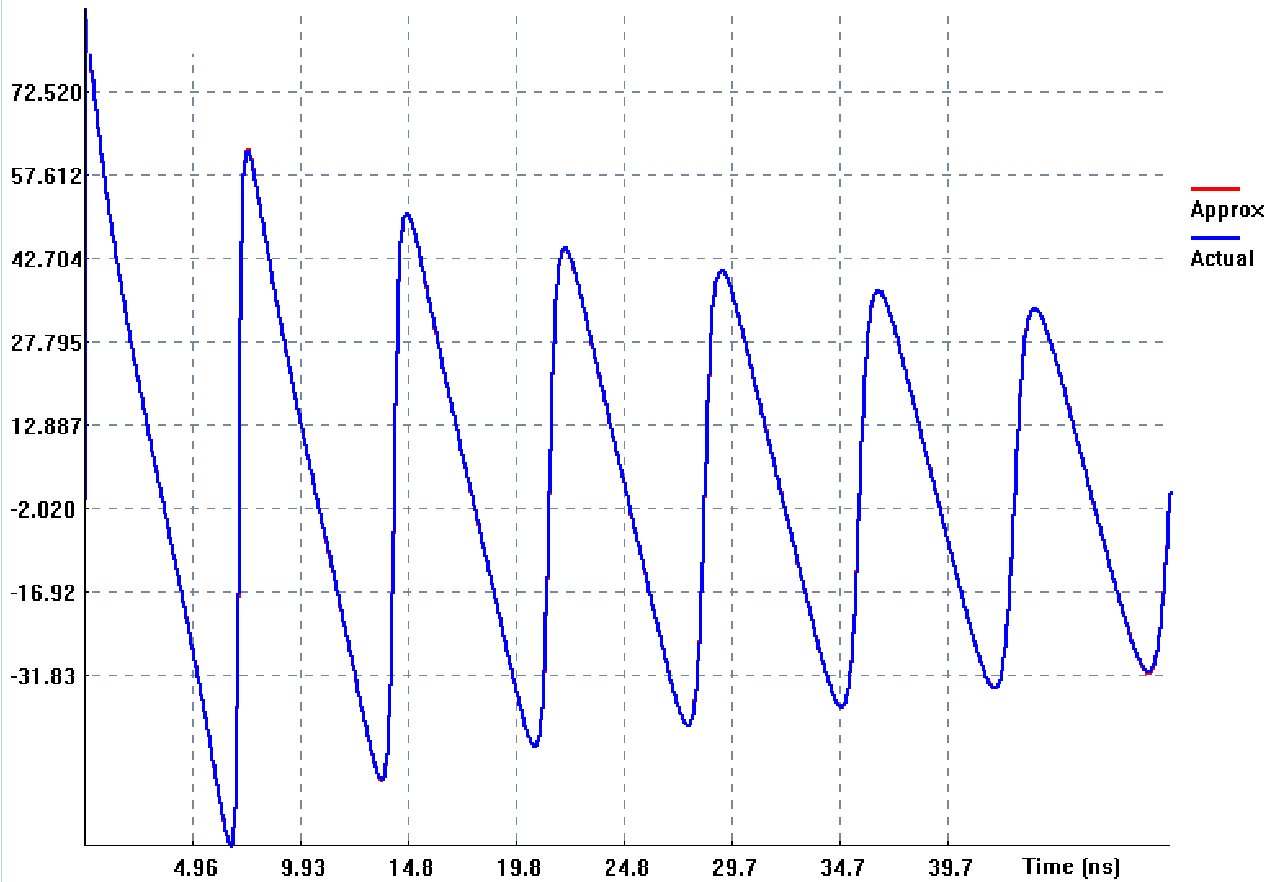
40-Port Passivity Enforced VF

Phase of S_{1-21}



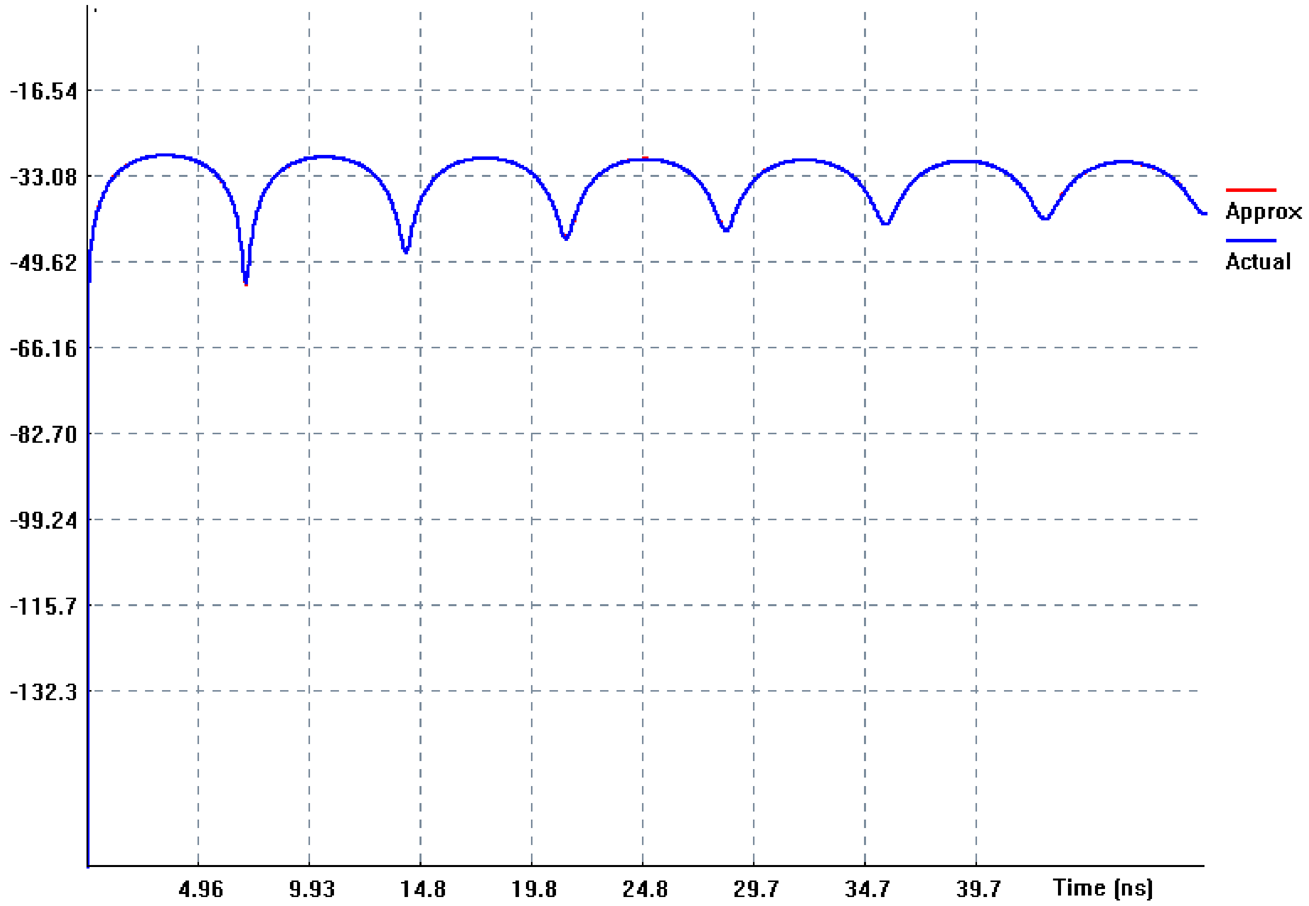
40-Port Passivity Enforced VF

Phase of S_{21}

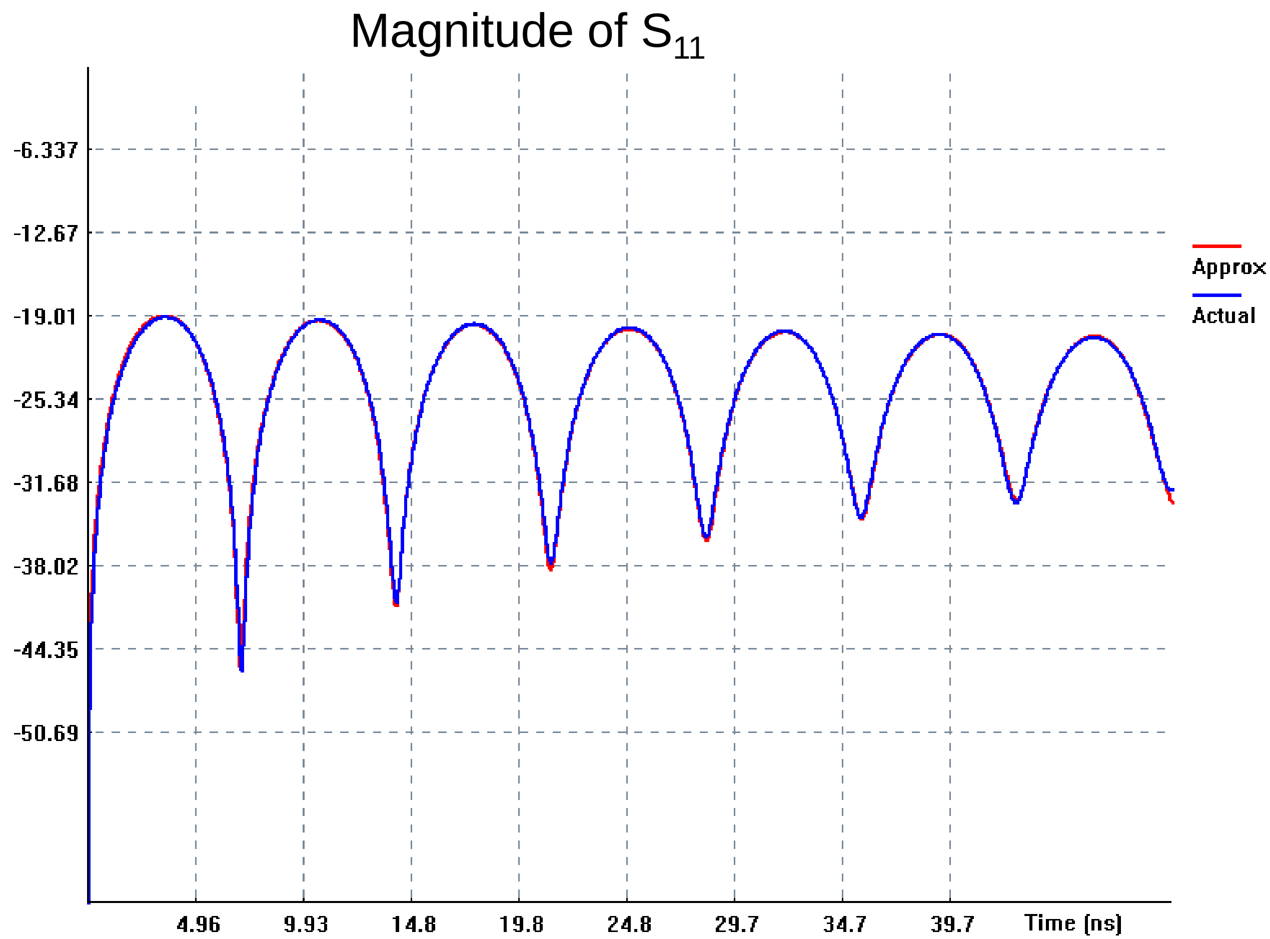


40-Port Passivity Enforced VF

Magnitude of S_{21}

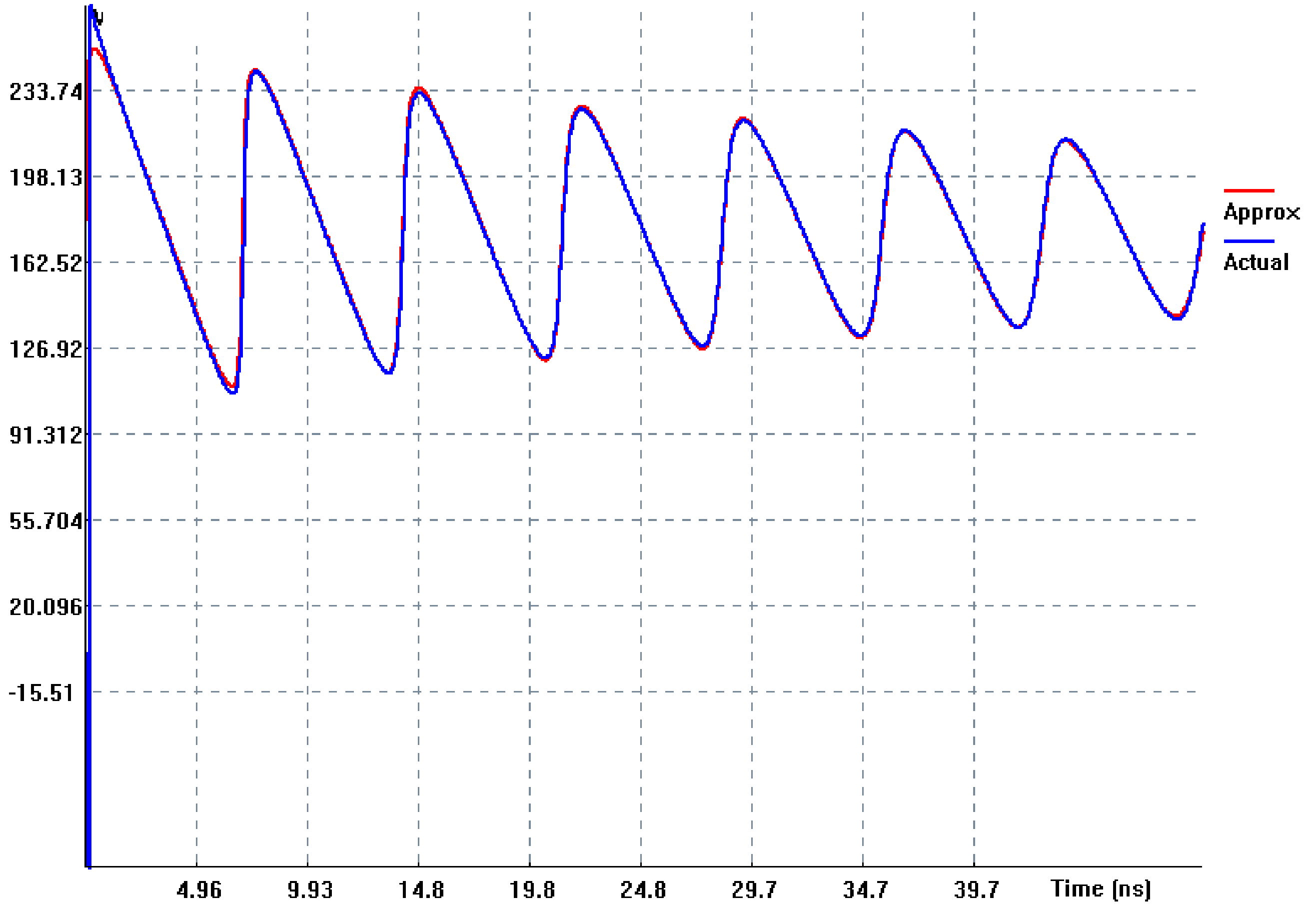


40-Port Passivity Enforced VF

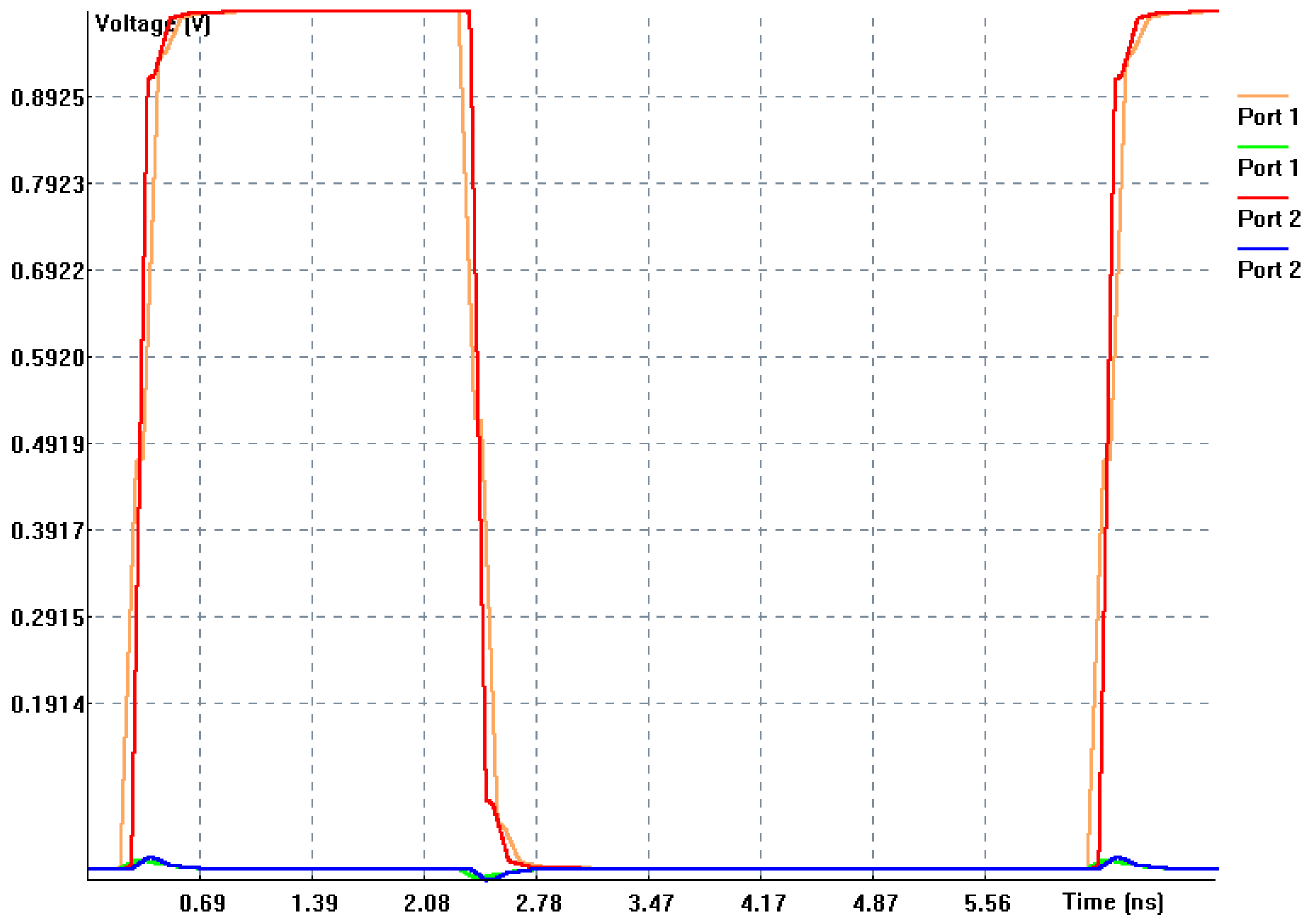


40-Port Passivity Enforced VF

Phase of S_{11}



40-Port Time-Domain Simulation



40-Port Time-Domain Simulation