ECE 453
Wireless Communication Systems

Power Definitions

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Power Definitions

\[ P_{in} : \text{Power delivered to input of 2-port} \]

\[ P_{out} : \text{Power delivered to the load} \]

\[ P_{avs} : \text{Power available from the source} \]
Power Gain Definitions

Operating Power Gain

\[ G = \frac{\text{Power delivered to load}}{\text{Power delivered to input of 2-port}} = \frac{P_{out}}{P_{in}} \]

Transducer Power Gain

\[ G_T = \frac{\text{Power delivered to load}}{\text{Power available from source}} = \frac{P_{out}}{P_{avs}} \]

Available Power Gain

\[ G_A = \frac{\text{Power available from output}}{\text{Power available from source}} = \frac{P_{avo}}{P_{avs}} \]
Power Available from a Source

\[ Z_S = R_S + jX_S \quad \quad \quad Z_L = R_L + jX_L \]

\[ P_{avs} = \frac{|V_S|^2}{8R_S} \]
Transducer Gain with Z-Parameters

\[
G_T = 4 \left( \frac{\left| Z_{21} \right|^2 R_L R_S}{\left| (Z_{11} + Z_S)(Z_{22} + Z_L) - Z_{12}Z_{21} \right|^2} \right)
\]
Linear Amplifiers

The transducer power gain is defined as the power delivered to the load divided by the power available from the source.

\[ P_{avS} = \frac{|b_s|^2}{1 - |\Gamma_s|^2} \]

\[ \Gamma_s = \frac{Z_s - Z_o}{Z_s + Z_o} \]

\[ \Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} \]

\( Z_o \) is reference impedance for S parameters
Transducer Gain

Definition of transducer gain

\[
G_T = \frac{P_{\text{del}}}{P_{\text{avs}}} = \frac{|b_2|^2 \left(1 - |\Gamma_L|^2\right)}{|b_s|^2 \left(1 - |\Gamma_S|^2\right)}
\]

In terms of two-port scattering parameters

\[
G_T = \frac{|S_{21}|^2 \left(1 - |\Gamma_S|^2\right) \left(1 - |\Gamma_L|^2\right)}{\left|\left(1 - S_{11}\Gamma_S\right)\left(1 - S_{22}\Gamma_L\right) - S_{21}S_{12}\Gamma_S\Gamma_L\right|^2}
\]
Linear Amplifiers

If we assume that the network is unilateral, then we can neglect $S_{12}$ and get the unilateral transducer gain for $S_{12}=0$.

$$G_{TU} = |S_{21}|^2 \frac{(1-|\Gamma_S|^2)}{|1-S_{11}\Gamma_S|^2} \frac{(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2}$$

The first term ($|S_{21}|^2$) depends on the transistor. The other 2 terms depend on the source and the load.
Linear Amplifiers

$G_s$ affects the degree of mismatch between the source and the input reflection coefficient of the two-port.
Linear Amplifiers

\( G_L \) affects the degree of mismatch between the load and the output reflection coefficient of the 2-port.
Linear Amplifiers

$G_o$ depends on the device and bias conditions
Linear Amplifiers

Maximum unilateral transducer gain can be accomplished by choosing impedance matching networks such that.

\[ \Gamma_S = S_{11}^* \]
\[ \Gamma_L = S_{22}^* \]

\[ G_{\text{UMAX}} = \frac{1}{1 - \left| S_{11} \right|^2} \cdot \left| S_{21} \right|^2 \cdot \frac{1}{1 - \left| S_{22} \right|^2} \]
Linear Amplifiers

\[ G_{UMAX} (dB) = G_{S_{\text{max}}} (dB) + G_o (dB) + G_{L_{\text{max}}} (dB) \]

For \( \Gamma_S = S_{11}^* \), \( G_S \) is a maximum

For \( |\Gamma_S| = 1 \), \( G_S \) is 0
Dissipated Power

\[ P_d = \frac{1}{2} a^T (U - S^T S^*) a^* \]

The dissipation matrix \( D \) is given by:

\[ D = U - S^T S^* \]

Passivity insures that the system will always be stable provided that it is connected to another passive network.

For passivity
- (1) the determinant of \( D \) must be \( \geq 0 \)
- (2) the determinant of the principal minors must be \( \geq 0 \)
Dissipated Power

When the dissipation matrix is 0, we have a lossless network ➔

\[ S^T S^* = U \]

The S matrix is unitary.

For a lossless two-port:

\[ |S_{11}|^2 + |S_{21}|^2 = 1 \]
\[ |S_{22}|^2 + |S_{12}|^2 = 1 \]

If in addition the network is reciprocal, then

\[ S_{12} = S_{21} \quad \text{and} \quad |S_{11}| = |S_{22}| = \sqrt{1 - |S_{12}|^2} \]