ECE 453
Wireless Communication Systems

Stability

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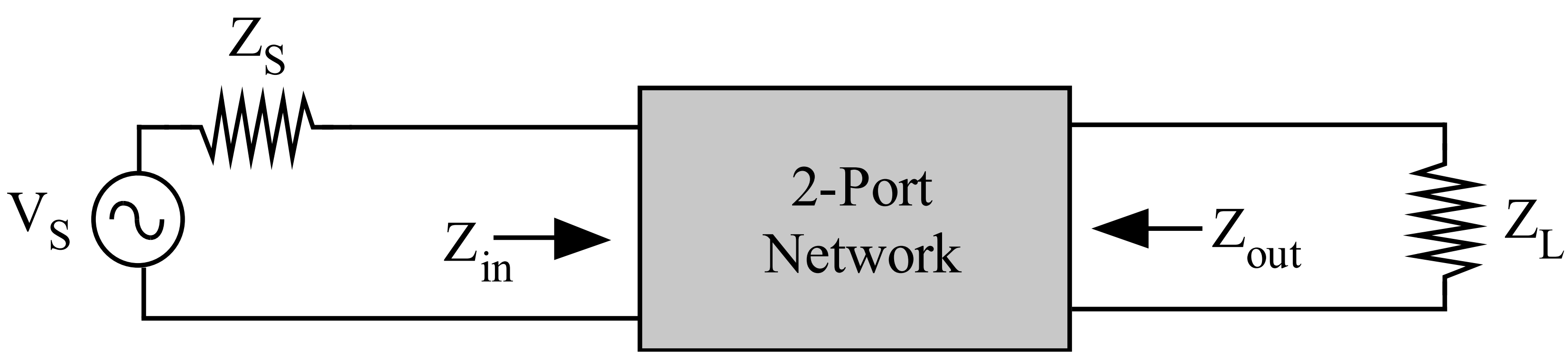
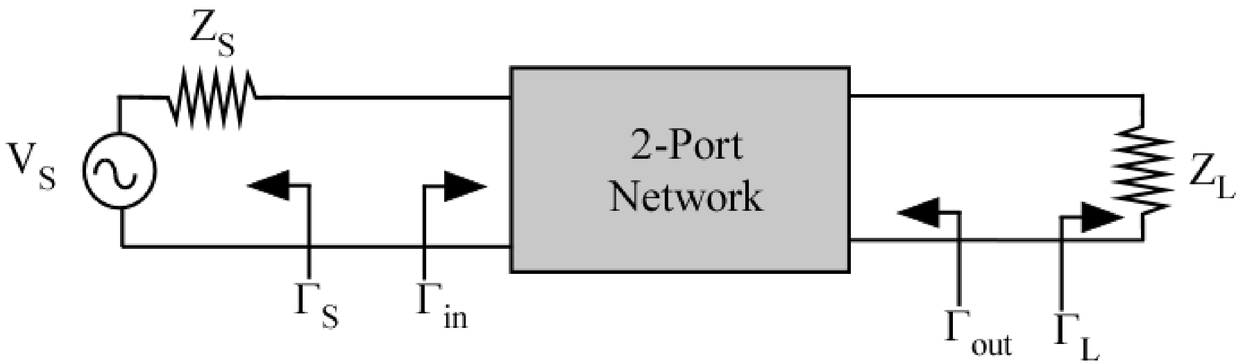
Stability Considerations

Before maximizing transducer gain, and perform conjugate match, it is necessary to study stability of two-port



Reflection Coefficients



Input reflection coefficient associated with Z_{in}

$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

Output reflection coefficient associated with Z_{out}

$$\Gamma_{out} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$$

Stability

A network is **conditionally stable** if the real part of Z_{in} and Z_{out} is greater than zero for ***some*** positive real source and load impedances at a specific frequency

A network is **unconditionally stable** if the real part of Z_{in} and Z_{out} is greater than zero for ***all*** positive real source and load impedances at a specific frequency

Stability Factor

Positive real source and load impedances imply that

$$|\Gamma_S| \; and \; |\Gamma_L| \le 1$$

If we want to match input and output for maximum power transfer, we have

$$\Gamma_S = \Gamma_{in}^* \qquad \Gamma_L = \Gamma_{out}^*$$

The *K* or *Rollet Stability Factor* for stability requires that

$$K = \frac{1 + |S_{11}S_{22} - S_{12}S_{21}|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}||S_{21}|} > 1$$

***K* factor must not be considered alone**

Stability Circle

$$\left|\Gamma_{in}\right| = \left| S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} \right| = 1$$

The solution for Γ_L will lie on a circle

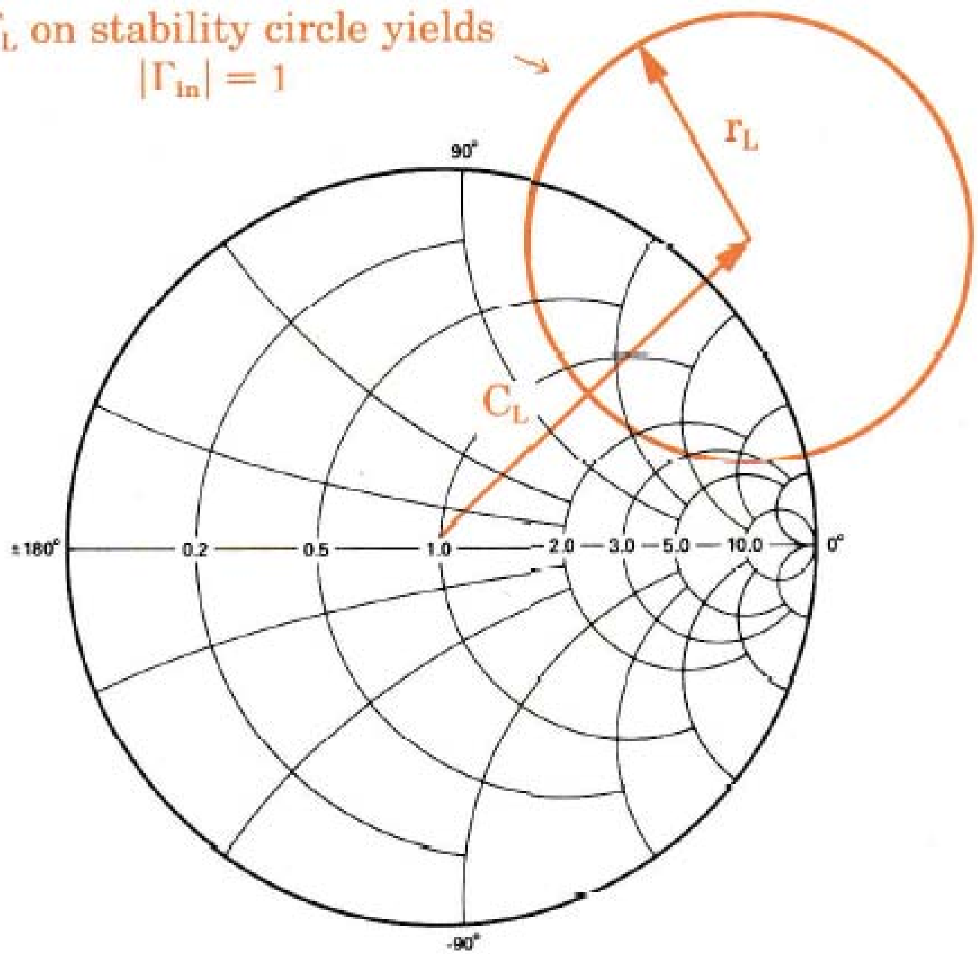
$$radius = r_L = \left| \frac{S_{21}S_{12}}{\left|S_{22}\right|^2 - \left|\Delta\right|^2} \right|$$

$$center = C_L = \frac{\left(S_{22} - \Delta S_{11}^*\right)^*}{\left|S_{22}\right|^2 - \left|\Delta\right|^2}$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

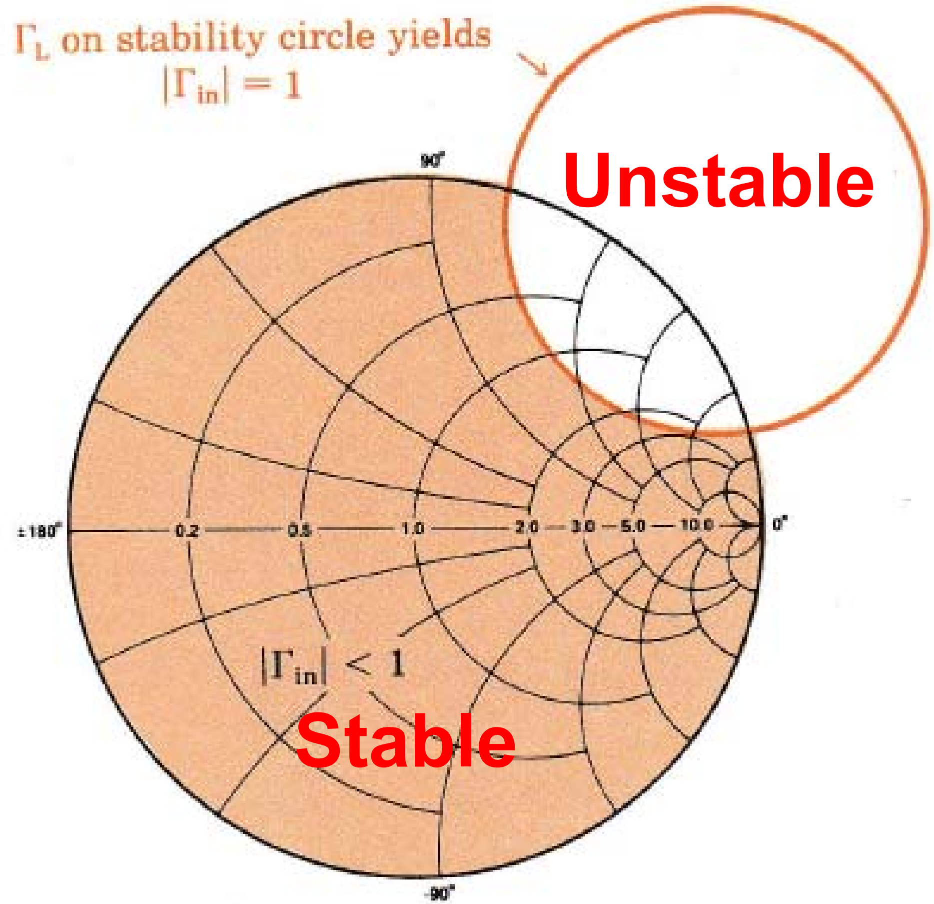
Stability Circle for Γ_L

Area inside or outside stability circle will represent a stable operating condition



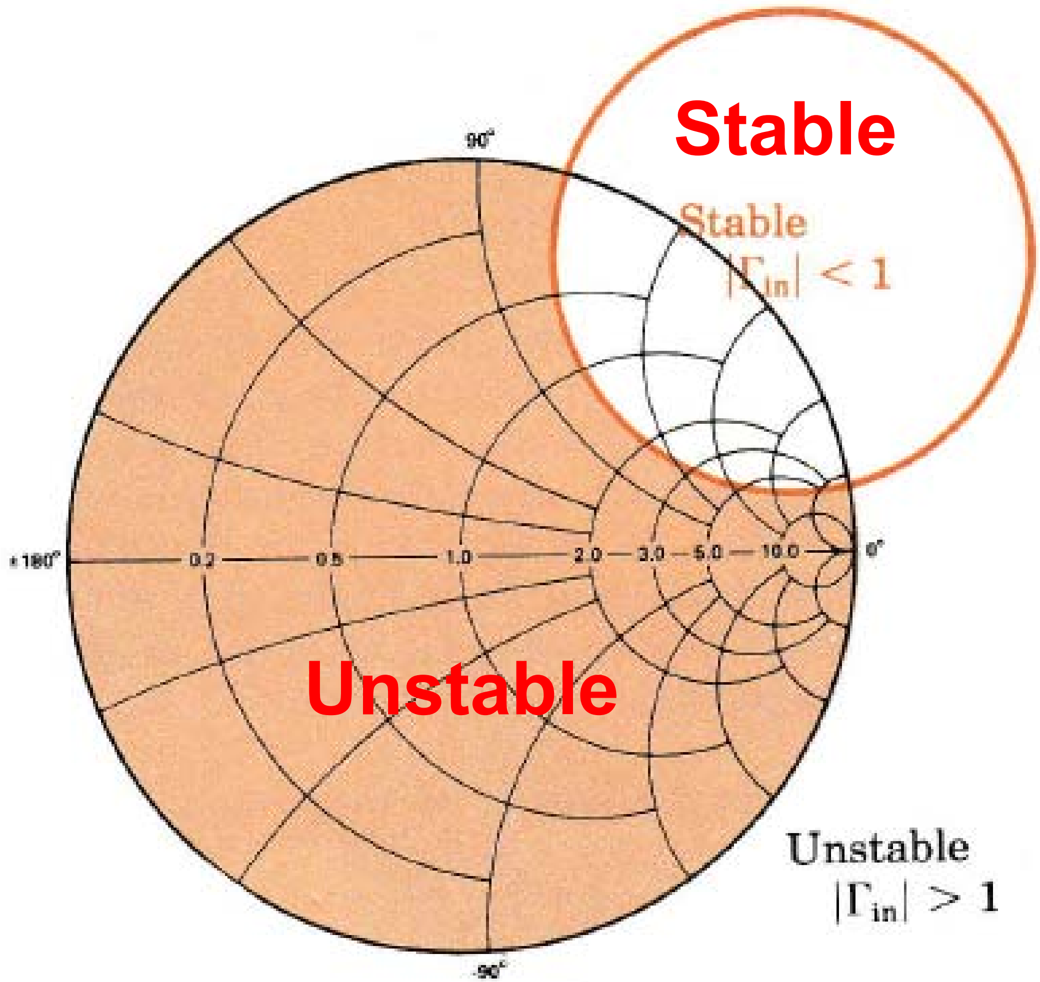
Stability Circle for Γ_L

To determine stable area, make $Z_L = Z_o$ or $\Gamma_L = 0$. If $|\Gamma_{in}| < 1$, then area corresponding to center of Smith chart is stable.

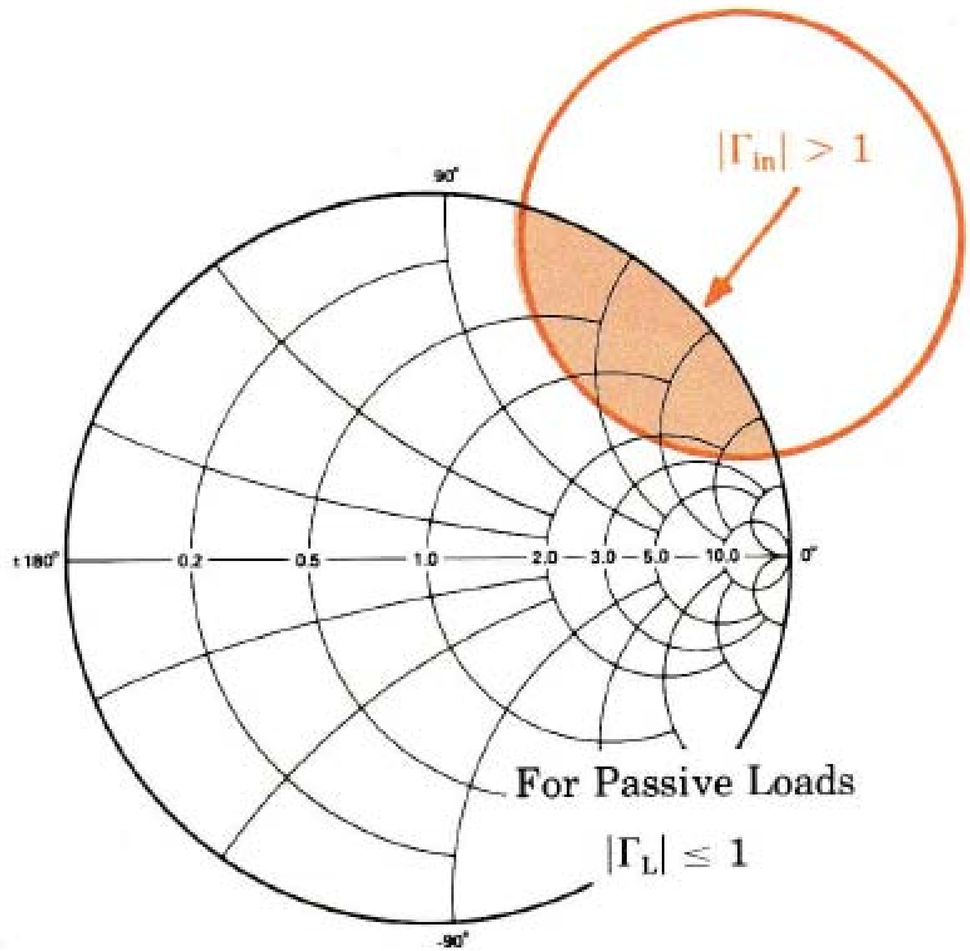


Stability Circle for Γ_L

To determine unstable area, make $Z_L = Z_o$ or $\Gamma_L = 0$. If $|\Gamma_{in}| > 1$, then area corresponding to center of Smith chart is unstable.

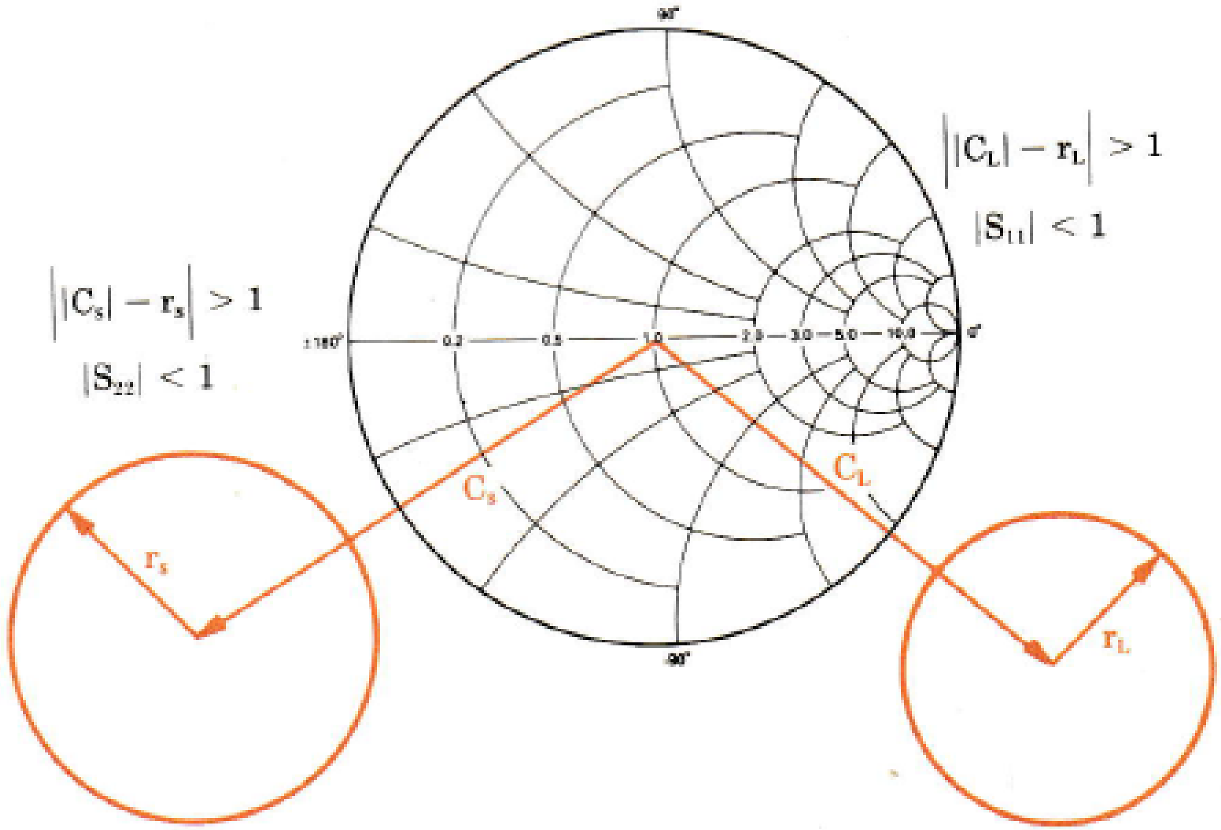


Stability Circle for Γ_L



Unconditional Stability

To insure unconditional stability for any passive load, stability circles must lie completely out of the Smith chart.



Unconditional Stability

1. Case where center of Smith chart is outside of stability circle

$$|S_{22}|^2 - |\Delta|^2 > 0$$

$$\frac{|S_{22}^* - \Delta^* S_{11}| - |S_{12}S_{21}|}{\left||S_{22}|^2 - |\Delta|^2\right|} > 1$$

2. Case where center of Smith chart is inside of stability circle

$$|S_{22}|^2 - |\Delta|^2 < 0$$

$$\frac{|S_{12}S_{21}| - |S_{22}^* - \Delta^* S_{11}|}{\left||S_{22}|^2 - |\Delta|^2\right|} > 1$$

Unconditional Stability

Both cases can be combined into a single inequality

$$\frac{\left|S_{22}^{*}-\Delta^{*}S_{11}\right|-\left|S_{12}S_{21}\right|}{\left|\left|S_{22}\right|^{2}-\left|\Delta\right|^{2}\right|}>1$$

which is valid for either case

Unconditional Stability

Criteria for unconditional stability

$$K > 1, \quad |S_{12}S_{21}| < 1 - |S_{11}|^2$$

$$K > 1, \quad |S_{12}S_{21}| < 1 - |S_{22}|^2$$

$$K > 1, \quad B_1 > 0$$

$$K > 1, \quad B_2 > 0$$

$$K > 1, \quad |D| < 1$$

$$\mu_{ES} = \frac{1 - |S_{11}|^2}{|S_{22} - S_{11}^* D| + |S_{12}S_{21}|} > 1$$

$$\mu'_{ES} = \frac{1 - |S_{22}|^2}{|S_{11} - S_{22}^* D| + |S_{12}S_{21}|} > 1$$

$$B_1 = 1 + |S_{11}|^2 - |D|^2 - |S_{22}|^2$$

$$B_2 = 1 + |S_{22}|^2 - |D|^2 - |S_{11}|^2$$

$$D = S_{11}S_{22} - S_{12}S_{21}$$

$$K = \frac{1 + |S_{11}S_{22} - S_{12}S_{21}|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}||S_{21}|} > 1$$

Stability Circle for Γ_L

Stability circles are functions of frequency.