# ECE 453
# Wireless Communication Systems

# Oscillators

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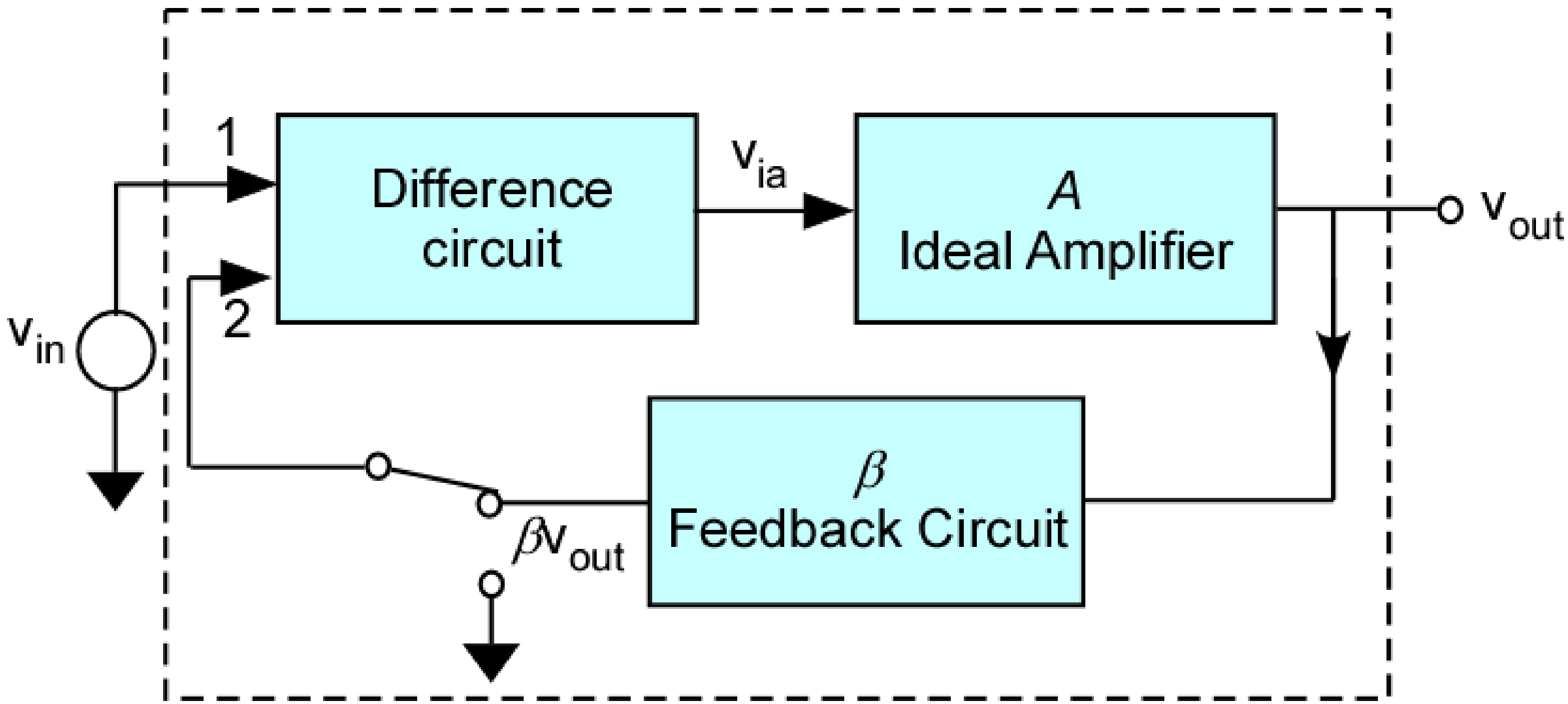
# Feedback – Basic Concept



# Feedback and Frequency Dependence

1. The closed-loop transfer function is a function of frequency

2. The manner in which the loop gain varies with frequency determines the stability or instability of the feedback amplifier

3. The frequency at which the phase of the transfer function is equal to $180^o$ will be unstable if the magnitude is greater than unity

# Feedback and Stability

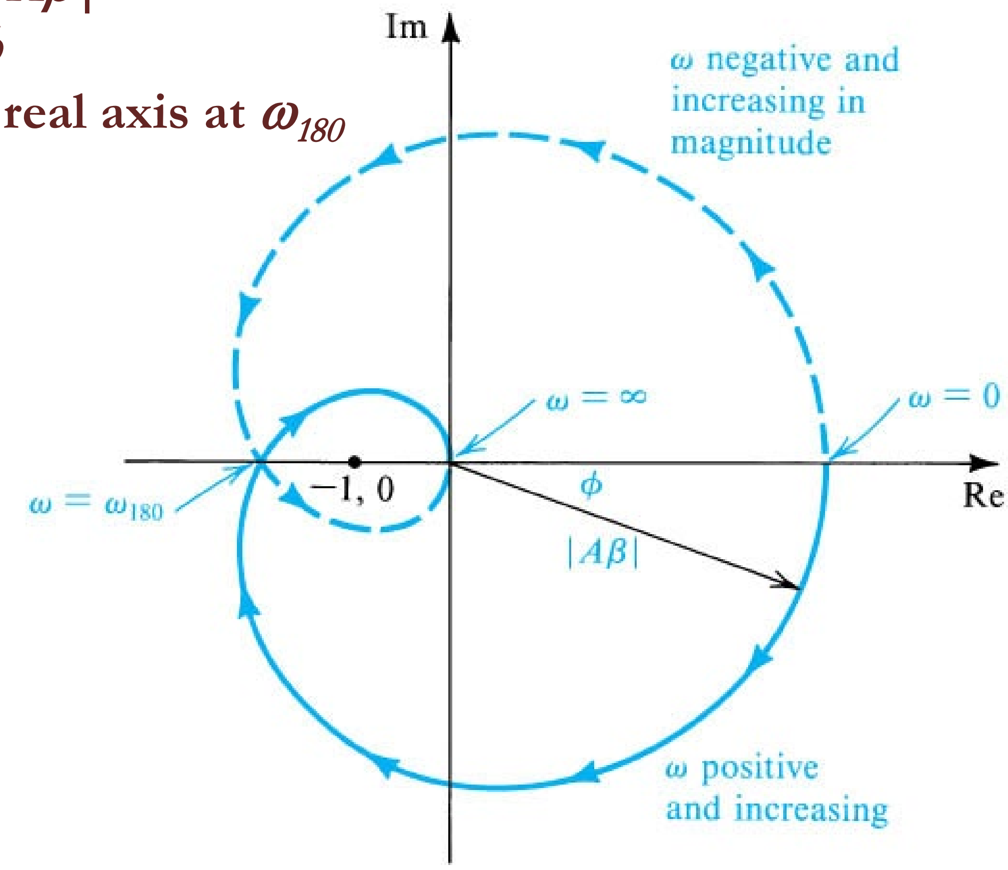
$$A_f(s) = \frac{A(s)}{1 + A(s)\beta(s)}$$

$$A_f(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)\beta(j\omega)}$$

**When loop gain $A(j\omega)\beta(j\omega)$ has 180° phase, we have positive feedback**

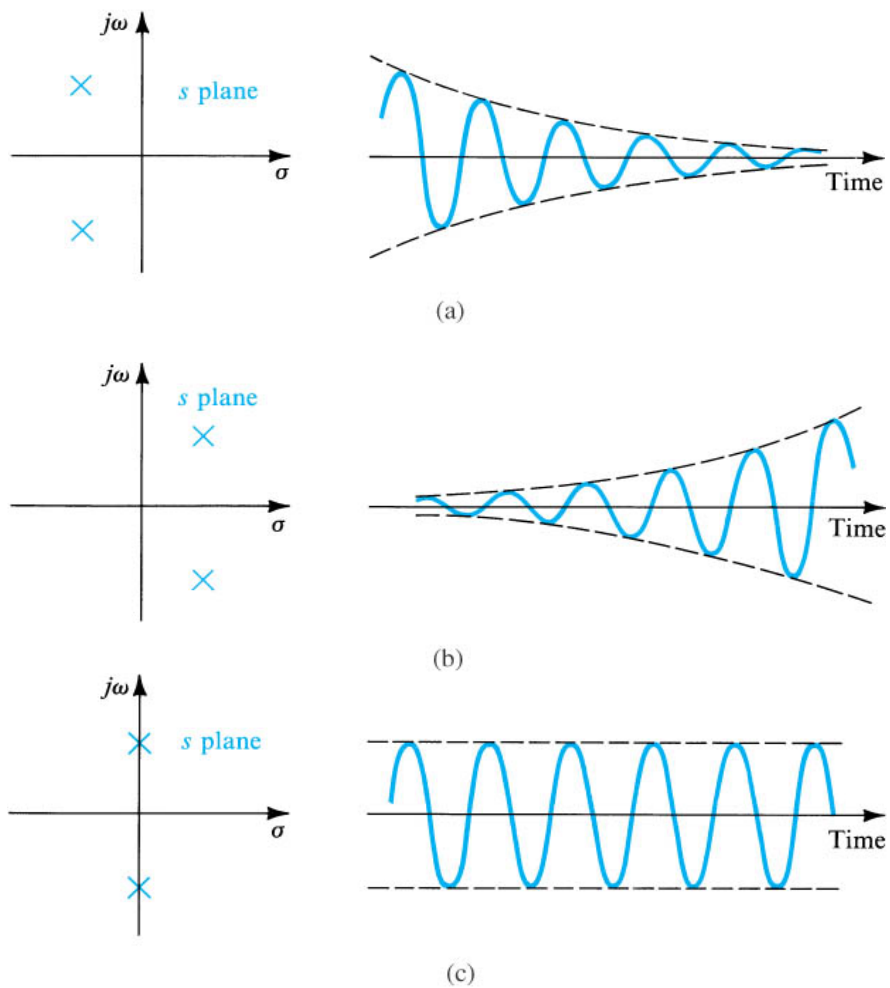
# Nyquist Plot

- Radial distance is $|A\beta|$
- Angle is phase of $\phi$
- Intersects negative real axis at $\omega_{180}$

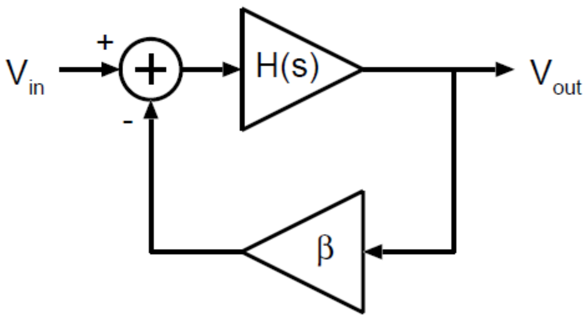


# Stability and Pole Location

$$v(t) = e^{\sigma_o t}\left[e^{+j\omega t} + e^{-j\omega t}\right] = 2e^{\sigma_o t}\cos\left(\omega_n t\right)$$



# Oscillator




- Closed-Loop Transfer function:

$$-\frac{V_{out}}{V_{in}}(s) = \frac{H(s)}{1+\beta H(s)}, where\ s = j\omega$$

- Barkhausen's criteria for oscillation:

$\Rightarrow |\beta H(j\omega_0)| = 1$

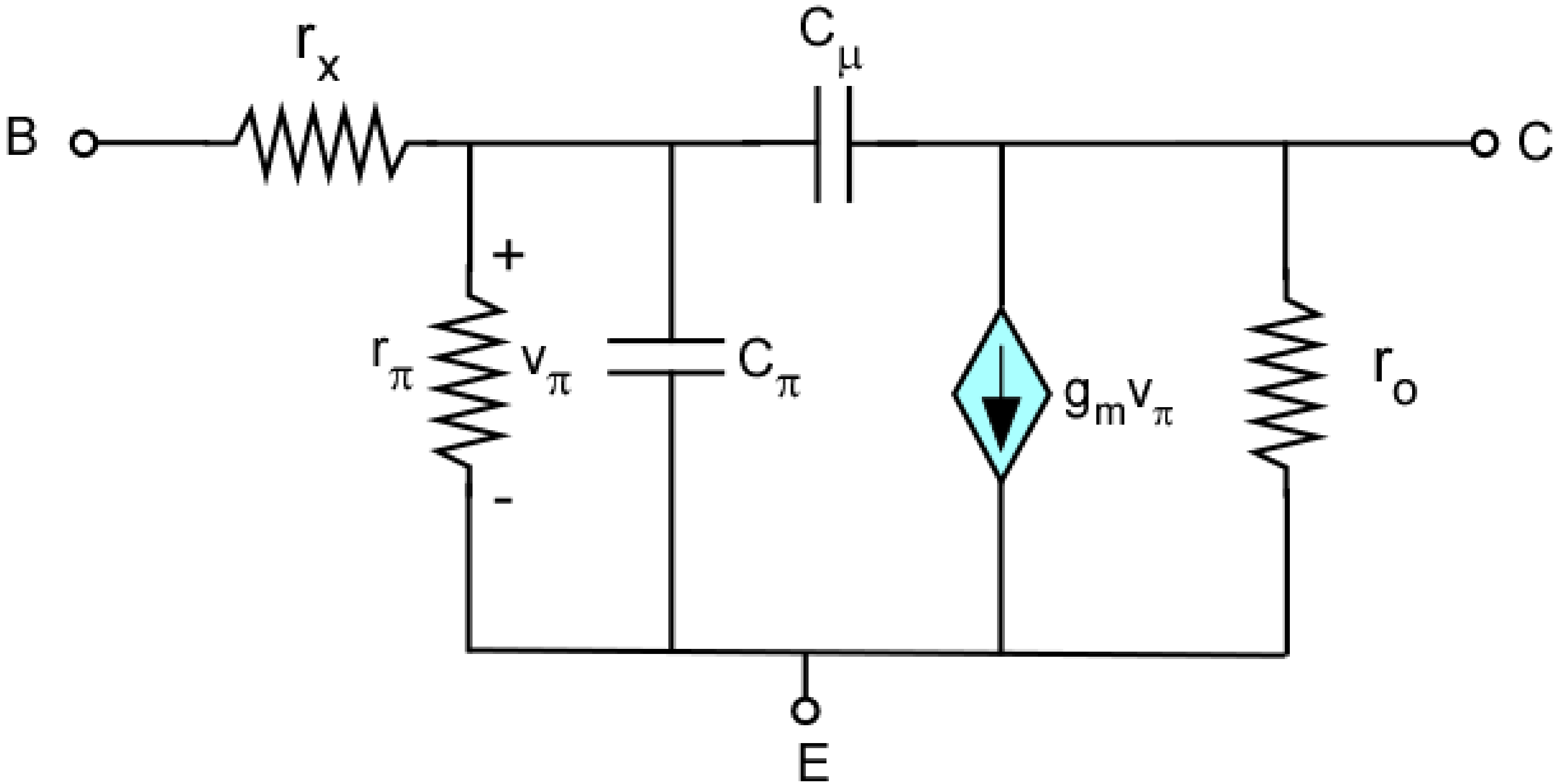
$\Rightarrow \arg\big(\beta H(j\omega_0)\big) = -180^0.$

- $\omega_0$ = oscillation-frequency.

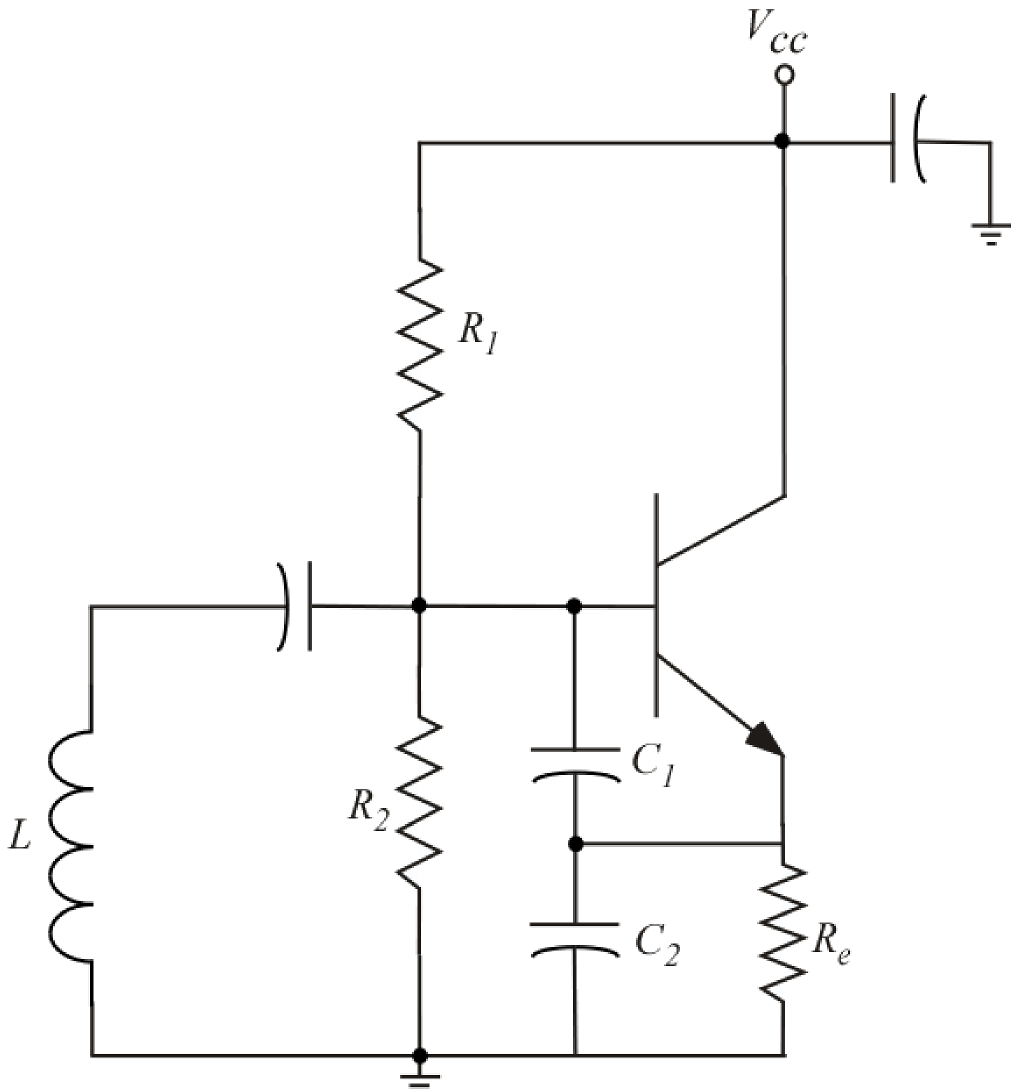
# Oscillator Topologies

- **Common-base topology is usually preferred because**
  - Feedback within transistor is minimized (low Miller)
  - External feedback is dominant
  - Current gain has little phase shift and is constant with $f$
- **Oscillator built around 2 major requirements:**
  - Oscillation frequency $f_o$
  - Power $P_o$ that must be delivered to a load
- **Other criteria**
  - Efficiency
  - Spectral purity
  - Frequency stability

# BJT – High-Frequency Model



# Common-Collector Colpitts

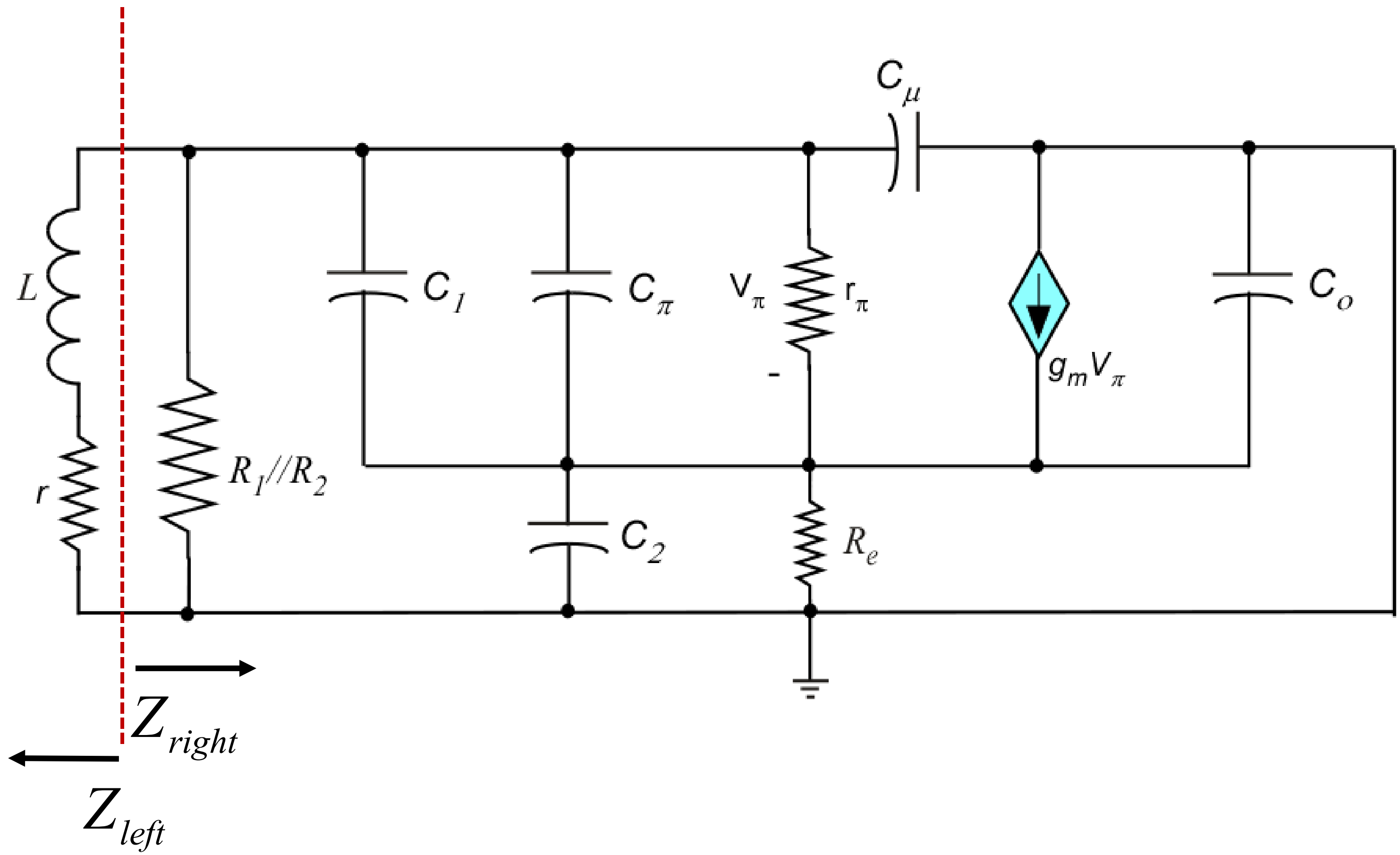



- Minimize circuit dependence on transistor internal parameters

- Choose

$$C_1 \gg C_\pi$$
$$C_2 \gg C_o$$

# Incremental Model




**For Oscillation, we want $Z_{left} = -Z_{right}$**

# Oscillation Criteria

Condition gives

$$\frac{1}{\omega^2 C_1^2 r_\pi} + \frac{1}{\omega^2 C_2^2 R_e} + r + g_m \left( \frac{1}{\omega^4 C_1^2 C_2^2 r_\pi R_e} - \frac{1}{\omega^2 C_1 C_2 R_e} \right) = 0$$

Term in $\omega^{-4}$ can be neglected. This gives

$$\frac{g_m}{\omega^2 C_1^2 \beta} + \frac{1}{\omega^2 C_2^2 R_e} + r - \frac{g_m}{\omega^2 C_1 C_2 R_e} = 0$$

Gives transconductance $g_{ms}$ to set net resistance to zero

$$g_{ms} = \frac{\omega^2 C_1 C_2 r + \dfrac{C_1}{C_2 R_e}}{1 - \dfrac{C_2}{C_1 \beta}}$$

# Oscillation Conditions

$g_{ms}$ is the transconductance needed to support steady-state oscillations

If $g_m \ll \omega C_1 \beta$ and assuming that $\beta$ is large,

$$g_{ms} \simeq \omega^2 C_1 C_2 r + \frac{C_1}{C_2}\frac{1}{R_e}$$

In terms of the inductor quality factor $Q_L$

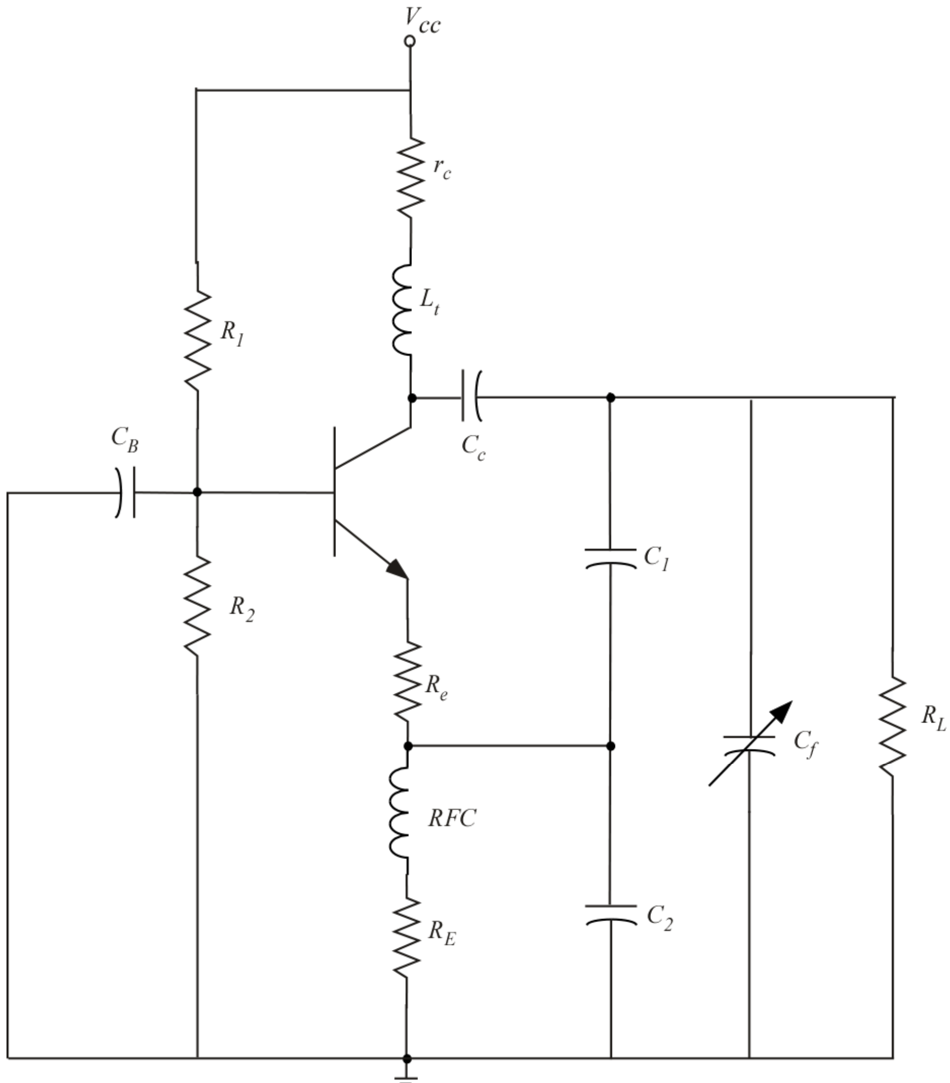
$$g_{ms} \simeq \frac{\omega_o \left(C_1 + C_2\right)}{Q_L} + \frac{C_1}{C_2}\frac{1}{R_e}$$

# CC-Colpitts - Observation

- **To implement tunable oscillator**
  - Use variable capacitor in parallel or is series with inductor
- **To extract power from oscillator**
  - Place resistance in shunt with inductor
  - Account for loading effect in analysis and design

1. **Oscillation amplitude builds up and $v_{be}$ increases**
2. **Transistor moves out of linear range and operation becomes nonlinear. Transconductance decreases.**
3. **When transconductance reaches $g_{ms}$, steady state operation is achieved.**

# Common-Base Colpitts




$C_f$: tuning capacitor

$C_1$ & $C_2$: feedback ratio

*RFC*: RF choke to prevent power dissipation in $R_E$

$R_P$ is equivalent resistance of coil $L_t$.

## DEFINITIONS

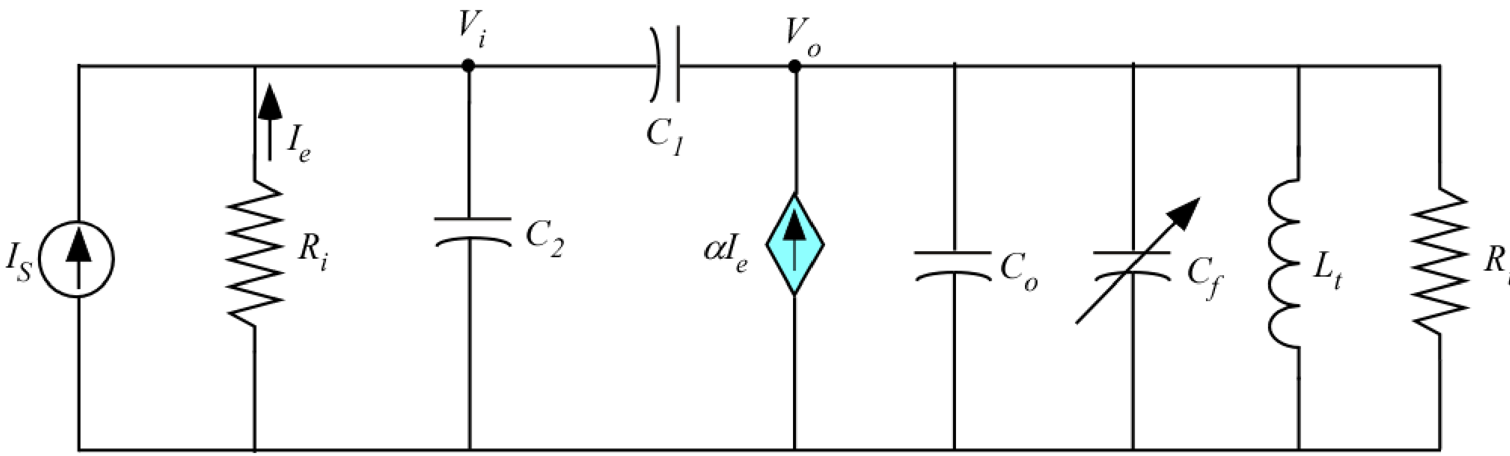
$$R_t = R_L \parallel R_p = \frac{R_L R_p}{R_L + R_p}$$

$$R_i = R_e + r_e$$

$$g_i = 1 / R_i$$

$$g_t = 1 / R_t$$

# CB-Colpitts - Incremental Model




$$\begin{bmatrix} I_S \\ 0 \end{bmatrix} = \begin{bmatrix} g_i + s\left(C_1 + C_2\right) & -sC_1 \\ -\alpha g_i - sC_1 & g_t + s\left(C_1 + C_o + C_f\right) + \dfrac{1}{sL_t} \end{bmatrix} \begin{bmatrix} V_i \\ V_o \end{bmatrix}$$

**Setting the determinant to zero provides the criterion for the onset of oscillation.**

# CB-Colpitts - Oscillation Conditions

**Setting the determinant to zero gives.**

$$\Delta(s) = g_i + s\left(L_t g_t g_i + C_a\right) + s^2\left(L_t g_i C_b + L_t g_t C_a - L_t C_1 \alpha g_i\right) + s^3\left(L_t C_a C_b - L_t C_1^2\right) = 0$$

**Defining** $C_a = C_1 + C_2$ **and** $C_b = C_1 + C_o + C_f$

**we get a complex conjugate pair of roots. This leads to 2 equations for real and imaginary parts that must be equal to zero separately**

$$\mathrm{Re}\left[\Delta(j\omega)\right] = g_i - \omega^2 L_t\left(C_b g_i + C_a g_t - C_1 \alpha g_i\right) = 0$$

$$\mathrm{Im}\left[\Delta(j\omega)\right] = L_t g_t g_i + C_a - \omega^2 L_t\left(C_a C_b - C_1^2\right) = 0$$

# CB-Colpitts - Oscillation Conditions

**From the equation for the imaginary part, we get**

$$\omega_o^2 = \frac{g_t g_i + s\left(C_a / L_t\right)}{C_a C_b - C_t^2}$$

**or**

$$\omega_o^2 = \frac{1}{\underbrace{L_t\left[C_o + C_f + \left(C_1 C_2 / \left(C_1 + C_2\right)\right)\right]}_{LC\ tank\ circuit}} + \frac{1}{\underbrace{R_t R_i\left[\left(C_f + C_o\right)\left(C_1 + C_2\right) + C_1 C_2\right]}_{transistor\ and\ load}}$$

**First term should predominate**

# CB-Colpitts - Oscillation Conditions

**In that case**

$$L_t \ll R_t R_i \left( C_1 + C_2 \right)$$

**which leads to** $$\omega_o^2 = \frac{1}{L_t \left[ C_o + C_f + C_1 C_2 / \left( C_1 + C_2 \right) \right]}$$

**Circuit oscillates at frequency determined by $L_t$ and equivalent capacitance.**

# CB-Colpitts - Oscillation Conditions

**The solution of the real part using $\omega = \omega_o$ gives**

$$\alpha_{\min} = 1 + \frac{C_f + C_o}{C_1} + \frac{R_i}{R_t}\left(1 + \frac{C_2}{C_1}\right) - \frac{1}{\omega_o^2 L_t C_1}$$

$$\alpha_{\min} \approx 1 - \frac{C_2}{C_1 + C_2} + \frac{R_i}{R_t}\left(1 + \frac{C_2}{C_1}\right) = \frac{1}{1 + \left(C_2 / C_1\right)} + \frac{R_i}{R_t}\left(1 + \frac{C_2}{C_1}\right)$$

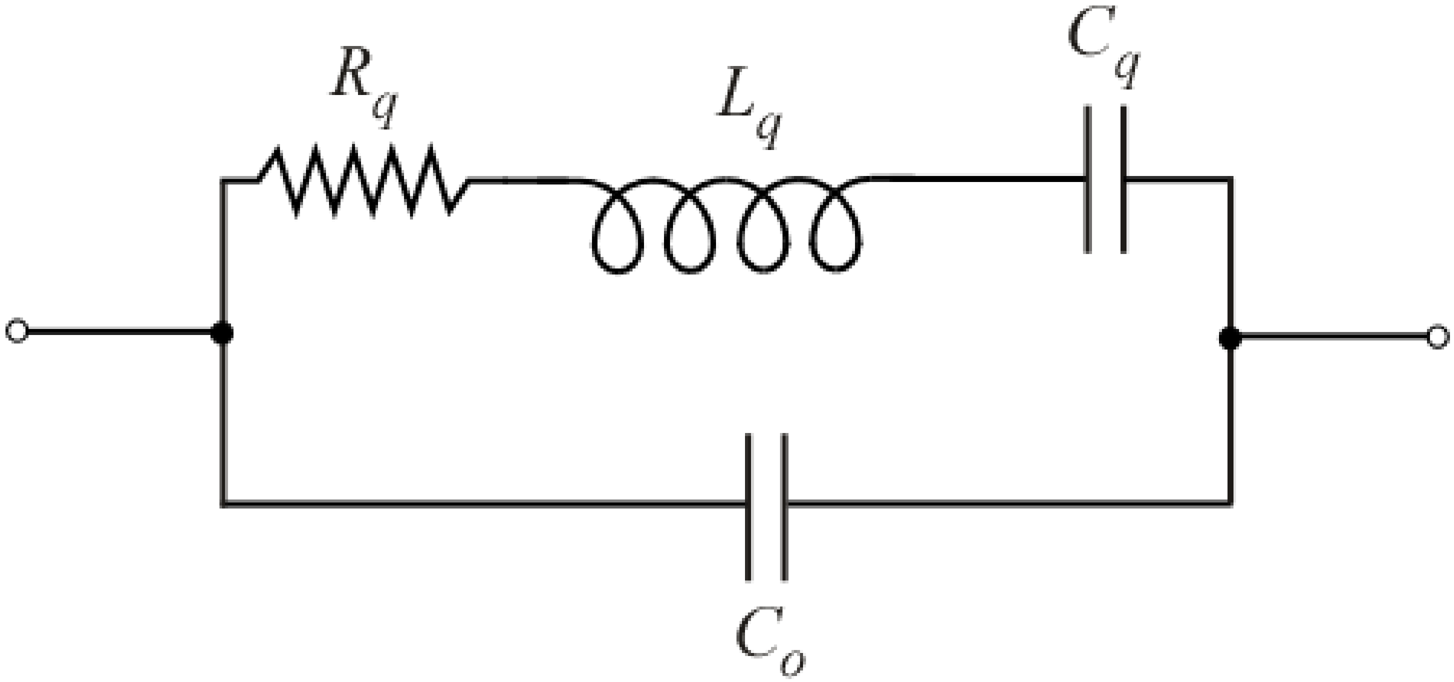
For oscillations to start, $\alpha$ of the transistor should be greater than $\alpha_{min}$.

# Crystal Oscillator

- **Quartz exhibits piezoelectric effect**
  - Reciprocal relationship between mechanical deformation and appearance of electrical potential
  - Deforming crystal produces voltage
  - Applying voltage produces deformation
- **When applied voltage is sinusoidal**
  - Mechanical oscillations that exhibit resonances at multiple frequencies
  - Extremely high Q that can reach $10^5$ and $10^6$
  - Do not work above 250 MHz

# Crystal Resonator - Circuit Model

$R_q$, $L_q$ and $C_q$ describe mechanical resonance behavior
$C_0$ is capacitance due to external contacting




- $L_q$ ,$C_q$, $R_q$ describe mechanical resonance
- $C_0$ *is* due to external contacting

# Crystal Resonance Frequencies

**Admittance from terminals is**

$$Y = j\omega C_0 + \frac{1}{R_q + j\left[\omega L_q - 1/\left(\omega C_q\right)\right]} = G + jB$$

**Resonance condition is expressed by**

$$\omega_0 C_o - \frac{\omega_0 L_q - 1/\left(\omega_0 C_q\right)}{R_q^2 + \left[\omega_0 L_q - 1/\left(\omega_0 C_q\right)\right]^2} = 0$$

# Crystal Resonance Frequencies

## Series resonance frequency

$$\omega_0 = \omega_S \approx \omega_{S0}\left[1+\frac{R_q^2}{2}\left(\frac{C_0}{L_q}\right)\right]$$

## Parallel resonance frequency

$$\omega_0 = \omega_P \approx \omega_{P0}\left[1-\frac{R_q^2}{2}\left(\frac{C_0}{L_q}\right)\right]$$

## where

$$\omega_{S0} = 1/\sqrt{L_q C_q} \qquad \omega_{P0} = \sqrt{\left(C_q + C_0\right)/\left(L_q C_q C_0\right)}$$

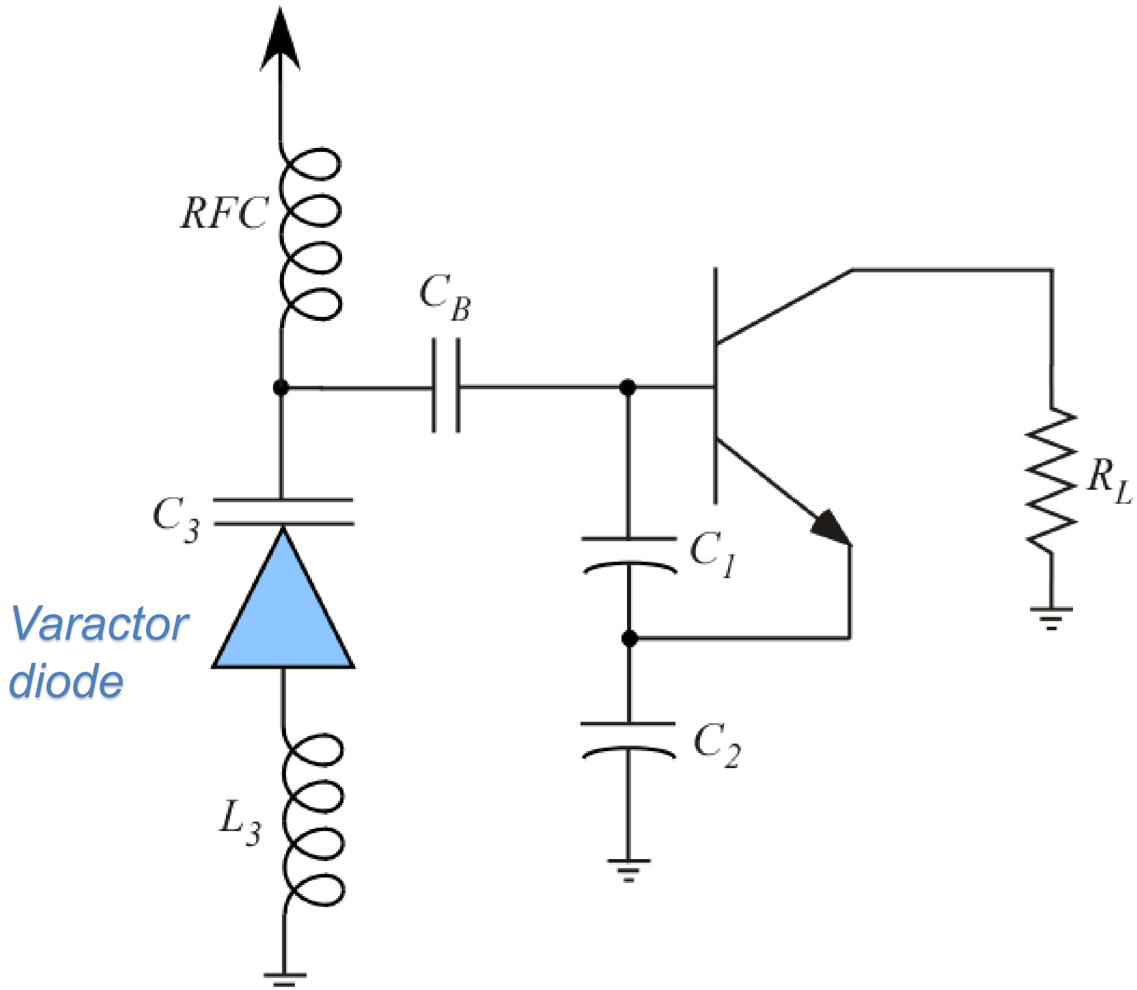
# Crystal Oscillator - Example

**Crystal is characterized by $L_q$ = 0.1H, $R_q$ = 25Ω, $C_q$ = 0.3pF, and $C_0$ = 1 pF. Find series and parallel resonance frequencies and compare them against the susceptance formula.**

$$f_S = f_{S0}\left[1+\frac{R_q^2}{2}\left(\frac{C_0}{L_q}\right)\right] = \frac{1}{2\pi\sqrt{L_q C_q}}\left[1+\frac{R_q^2}{2}\left(\frac{C_0}{L_q}\right)\right] = 0.919\ MHz$$

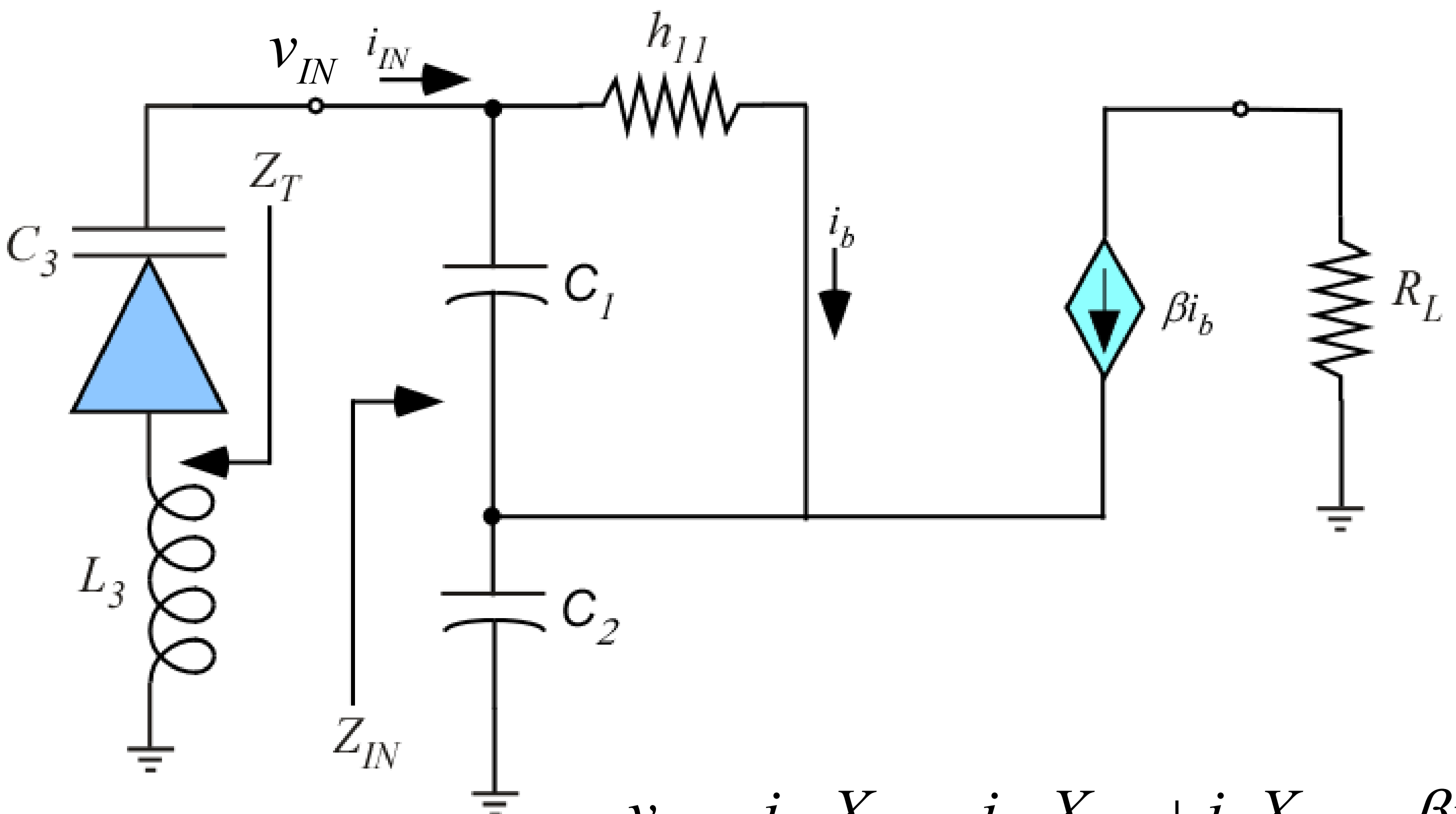
$$f_P = f_{P0}\left[1-\frac{R_q^2}{2}\left(\frac{C_0}{L_q}\right)\right] = \frac{1}{2\pi}\sqrt{\frac{C_q+C_0}{L_q C_q C_0}}\left[1-\frac{R_q^2}{2}\left(\frac{C_0}{L_q}\right)\right] = 1.048\ MHz$$

# Voltage Controlled Oscillator




***Varactor diode exhibits large change in capacitance in response to an applied bias voltage***

# Voltage Controlled Oscillator




$$v_{IN} - i_{IN}X_{C1} - i_{IN}X_{C2} + i_B X_{C1} - \beta i_B X_{C2} = 0$$

$$h_{11}i_B + i_B X_{C1} - i_{IN}X_{C1} = 0$$

# Voltage Controlled Oscillator

## Finding input impedance

$$Z_{IN} = \frac{1}{h_{11} + X_{C1}} \left[ h_{11}\left(X_{C1} + X_{C2}\right) + X_{C1}X_{C2}\left(1+\beta\right)\right]$$

## Re-write as (assuming $h_{11} >> X_{C1}$ and $\beta+1 \sim \beta$)

$$Z_{IN} = \frac{1}{j\omega}\left[\frac{1}{C_1} + \frac{1}{C_2}\right] - \frac{\beta}{h_{11}}\left(\frac{1}{\omega^2 C_1 C_2}\right)$$

## Using $g_m = \beta/h_{11}$

$$R_{IN} = -\frac{g_m}{\omega^2 C_1 C_2}$$

$$X_{IN} = \frac{1}{j\omega C_{IN}}$$

*Negative resistance*

# VCO – Resonance Frequency

$$C_{IN} = \frac{C_1 C_2}{C_1 + C_2}$$

**Resonance frequency follows from condition:**

$$X_{IN} = -X_{VARACTOR}$$ **where** $$X_{VARACTOR} = j\left(\omega_0 L_3 - \frac{1}{\omega_0 C_3}\right)$$

**which gives**

$$-j\left(\omega_0 L_3 - \frac{1}{\omega_0 C_3}\right) - \frac{1}{j\omega_0}\left[\frac{1}{C_1} + \frac{1}{C_2}\right] = 0$$

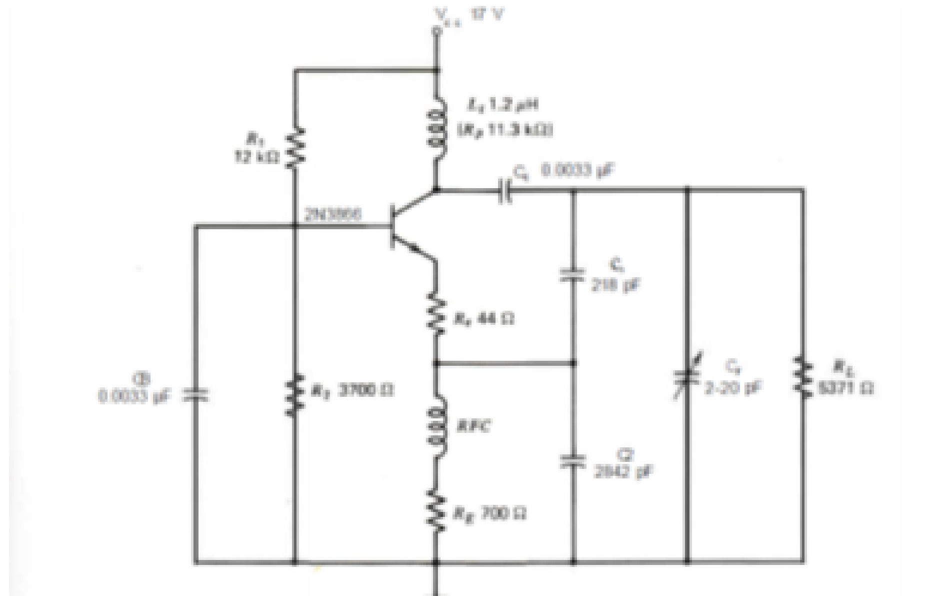
**and**

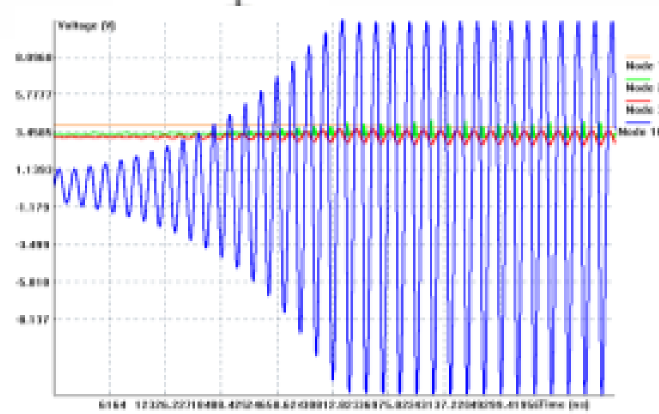
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{L_3}\left(\frac{1}{C_3} + \frac{1}{C_2} + \frac{1}{C_1}\right)}$$

***Combined resistance of varactor diode must be equal to or less than $|R_{IN}|$ to create sustained oscillations***

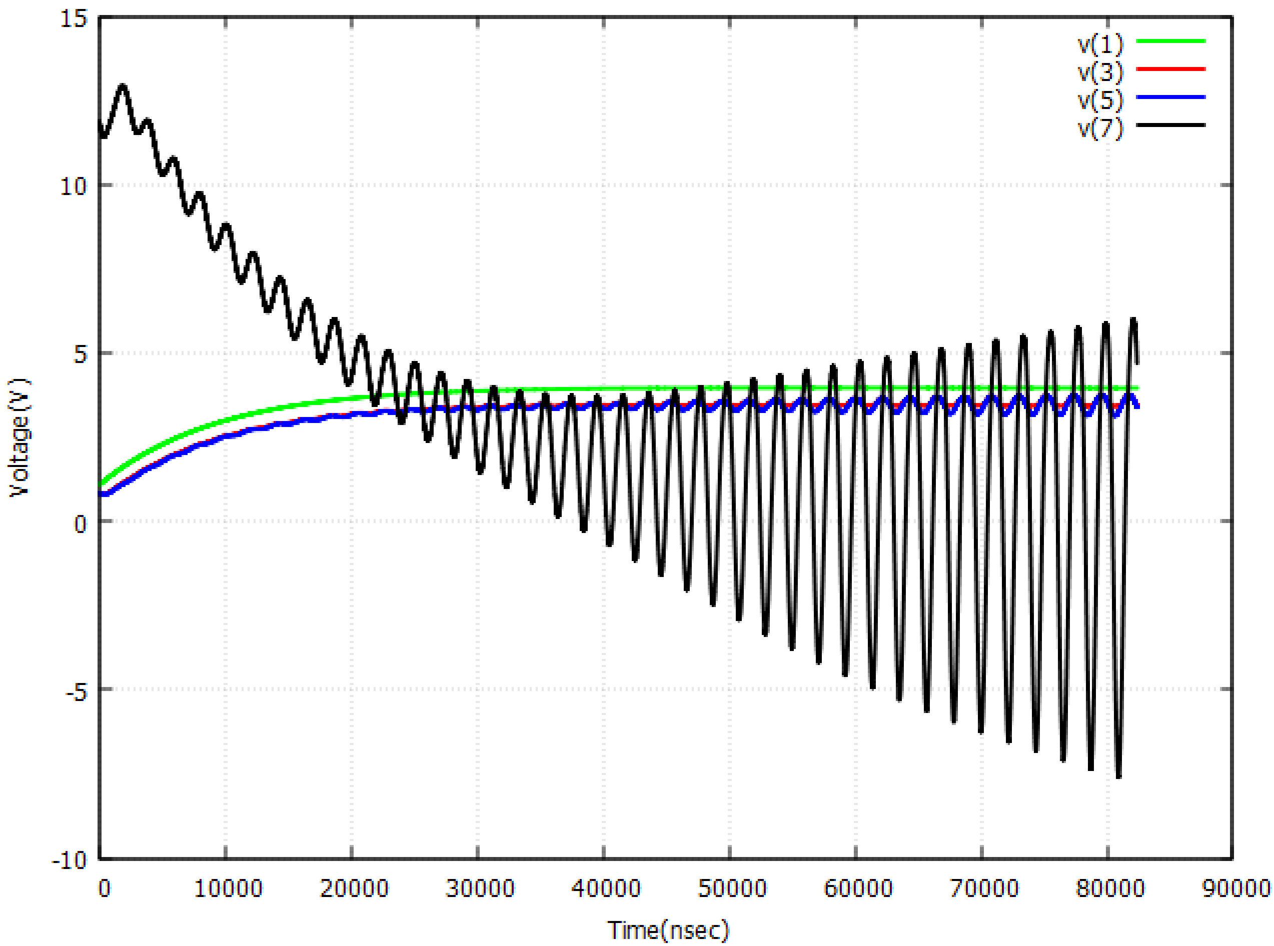
# Simulation - Colpitts Oscillator

```
*BJT Circuit
q1 2 1 3 qmod
vcc 4 0 17.0
r1 4 1 12000
lt 4 2 1.2e-03
rp 4 2 11300
r2 1 0 3700
re 3 5 44.0
lrfc 5 6 100.8e-03
ree 6 0 700.0
cc 2 7 3.3e-09
c1 7 5 218.0e-12
c2 5 0 2.842e-09
cf 7 0 18.0e-12
rl 7 0 5371
cb 1 0 3.3e-09
*vin 5 0 pulse(0 8 0.1e-09 0.1e-09 0.1e-
09 0.1e-09  40.0e-09)
*vin 5 0 sin (0 8.0 10.0e+06 0.0 0.0 )
.tran 9.1490e-12   62000.4e-09
.print tran  v(1) v(3) v(5) v(7)
.plot tran  v(1) v(3) v(5) v(7)
.model qmod npn level=1 bf=100 vaf=50
is=8.0e-12 rb=100.0 cjc=0.5e-12
tf=0.06e-09
.LIM C=1.010e-12 L=0.1e-09 G=1.0e-20
P=5000 O=200
.end
```



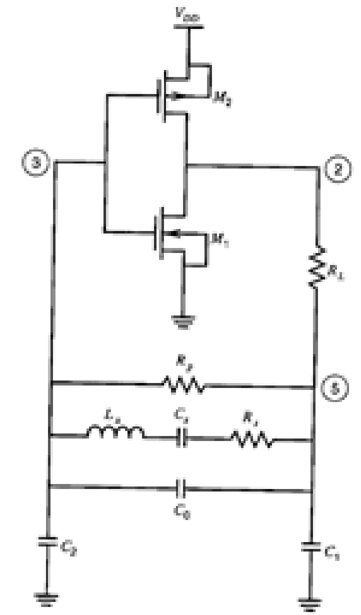



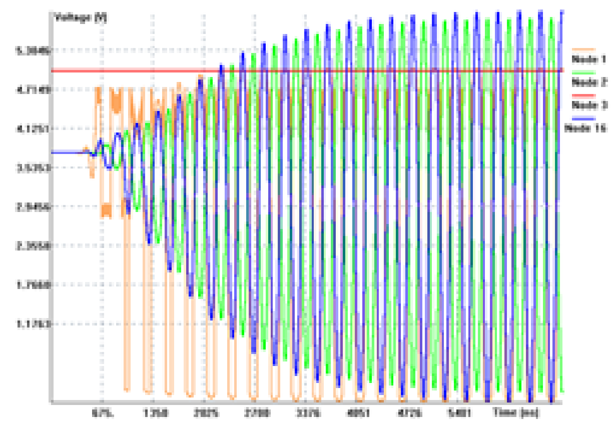
# Simulation - Colpitts Oscillator



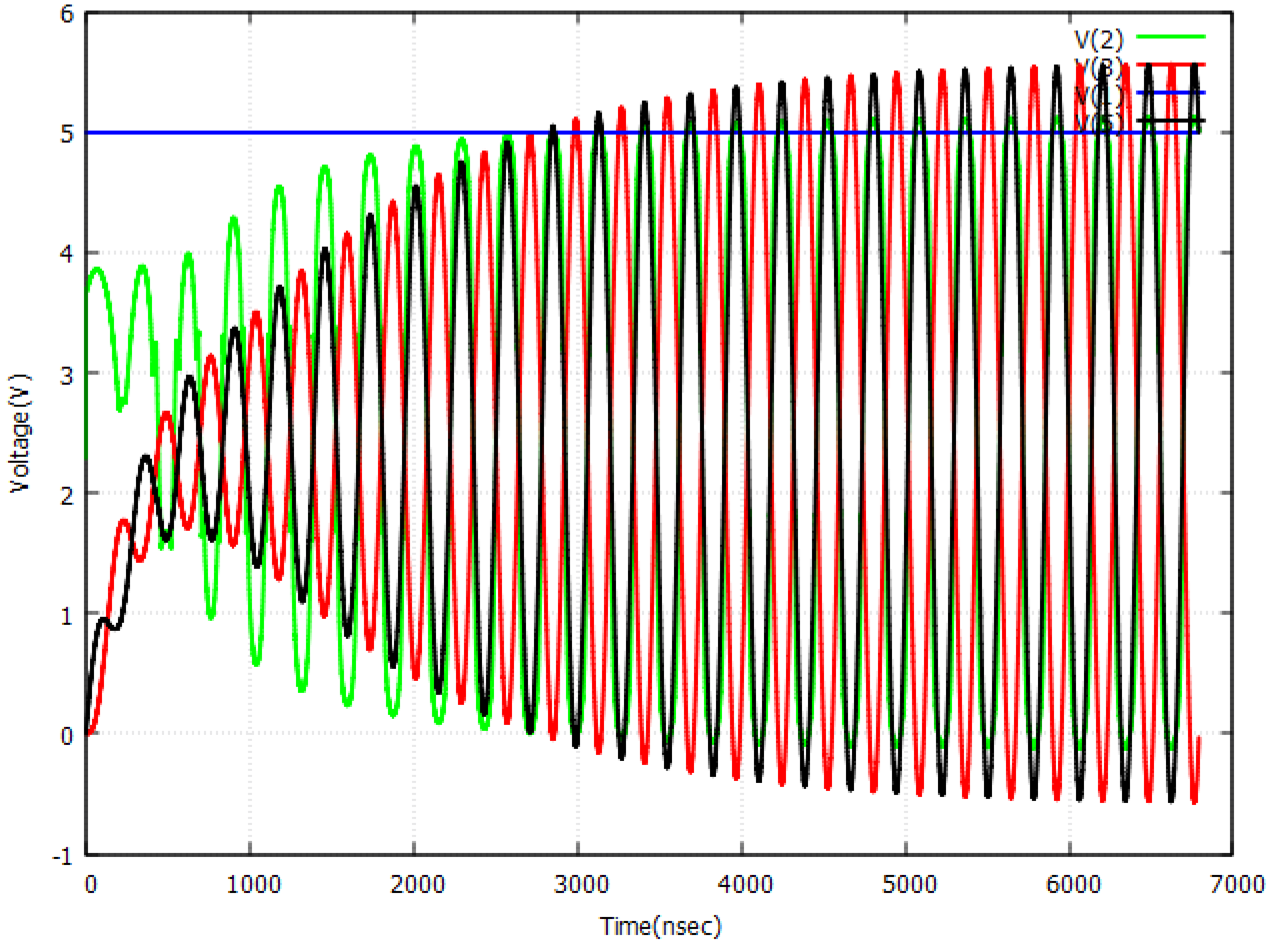
# Simulation - Pierce Oscillator

```
*PIERCE XTAL OSCILLATOR WITH CMOS
M1 2 3 0 0 NMOS W=40U L=10U
M2 2 3 1 1 PMOS W=80U L=10U
RL 2 5 10000
C1 5 0 22.0e-12
C2 3 0 22.0e-12
RP 3 5 22.0e+06
LEFF 3 5 0.18e-03
VDD 1 0 5
.MODEL NMOS NMOS LEVEL=1 VTO=1 KP=20U
LAMBDA=0.02
.MODEL PMOS PMOS LEVEL=1 VTO=-1 KP=10U
LAMBDA=0.02
.tran 0.001e-09   6793.0e-09e-08
.LIM C=0.015e-12 L=0.1e-09 G=1.0e-20
.PRINT TRAN V(2) V(3) V(1) V(5)
.PLOT TRAN V(2) V(3) V(1) V(5)
.END
```



# Simulation - Pierce Oscillator



# Simulation - Ring Oscillator

```
*NMOS RING OSCILLATOR SPICEBOOK PAGE 183
VDD 11 0 5
M1 1 3 0 0 ENH L=10U W=40U
M2 2 1 0 0 ENH L=10U W=40U
M3 3 2 0 0 ENH L=10U W=40U
M4 11 1 1 0 DEP L=10U W=5U
M5 11 2 2 0 DEP L=10U W=5U
M6 11 3 3 0 DEP L=10U W=5U
*VINN 1 0  PULSE (0  5.0 0.2e-09  .5e-09
.5e-09  1.0e-09  6.0e-06)
*VANN 2 0  PULSE (4.86  0.0 0.2e-09  .5e-09
.5e-09  1.0e-09  6.0e-06)
.TRAN .15e-09 888.0e-09
.MODEL DEP NMOS LEVEL=1 VTO=-3 LAMBDA=.001
KP=.4E-4
.MODEL ENH NMOS LEVEL=1 VTO=1.8 CGSO=20N
LAMBDA=.001 KP=.4E-4
.PRINT TRAN V(1) V(2) V(3) V(11)
.PLOT TRAN V(1) V(2) V(3) V(11)
.LIM C=10.0e-12 L=0.01e-09 P=20 O=10 D=0
.END
```

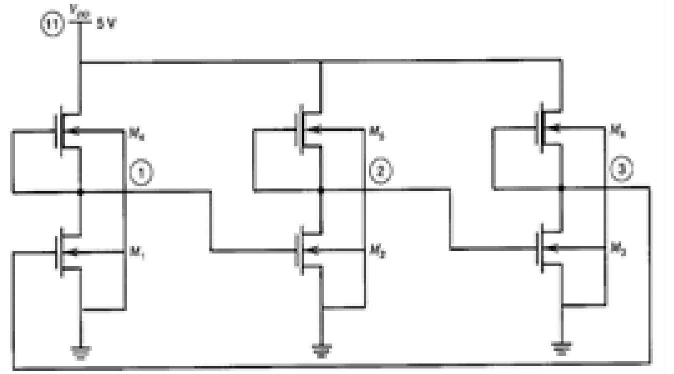



Figure 6.9 NMOS ring oscillator.