ECE 451
Automated Microwave Measurements

TRL Calibration

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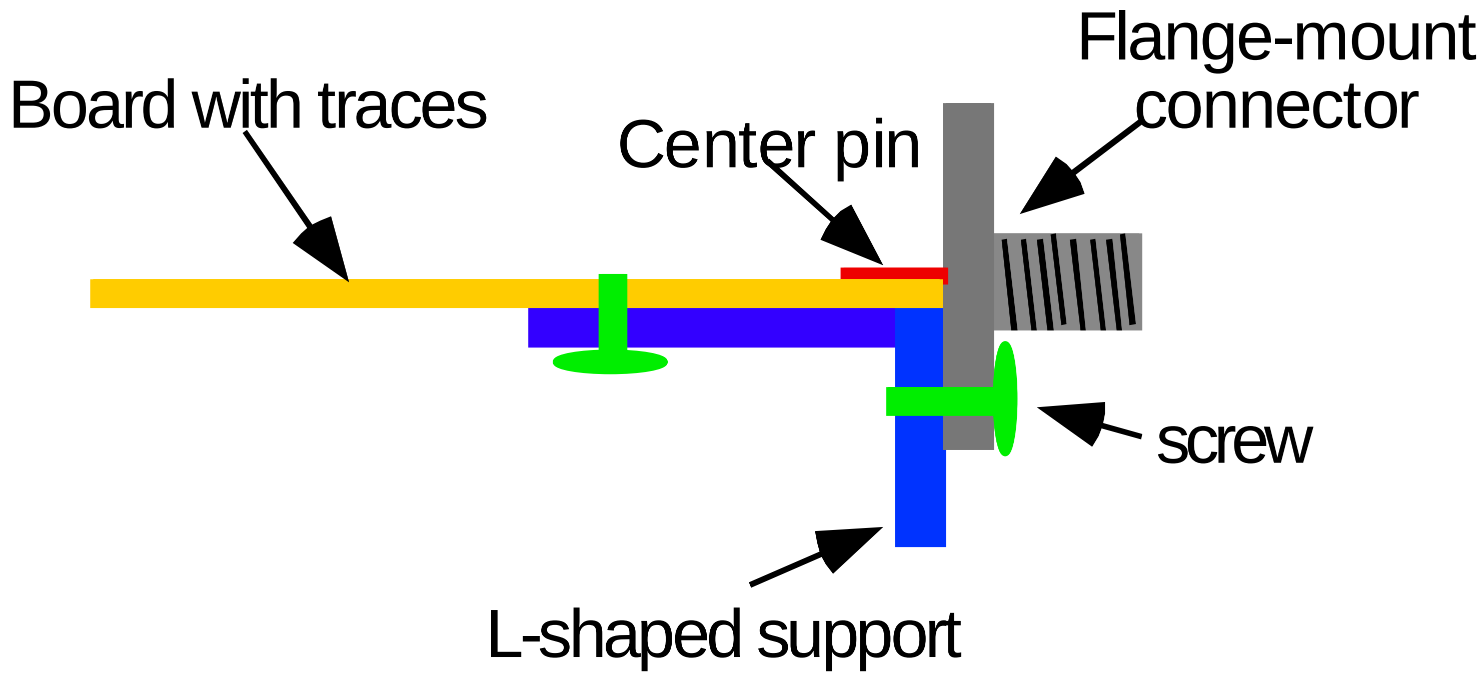
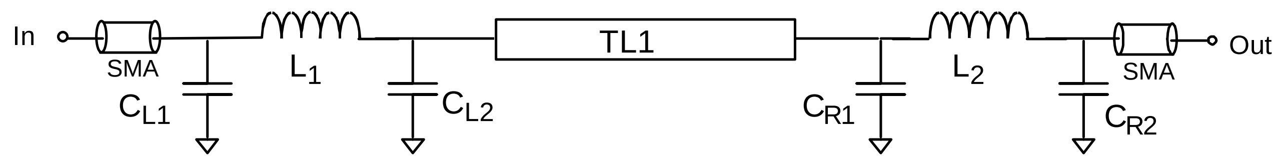
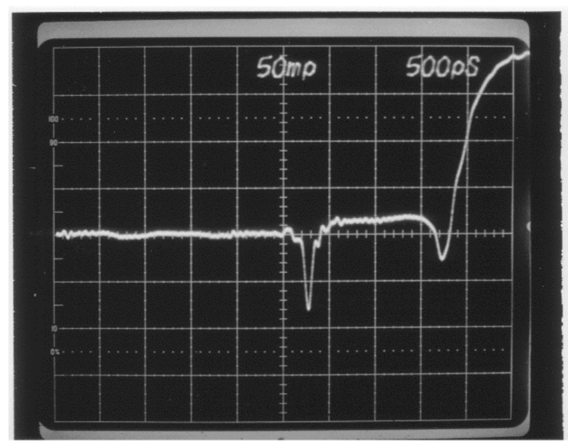
Coaxial-Microstrip Transition



Coaxial-Microstrip Transition



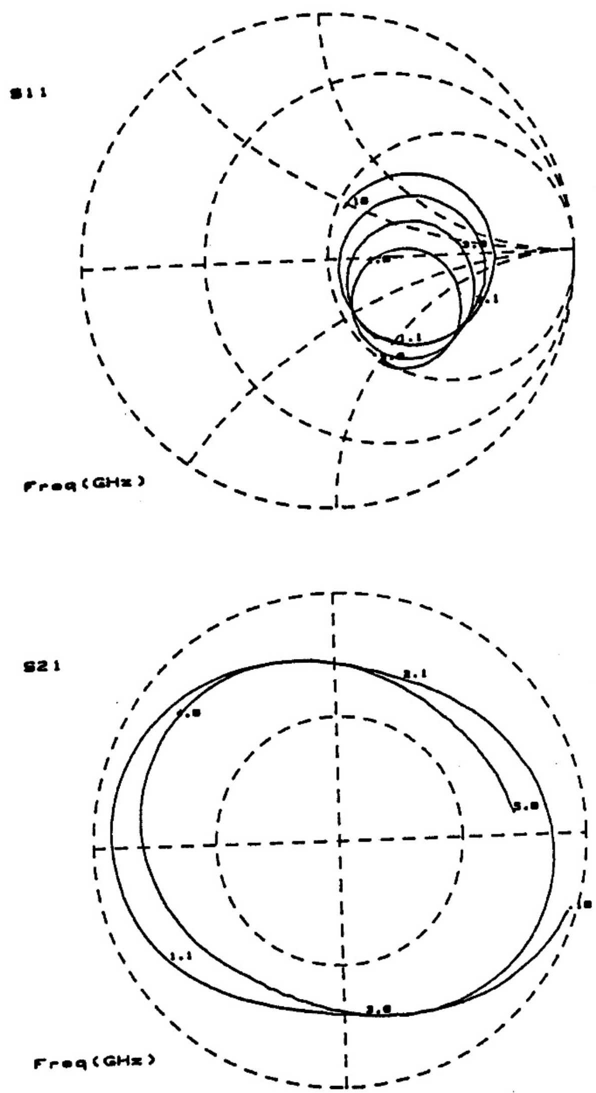
Equivalent Circuit

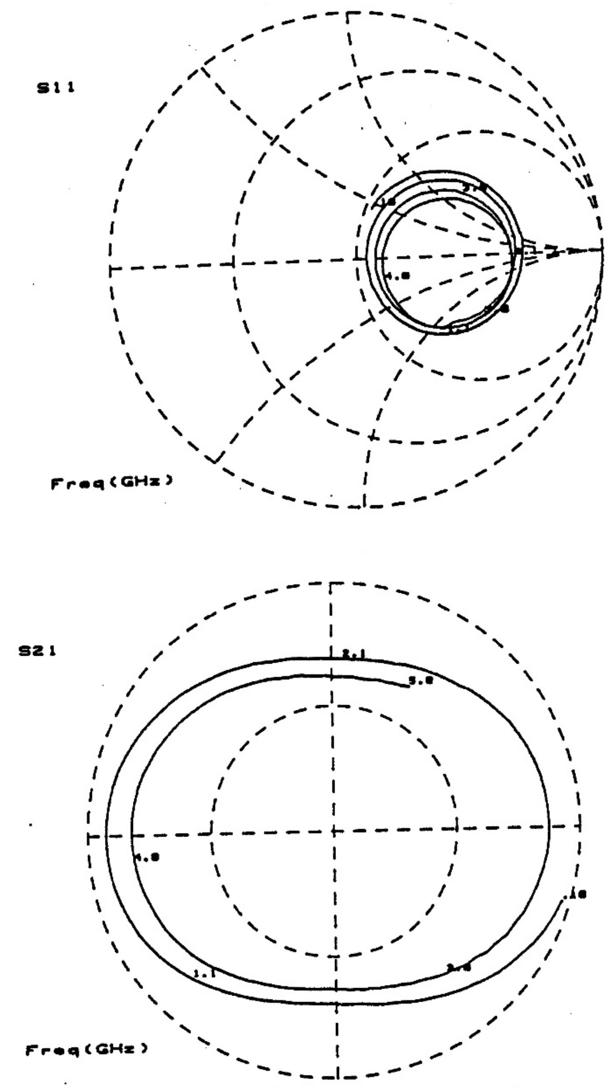


TDR Plot

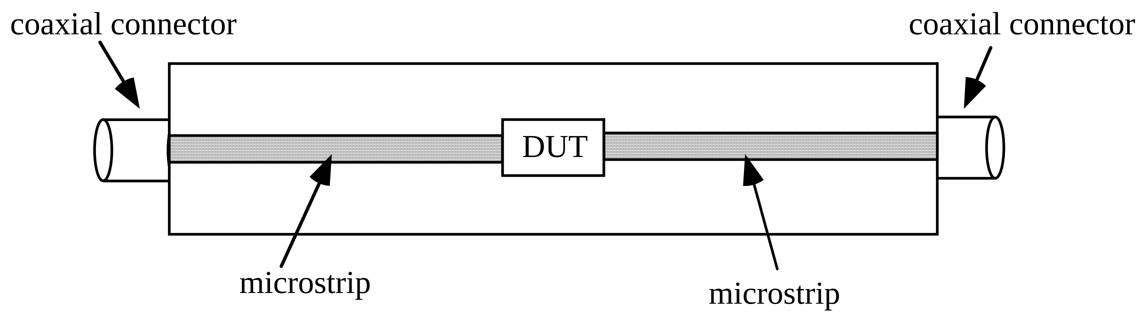
With parasitics

No parasitics





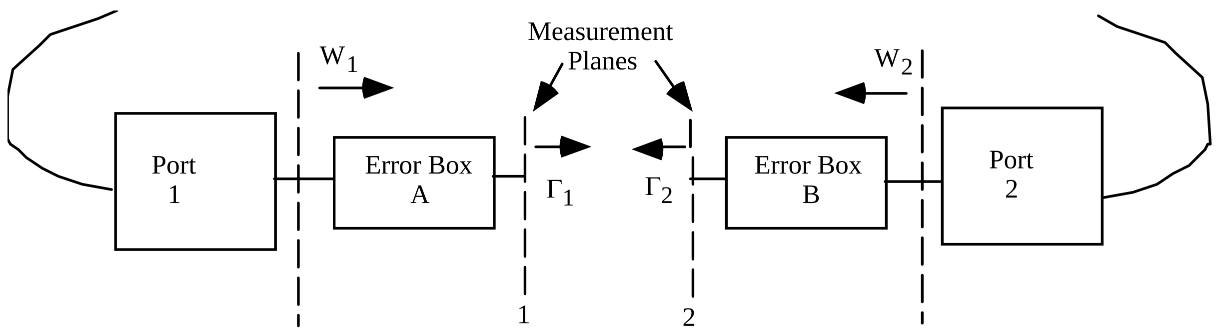
TRL CALIBRATION SCHEME



Want to measure DUT only and need to remove the effect of coax-to-microstrip transitions. Use TRL calibration

TRL Error Box Modeling

A model for the different error boxes can be implemented

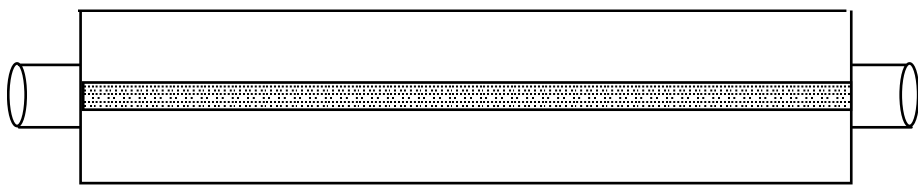


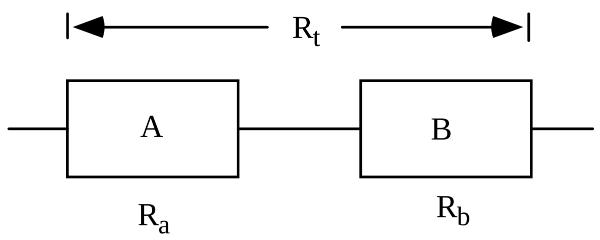
Error boxes A and B account for the transition parasitics and the electrical lengths of the microstrip.

Make three standards: Thru, Line and Reflect

Step 1 - THRU Calibration

connect thru

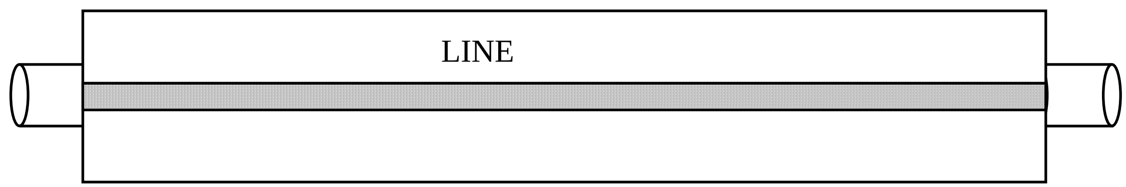


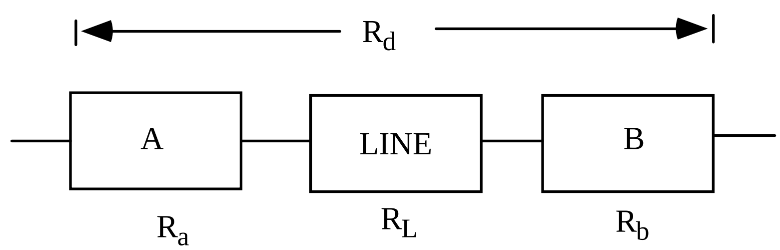


$$R_t = R_a R_b$$

Step 2 - LINE Calibration

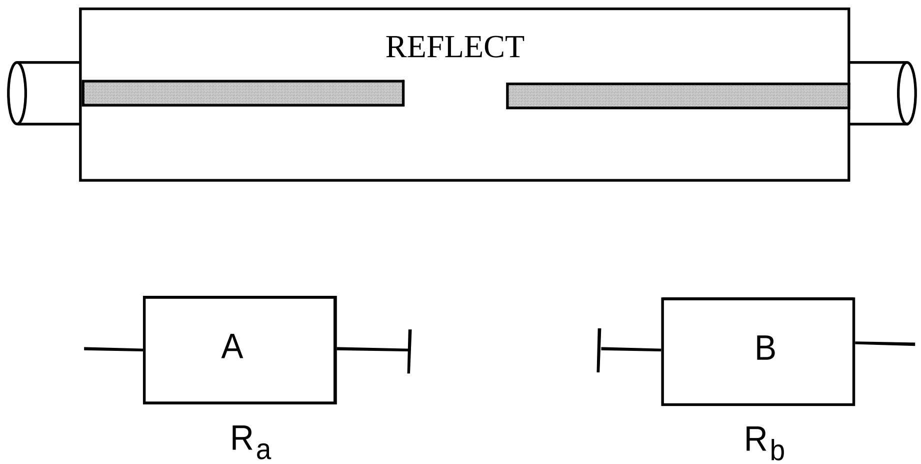
connect line (Note: difference in length between thru and line)





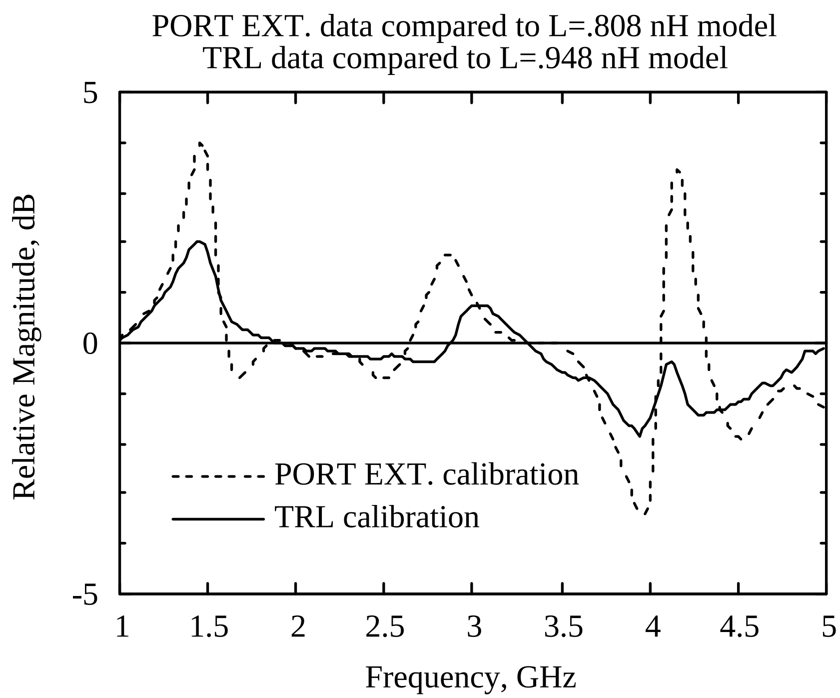
Step 3 - REFLECT Calibration

connect reflect

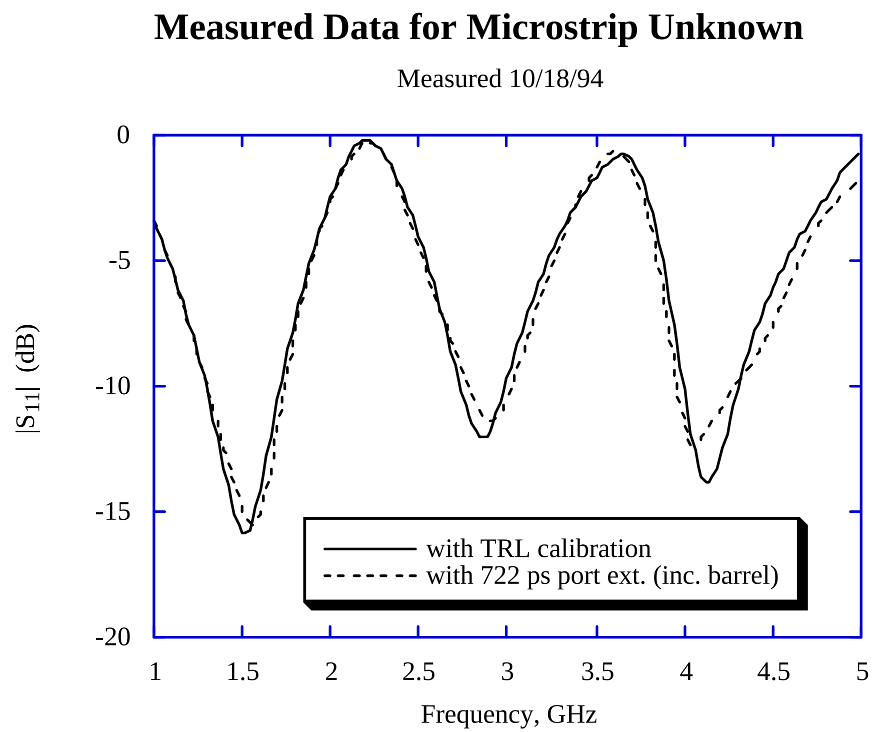


TRL – Measurement Comparison

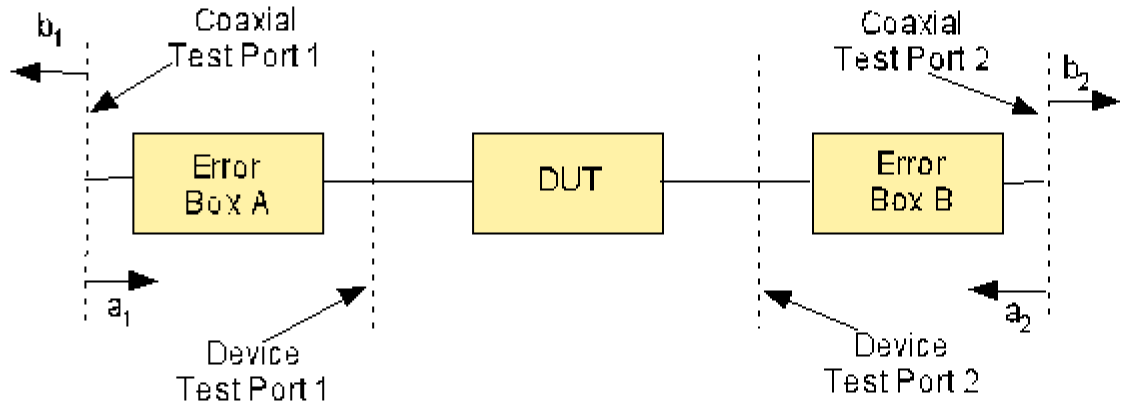
Measured $|S_{11}|$ of Microstrip Unknown Relative to TOUCHSTONE Models



TRL – Measurement Comparison



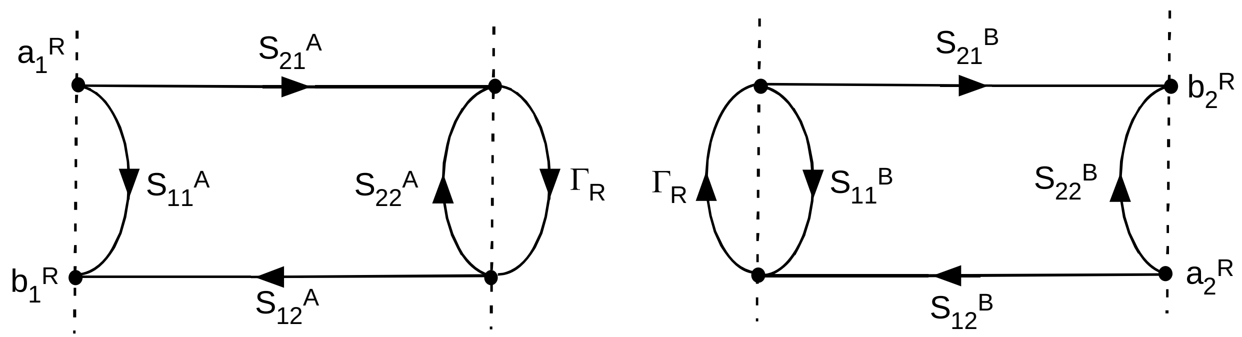
TRL Derivation



TRL Objectives

- **Obtain network parameters of error boxes A and B**
- **Remove their effects in subsequent measurements**

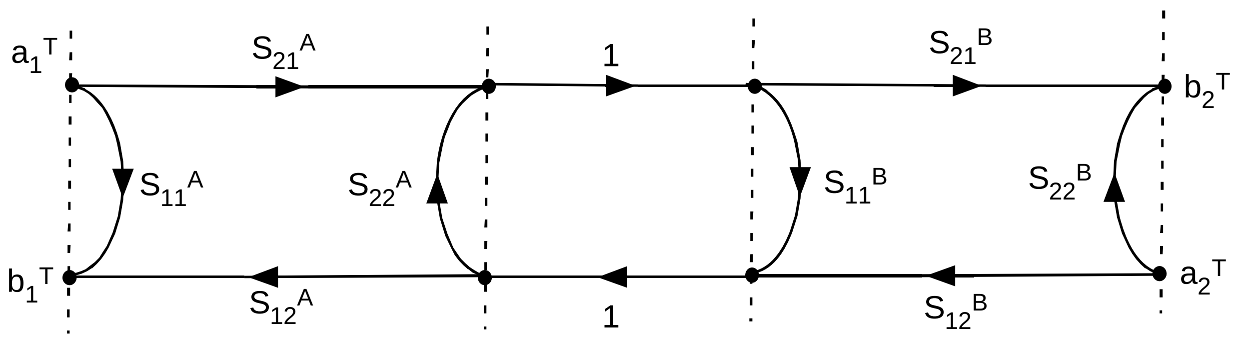
Model for Reflect



$$\left.\frac{b_1^R}{a_1^R}\right|_{a_2^R=0} \qquad \left.\frac{b_2^R}{a_2^R}\right|_{a_2^R=0}$$

2 Measurements

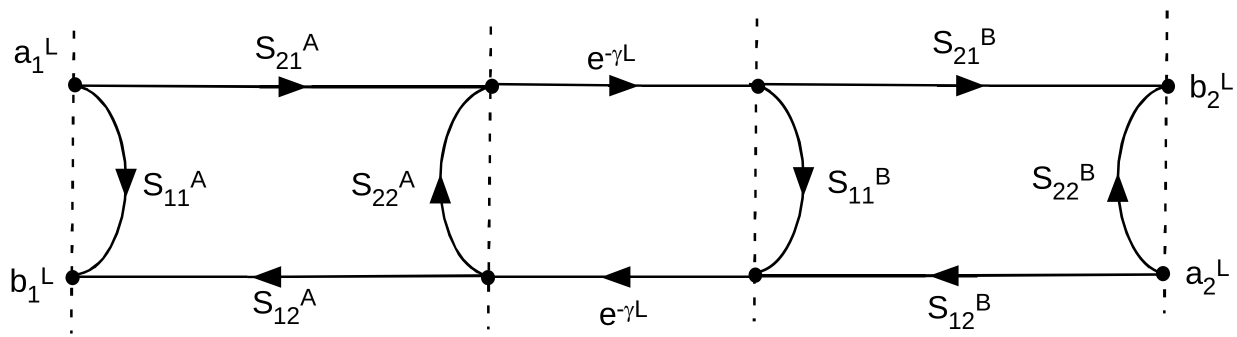
Model for Thru



$$\left.\frac{b_1^T}{a_1^T}\right|_{a_2^T=0} \qquad \left.\frac{b_2^T}{a_1^T}\right|_{a_2^T=0} \qquad \left.\frac{b_2^T}{a_2^T}\right|_{a_1^T=0} \qquad \left.\frac{b_1^T}{a_2^T}\right|_{a_1^T=0}$$

4 Measurements

Model for Line



$$\left.\frac{b_1^L}{a_1^R}\right|_{a_2^L=0} \qquad \left.\frac{b_2^L}{a_1^L}\right|_{a_2^L=0} \qquad \left.\frac{b_2^L}{a_1^L}\right|_{a_1^L=0} \qquad \left.\frac{b_1^L}{a_2^L}\right|_{a_1^L=0}$$

4 Measurements

Use R (or T) Parameters

Using R parameters (same as T transfer parameters), we can show that if

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

$$\begin{pmatrix} b_1 \\ a_1 \end{pmatrix} = \frac{1}{S_{21}} \begin{pmatrix} -\Delta & S_{11} \\ -S_{22} & 1 \end{pmatrix} \begin{pmatrix} b_2 \\ a_2 \end{pmatrix}$$

$$\Delta = S_{12}S_{21} - S_{11}S_{22}$$

$$R = \frac{1}{S_{21}} \begin{pmatrix} -\Delta & S_{11} \\ -S_{22} & 1 \end{pmatrix} \begin{pmatrix} b_2 \\ a_2 \end{pmatrix}$$

TRL Derivation

The measurement matrix R_M is just the product of the matrices of the error boxes and the unknown DUT

$$R_M = R_A R R_B$$

or

$$R = R_A^{-1} R_M R_B^{-1}$$

Let R_A be written as

$$R_A = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} = r_{22} \begin{bmatrix} a & b \\ c & 1 \end{bmatrix}$$

R_B is similarly written as

$$R_B = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} = \rho_{22} \begin{bmatrix} \alpha & \beta \\ \gamma & 1 \end{bmatrix}$$

The inverse of R_A is

$$R_A^{-1} = \frac{1}{r_{22}} \frac{1}{a - bc} \begin{bmatrix} 1 & -b \\ -c & a \end{bmatrix}$$

TRL Derivation

And the inverse of R_B is

$$R_B^{-1} = \frac{1}{\rho_{22}} \frac{1}{\alpha - \beta\gamma} \begin{bmatrix} 1 & -\beta \\ -\gamma & \alpha \end{bmatrix}$$

The matrix of the DUT is then found from

$$R = \frac{1}{r_{22}\rho_{22}} \frac{1}{a\alpha} \frac{1}{1 - b\frac{c}{a}} \frac{1}{1 - \gamma\frac{\beta}{\alpha}} \begin{bmatrix} 1 & -b \\ -c & a \end{bmatrix} R_M \begin{bmatrix} 1 & -\beta \\ -\gamma & \alpha \end{bmatrix}$$

Note that although there are eight terms in the error boxes, only seven quantities are needed to find R. They are a, b, c, α, β, γ, and $r_{22}\rho_{22}$

From the measurement of the through and of the line, seven quantities will be found. They are b, c/a, β/α, γ, $r_{22}\rho_{22}$, αa and $e^{2\gamma l}$

In addition to the seven quantities, if a were found, the solution would be complete. Let us first find the above seven quantities.

The ideal through has an R matrix which is the 2 x 2 unit matrix. The measured R matrix with the through connected will be denoted by R_T and is given by

$$R_T = R_A R_B$$

Where R_A and R_B are the R matrices of the error box A and B respectively. With the line connected, the measured R matrix will be denoted by R_D and is equal to

TRL Derivation

$$R_D = R_A R_L R_B$$

NOTE: quantities shown in RED are known

where R_L is the R matrix of the line

Now $R_B = R_A^{-1} R_T$

so that $R_D = R_A R_L R_A^{-1} R_T$

$$R_D R_T^{-1} R_A = R_A R_L$$

Define $T = R_D R_T^{-1}$ Which when substituted into the above equations results in

$$T R_A = R_A R_L$$

The matrix T is known from measurements and will be written as

$$T = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}$$

$$R_L = \begin{bmatrix} e^{-\gamma l} & 0 \\ 0 & e^{+\gamma l} \end{bmatrix}$$, since the line is non-reflecting

TRL Derivation

R_A is unknown and was written as

$$R_A = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} = r_{22} \begin{bmatrix} a & b \\ c & 1 \end{bmatrix}$$

R_B similarly was written as

$$R_B = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} = \rho_{22} \begin{bmatrix} \alpha & \beta \\ \gamma & 1 \end{bmatrix}$$

Recalling $TR_A = R_A R_L$ and writing the matrices results in

$$\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} a & b \\ c & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & 1 \end{bmatrix} \begin{bmatrix} e^{-\gamma l} & 0 \\ 0 & e^{+\gamma l} \end{bmatrix}$$

Next, writing out the four equations gives:

TRL Derivation

$$t_{11}a + t_{12}c = ae^{-\gamma l}$$

$$t_{21}a + t_{22}c = ce^{-\gamma l}$$

$$t_{11}b + t_{12} = be^{+\gamma l}$$

$$t_{21}b + t_{22} = be^{+\gamma l}$$

Dividing the first of the above equation by the second results in

$$\frac{t_{11}a + t_{12}c}{t_{21}a + t_{22}c} = \frac{c}{a} = \frac{t_{11}\dfrac{a}{c} + t_{12}}{t_{21}\dfrac{a}{c} + t_{22}}$$

which gives a quadratic equation for *a/c*

$$t_{21}\left(\frac{a}{c}\right)^2 + \left(t_{22} - t_{11}\right)\frac{a}{c} - t_{12} = 0$$

Dividing the third equation in the group by the fourth results in

TRL Derivation

$$\frac{t_{11}b+t_{12}}{t_{21}b+t_{22}}=b$$ which gives the analogous quadratic equation for b as

$$t_{21}b^2+\left(t_{22}-t_{11}\right)b-t_{12}=0$$

Dividing the fourth equation in the group by the second results in

$$e^{2\gamma L}=c\frac{t_{21}b+t_{22}}{t_{21}a+t_{22}c}=\frac{t_{21}b+t_{22}}{t_{21}\frac{a}{c}+t_{22}}$$

Since $e^{2\gamma L}$ is not equal to 1, *b* and *c/a* are distinct roots of the quadratic equation. The following discussion will enable the choice of the root. Now $b=r_{12}/r_{22}=S_{11}$ and

$$\frac{a}{c}=\frac{r_{11}}{r_{21}}=S_{11}-\frac{S_{12}S_{21}}{S_{22}}$$

TRL Derivation

For a well designed transition between coax and the non-coax $|S_{22}|$, $|S_{11}| << 1$ which yields $|b| << 1$ and $|a/c| >> 1$. Therefore,

$$|b| \ll \left|\frac{a}{c}\right|$$ which determines the choice of the root

Recalling $$TR_A = R_A R_L$$

$$(det\ T)(det\ R_A) = (det\ R_A)(det\ R_L)$$

or

$$(det\ T) = (det\ R_L) = 1$$

so that

$$t_{11}t_{22} - t_{12}t_{21} = 1$$

which implies that there are only three independent T_{ij}. Then there are only three independent results, e.g. b, a/c, and $e^{2\gamma L}$.

TRL Derivation

Now let us find four more quantities

$$r_{22}\rho_{22}\begin{bmatrix} a & b \\ c & 1 \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & 1 \end{bmatrix} = R_A R_B = R_T = g\begin{bmatrix} d & e \\ f & 1 \end{bmatrix}$$

Now

$$\begin{bmatrix} a & b \\ c & 1 \end{bmatrix}^{-1} = \frac{1}{a-bc}\begin{bmatrix} 1 & -b \\ -c & a \end{bmatrix}$$

So that

$$r_{22}\rho_{22}\begin{bmatrix} \alpha & \beta \\ \gamma & 1 \end{bmatrix} = \frac{g}{a-bc}\begin{bmatrix} 1 & -b \\ -c & a \end{bmatrix}\begin{bmatrix} d & e \\ f & 1 \end{bmatrix}$$

or

$$r_{22}\rho_{22}\begin{bmatrix} \alpha & \beta \\ \gamma & 1 \end{bmatrix} = \frac{g}{a-bc}\begin{bmatrix} d-bf & e-b \\ af-cd & a-ce \end{bmatrix}$$

TRL Derivation

from which we can extract

$$r_{22}\rho_{22} = g\frac{a-ce}{a-bc} = g\frac{1-e\dfrac{c}{a}}{1-b\dfrac{c}{a}}$$

We also have

$$\begin{bmatrix} \alpha & \beta \\ \gamma & 1 \end{bmatrix} = \frac{1}{a-ce}\begin{bmatrix} d-bf & e-b \\ af-cd & a-ce \end{bmatrix}$$

from which we obtain

$$\gamma = \frac{f-\dfrac{c}{a}d}{1-\dfrac{c}{a}e}$$

and

$$\frac{\beta}{\alpha} = \frac{e-b}{d-bf}$$

TRL Derivation

and

$$\alpha a = \frac{d - bf}{1 - \frac{c}{a} e}$$

The additional four quantities found are β/α, γ, $r_{22}\rho_{22}$ and αa. To complete the solution, one needs to find *a*. Let the reflection measurement through error box A be w_1. Then

$$w_1 = \frac{a\Gamma_R + b}{c\Gamma_R + 1}$$

which may be solved for *a* in terms of the known *b* and *a/c* as

$$a = \frac{w_1 - b}{\Gamma_R\left(1 - w_1 \frac{c}{a}\right)}$$

We need a method to determine *a*. Use the measurement for the reflect from through the error box B. Let w_2 denote the measurement

$$w_2 = S_{22} + \frac{S_{12}S_{21}\Gamma_R}{1 - S_{11}\Gamma_R} = \frac{S_{22} - \Delta\Gamma_R}{1 - S_{11}\Gamma_R}$$

TRL Derivation

$$w_2 = \frac{-\frac{\rho_{21}}{\rho_{22}} + \frac{\rho_{11}}{\rho_{22}}\Gamma_R}{1 - \frac{\rho_{12}}{\rho_{22}}\Gamma_R}$$

or

$$w_2 = -\frac{\alpha\Gamma_R - \gamma}{\beta\Gamma_R - 1}$$

α may be found in terms of γ and β/α as

$$\alpha = \frac{w_2 + \gamma}{\Gamma_R\left(1 + w_2\frac{\beta}{\alpha}\right)}$$

Recall $$a = \frac{w_1 - b}{\Gamma_R\left(1 - w_1\frac{c}{a}\right)}$$

TRL Derivation

so that

$$\frac{a}{\alpha} = \frac{w_1 - b}{w_2 + \gamma} \times \frac{1 + w_2 \frac{\beta}{\alpha}}{1 - w_1 \frac{c}{a}}$$

From earlier $\alpha a = \dfrac{d - bf}{1 - \frac{c}{a} e}$

so that

$$a^2 = \frac{w_1 - b}{w_2 + \gamma} \frac{1 + w_2 \frac{\beta}{\alpha}}{1 - w_1 \frac{c}{a}} \frac{d - bf}{1 - \frac{c}{a} e}$$

or

$$a = \pm \left(\frac{w_1 - b}{w_2 + \gamma} \times \frac{1 + w_2 \frac{\beta}{\alpha}}{1 - w_1 \frac{c}{a}} \times \frac{d - bf}{1 - \frac{c}{a} e} \right)^{\frac{1}{2}}$$

which determines *a* to within a ± sign.

TRL Derivation

$$\Gamma_R = \frac{w_1 - b}{a\left(1 - w_1 \frac{c}{a}\right)}$$

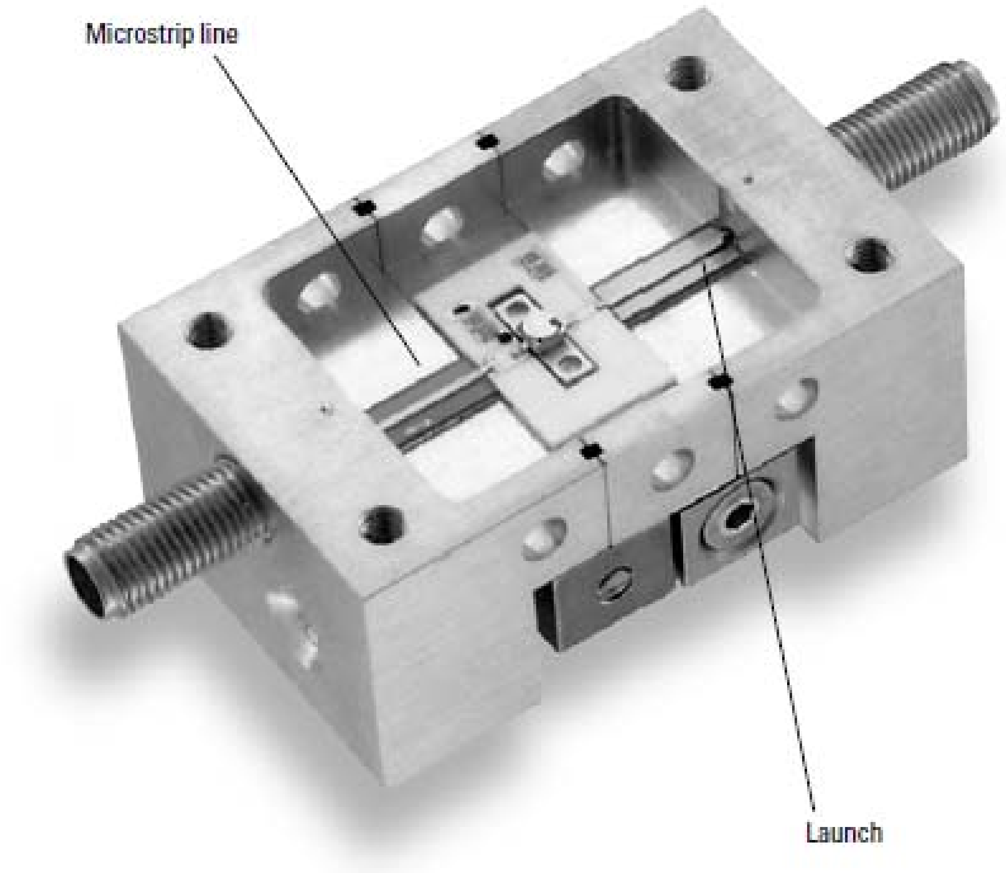
So if Γ_R is known to within ± then a may be determined as well. Calibration is complete and we can now proceed to the measurement of the DUT.

From earlier, the matrix of the DUT is found from

$$R = \frac{1}{r_{22}\rho_{22}} \frac{1}{a\alpha} \frac{1}{1 - b\frac{c}{a}} \frac{1}{1 - \gamma\frac{\beta}{\alpha}} \begin{bmatrix} 1 & -b \\ -c & a \end{bmatrix} R_M \begin{bmatrix} 1 & -\beta \\ -\gamma & \alpha \end{bmatrix}$$

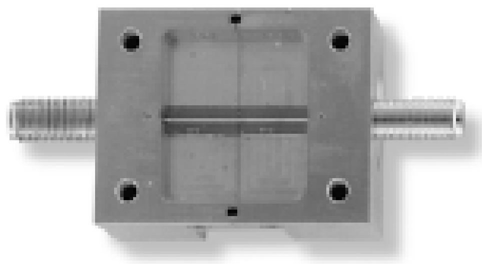
in which all the terms have now been determined.

TRL Application

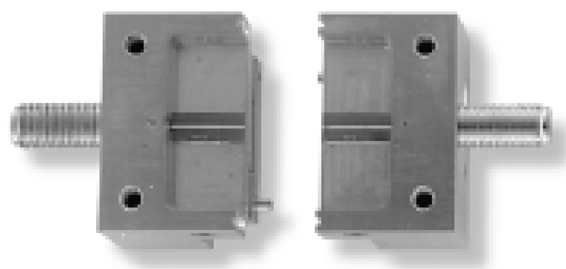


TRL Application

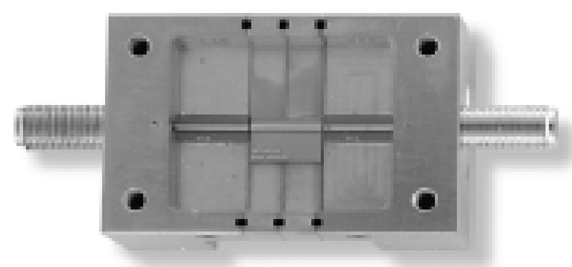
Thru



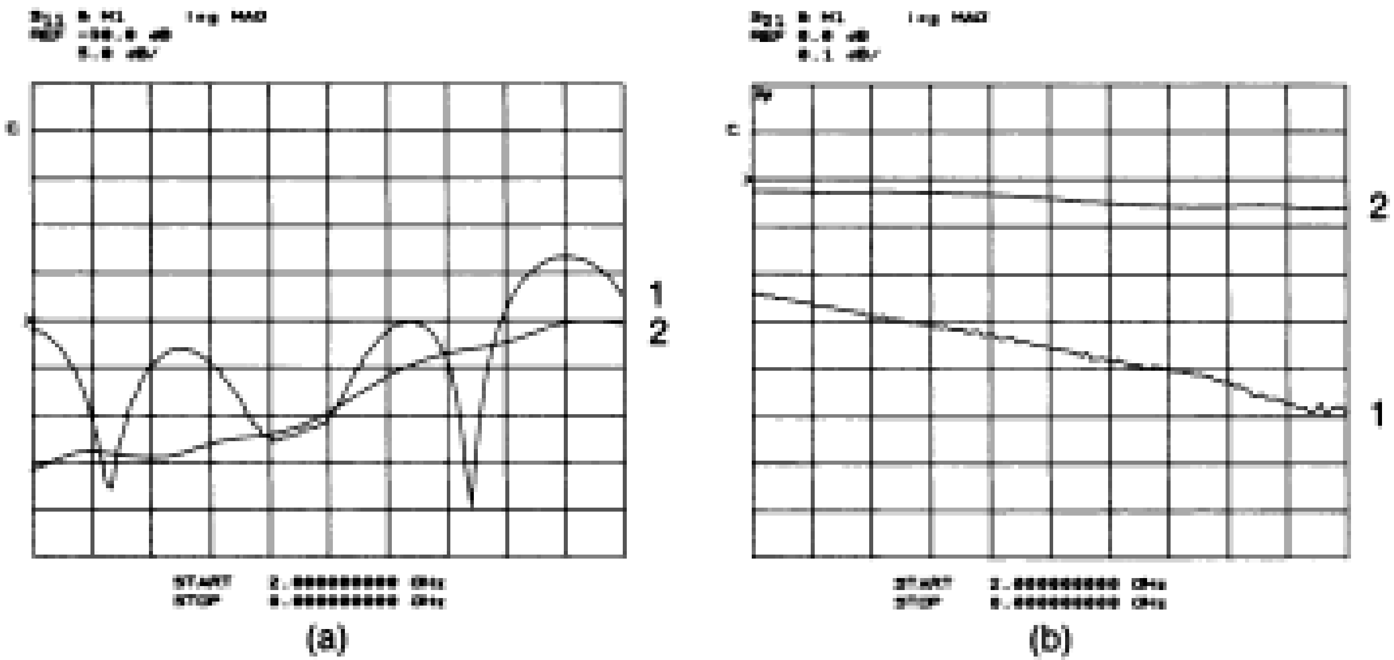
Reflect



Line

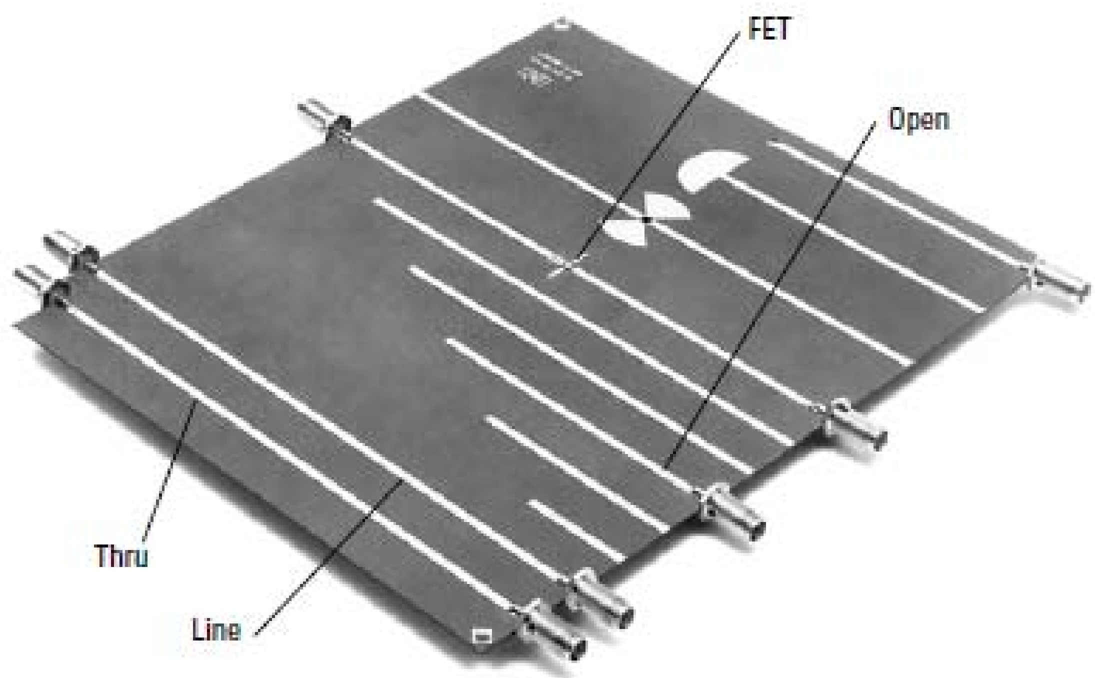


TRL Application



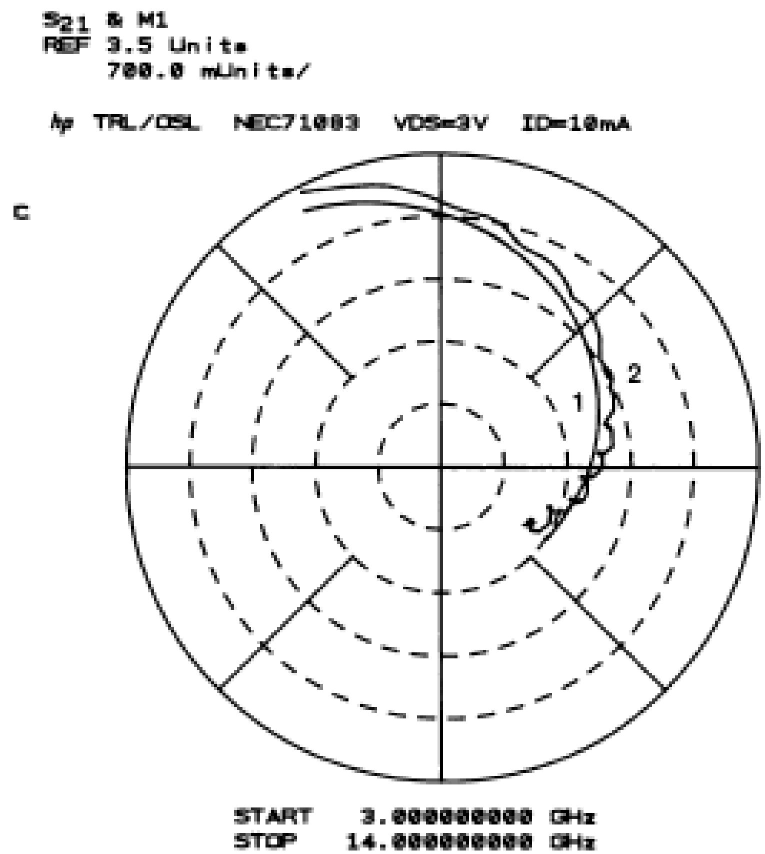
Example measurement (a) return loss of microstrip transmission line (b) insertion loss of a microstrip transmission line (1) calibrated at the coaxial ports of the fixture (2) calibrated in-fixture with TRL.

TRL Application



Microstrip PC board as a test fixture including separate transmission lines as the THRU and LINE, an open circuit, and a test line for insertion of a test device.

TRL Application



Example measurement of a linear FET on the microstrip PC board compared to measurement in a de-embedded test fixture (Agilent 85041A). (1) de-embedded measurement (2) TRL calibration using PC board standards.

Standard	Requirements
REFLECT	Reflection coefficient G magnitude (optimally 1.0) need not be known Phase of G must be known within ±1/4 wavelength[1] Must be the same G on both ports May be used to set the reference plane if the phase response of the REFLECT is well-known and specified
Zero Length THRU	S_{21} and S_{12} are defined equal to 1 at 0 degrees (typically used to set the reference plane) S_{11} and S_{22} are defined equal to zero[2]
Non-Zero Length THRU	Characteristic impedance Z_0 of the THRU and LINE must be the same[4,5] Attenuation of the THRU need not be known Insertion phase or electrical length must be specified if the THRU is used to set the reference plane[3]
LINE	Z_0 of the LINE establishes the reference impedance after error correction is applied[5] Insertion phase of the LINE must never be the same as that of the THRU (zero or non-zero length)[6] Optimal LINE length is 1/4 wavelength or 90 degrees relative to the THRU at the center frequency[7] Useable bandwidth of a single THRU/LINE pair is 8:1 (frequency span/start frequency) Multiple THRU/LINE pairs (Z_0 assumed identical) can be used to extend the bandwidth to the extent transmission lines are realizable Attenuation of the LINE need not be known insertion phase or electrical length need only be specified within 1/4 wavelength
MATCH	Assumes same Z_0 on both ports Z_0 of the MATCH standards establishes the reference impedance after error correction is applied No frequency range limitations (MATCH may be used instead of LOWBAND REFLECTION cal steps)

1. The phase response need only be specified within a 1/4 wavelength ±90 degrees either way. During computation of the error model, the root choice in the solution of a quadratic equation is made based on the reflection data. An error in definition would show up as a 180-degree error in the measured phase.
2. A zero-length THRU has no loss and has no characteristic impedance.
3. If a non-zero-length THRU is used but specified to have zero delay, the reference plane will be established in the middle of the THRU.
4. When the Z_0 of the THRU and LINE are not the same, the average impedance is used.
5. S_{11} and S_{22} of the LINE are also defined to be zero. With this assumption, the system impedance is set to the characteristic impedance of the LINE. If the Z_0 is known but not the desired value, the impedance of the LINE can be specified when defining the calibration standards.
6. The insertion phase difference between the THRU and LINE must be between (20 and 160 degrees) $\pm n \times 180$ degrees. Measurement uncertainty will increase significantly when the insertion phase nears 0 or an integer multiple of 180 degrees.
7. The optimal length of a LINE is 1/4 wavelength or 90 degrees of insertion phase in the middle or the geometric mean of the desired frequency span.

References

Paul W. Klock, "*The Theory of Reflectometers*", *1995*

Agilent Network Analysis, "*Applying the 8510 TRL Calibration for Non-Coaxial Measurements*", Product Note 8510-8A