ECE 453
Wireless Communication Systems

Noise

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Noise

Random fluctuations of voltage and current

Properties

- Establishes minimum detectable signal
- Sets bounds on range of signals that can be processed
- Comes in different types (thermal, flicker, shot)
- Has frequency and time-domain characteristics
- Not fully understood

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Sources of Noise

Shot Noise

- Associated with direct current flows
- Present in diodes, MOS and BJTs
- Passage of carriers through junction is a random event
- External current composed of large independent pulses
- Instantaneous value of waveform cannot be predicted at any time
- Only information available concerns mean square value of signal
Sources of Noise

Thermal Noise

- Present in linear resistors
- Caused by thermal random motion of electrons
- Unaffected by presence or absence of direct current
- Proportional to absolute temperature $T$
- RMS noise voltage varies as square root of bandwidth
Sources of Noise

Flicker (1/f) Noise

- Found in all active devices
- Present in discrete passive devices (carbon resistors)
- Caused by traps associated with contaminations and crystal defects
- Traps capture and release carriers in random fashion
- Noise spectral density has 1/f behavior
Noise in Transistors

\[
\bar{i}_b^2 = 2qI_B \Delta f + K_1 \frac{I_B}{f} \Delta f + K_1 \frac{I_B}{f_c} \Delta f \left(1 + \left(\frac{f}{f_c}\right)^2\right)
\]

Shot noise and flicker noise asymptotes meet at corner frequency \( f_a \)
Effects of Noise

Output voltage waveform for input level comparable to noise level*

Noise from a Resistor

Random fluctuations of charge carriers in a conductor will produce thermal noise

At room temperature a resistor has available noise power $\sim -174$ dBm in 1 Hz of bandwidth
Nyquist Noise Formula

The spectral density of noise voltage at the terminals of a resistor is given by

\[ S_n(f) = 4R \frac{hf}{e^{hf/kT} - 1} \text{ (Volts}^2 / \text{Hz}) \]

\( f = \text{frequency in Hz} \)
\( h = \text{Plank’s constant} = 6.62 \times 10^{-34} \text{ joules-sec} \)
\( k = \text{Boltzman’s constant} = 1.38 \times 10^{-23} \text{ joules/°K} \)
\( T = \text{absolute temperature °K} \)
\( R = \text{resistance in ohms} \)
Noise from a Resistor

If $f \ll kT/h$, then $S_n$ can be approximated as:

$$S_n = 4kTR \text{ (Volts}^2 / \text{Hz})$$

Noise voltage at the terminals of a resistor is given by

$$V_n^2 = 4kTBR$$

$B = \text{bandwidth in Hz}$
Noise from a Resistor

AC Voltmeter

Input resistance = 10 MΩ
Bandwidth = 40 MHz

\[ V_n^2 = 4 \times 1.38 \times 10^{-23} \times 300 \times 4 \times 10^6 \times 10^7 \]
\[ V_n^2 = 66.24 \times 10^{-8} \]
\[ V_n \approx 8 \times 10^{-4} = 0.8 \text{ mV} \]
Resistors in Series

\[ V_{n1}^2 = \frac{1}{T} \int_0^T v_{n1}^2(t) \, dt \]

\[ V_{n2}^2 = \frac{1}{T} \int_0^T v_{n2}^2(t) \, dt \]

\[ V_{nT}^2 = \frac{1}{T} \int_0^T \left[ v_{n1}(t) + v_{n2}(t) \right]^2 \, dt \]

\[ V_{nT}^2 = \frac{1}{T} \int_0^T v_{n1}^2(t) \, dt + \frac{1}{T} \int_0^T 2v_{n1}(t)v_{n2}(t) \, dt + \frac{1}{T} \int_0^T v_{n2}^2(t) \, dt \]

0 if \( v_{n1} \) and \( v_{n2} \) are uncorrelated

\[ V_{nT}^2 = V_{n1}^2 + V_{n2}^2 = 4kTBR_1 + 4kTBR_2 \]
Resistors in Series

\[ V_{n_{T}} = V_{n_{1}} + V_{n_{2}} \text{ for 2 uncorrelated noise sources} \]
Effective Value of Noise Sources

\[ V_{nT}^2 = V_n^2 + V_s^2 \]
Bandwidth

\[ \frac{V_o}{V_{in}} = \frac{1}{1 + j\omega RC} \]

The output voltage drops to 0.707 of its low-frequency value when \( \omega RC = 1 \)

The half-power frequency is given by \( 2\pi f_2 RC = 1 \)

\[ f_2 = \frac{1}{2\pi RC} \]
Noise from a Circuit

\[ V_n^2 = 4kT \left[ \int_0^\infty R(f) df \right] = BR \]

\( R(f) \) is the real part of the impedance at each frequency.

\[ V_n = \sqrt{4kTBR} \]
Noise from a Circuit

For an RC circuit

For coherent signals, the bandwidth of an RC circuit is

\[ f_2 = \frac{1}{2\pi RC} \]

\(|Z|\) is 0.707R when \(\omega R = 1/R \Rightarrow 2\pi f_2 = 1/RC\)
RC Circuit

The noise from the RC circuit is given by

\[ V_n^2 = 4kT \int_0^\infty R(f) \, df \]

\[ R(f) = Re(Z) = \frac{G}{G^2 + \omega^2 C^2} \]

In terms of \( \omega_2 = 1/RC \),

\[ R(f) = R \left[ \frac{1}{R^2 + \frac{\omega^2}{\omega_2^2 R^2}} \right] \]
RC Circuit

The noise from the RC circuit is given by

\[ V_n^2 = 4kT \int_0^\infty R(f) df = \frac{4kT}{2\pi} \int_0^\infty \frac{R}{1 + \frac{\omega^2}{\omega_2^2}} d\omega \]

Using \[ \int \frac{dx}{a^2 + b^2 x^2} = \frac{1}{ab} \tan^{-1} \frac{bx}{a} \]

\[ V_n^2 = \frac{4kTR}{2\pi} \frac{\pi}{\omega_2} \frac{\pi}{2} = 4kTRf_2 \frac{\pi}{2} \]

The coherent signal bandwidth is from 0 to \( f_2 \)
For an RC circuit,

\[ V_n^2 = \frac{4kTR}{2\pi} \omega_2 \frac{\pi}{2} = 4kTRf_2 \frac{\pi}{2} \]

where \( f_2 = \frac{1}{2\pi RC} \)

Consequently,

\[ V_n^2 = 4kTR \frac{\pi}{2\pi RC^2} = \frac{kT}{C} \]

Independent of resistance!
Noise Factor

The noise at the input $N_{in}$ is given by

$$N_{in} = \frac{V_n^2}{R_{in}} = \frac{4kTBR}{(2)^2 R} = kTB$$

$S_{in}$ – Signal power into the amplifier

$N_{in}$ – Noise power into the amplifier

$S_o$ – Signal power at the output

$N_o$ – Noise power at the output
Noise Factor

Let $N_{\text{int}}$ be the noise power generated in the amplifier referenced to the amplifier input.

The total noise power at the amplifier output is:

$$N_o = N_{\text{in}}G + N_{\text{int}}G$$

where $G$ is the power gain. We define the noise factor as:

$$\text{Noise Factor} = F = \frac{S_{\text{in}} / N_{\text{in}}}{S_o / N_o} > 1$$

if noise is generated by the amplifier.
Noise Figure

\[ F = \frac{S_{in} / N_{in}}{S_{o} / N_{o}} = \frac{S_{in}}{N_{in}} \times \frac{N_{o}}{S_{o}} = \frac{S_{in}}{N_{in}} \times \frac{N_{o}}{S_{in}G} \]

\[ F = \frac{N_{o}}{kTBG} = \frac{N_{in}G + N_{int}G}{kTBG} \]

\[ F(kTB) = kTB + N_{int} \]

\[ N_{int} = (F - 1)kTB \]

The Noise Figure is the noise factor expressed in decibels

\[ NF = 10\log(F) \]
Noise Figure

\[ F = \frac{S_{in}}{S_o} \cdot \frac{N_o}{N_{in}} = 1 \times \left[ \frac{(N_{in} + N_{int})G}{N_{in}} \right] \]

\[ F = \frac{1}{G} \times \left[ \frac{(kTB + N_{int})G}{kTB} \right] \]

\[ N_{in} = kTB \text{ if matched} \]

To have a standard value for \( F \), we use \( T = 290^\circ \) for the temperature of the resistor giving the \( N_{in} \)

\[ F(G)kTB = (kTB + N_{int})G \]

\[ (F - 1)kTB = N_{int} \]
Noise Figure - Example

**Receiver**

\( NF=9 \text{ dB}, \quad BW=3 \text{ kHz}, \quad R_o = 500 \, \Omega, \quad R_{in} = 50 \, \Omega, \quad G = 10^8 \)

\[ NF = 10 \log_{10} \frac{S_{in} / N_{in}}{S_o / N_o} \]

What input signal voltage do we need to get \( S_o/N_o = 10 \)?

\[ S_o = GS_{in} \quad \quad \quad N_o = G N_o \bigg|_{in} \]

\[ N_o = kTBG + N_{int} G = kTBG + (F \, - \, 1) kTBG \]

\[ N_o \big|_{in} = F(kTB) = 8 \times 1.38 \times 10^{-23} \times 290 \times 3 \times 10^3 = 9.936 \times 10^{-17} \text{ watts} \]
Noise Figure - Example

For $S_o/N_o = 10$, we need

$$S_{in} = 10N_o\left|_{in}\right. = \frac{V_{in}^2}{R_{in}} = \frac{V_{in}^2}{50\Omega} = 10 \times 9.93 \times 10^{-17} \text{ watts}$$

$$V_{in} = 0.223 \, \mu V$$
Cascaded Stages

What is the overall Noise Factor?

\[ N_{output} = kTB G_A G_B G_C + (F_A - 1)kTB G_A G_B G_C \]
\[ \quad + (F_B - 1)kTB G_B G_C + (F_C - 1)kTB G_C \]

\[ F = \frac{N_o}{GN_{in}} = \frac{N_o}{kTB G_A G_B G_C} \]
\[ \quad = 1 + (F_A - 1) + \frac{(F_B - 1)}{G_A} + \frac{(F_C - 1)}{G_A G_B} \]
Cascaded Stages

Noise Factor of 3 stages

\[ F = F_A + \frac{(F_B - 1)}{G_A} + \frac{(F_C - 1)}{G_A G_B} \]

Frii’s formula for \( m \) cascaded stages

\[ F^T = F_1 + \sum_{n=2}^{m} \frac{F_n - 1}{\prod_{i=2}^{n} G_{i-1}} \]
Antenna

We can write:

\[ N_o = N_{int} G + N_A G \]

\[ N_A = kT_A B \]

\[ T_A = \text{equivalent temperature of the antenna} \]
200-Ω antenna exhibiting an rms noise voltage of 0.1 µV with B=10^4 Hz

\[ V^2 = 4kT_A RB \]

\[ T_A = \frac{V^2}{4kRB} = \frac{10^{-14}}{1.38 \times 10^{-23} \times 200 \times 10^4} = 90.6K \]

Noise equivalent to that of a 200-Ω resistor at temperature of 90.6K
Antenna

Antenna temperatures

- $T_A$ looking at earth = 300 K
- $T_A$ looking at the moon = 450 K
- $T_A$ looking at the sun = 3000 K

\[ N_o = kT_{eq}BG \]

where $T_{eq}$ is the temperature of the system referred to the input of the amplifier

\[ N_o = kT_ABG + N_{int}G \]

\[ kT_{eq}BG = kT_ABG + (F - 1)kT_{stan}BG \]
Antenna

\[ T_{eq} = T_A + \left( NF - 1 \right) \frac{T_{290}BG}{T_{rec}} \]

\[ F = \frac{S_{in}}{N_{int}} / \frac{S_o}{N_o} \]

For \( F = 1 \), the amplifier is noiseless and \( T_{rec} = 0 \)
RG8v cable given 3dB loss per 100 ft at 200 MHz

**Cable**

- Attenuates signal
- Attenuates noise from antenna
- Generates noise
Cable

Let \( L = \text{loss factor of cable} \)

For 6-dB loss \( L = 4 \)

\[ N \text{ into the amplifier} = kT_{\text{cable}}B \]

Also, \( N \text{ into the amplifier} = \frac{kT_{\text{cable}}B}{L} + \text{Noise generated in the finite length of cable} \)

\[ L = \frac{P_{\text{in}}}{P_{\text{out}}} \]
Cable

Let $L = \text{loss factor of cable}$  

For 6-dB loss $L = 4$

$N \text{ into the amplifier} = kT_{\text{cable}}B$

Also, $N \text{ into the amplifier} = \frac{kT_{\text{cable}}B}{L} + \text{Noise generated in the finite length of cable}$
Cable

Noise generated in the finite length of cable

\[ = kT_{\text{cable}} B - \frac{kT_{\text{cable}} B}{L} = kT_{\text{cable}} \left[ 1 - \frac{1}{L} \right] B \]

The equivalent noise temperature of a finite length of cable =

\[ T_{\text{cable}} \left[ 1 - \frac{1}{L} \right] \]

For a short cable \( L = 1 \) and \( T_{eq} \) of cable =0

For a very long cable \( L \to \infty \) and \( T_{eq} \) of cable = \( T_{\text{cable}} \)

\[ T_{\text{system}} \left|_{\text{referred to amp input}} \right. = \frac{T_A}{L} + T_{\text{cable}} \left[ 1 - \frac{1}{L} \right] + T_{\text{amp}} \]

\[ N_{o|\text{input}} = kT_{\text{system}} B \]