ECE 451
Advanced Microwave Measurements

Requirements of Physical Channels

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Issues

- Frequency and time limitations
- Minimum phase characteristics
- Reality
- Stability
- Causality
- Passivity

Complex Plane

– An arbitrary network's transfer function can be described in terms of its s-domain representation

– s is a complex number $s = \sigma + j\omega$

– The impedance (or admittance) or transfer function of networks can be described in the s domain as

$$T(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \ldots + a_1 s + a_0}{s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0}$$

Transfer Functions

$$T(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \ldots + a_1 s + a_0}{s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0}$$

The coefficients *a* and *b* are real and the order *m* of the numerator is smaller than or equal to the order *n* of the denominator

A stable system is one that does not generate signal on its own.

For a stable network, the roots of the denominator should have negative real parts

Transfer Functions

The transfer function can also be written in the form

$$T(s) = a_m \frac{(s - Z_1)(s - Z_2)\ldots(s - Z_m)}{(s - P_1)(s - P_2)\ldots(s - P_m)}$$

$Z_1, Z_2, \ldots Z_m$ are the **zeros** of the transfer function

$P_1, P_2, \ldots P_m$ are the **poles** of the transfer function

For a stable network, the poles should lie on the left half of the complex plane

Fourier Transform Pairs

$a_{re}(t)$: real part of even time-domain function
$a_{ie}(t)$: imaginary part of even time-domain function
$a_{ro}(t)$: real part of odd time-domain function
$a_{io}(t)$: imaginary part of odd time-domain function

$$a(t) = a_{re}(t) + ja_{ie}(t) + a_{ro}(t) + ja_{io}(t)$$

In the frequency domain accounting for all the components, we can write:

$A_{RE}(\omega)$: real part of even function in the frequency domain
$A_{IE}(\omega)$: imaginary part of even function in the frequency domain
$A_{RO}(\omega)$: real part of odd function in the frequency domain
$A_{IO}(\omega)$: imaginary part of odd function in the frequency domain

$$A(\omega) = A_{RE}(\omega) + jA_{IE}(\omega) + A_{RO}(\omega) + jA_{IO}(\omega)$$

Fourier Transform Pairs

We also have the Fourier-transform-pair relationships:

$$
\begin{array}{lccccc}
\text{Time Domain}: & a(t) = & a_{re}(t) + & ja_{ie}(t) + & a_{ro}(t) + & ja_{io}(t) \\
 & \updownarrow & \updownarrow & \updownarrow & \multicolumn{2}{c}{\nwarrow \nearrow \quad \swarrow \searrow} \\
\text{Freq Domain}: & A(\omega) = & A_{RE}(\omega) + & jA_{IE}(\omega) + & A_{RO}(\omega) + & jA_{IO}(\omega)
\end{array}
$$

$$B(\omega) = S(\omega)\left[A_{RE}(\omega) + jA_{IE}(\omega) + A_{RO}(\omega) + jA_{IO}(\omega)\right]$$

In the time domain, this corresponds to:

$$b(t) = s(t) * \left[\left(a_{re}(t) + a_{ro}(t)\right) + j\left(a_{ie}(t) + a_{io}(t)\right)\right]$$

Fourier Transform Pairs

We now impose the restriction that in the time domain, the function must be real. As a result,

$$a_{ie}(t) = a_{io}(t) = 0 \quad \text{which implies that:} \quad A_{IE}(\omega) = A_{RO}(\omega) = 0$$

The Fourier-transform pair relationship then becomes:

$$\begin{array}{lcccc} \text{Time Domain}: & a(t) = & a_{re}(t) & + & a_{ro}(t) \\ & \uparrow & \uparrow & & \uparrow \\ & \downarrow & \downarrow & & \downarrow \\ \text{Freq Domain}: & A(\omega) = & A_{RE}(\omega) & + & jA_{IO}(\omega) \end{array}$$

The frequency-domain relations reduce to:

$$B(\omega) = S(\omega)\left[A_{RE}(\omega) + jA_{IO}(\omega)\right]$$

Fourier Transform Pairs

In summary, the general relationship is:

$$\text{Time Domain}: \quad b(t) = b_{re}(t) + jb_{ie}(t) + b_{ro}(t) + jb_{io}(t)$$

$$\begin{array}{cccc} \uparrow & \uparrow & \uparrow & \nwarrow \; \nearrow \\ \downarrow & \downarrow & \downarrow & \swarrow \; \searrow \end{array}$$

$$\text{Freq Domain}: \quad B(\omega) = B_{RE}(\omega) + jB_{IE}(\omega) + B_{RO}(\omega) + jB_{IO}(\omega)$$

But for a real system:

$$\text{Time Domain}: \quad b(t) = b_{re}(t) + jb_{ie}(t) + b_{ro}(t) + jb_{io}(t)$$

$$\begin{array}{cccc} \uparrow & \uparrow & \uparrow & \nwarrow \; \nearrow \\ \downarrow & \downarrow & \downarrow & \swarrow \; \searrow \end{array}$$

$$\text{Freq Domain}: \quad B(\omega) = B_{RE}(\omega) + jB_{IE}(\omega) + B_{RO}(\omega) + jB_{IO}(\omega)$$

Fourier Transform Pairs

So, in summary

$$\text{Time Domain}: \quad b(t) = b_e(t) + b_o(t)$$

$$\uparrow \qquad \uparrow \qquad \uparrow$$

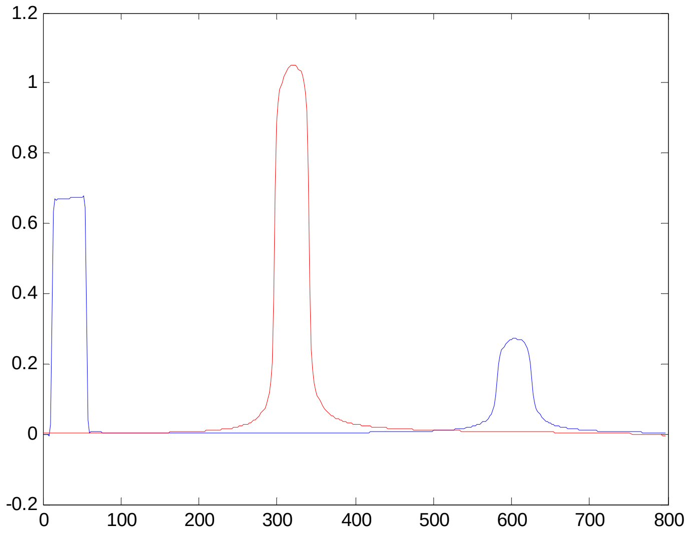
$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\text{Freq Domain}: \quad B(\omega) = B_R(\omega) + jB_I(\omega)$$

The real part of the frequency-domain transfer function is associated with the even part of the time-domain response

The imaginary part of the frequency-domain transfer function is associated with the odd part of the time-domain response

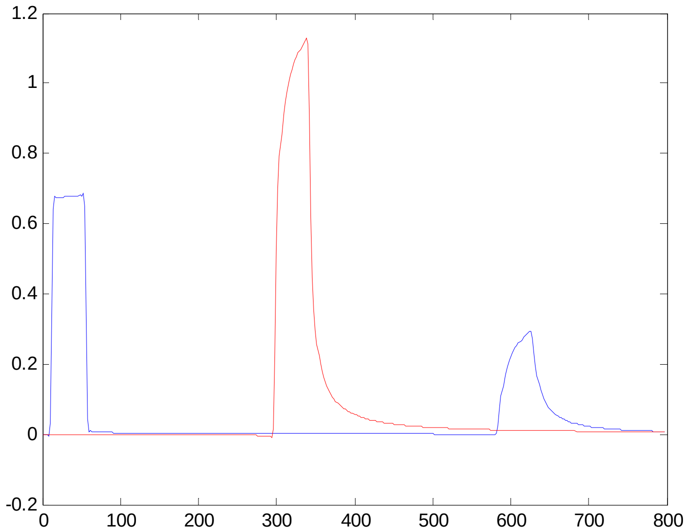
Causality Violations



NON-CAUSAL

$$Z(f) = R_o\sqrt{f} + jL\omega$$

Near (blue) and Far (red) end responses of lossy TL



CAUSAL

$$Z(f) = R_o\sqrt{f} + jR_o\sqrt{f} + jL\omega$$

Causality Principle

Consider a function *h(t)*

$$h(t) = 0, \quad t < 0$$

Every function can be considered as the sum of an even function and an odd function

$$h(t) = h_e(t) + h_o(t)$$

$$h_e(t) = \frac{1}{2}\left[h(t) + h(-t)\right]$$ **Even function**

$$h_o(t) = \frac{1}{2}\left[h(t) - h(-t)\right]$$ **Odd function**

$$h_o(t) = \begin{cases} h_e(t), & t > 0 \\ -h_e(t), & t < 0 \end{cases}$$

$$h_o(t) = \operatorname{sgn}(t) h_e(t)$$

Hilbert Transform

$$h(t) = h_e(t) + \mathrm{sgn}(t) h_e(t)$$

In frequency domain this becomes

$$H(f) = H_e(f) + \frac{1}{j\pi f} * H_e(f)$$

$$H(f) = H_e(f) - j\hat{H}_e(f)$$

➔Imaginary part of transfer function is related to the real part through the Hilbert transform

$\hat{H}_e(f)$ is the Hilbert transform of $H_e(f)$

$$\hat{x}(t) = x(t) * \frac{1}{\pi t} = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau} d\tau$$

Discrete Hilbert Transform

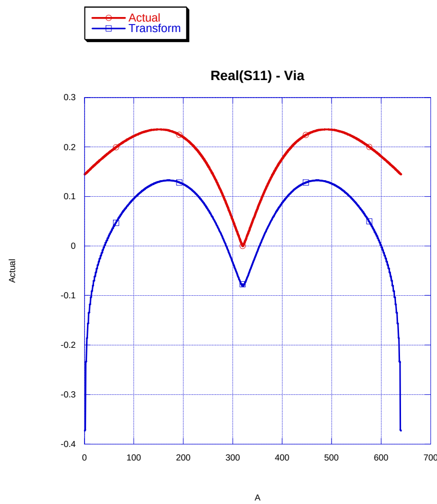
➔Imaginary part of transfer function can be recovered from the real part through the Hilbert transform

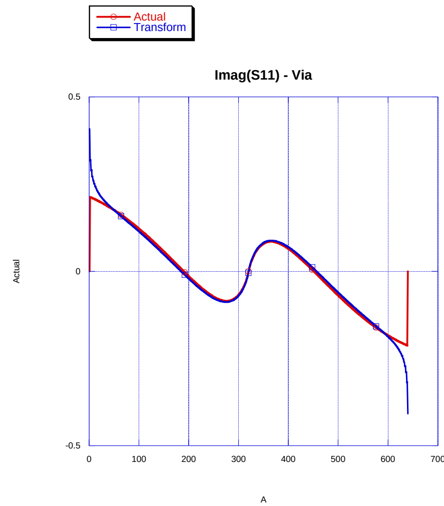
➔If frequency-domain data is discrete, use discrete Hilbert Transform (DHT)*

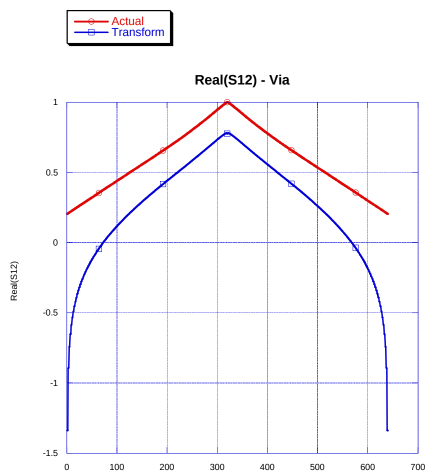
$$H(f_n) = \hat{f}_k = \begin{cases} \dfrac{2}{\pi} \displaystyle\sum_{n \text{ odd}} \dfrac{f_n}{k-n}, & k \text{ even} \\ \dfrac{2}{\pi} \displaystyle\sum_{n \text{ even}} \dfrac{f_n}{k-n}, & k \text{ odd} \end{cases}$$

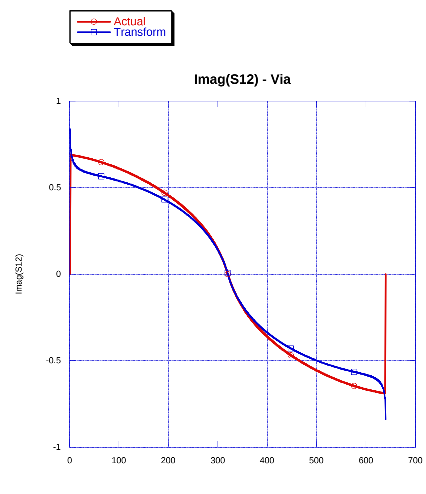
*S. C. Kak, "The Discrete Hilbert Transform", Proceedings of the IEEE, pp. 585-586, April 1970.

HT for Via: 1 MHz – 20 GHz





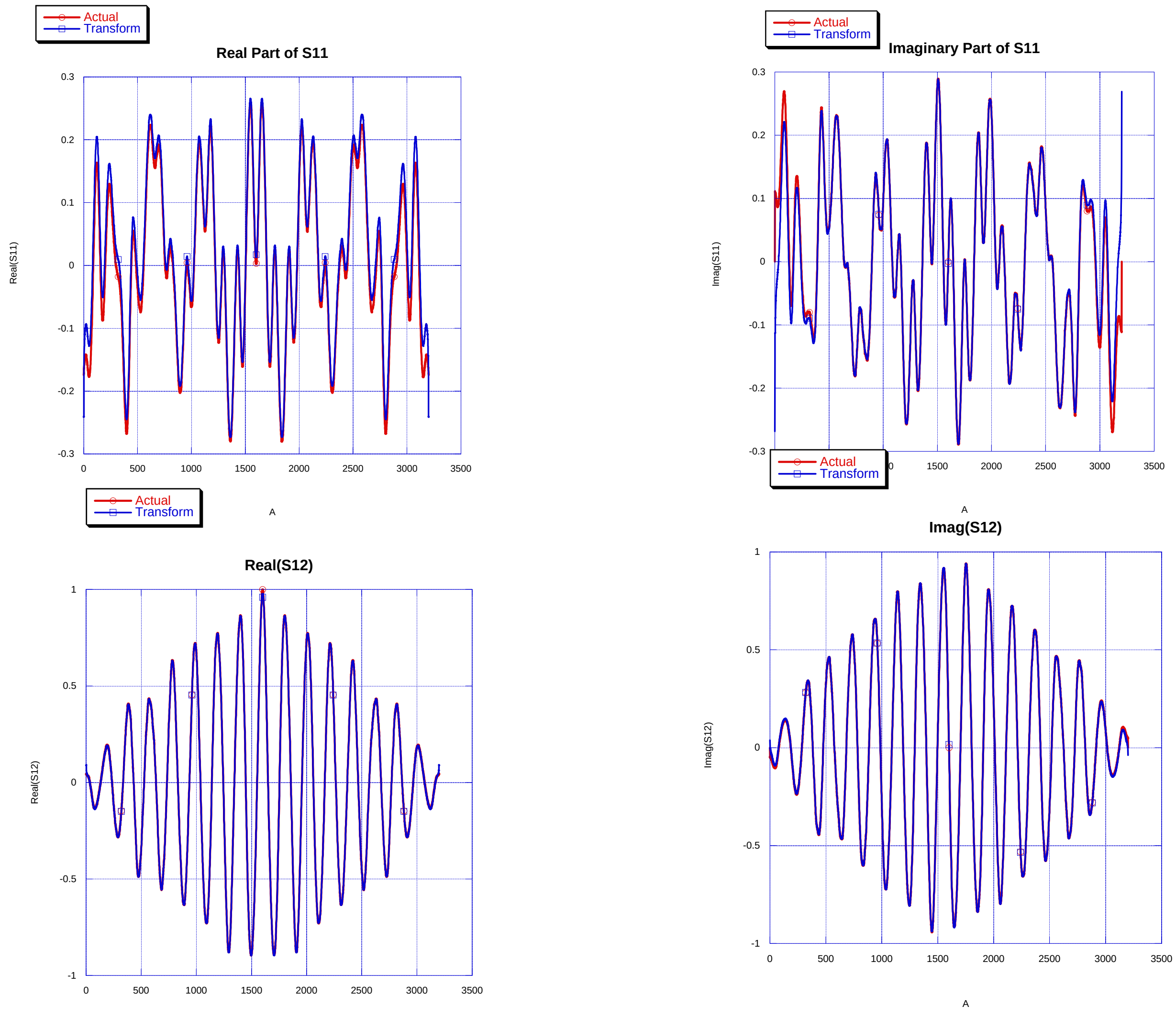




Actual is red, **HT is blue**

Observation: Poor agreement (because frequency range is limited)

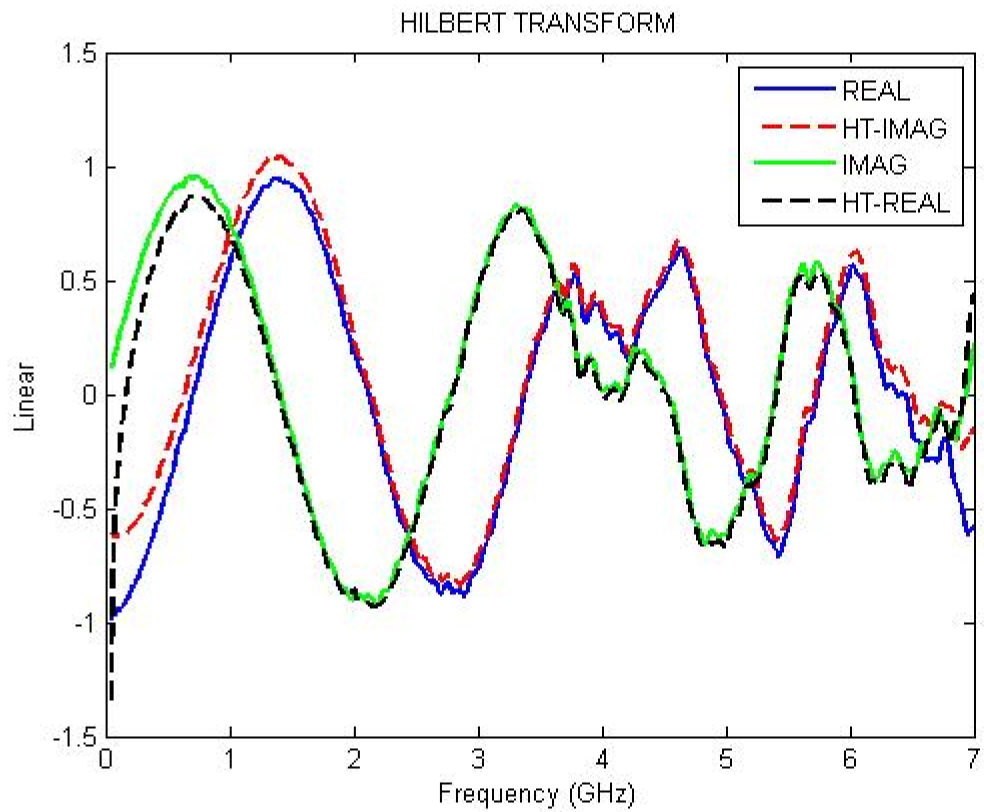
Example: 300 KHz – 6 GHz



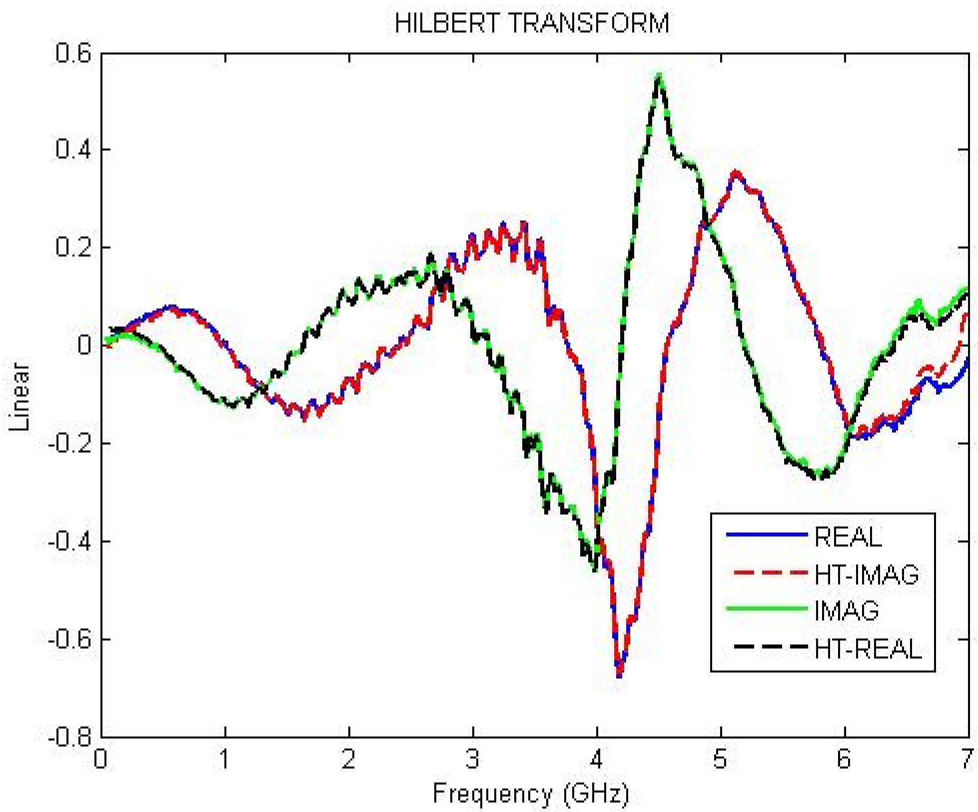
Actual is red, HT is blue

Observation: Good agreement

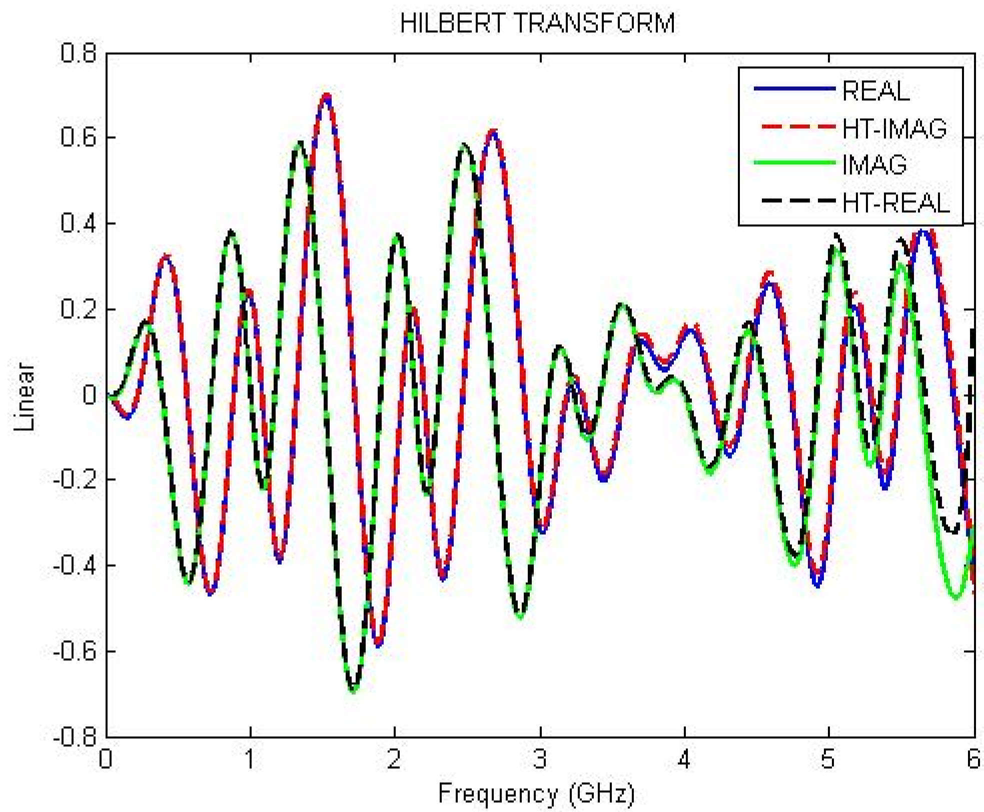
Microstrip Line S11



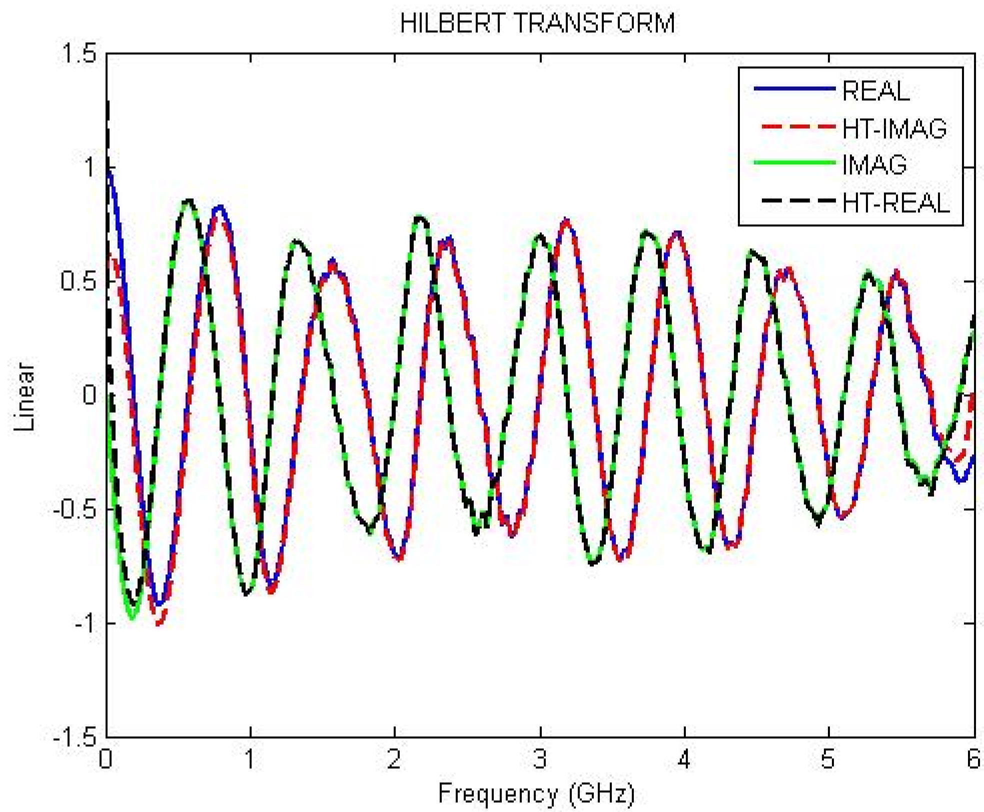
Microstrip Line S21



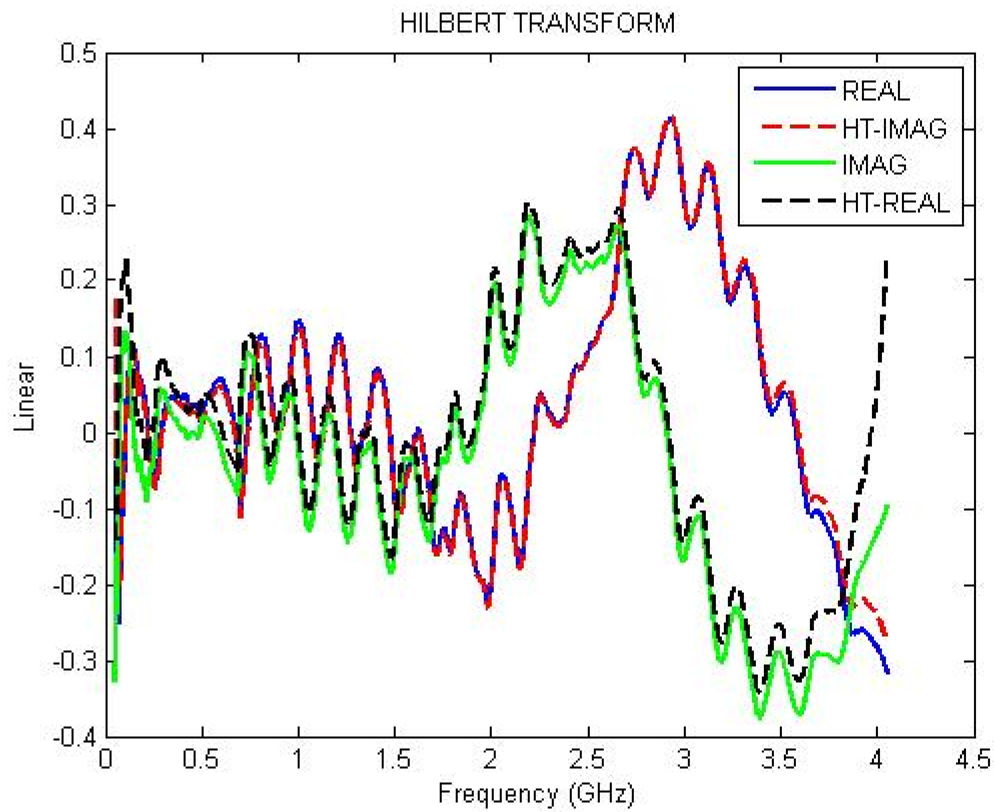
Discontinuity S11



Discontinuity S21



Backplane S11



Backplane S21