ECE 453
Wireless Communication Systems

Stability

Jose E. Schutt-Aine
Electrical & Computer Engineering
University of Illinois
jesa@illinois.edu
Stability Considerations

Before maximizing transducer gain, and perform conjugate match, it is necessary to study stability of two-port
Reflection Coefficients

Input reflection coefficient associated with $Z_{in}$

$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}$$

Output reflection coefficient associated with $Z_{out}$

$$\Gamma_{out} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S}$$
Stability

A network is **conditionally stable** if the real part of $Z_{in}$ and $Z_{out}$ is greater than zero for *some* positive real source and load impedances at a specific frequency.

A network is **unconditionally stable** if the real part of $Z_{in}$ and $Z_{out}$ is greater than zero for *all* positive real source and load impedances at a specific frequency.
Stability Factor

Positive real source and load impedances imply that

\[ |\Gamma_s| \text{ and } |\Gamma_L| \leq 1 \]

If we want to match input and output for maximum power transfer, we have

\[ \Gamma_s = \Gamma_{in}^* \quad \Gamma_L = \Gamma_{out}^* \]

The \( K \) or Rollet Stability Factor for stability requires that

\[
K = \frac{1 + |S_{11}S_{22} - S_{12}S_{21}|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}||S_{21}|} > 1
\]

\( K \) factor must not be considered alone
Stability Circle

\[ |\Gamma_{in}| = \left| S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} \right| = 1 \]

The solution for \( \Gamma_L \) will lie on a circle

\[ \text{radius} = r_L = \left| \frac{S_{21}S_{12}}{|S_{22}|^2 - |\Delta|^2} \right| \]

\[ \text{center} = C_L = \frac{\left( S_{22} - \Delta S_{11}^* \right)^*}{|S_{22}|^2 - |\Delta|^2} \]

\[ \Delta = S_{11}S_{22} - S_{12}S_{21} \]
Stability Circle for $\Gamma_L$

Area inside or outside stability circle will represent a stable operating condition
Stability Circle for $\Gamma_L$

To determine stable area, make $Z_L = Z_o$ or $\Gamma_L = 0$. If $|\Gamma_{in}| < 1$, then area corresponding to center of Smith chart is stable.
To determine unstable area, make $Z_L = Z_o$ or $\Gamma_L = 0$. If $|\Gamma_{in}| > 1$, then area corresponding to center of Smith chart is unstable.
Stability Circle for $\Gamma_L$
Unconditional Stability

To insure unconditional stability for any passive load, stability circles must lie completely out of the Smith chart.
Unconditional Stability

1. Case where center of Smith chart is outside of stability circle

\[ |S_{22}|^2 - |\Delta|^2 > 0 \]

\[ \left| S_{22}^* - \Delta^* S_{11} - |S_{12} S_{21}| \right| > 1 \]

2. Case where center of Smith chart is inside of stability circle

\[ |S_{22}|^2 - |\Delta|^2 < 0 \]

\[ \left| S_{12} S_{21} - |S_{22}^* - \Delta^* S_{11}| \right| > 1 \]
Unconditional Stability

Both cases can be combined into a single inequality

\[
\frac{|S_{22}^* - \Delta^* S_{11} - S_{12} S_{21}|}{|S_{22}|^2 - |\Delta|^2} > 1
\]

which is valid for either case
Unconditional Stability

Criteria for unconditional stability

\[ K > 1, \quad |S_{12}S_{21}| < 1 - |S_{11}|^2 \]
\[ K > 1, \quad |S_{12}S_{21}| < 1 - |S_{22}|^2 \]
\[ K > 1, \quad B_1 > 0 \]
\[ K > 1, \quad B_2 > 0 \]
\[ K > 1, \quad |D| < 1 \]

\[ \mu_{ES} = \frac{1 - |S_{11}|^2}{|S_{22} - S_{11}^*D| + |S_{12}S_{21}|} > 1 \]

\[ \mu_{ES}' = \frac{1 - |S_{22}|^2}{|S_{11} - S_{22}^*D| + |S_{12}S_{21}|} > 1 \]

\[ B_1 = 1 + |S_{11}|^2 - |D|^2 - |S_{22}|^2 \]
\[ B_2 = 1 + |S_{22}|^2 - |D|^2 - |S_{11}|^2 \]
\[ D = S_{11}S_{22} - S_{12}S_{21} \]

\[ K = \frac{1 + |S_{11}S_{22} - S_{12}S_{21}|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}||S_{21}|} > 1 \]
Stability Circle for $\Gamma_L$

Stability circles are functions of frequency.