ECE 546
Lecture 04
Resistance, Capacitance, Inductance

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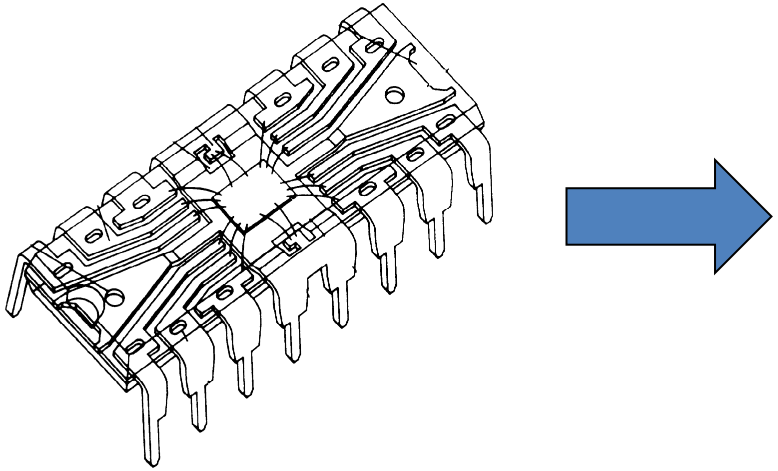
What is Extraction?



Process in which a complex arrangement of conductors and dielectrics is converted into a netlist of elements in a form that is amenable to circuit simulation.

Need Field Solvers

Electromagnetic Modeling Tools

We need electromagnetic modeling tools to analyze:

Transmission line propagation
Reflections from discontinuities
Crosstalk between interconnects
Simultaneous switching noise

So we can provide:

Improved design of interconnects
Robust design guidelines
Faster, more cost effective design cycles

Field Solvers – History

♦1960s
Conformal mapping techniques
Finite difference methods (2-D Laplace eq.)
Variational methods

♦1970s
Boundary element method
Finite element method (2-D)
Partial element equivalent circuit (3-D)

♦1980s
Time domain methods (3-D)
Finite element method (3-D)
Moment method (3-D)
rPEEC method (3-D)

♦1990s
Adapting methods to parallel computers
Including methods in CAD tools

♦2000s
Incorporation of Passivity
Incorporation of Causality

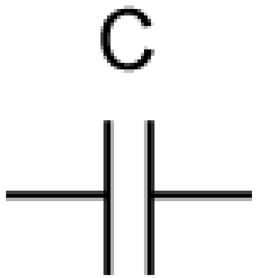
♦2010s
Stochastic Techniques
Multiphysics Tools

Categories of Field Solvers

- Method of Moments (MOM)
- Application to 2-D Interconnects
- Closed-Form Green's Function
- Full-Wave and FDTD
- Parallel FDTD
- Applications

Capacitance

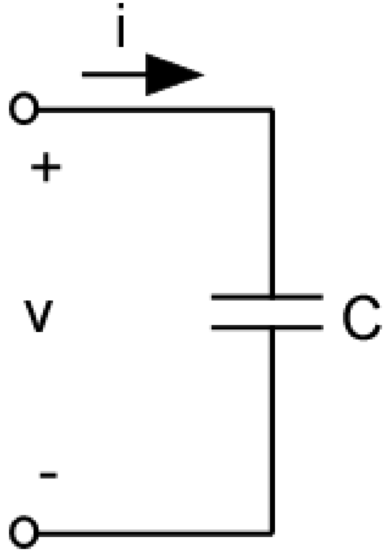
Relation: $Q = Cv$



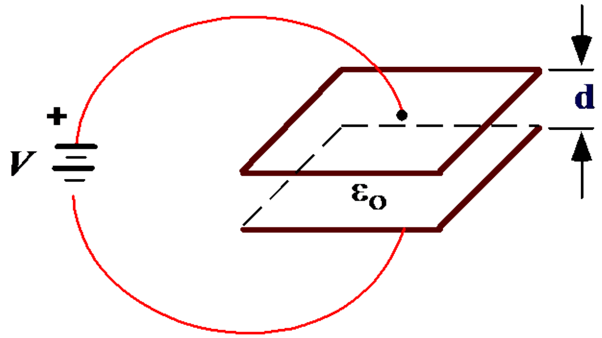
Q: charge stored by capacitor
C: capacitance
v: voltage across capacitor
i: current into capacitor

$$i(t) = C\frac{dv}{dt} = \frac{dQ}{dt}$$

$$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$$



Capacitance



$$C = \frac{\varepsilon_o A}{d}$$

A : area

ε_o : permittivity

For more complex capacitance geometries, need to use numerical methods

Potential and Charge Distribution

How do we find the potential due to a charge distribution?

$$\nabla^2 \phi = -\frac{\rho}{\varepsilon}$$

← **Poisson's Equation**

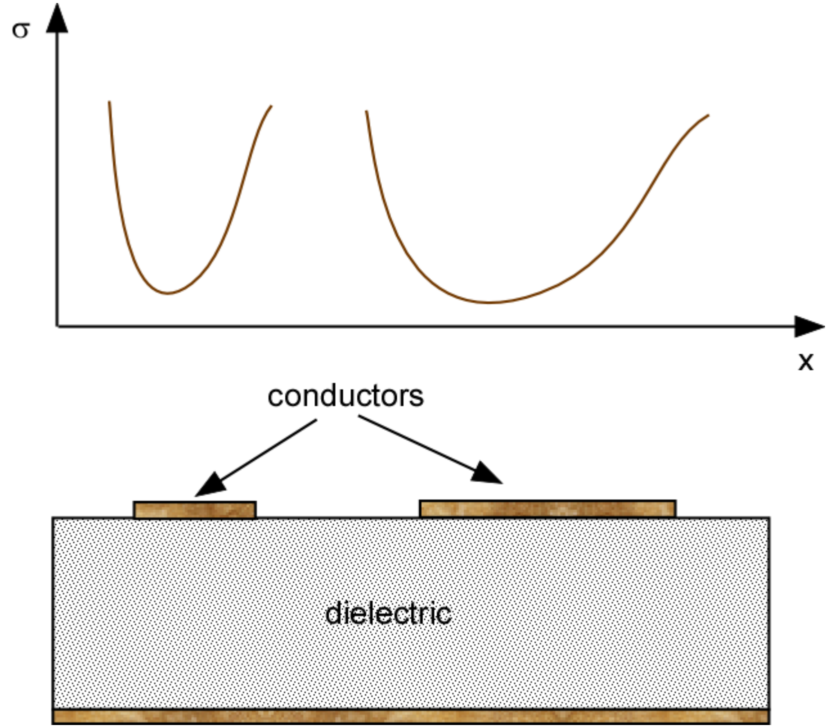
First find solution for infinitely small point source at origin

$$\nabla^2 \phi = \delta(r)$$

Solution is Green's function *g(r, r')*. Potential is then found via superposition.

$$\phi(x,y,z) = \iiint \frac{\rho(x',y',z')}{4\pi\varepsilon R} dx'dy'dz'$$

Capacitance Calculation



$$\phi(r) = \int g(r,r')\sigma(r')dr'$$

$$\phi(r) = potential\ (known)$$

$$g(r,r') = Green's\ function\ (\text{known})$$

$$\sigma(r') = \text{charge distribution}\ (unknown)$$

Once the charge distribution is known, the total charge Q can be determined. If the potential $\phi=V$, we have

$$Q=CV$$

To determine the charge distribution, use the moment method

Method of Moments

Operator equation

$$L(\mathrm{f}) = \mathrm{g}$$

L = integral or differential operator

f = unknown function

g = known function

Expand unknown function f

$$\mathrm{f} = \sum_n \alpha_n \mathrm{f}_\mathrm{n}$$

Method of Moments

in terms of basis functions f_n, with unknown coefficients α_n to get

$$\sum_n \alpha_n L(f_n) = g$$

Finally, take the scalar or ***inner product*** with testing of weighting functions w_m:

$$\sum_n \alpha_n \langle w_m, Lf_n \rangle = \langle w_m, g \rangle$$

with

$$\langle w_m, g \rangle = \int w_m(r') g(r, r') dr'$$

Matrix equation $\quad [l_{mn}][\alpha_n] = [g_m]$

Method of Moments

$$[l_{mn}] = \begin{bmatrix} \langle w_1, Lf_1 \rangle & \langle w_1, Lf_2 \rangle & \ldots \\ \langle w_2, Lf_1 \rangle & \langle w_2, Lf_2 \rangle & \ldots \\ \ldots\ldots\ldots & \ldots\ldots\ldots\ldots & \ldots \end{bmatrix}$$

$$[\alpha_n] = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ . \end{bmatrix} \qquad [g_m] = \begin{bmatrix} \langle w_1, g \rangle \\ \langle w_2, g \rangle \\ . \end{bmatrix}$$

Solution for weight coefficients

$$[\alpha_n] = \left[l_{nm}^{-1} \right] [g_m]$$

Moment Method Solution

$$\nabla \cdot D = \rho$$

$$E = -\nabla \phi$$

$$\nabla^2 \phi = -\frac{\rho}{\varepsilon}$$

$$L\phi = -\frac{\rho}{\varepsilon}$$

$$\phi(x,y,z) = \iiint \frac{\rho(x',y',z')}{4\pi\varepsilon R} dx'dy'dz'$$

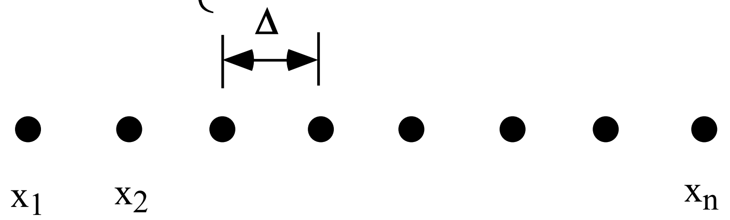
$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

Green's function G: $LG = \delta$

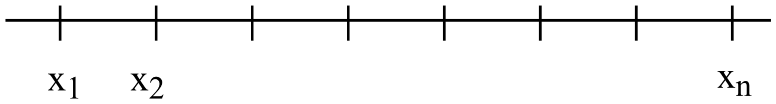
Basis Functions

Subdomain bases

$$P(x_n) = \begin{cases} 1 & x_n - \dfrac{\Delta}{2} < x < x_n + \dfrac{\Delta}{2} \\ 0 & otherwise \end{cases}$$

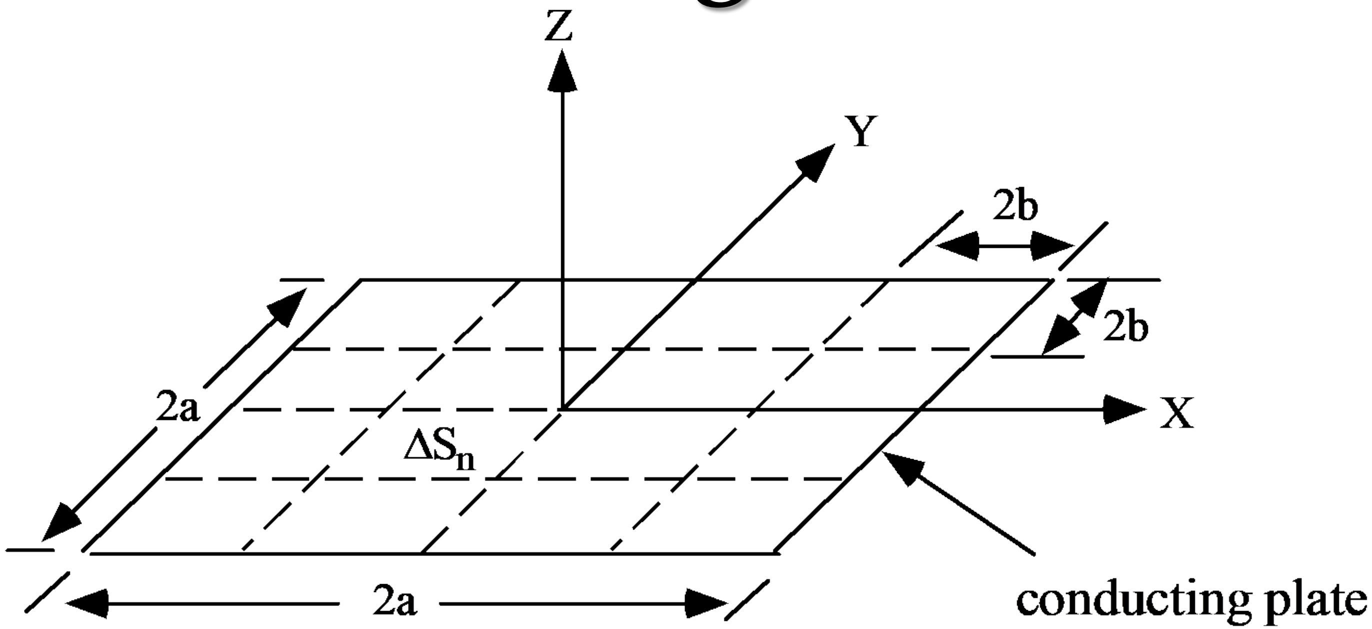


$$T(x_n) = \begin{cases} 1 - |x| & x_n - \dfrac{\Delta}{2} < x < x_n + \dfrac{\Delta}{2} \\ 0 & otherwise \end{cases}$$



Testing functions often (not always) chosen same as basis function.

Conducting Plate



$$\phi(x,y,z) = \int_{-a}^{a} dx' \int_{-a}^{a} dy' \frac{\sigma(x',y',z')}{4\pi\varepsilon R}$$

σ = charge density on plate

Conducting Plate

Setting ϕ = V on plate

$$R = \sqrt{(x-x')^2 + (y-y')^2}$$

$$V = \int_{-a}^{a} dx' \int_{-a}^{a} dy' \frac{\sigma(x',y',z')}{4\pi\varepsilon\sqrt{(x-x')^2 + (y-y')^2}}$$

for $|x| < a$; $|y| < a$

Capacitance of plate: $C = \dfrac{q}{V} = \dfrac{1}{V}\int_{-a}^{a} dx' \int_{-a}^{a} dy' \sigma(x',y',z')$

Conducting Plate

Basis function P_n

$$P_n(x_m, y_n) = \begin{cases} 1 & x_m - \frac{\Delta s}{2} < x < x_m + \frac{\Delta s}{2} \\ 1 & y_n - \frac{\Delta s}{2} < y < y_n + \frac{\Delta s}{2} \\ 0 & otherwise \end{cases}$$

Representation of unknown charge

$$\sigma(x, y) = \sum_{n=1}^{N} \alpha_n \mathrm{f}_\mathrm{n}$$

Conducting Plate

Matrix equation:

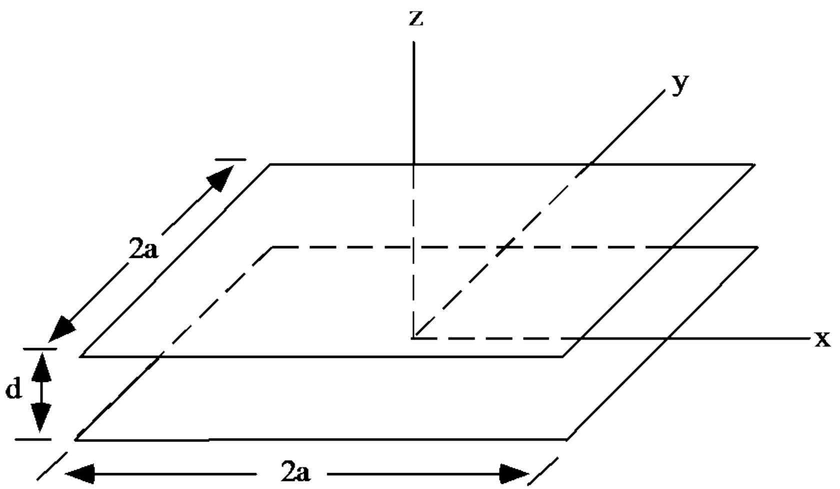
$$V = \sum_{n=1}^{N} l_{mn} f_n$$

Matrix element:

$$l_{mn} = \int_{\Delta x_m} dx' \int_{\Delta y_n} dy' \frac{1}{4\pi\varepsilon\sqrt{\left(x_m - x'\right)^2 + \left(y_n - x'\right)^2}}$$

$$C = \frac{1}{V}\sum_{n=1}^{N} \alpha_n \Delta s_n = \sum_{mn} l_{mn}^{-1} \Delta s_n$$

Parallel Plates



Using N unknowns per plate, we get 2N × 2N matrix equation:

$$[l] = \begin{bmatrix} [l^{tt}] & [l^{tb}] \\ [l^{bt}] & [l^{bb}] \end{bmatrix}$$

Subscript 't' for top and 'b' for bottom plate, respectively.

Parallel Plates

Matrix equation becomes $\left[l_{mn}^{tt} - l_{mn}^{tb} \right]\left[\alpha_n^t \right] = \left[g_m^t \right]$

Solution: $\left[\alpha_m^t \right] = \left[\left(l^{tt} - l^{tb} \right)_{mn}^{-1} \right]\left[g_n^t \right]$

$$\text{Capacitance } C = \frac{\text{charge on top plate}}{2\text{V}}$$

$$= \frac{1}{2V} \sum_{top} \alpha_n^t \Delta s_n$$

Using $\Delta s = 4b^2$ and all elements of $\left[g^t \right] = V$

$$C = 2b^2 \sum_{mn} \left(l^{tt} - l^{tb} \right)_{mn}^{-1}$$

Inductance - Definitions

Flux-based definition

$$L = \frac{\iint_S \vec{B} \cdot d\vec{s}}{i}$$

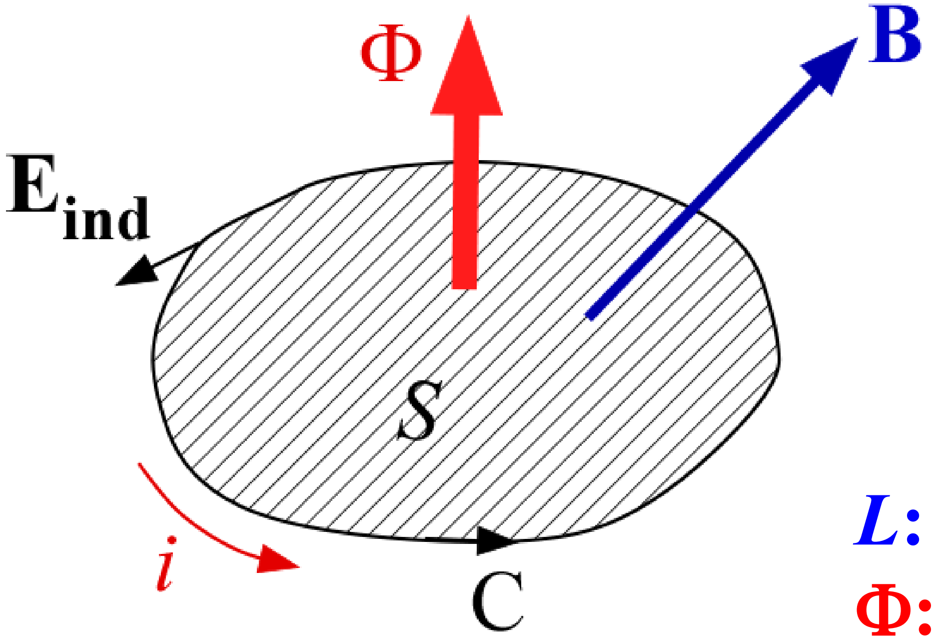
Field-based definition

$$L = \frac{\oint_C \vec{A} \cdot d\vec{l}}{i}$$

Energy-based definition

$$L = \frac{1}{i^2} \iiint_V \vec{B} \cdot \vec{H} dV$$

Inductance



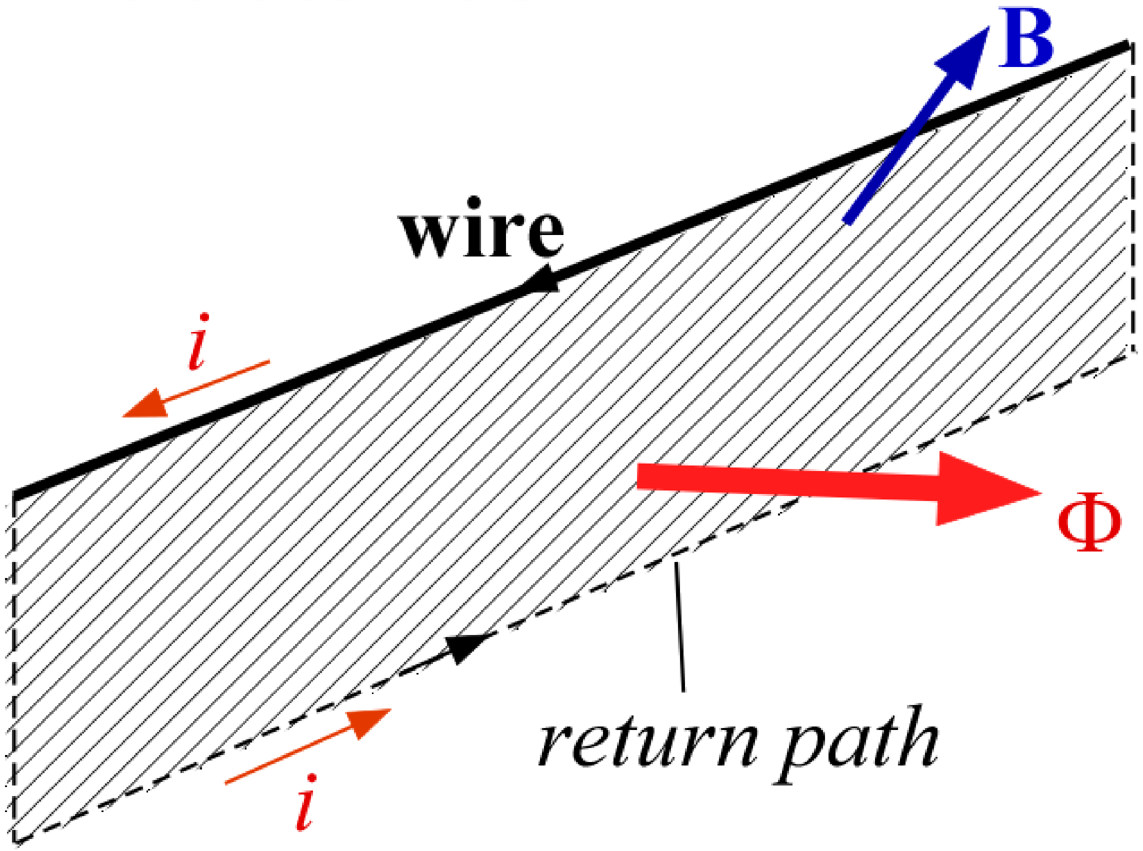
$$L = \frac{\Phi}{i}$$

L: inductance
Φ: magnetic flux
B: magnetic flux density
i: current

Current loop defines flux for inductance calculation

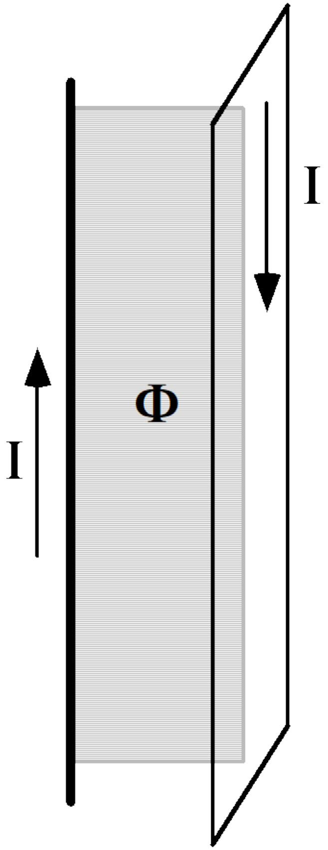
Inductance

$$L = \frac{\Phi}{i}$$



Inductance calculation requires a return path

Inductance – Wire over Ground Plane



Magnetic Flux

$$\Phi = \int_S \vec{B} \cdot d\vec{S}$$

$$\text{Inductance} = \frac{\text{Total flux linked}}{\text{Current}}$$

Inductance

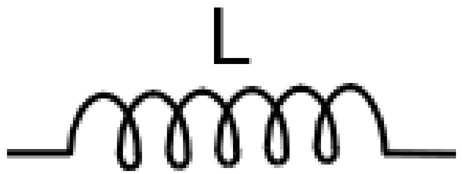
Relation: $\Phi = Li$

Φ: flux stored by inductor

L: inductance

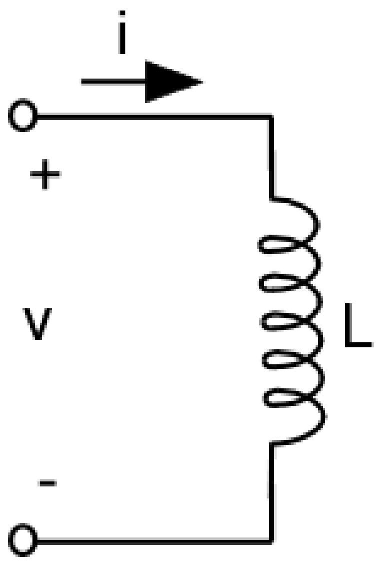
i: current through inductor

v: voltage across inductor

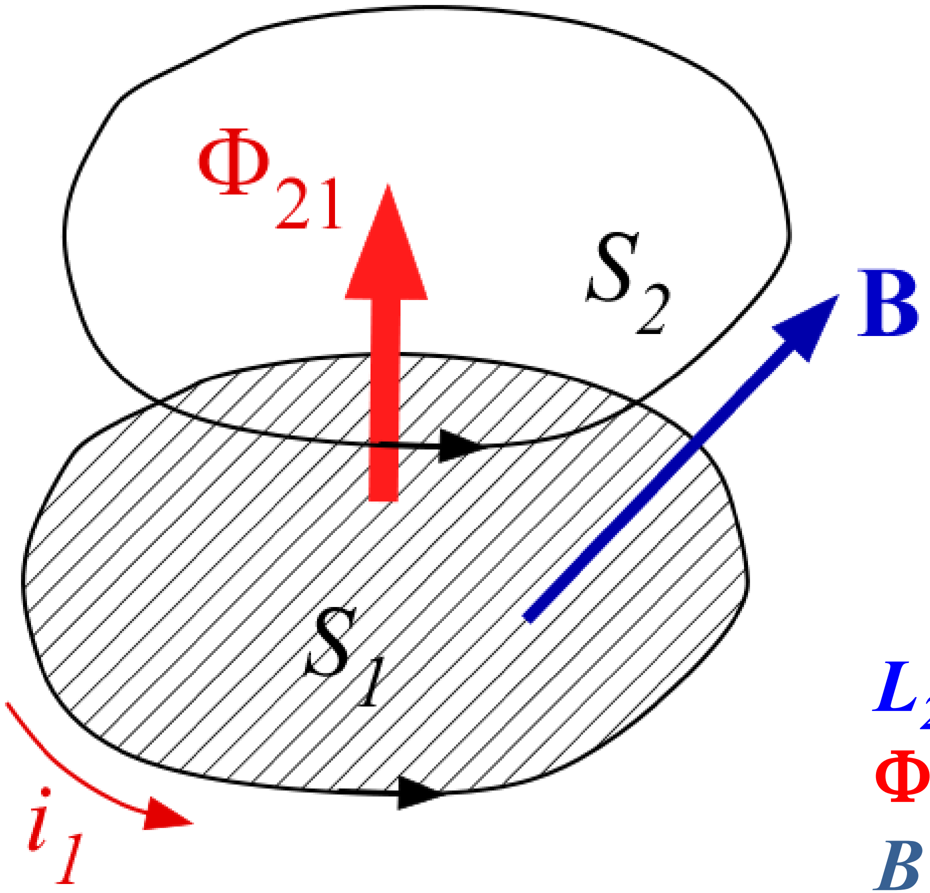


$$v(t) = L\frac{di}{dt} = \frac{d\Phi}{dt}$$

$$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau$$



Mutual Inductance



$$L_{21} = \frac{\Phi_{21}}{i_1}$$

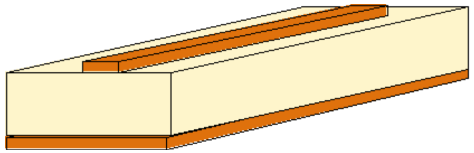
L_{21}: **mutual inductance**

Φ_{21}: **magnetic flux in loop2**

B: **magnetic flux density**

i_1: **loop 1 current**

2-D Isomorphism



Electrostatics

$$\left(\hat{z}\times\hat{n}\right)\cdot\nabla_t V_i = 0$$

$$\hat{n}\cdot\left(\varepsilon_{ri}\nabla_t V_i\right) = -\frac{q_s}{\varepsilon_o}$$

$$CV = Q$$

Magnetostatics

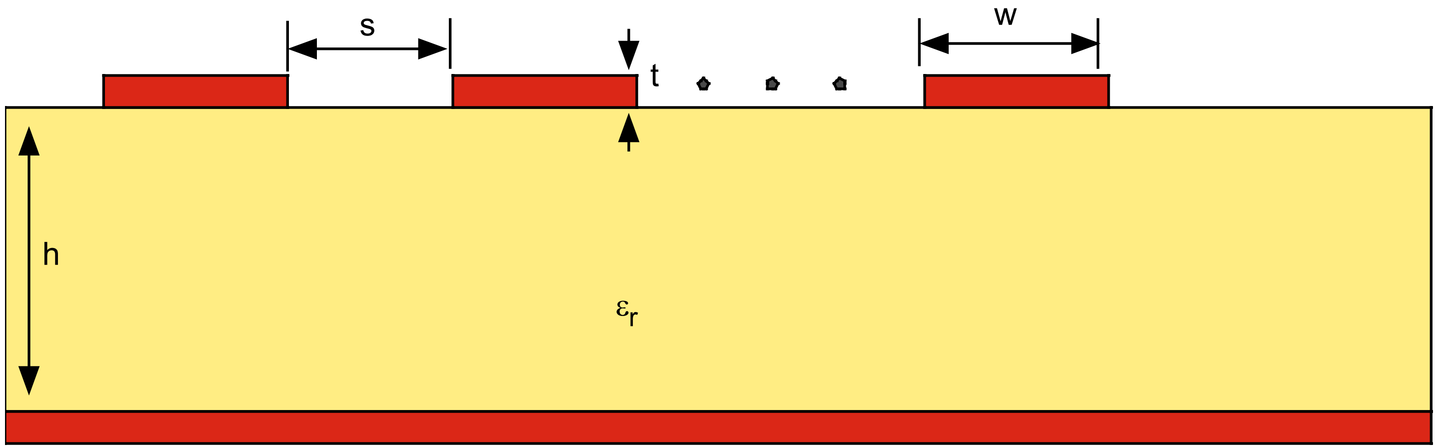
$$\left(\hat{z}\times\hat{n}\right)\cdot\nabla_t A_{zi} = 0$$

$$\hat{n}\cdot\left(\frac{1}{\mu_{ri}}\nabla_t A_{zi}\right) = -\mu_o J_z$$

$$LI = \psi$$

Consequence: 2D inductance can be calculated from 2D capacitance formulas

2-D N-line LC Extractor using MOM



- Symmetric signal traces
- Uniform spacing
- Lossless lines
- Uses MOM for solution

Output from MoM Extractor

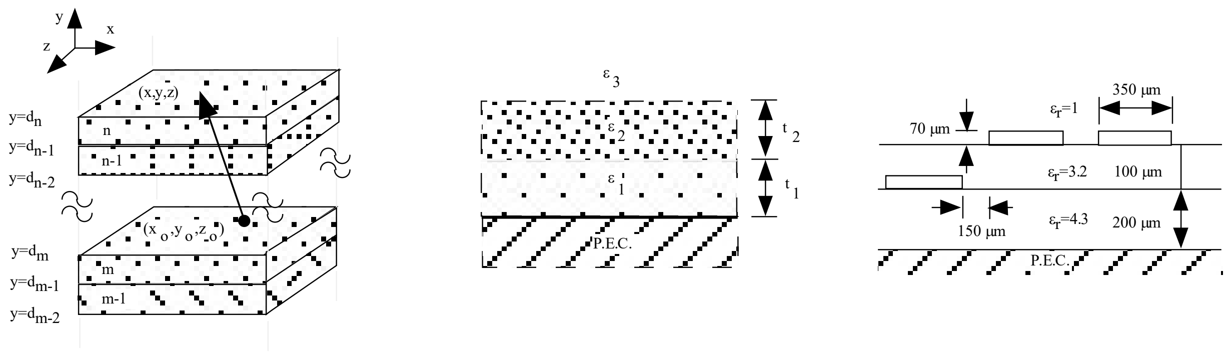
Capacitance (pF/m)

118.02299	-8.86533	-0.03030	-0.00011	-0.00000
-8.86533	119.04875	-8.86185	-0.03029	-0.00011
-0.03030	-8.86185	119.04876	-8.86185	-0.03030
-0.00011	-0.03029	-8.86185	119.04875	-8.86533
-0.00000	-0.00011	-0.03030	-8.86533	118.02299

Inductance (nH/m)

312.71680	23.42397	1.83394	0.14361	0.01128
23.42397	311.76042	23.34917	1.82812	0.14361
1.83394	23.34917	311.75461	23.34917	1.83394
0.14361	1.82812	23.34917	311.76042	23.42397
0.01128	0.14361	1.83394	23.42397	312.71680

RLGC: Formulation Method



Closed-Form Spatial Green's Function

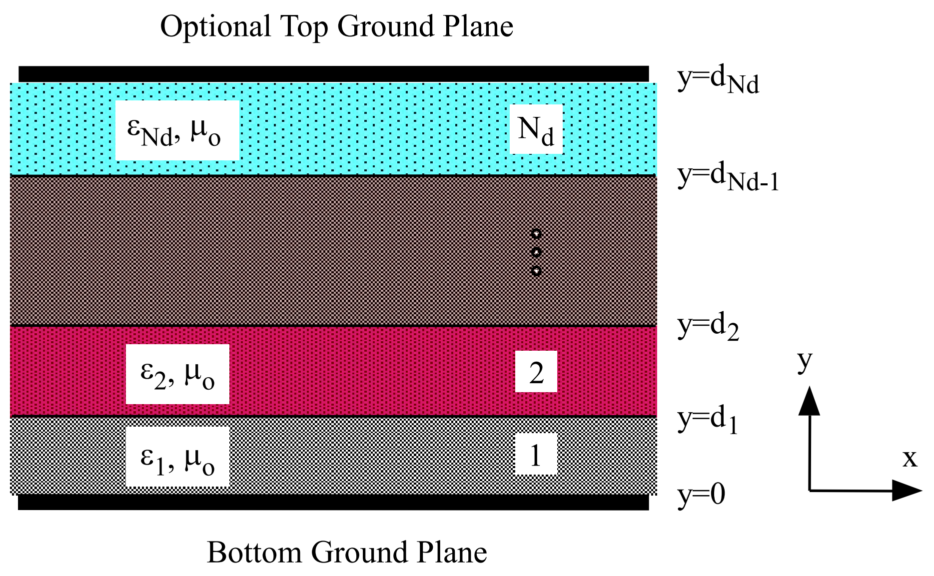
*** Computes 2-D and 3-D capacitance matrix in multilayered dielectric**

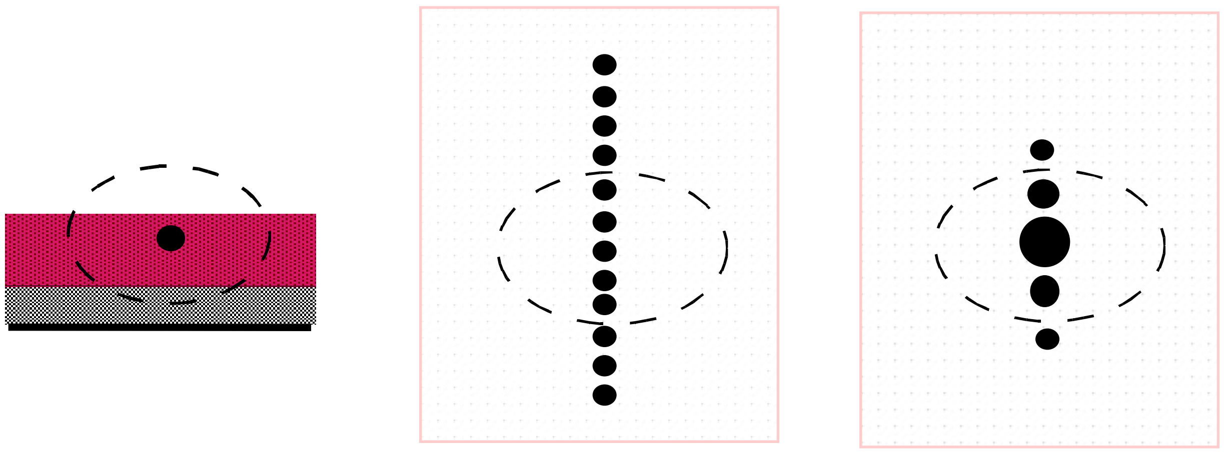
*** Method is applicable to arbitrary polygon-shaped conductors**

*** Computationally efficient**

- Reference
  - K. S. Oh, D. B. Kuznetsov and J. E. Schutt-Aine, "Capacitance Computations in a Multilayered Dielectric Medium Using Closed-Form Spatial Green's Functions," IEEE Trans. Microwave Theory Tech., vol. MTT-42, pp. 1443-1453, August 1994.

Multilayer Green's Function





Extraction Program: RLGC

RLGC computes the four transmission line parameters, viz., the capacitance matrix **C**, the inductance matrix **L**, the conductance matrix **G**, and the resistance matrix **R**, of a multiconductor transmission line in a multilayered dielectric medium. **RLGC** features the following list of functions:

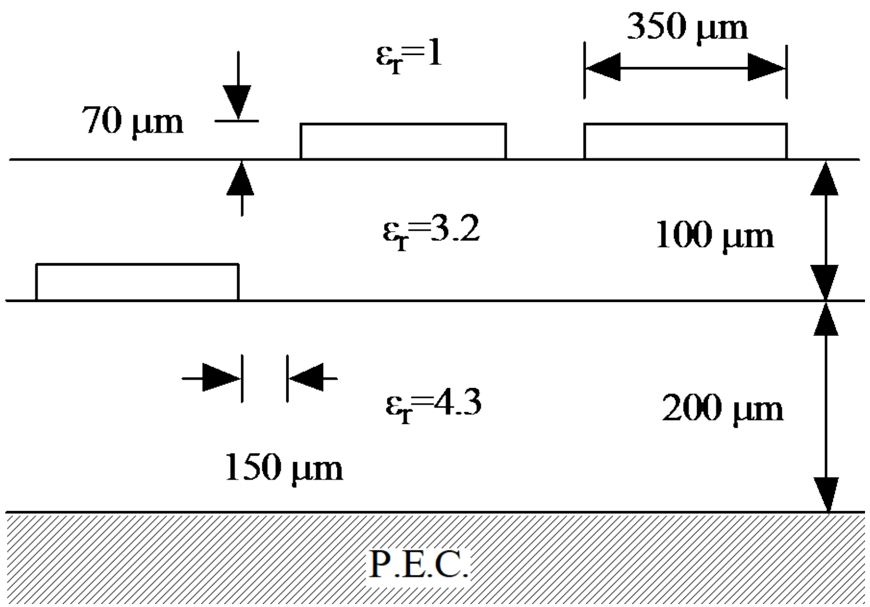
RLGC – Multilayer Extractor

- **Features**
  - Handling of dielectric layers with no ground plane, either top or bottom ground plane (microstrip cases), or both top and bottom ground planes (stripline cases)
  - Static solutions are obtained using the Method of Moment (MoM) in conjunction with closed-form Green’s functions: one of the most accurate and efficient methods for static analysis
  - Modeling of dielectric losses as well as conductor losses (including ground plane losses
  - The resistance matrix R is computed based on the current distribution - more accurate than the use of any closed-form formulae
  - Both the proximity effect and the skin effect are modeled in the resistance matrix R.
  - Computes the potential distribution
  - Handling of an arbitrary number of dielectric layers as well as an arbitrary number of conductors.
  - The cross section of a conductor can be arbitrary or even be infinitely thin

- **Reference**

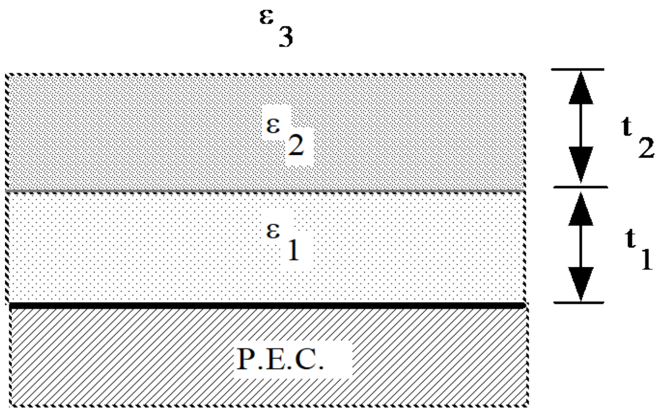
  - K. S. Oh, D. B. Kuznetsov and J. E. Schutt-Aine, "Capacitance Computations in a Multilayered Dielectric Medium Using Closed-Form Spatial Green's Functions," IEEE Trans. Microwave Theory Tech., vol. MTT-42, pp. 1443-1453, August 1994.

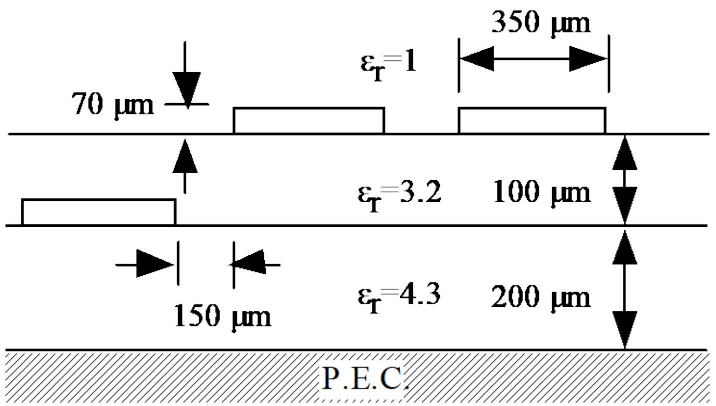
RLGC – General Topology



Three conductors in a layered medium. All conductor dimensions and spacing are identical. The loss tangents of the lower and upper dielectric layers are 0.004 and 0.001 respectively, the conductivity of each line is 5.8e7 S/m, and the operating frequency is 1 GHz

3-Line Capacitance Results





Capacitance Matrix (pF/m)

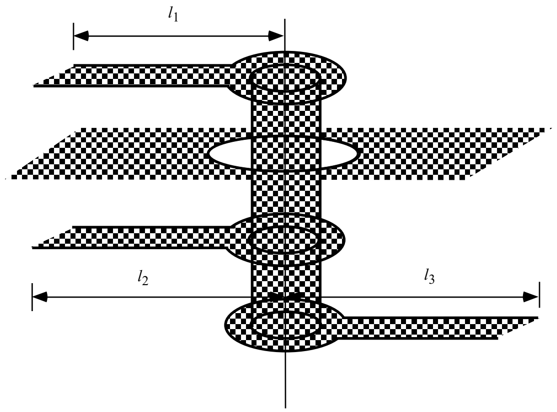
$$\begin{bmatrix} 142.09 & -21.765 & -0.8920 \\ -21.733 & 93.529 & -18.098 \\ -0.8900 & -18.097 & 87.962 \end{bmatrix}$$

Delabare et al.

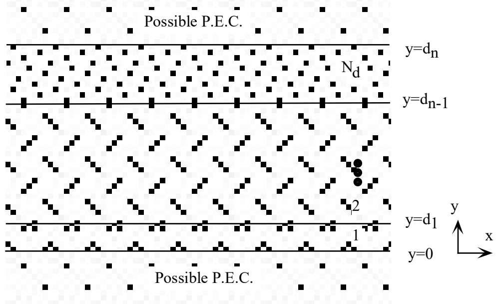
$$\begin{bmatrix} 145.33 & -23.630 & -1.4124 \\ -22.512 & 93.774 & -17.870 \\ -1.3244 & -17.876 & 87.876 \end{bmatrix}$$

RLGC Method

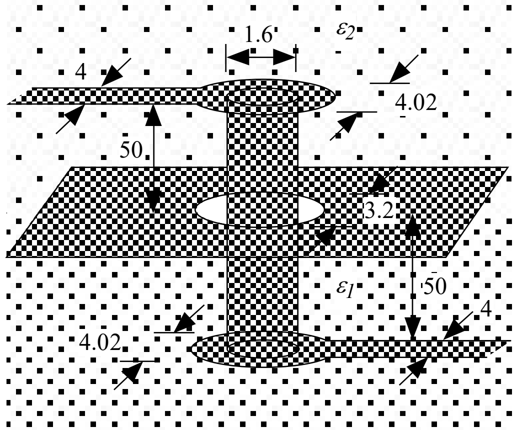
Modeling Vias



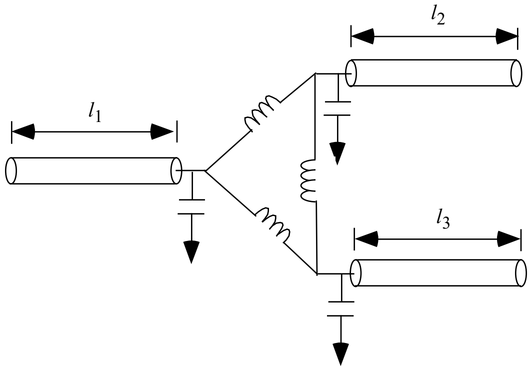
Via



Medium

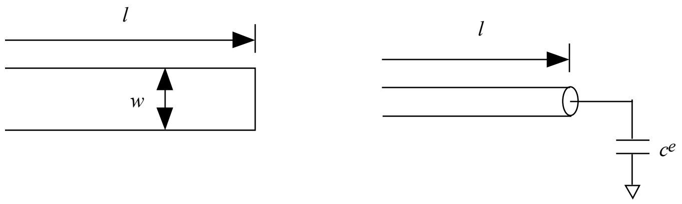


Via in multilayer medium

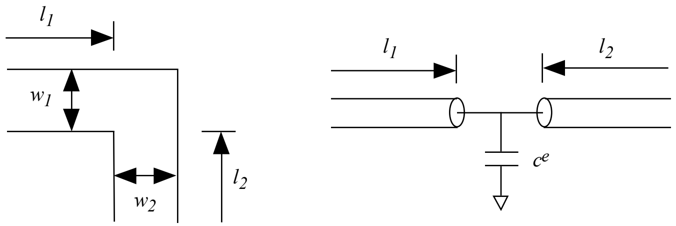


Equivalent circuit

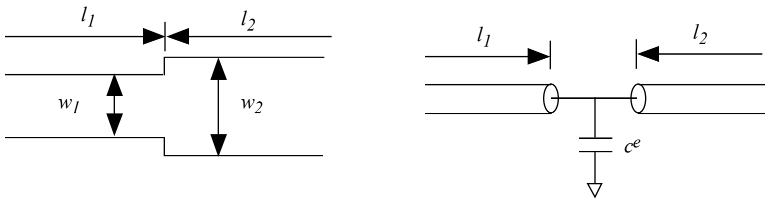
Modeling Discontinuities



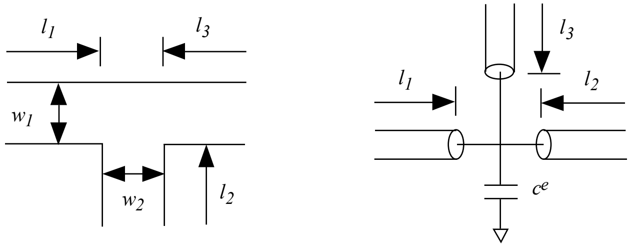
Open



Bend



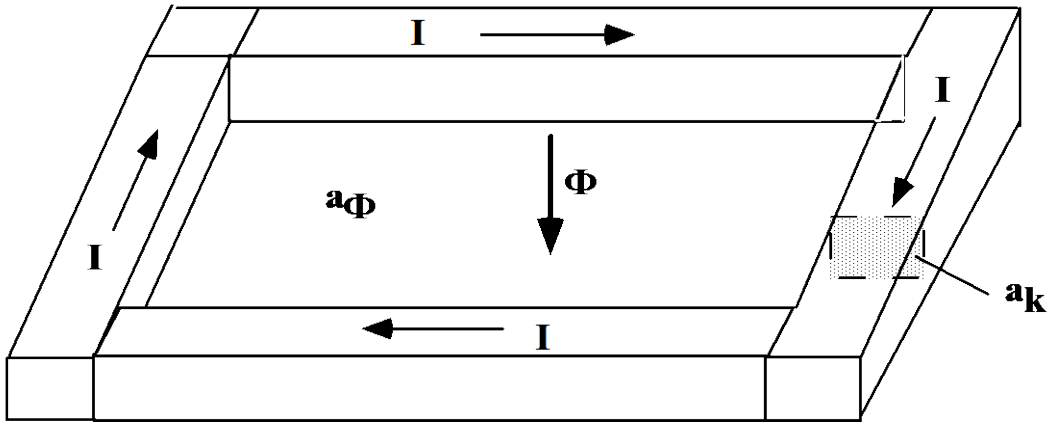
Step



T-Junction

3D Inductance Calculation

Loop Inductance



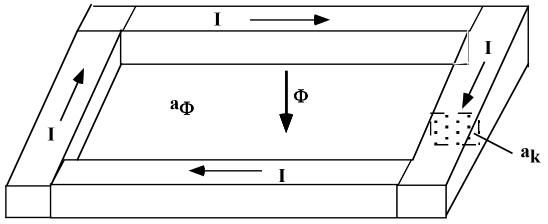
$$L_{loop} = \frac{\Phi}{I} = \frac{1}{I}\int_{a_\Phi} \vec{B}\cdot d\vec{a} = \frac{1}{I}\int_{a_\Phi} \left(\nabla \times \vec{A}\right)\cdot d\vec{a}$$

QUESTION: Can we associate inductance with piece of conductor rather than a loop? ➔PEEC Method*

*A. E. Ruehli, "Equivalent Circuit Models for Three-Dimensional Multiconductor Systems," in *IEEE Transactions on Microwave Theory and Techniques*, vol. 22, no. 3, pp. 216-221, Mar. 1974.

Partial Inductance (PEEC) Approach

QUESTION: Can we associate inductance with piece of conductor rather than a loop?



$$L_{loop} = \sum_{i=1}^{4}\sum_{j=1}^{4}\frac{1}{a_i a_j}\int_{a_i}\int_{a_j}\int_{l_i}\int_{l_j}\frac{\mu}{4\pi}\frac{d\vec{l}_i \cdot d\vec{l}_j}{\left|\vec{r}_i - \vec{r}_j\right|}da_i da_j$$

DEFINITION OF PARTIAL INDUCTANCE

$$L_{pij} = \frac{1}{a_i a_j}\frac{\mu}{4\pi}\int_{a_i}\int_{a_j}\int_{l_i}\int_{l_j}\frac{d\vec{l}_i \cdot d\vec{l}_j}{\left|\vec{r}_i - \vec{r}_j\right|}da_i da_j$$

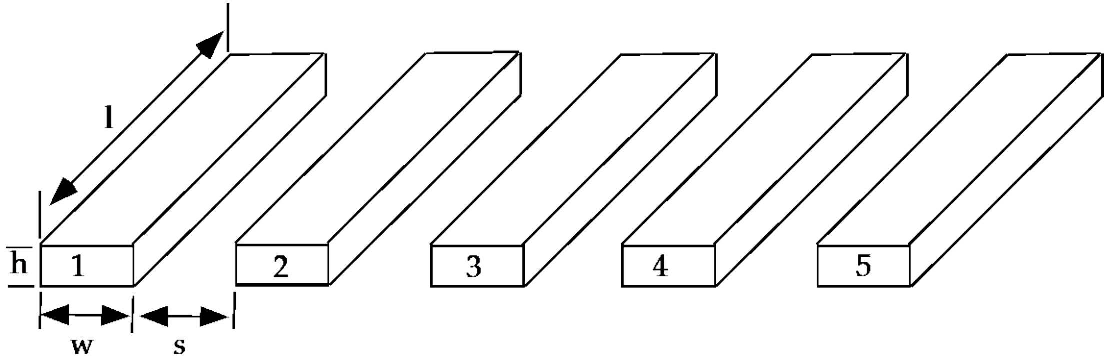
$$L_{loop} = \sum_{i=1}^{4}\sum_{j=1}^{4}s_{ij}L_{pij}$$

Circuit Element K

$$[K]=[L]^{-1}$$

- Better locality property
- Leads to sparser matrix
- Diagonally dominant
- Allows truncation of far off-diagonal elements
- Better suited for on-chip inductance analysis

Locality of K Matrix



$$[L]=\begin{bmatrix} 11.4 & 4.26 & 2.54 & 1.79 & 1.38 \\ 4.26 & 11.4 & 4.26 & 2.54 & 1.79 \\ 2.54 & 4.26 & 11.4 & 4.26 & 2.54 \\ 1.79 & 2.54 & 4.26 & 11.4 & 4.26 \\ 1.38 & 1.79 & 2.54 & 4.26 & 11.4 \end{bmatrix} \qquad [K]=\begin{bmatrix} 103 & -34.1 & -7.80 & -4.31 & -3.76 \\ -34.1 & 114 & -31.6 & -6.67 & -4.31 \\ -7.80 & -31.6 & 115 & -31.6 & -7.80 \\ -4.31 & -6.67 & -31.6 & 114 & -34.1 \\ -3.76 & -4.31 & -7.80 & -34.1 & 103 \end{bmatrix}$$

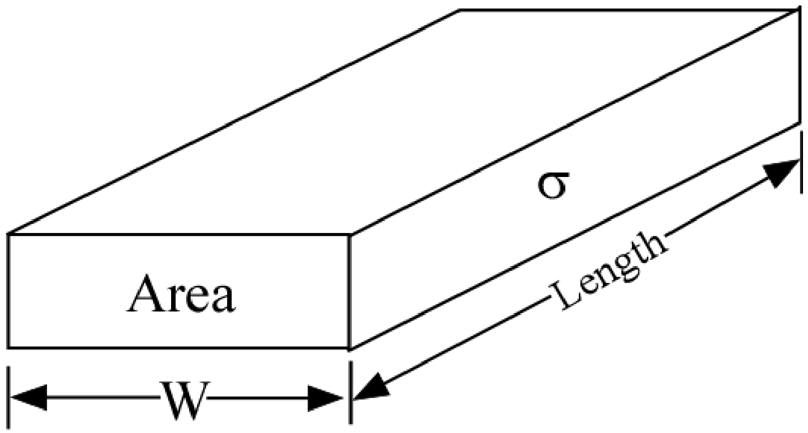
Package Inductance & Capacitance

Component	Capacitance (pF)	Inductance (nH)
68-pin plastic DIP pin†	4	35
68-pin ceramic DIP pin† †	7	20
68-pin SMT chip carrier†	2	7
68-pin PGA pin† †	2	7
256-pin PGA pin† †	5	15
Wire bond	1	1
Solder bump	0.5	0.1

† No ground plane; capacitance is dominated by wire-to-wire component.

†† With ground plane; capacitance and inductance are determined by the distance between the lead frame and the ground plane, and the lead length.

Metallic Conductors



Resistance: *R*

$$R = \frac{Length}{\sigma \cdot Area}$$

Resistance per unit length

Package level
W = 3 mils
R = 0.0045 Ω/mm

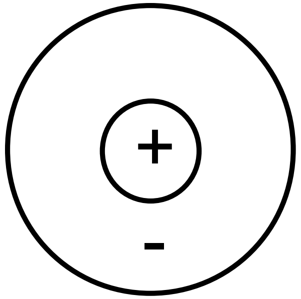
On-chip submicron
W = 0.25 microns
R = 422 Ω/mm

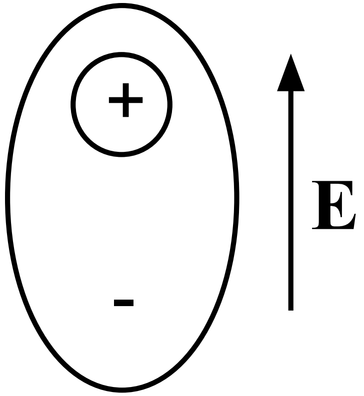
Metallic Conductors

Metal	Conductivity $\left(\Omega^{-1}\text{m}^{-1} \times 10^{-7}\right)$
Silver	6.1
Copper	5.8
Gold	3.5
Aluminum	1.8
Tungsten	1.8
Brass	1.5
Solder	0.7
Lead	0.5
Mercury	0.1

Dielectrics

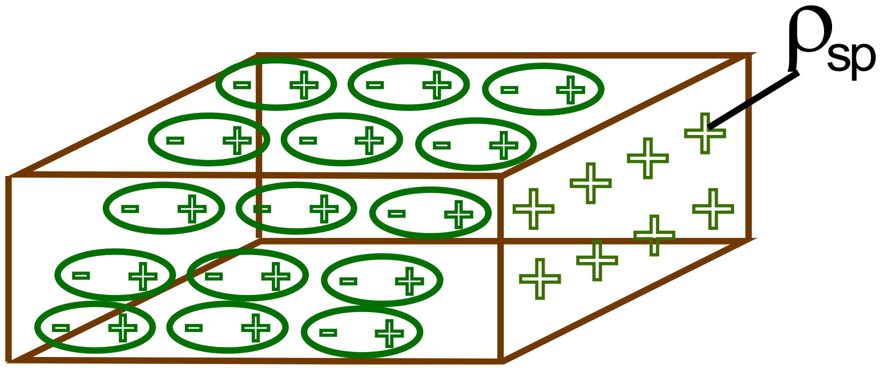
- Dielectrics contain charges that are tightly bound to the nuclei
- Charges can move a fraction of an atomic distance away from equilibrium position
- Electron orbits can be distorted when an electric field is applied





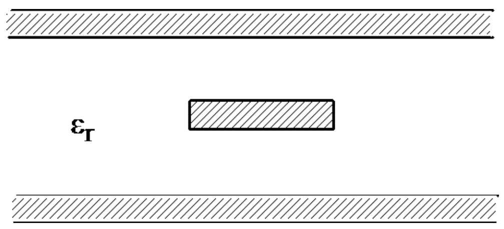
Dielectrics

- Charge density within volume is zero
- Surface charge density is nonzero



$$\mathbf{D}=\varepsilon_o(1+\chi_e)\mathbf{E}=\varepsilon\mathbf{E}$$

Dielectric Materials



$$v = \sqrt{\frac{1}{LC}}$$

Material	ε_r	Velocity (m/ns)
Polyimide	2.5 – 3.5	0.16-0.19
Silicon dioxide	3.9	0.15
Epoxy glass (FR4)	5.0	0.13
Alumina (ceramic)	9.5	0.10

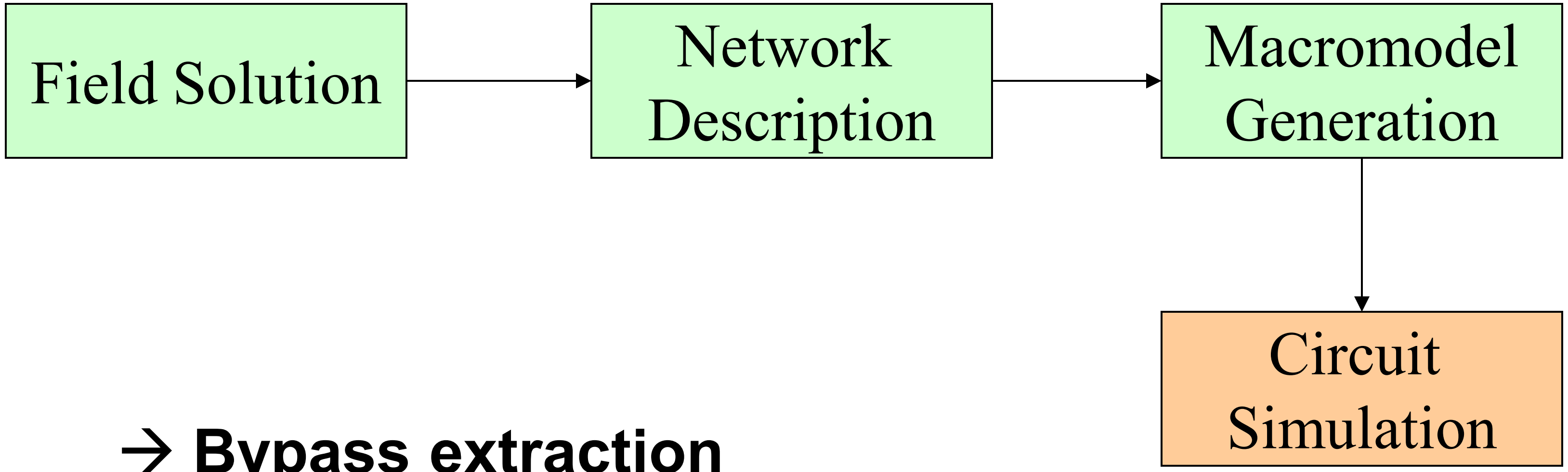
Conductivity of Dielectric Materials

$$\varepsilon = \varepsilon_r + j\varepsilon_i$$

Material	Conductivity $(\Omega^{-1}\text{m}^{-1})$
Germanium	2.2
Silicon	0.0016
Glass	$10^{-10} – 10^{-14}$
Quartz	0.5×10^{-17}

Loss tangent: $\tan\delta = \dfrac{\sigma}{\omega\varepsilon}$

Combining Field and Circuit Solutions



→ Bypass extraction procedure through the use of Y, Z, or S parameters (frequency domain)

Full-Wave Methods

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Faraday's Law of Induction

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Ampère's Law

$$\nabla \cdot \vec{D} = \rho$$

Gauss' Law for electric field

$$\nabla \cdot \vec{B} = 0$$

Gauss' Law for magnetic field

FDTD: Discretize equations and solve with appropriate boundary conditions

FDTD - Formulation

- FDTD solves Maxwell's equations in time-domain

$$\frac{\partial E}{\partial t} = \frac{1}{\varepsilon_0} \nabla \times H$$

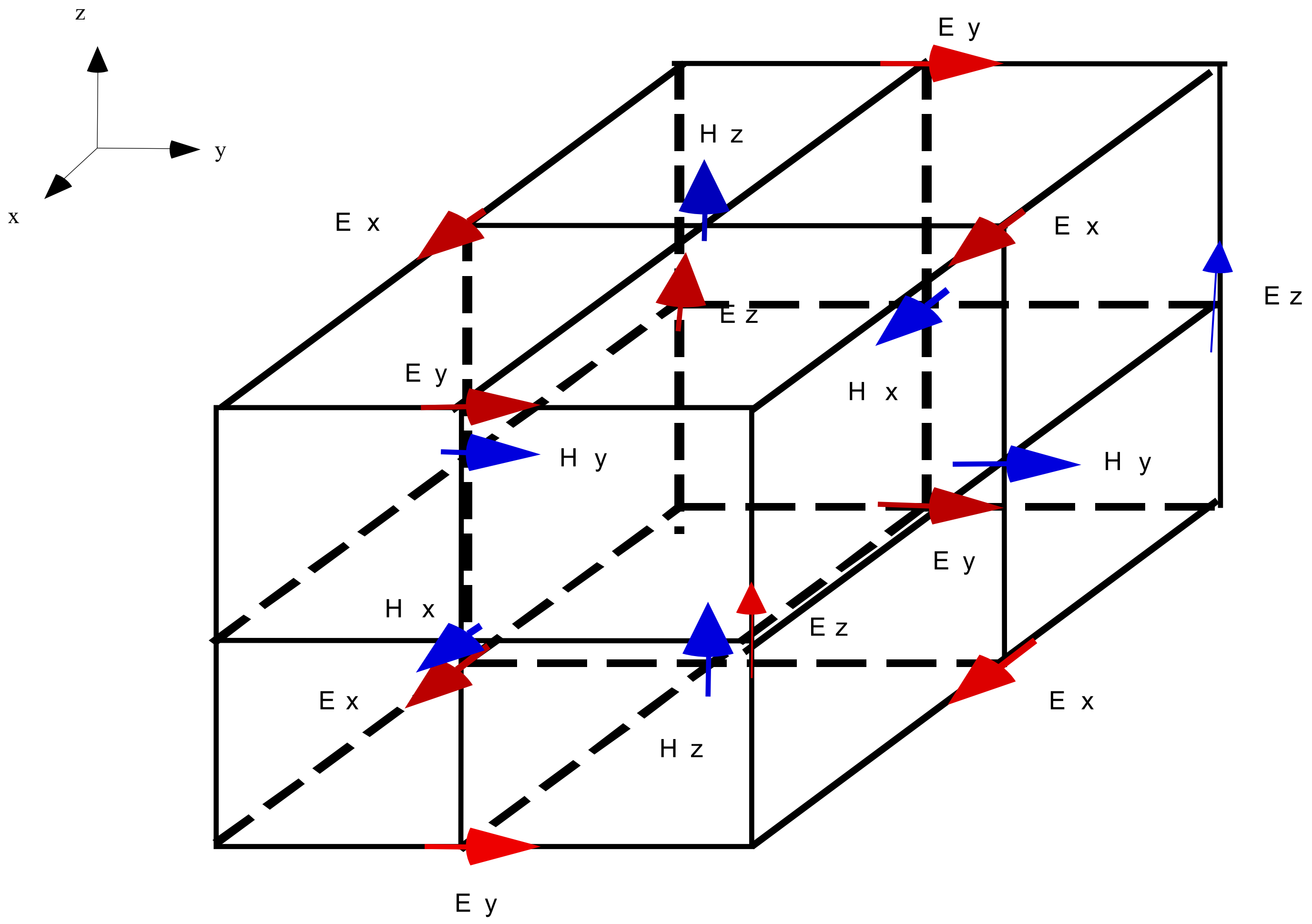
$$\frac{\partial H}{\partial t} = -\frac{1}{\mu_0} \nabla \times E$$

- Problem space is discretized
- Derivatives are approximated as

$$\frac{\partial u}{\partial v} \approx \frac{u(v_0 + \Delta v) - u(v_0 - \Delta v)}{2\Delta v}$$

- Time stepping algorithm
- Field values at all points of the grid are updated at each time step

Finite Difference Time Domain (FDTD)

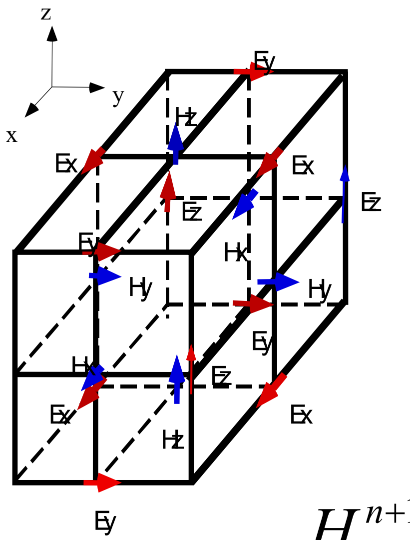


Space Discretization

FDTD – Yee Algorithm

$$E_x^n(i,j,k) = E_x^{n-1} + \frac{c}{\varepsilon}\frac{\Delta t}{\Delta y}\left(H_z^{n-1/2}(i,j,k) - H_z^{n-1/2}(i,j-1,k)\right)$$

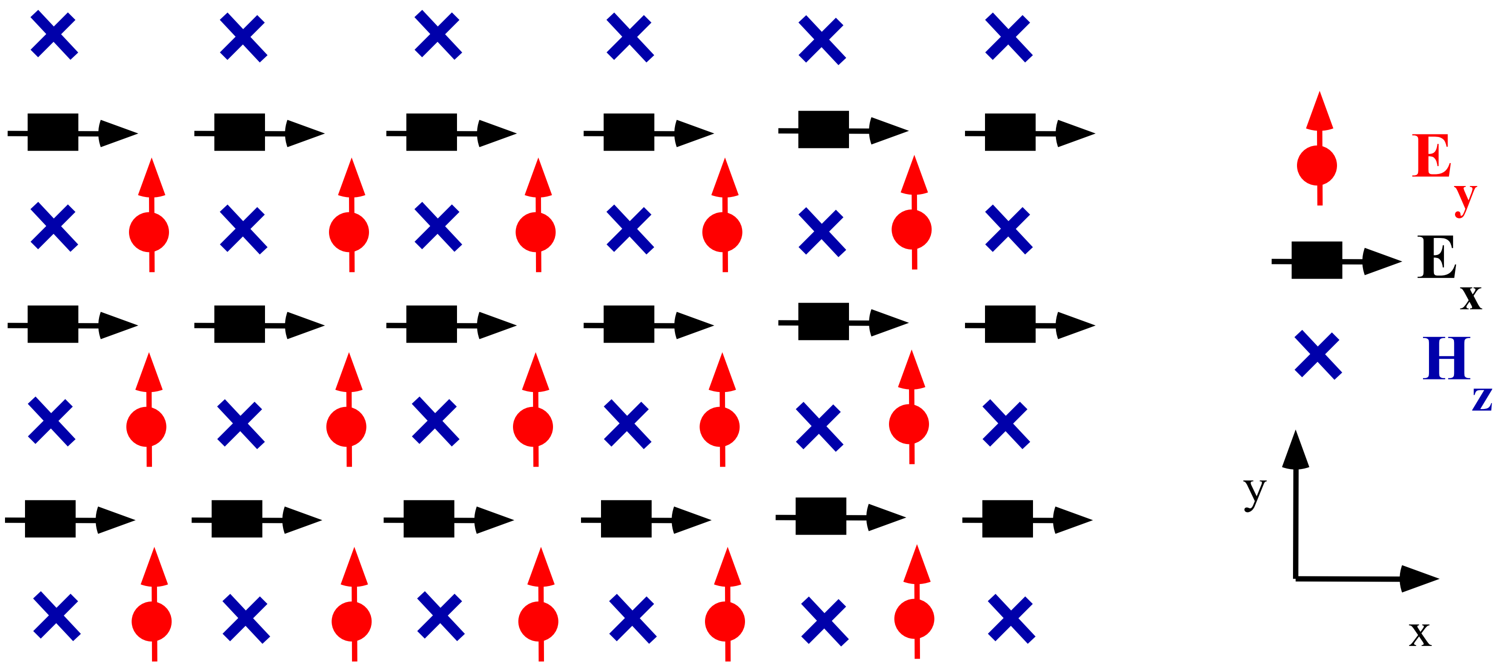
$$-\frac{c}{\varepsilon}\frac{\Delta t}{\Delta z}\left(H_y^{n-1/2}(i,j,k) - H_y^{n-1/2}(i,j,k-1)\right)$$



$$H_x^{n+1/2}(i,j,k) = H_x^{n-1/2} + \frac{c}{\mu}\frac{\Delta t}{\Delta y}\left(E_z^n(i,j+1,k) - E_z^n(i,j,k)\right)$$

$$+\frac{c}{\mu}\frac{\Delta t}{\Delta z}\left(E_y^{n-1/2}(i,j,k+1) - E_y^n(i,j,k)\right)$$

2D-FDTD



$$E_x^n\left(i+\frac{1}{2},j\right)=E_x^{n-1}\left(i+\frac{1}{2},j\right)+\frac{\Delta t}{\varepsilon_o \Delta y}\left[H_z^{n-1/2}\left(i+\frac{1}{2},j+\frac{1}{2}\right)-H_z^{n-1/2}\left(i+\frac{1}{2},j-\frac{1}{2}\right)\right]$$

$$E_y^n\left(i,j+\frac{1}{2}\right)=E_y^{n-1}\left(i,j+\frac{1}{2}\right)-\frac{\Delta t}{\varepsilon_o \Delta x}\left[H_z^{n-1/2}\left(i+\frac{1}{2},j+\frac{1}{2}\right)-H_z^{n-1/2}\left(i-\frac{1}{2},j+\frac{1}{2}\right)\right]$$

$$H_z^{n+1/2}\left(i+\frac{1}{2},j+\frac{1}{2}\right)=H_z^{n-1/2}\left(i+\frac{1}{2},j+\frac{1}{2}\right)+\frac{\Delta t}{\mu_o \Delta y}\left[E_x^n\left(i+\frac{1}{2},j+1\right)-E_x^n\left(i+\frac{1}{2},j\right)\right]$$

$$-\frac{\Delta t}{\mu_o \Delta x}\left[E_y^n\left(i+1,j+\frac{1}{2}\right)-E_x^n\left(i,j+\frac{1}{2}\right)\right]$$

Absorbing Boundary Condition: 2D-PML Formulation

Simulation Medium

$$\varepsilon_o \frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y}$$

$$\varepsilon_o \frac{\partial E_y}{\partial t} = -\frac{\partial H_z}{\partial x}$$

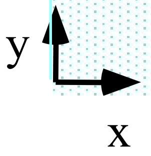
$$\mu_o \frac{\partial H_z}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}$$

PML Medium

$$\varepsilon_o \frac{\partial E_x}{\partial t} + \sigma E_x = \frac{\partial H_z}{\partial y}$$

$$\varepsilon_o \frac{\partial E_y}{\partial t} + \sigma E_y = -\frac{\partial H_z}{\partial x}$$

$$\mu_o \frac{\partial H_z}{\partial t} + \sigma * H_z = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}$$



$$\frac{\sigma}{\varepsilon_o} = \frac{\sigma *}{\mu_o}$$

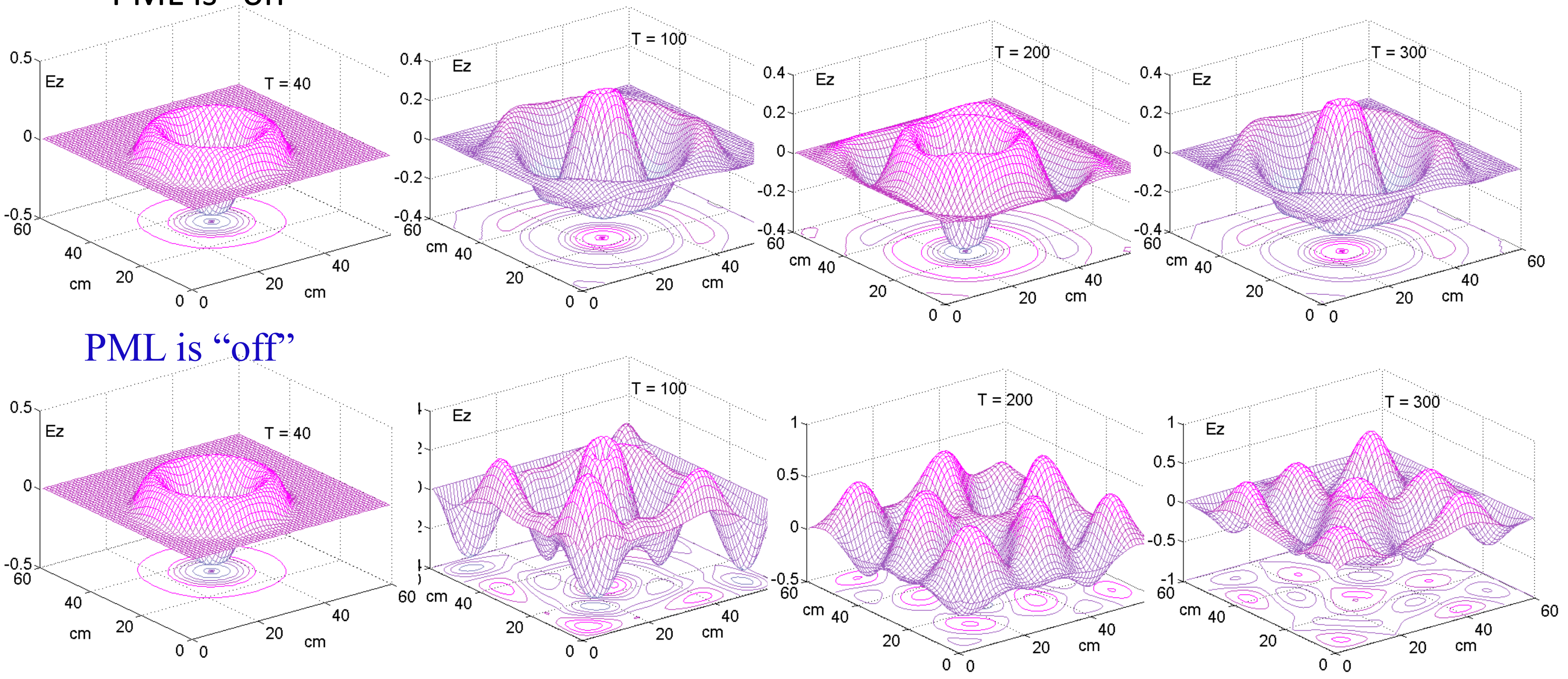
No reflection from PML interface

Importance of the PML

➤ Example: Simulation of the sinusoidal point source

PML is "on"

PML is "off"



Some Features of the FDTD

❖ Advantages

- FDTD is straightforward (fully explicit)
- Versatile (universal formulation)
- Time-domain (response at all frequencies can be obtained from a single simulation)
- EM fields can be easily visualized

❖ Issues

- Resource hungry (fields through the whole problem space are updated at each step)
- Discretization errors
- Time domain data is not immediately useful
- Problem space has to be truncated

Pros of The FDTD Method

- FDTD directly solves Maxwell's equations providing all information about the EM field at each of the space sells at every time-step
- Being a time-domain technique, FDTD directly calculates the impulse response of an electromagnetic system. Therefore? A single FDTD simulation can provide either ultrawideband temporal waveforms or the sinusoidal steady-state response at any frequency within the excitation spectrum
- FDTD uses no linear algebra
- Being a time-domain technique, FDTD directly calculates the nonlinear response of an electromagnetic system

Cons of The FDTD Method

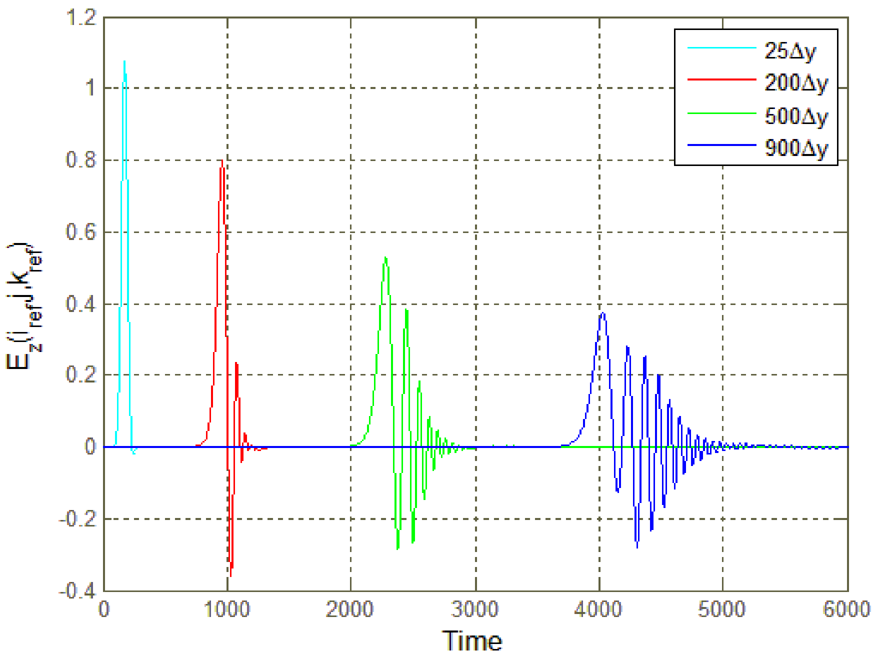
- Computationally expensive, requires large random access memory. At each time step values of the fields at each point in space are updated using values from the previous step

- FDTD works well with regular uniform meshes but the use of regular uniform meshes leads to staircasing. Implementation of nonuniform meshes, on the other hand, requires special mesh-generation software and can lead to additional computer operations and instabilities

- Requires truncation of the problem space in a way that does not create reflection errors

Numerical Dispersion

- Occurs because of the difference between the phase speed of the wave in the real world and the speed of propagation of the numerical wave along the grid

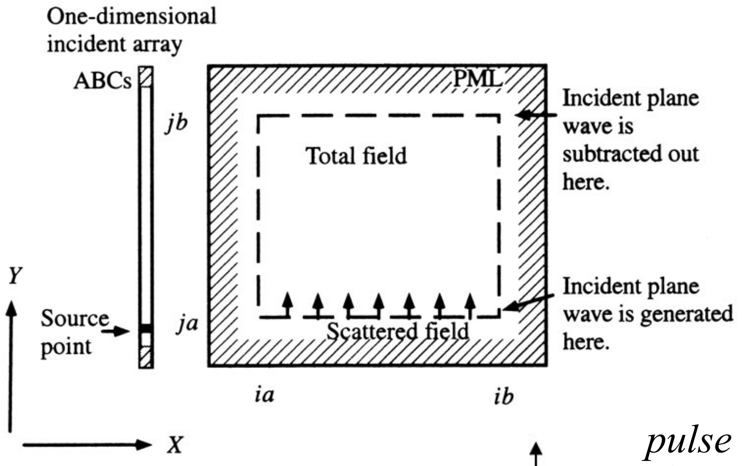


Distortion of the pulse propagating over the grid

(time domain data is recorded at different reference points)

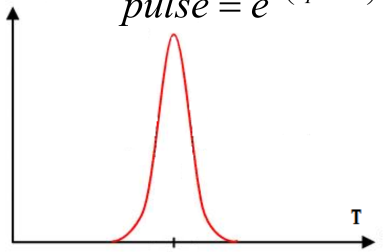
Setting Up a Simulation

- Main steps:
  - ✓ Discretize the problem space – create a mesh
  - ✓ Set up the source of the incident field
  - ✓ Truncate the problem space – create the absorbing boundary conditions (ABC)



- We are using (mainly):
  - ✓ Rectangular mesh
  - ✓ Plane wave source with Gaussian distribution
  - ✓ Perfectly matched layer (PML) for the ABC

$$pulse = e^{-\left(\frac{t_0 - T}{spread}\right)^2}$$

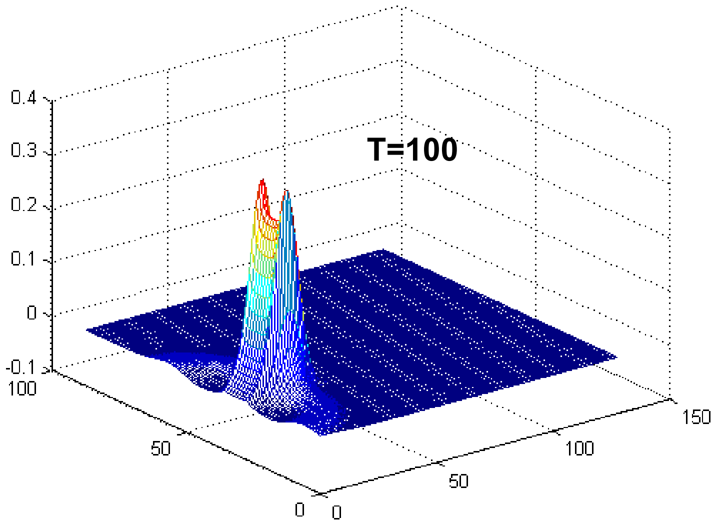


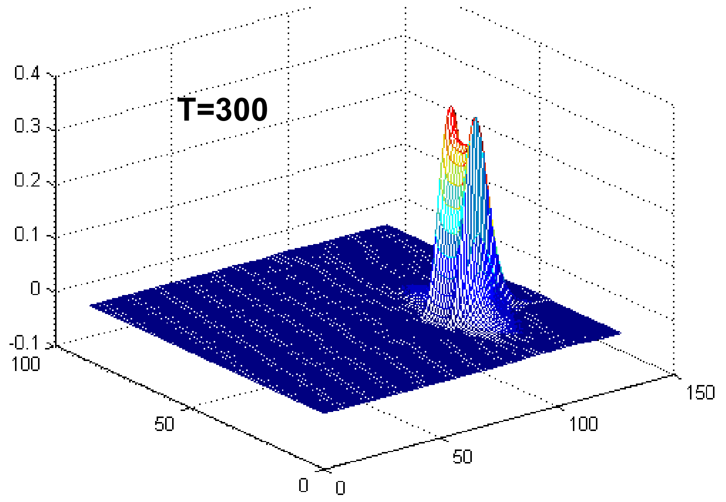
3D FDTD for Single Microstrip Line

Computational domain size: 90x130x20 cells

(in x, y, and z directions, respectively)

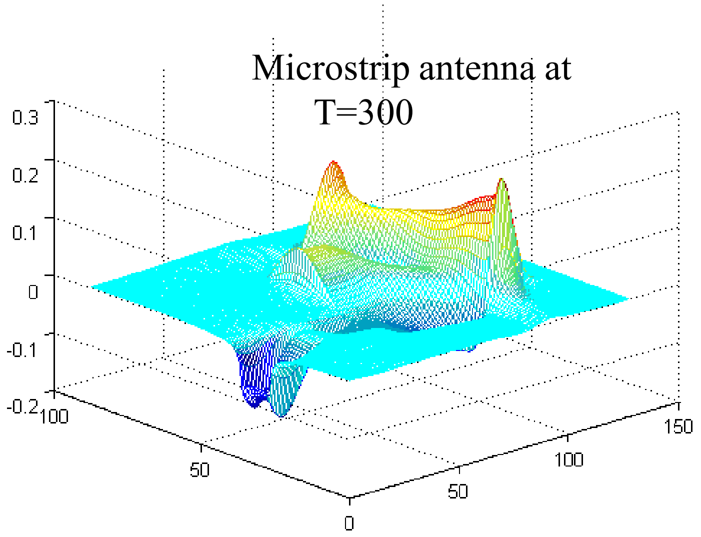
- *   Cell size 0.026 cm
- *   Source plane at y = 0
- *   Ground plane at z = 0
- *   Duroid substrate with relative permittivity 2.2. Electric field nodes on interface between duroid and free space use average permittivity of media to either side.
- *   Substrate 3 cells thick
- *   Microstrip 9 cells wide

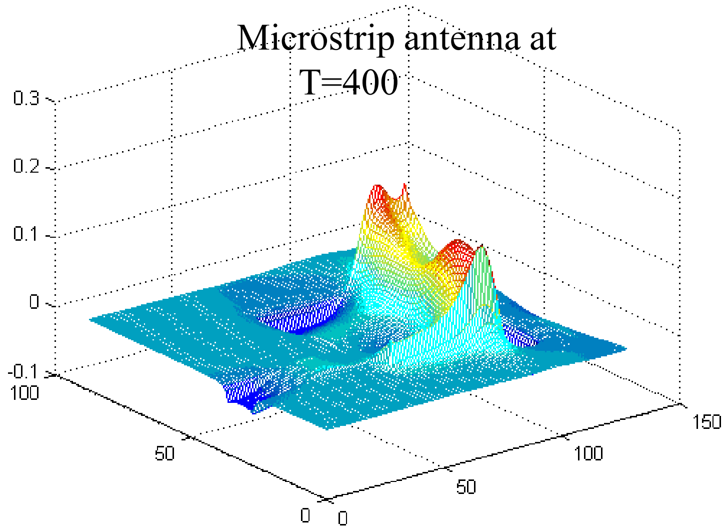




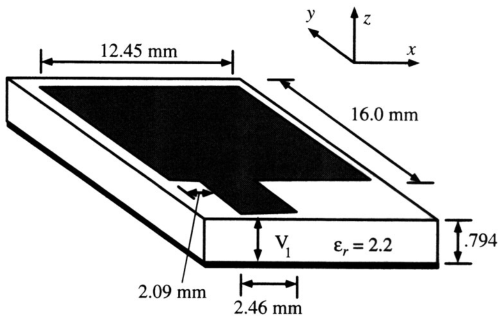
Figures on the left show a pulse propagating along the microstrip line. A Gaussian pulse is used for excitation. A voltage source is simulated by imposing the vertical *Ez* field in the area underneath the strip.

3D FDTD for Patch Antenna

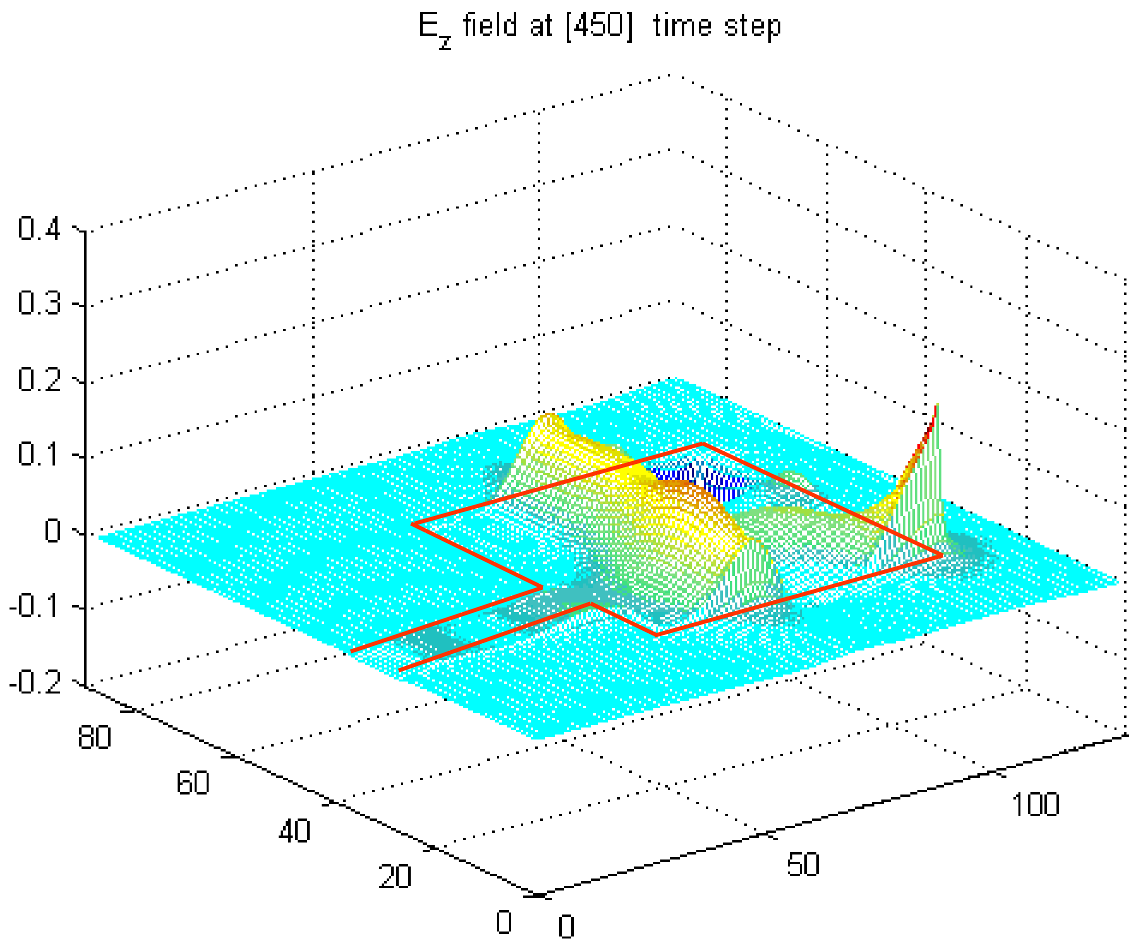




Patch dimensions 47 x 60 cells

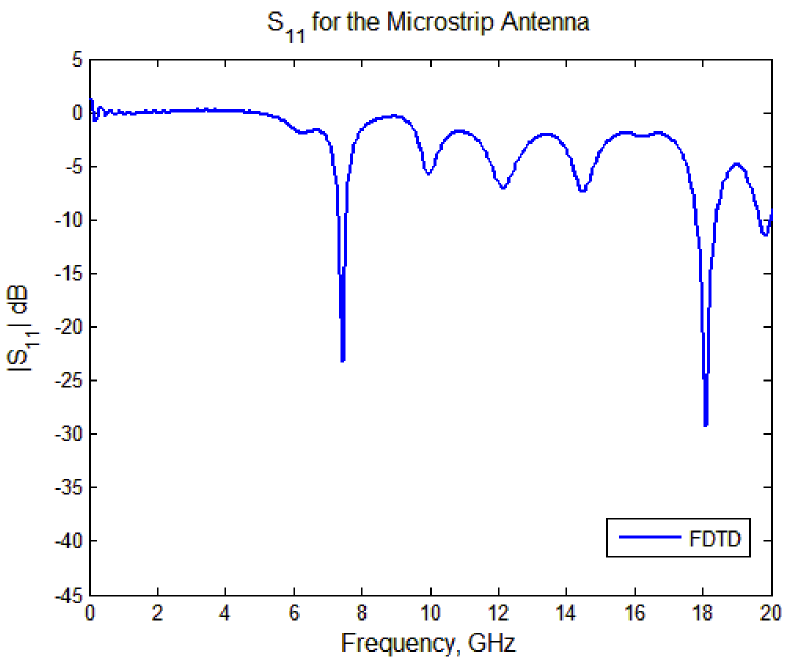


Simulation of the Microstrip Antenna

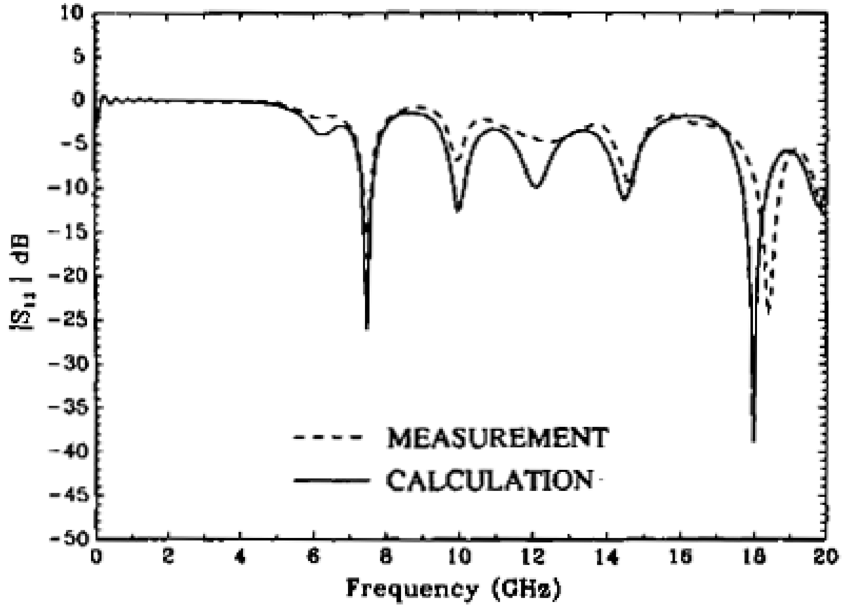


Frequency-Dependent Parameters

- S_{11} for the patch antenna $S_{11}(\omega) = 20 \cdot \log\left(\text{abs}\left(\dfrac{\text{fft}(inc)}{\text{fft}(ref)} \right) \right)$



Our simulation



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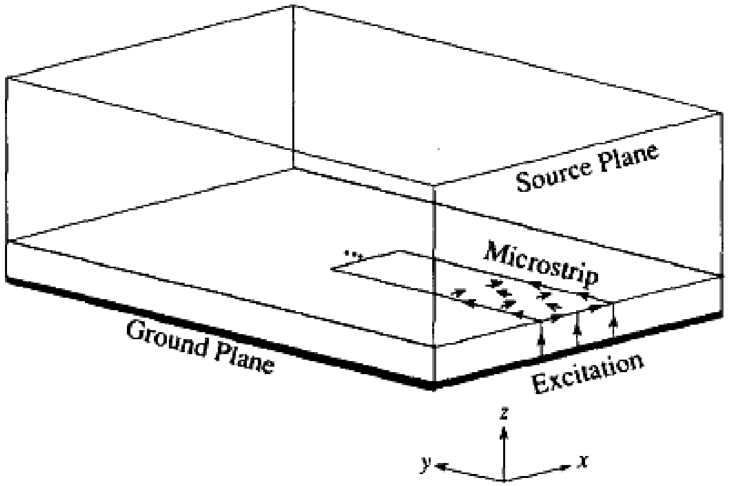
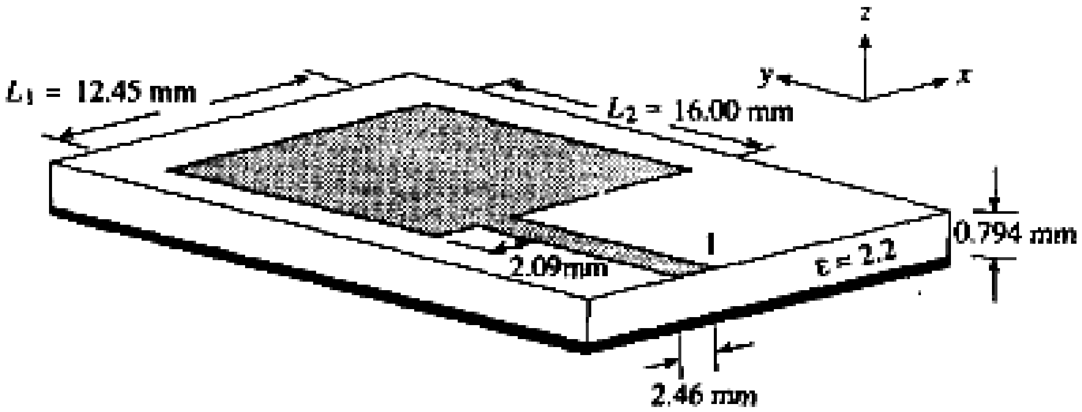
Simulation of Microstrip Structures

- Source setup:

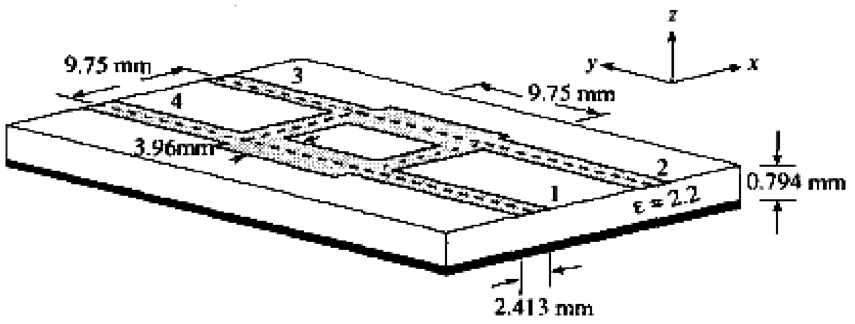


- Mic

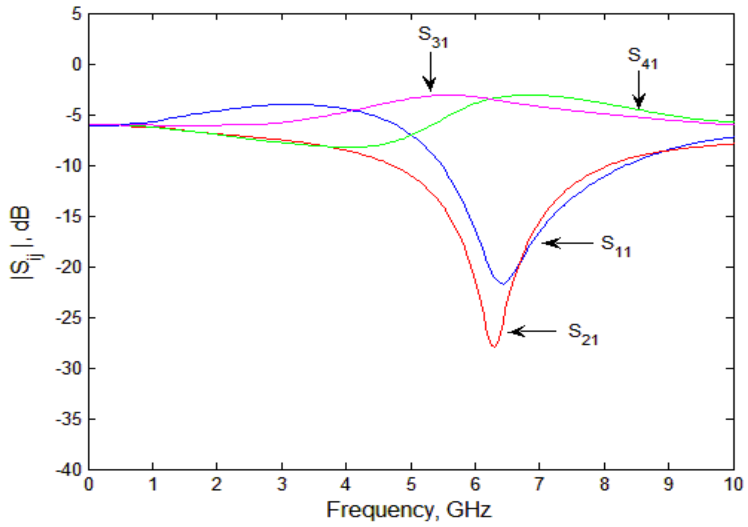


Microstrip Coupler

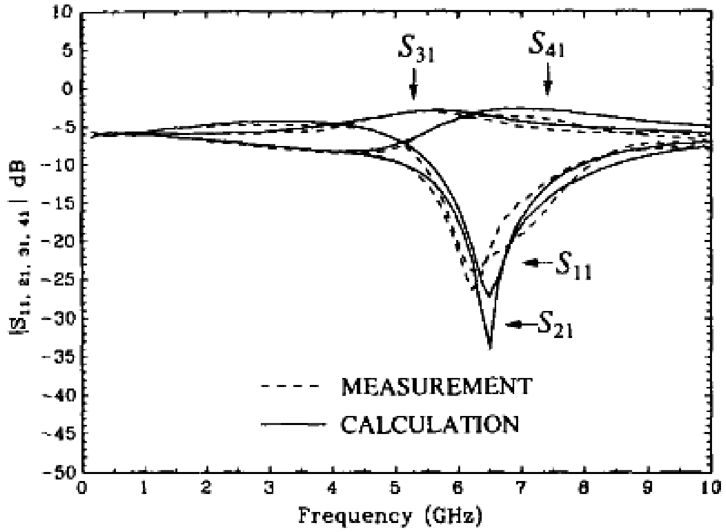
- Branch line coupler



- Scattering parameters of the branch line



Our simulation



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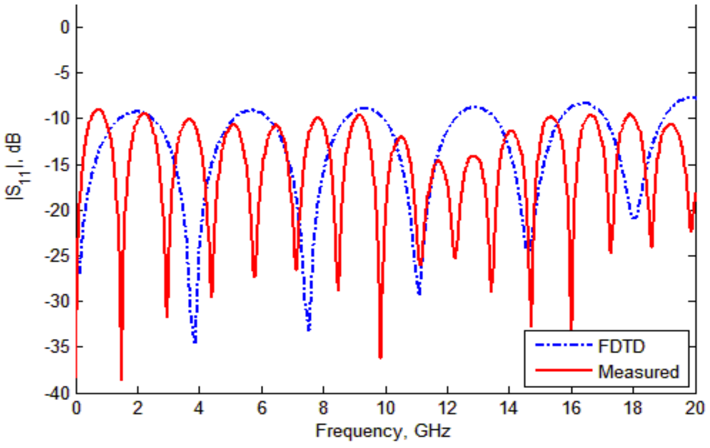
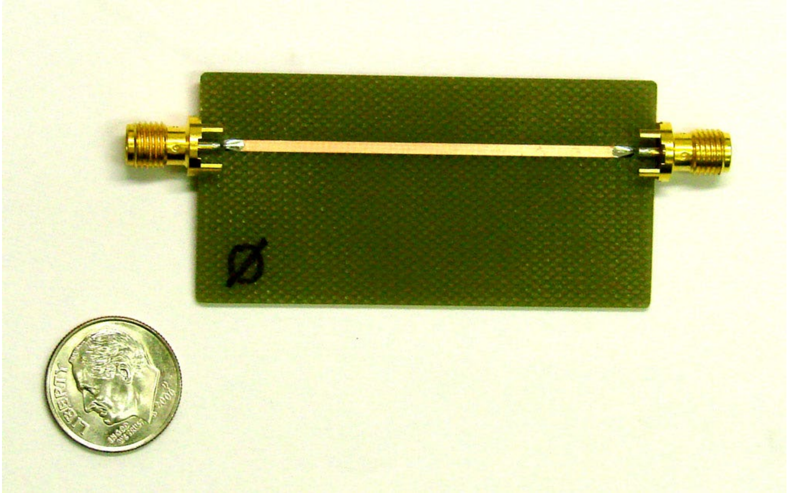
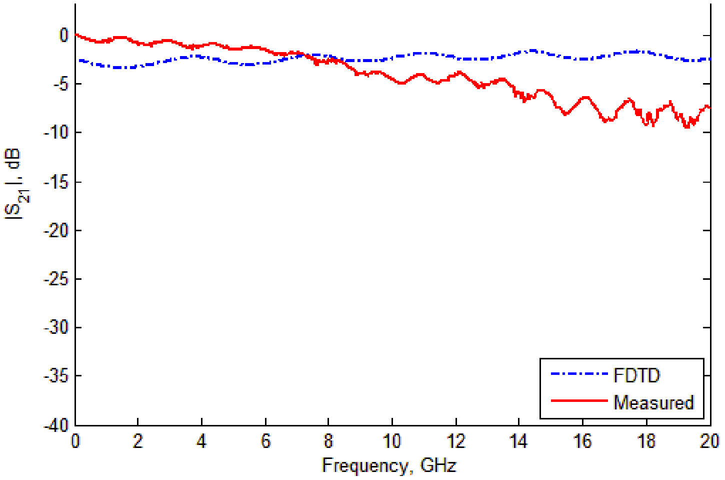
Single Straight Microstrip

- Comparison with measured data



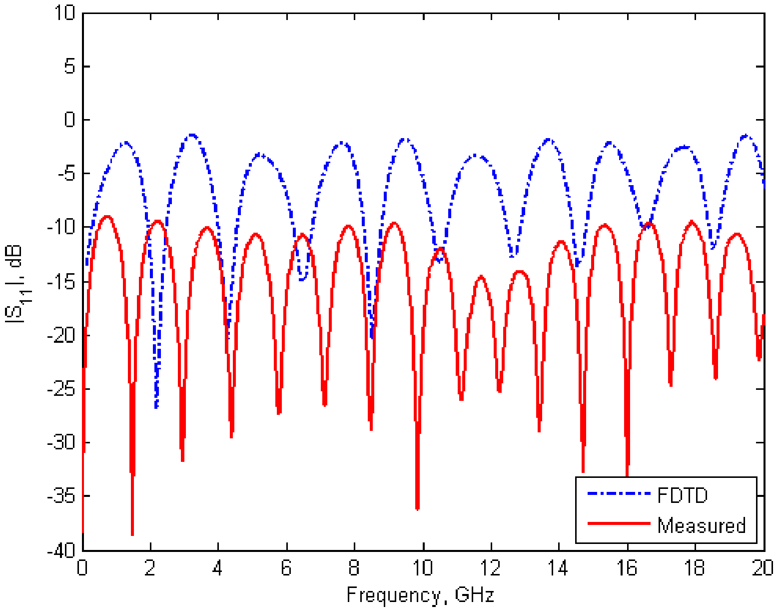


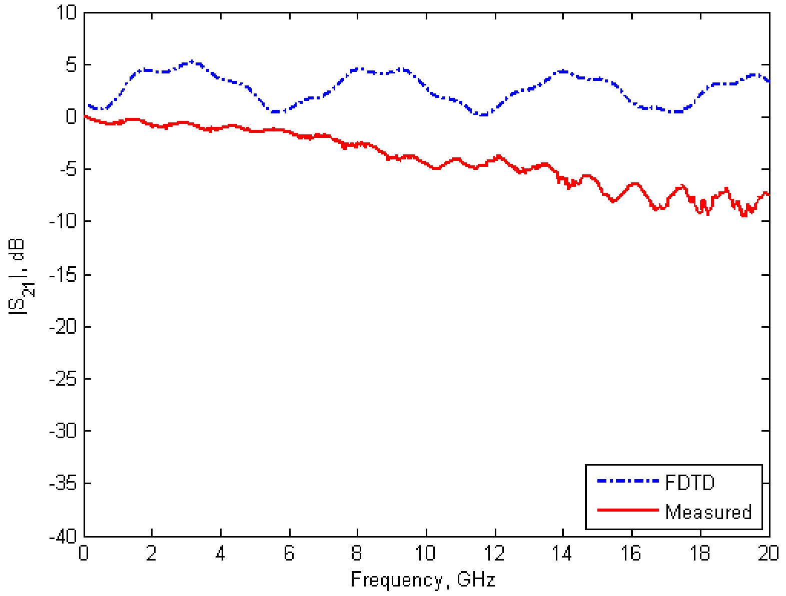


Comparison is only qualitative, since parameters used correspond to the line with (length/2)

Single Straight Microstrip

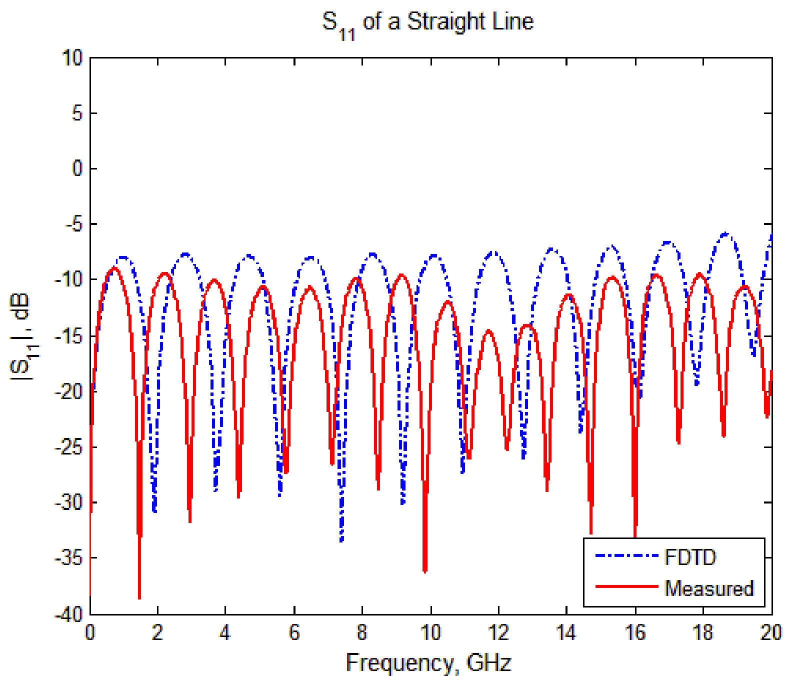
- Simulation with length doubled (example of what happens when the mesh is bad)

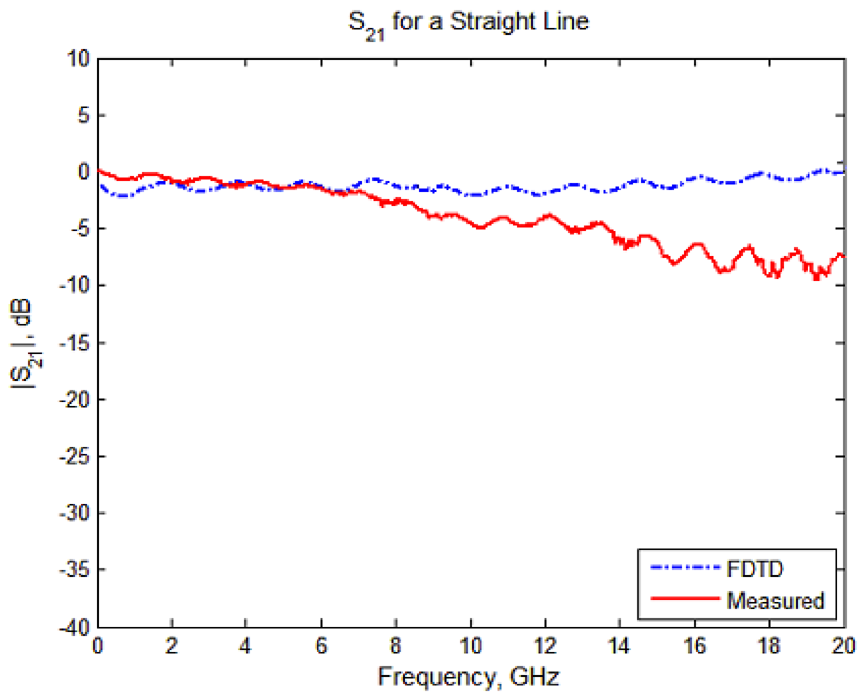




Single Straight Microstrip

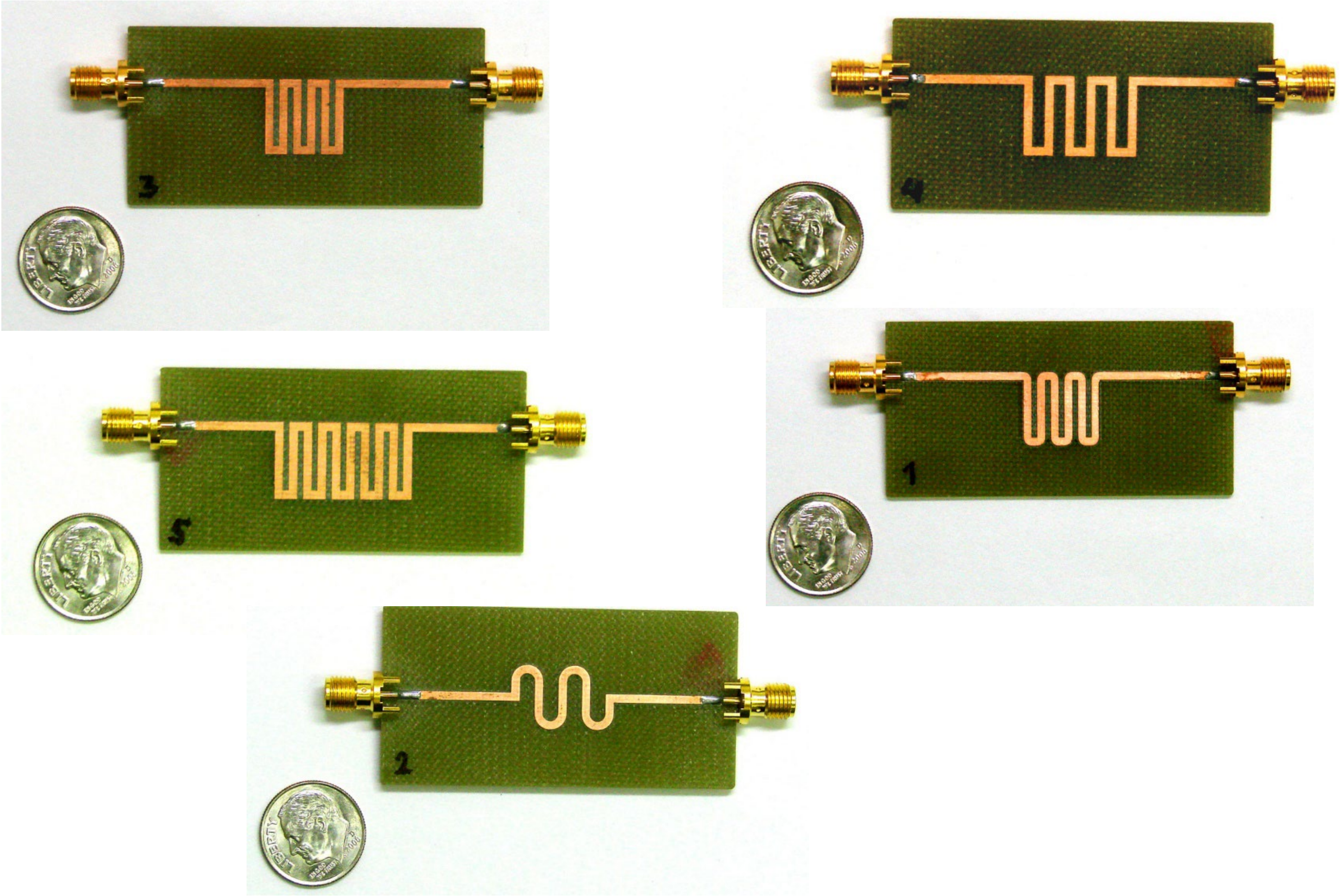
- Simulation with the adjusted mesh





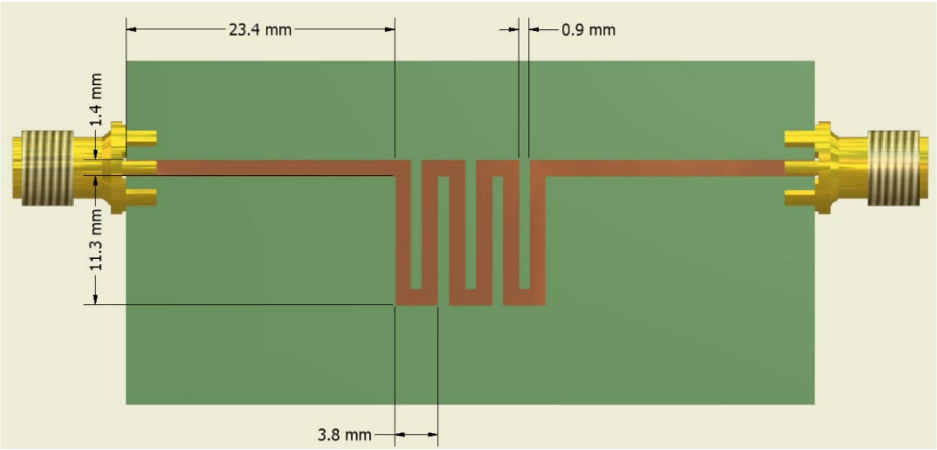
Meandered Microstrip Lines

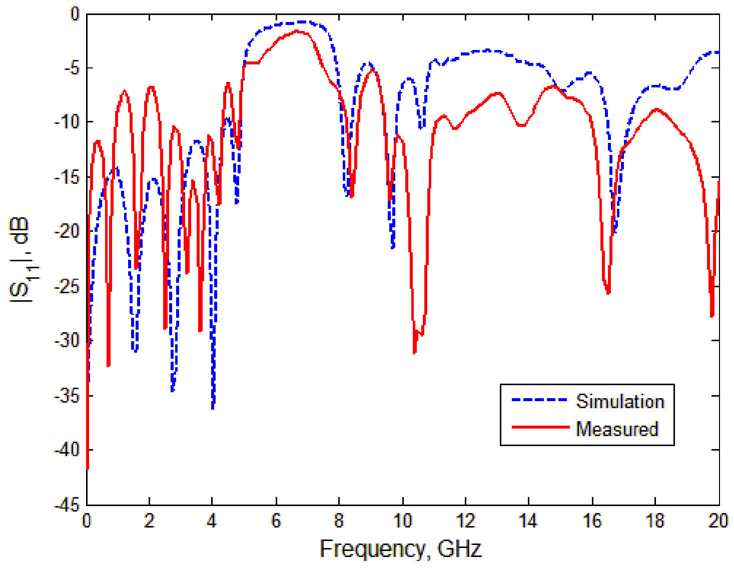
- Test boards were fabricated

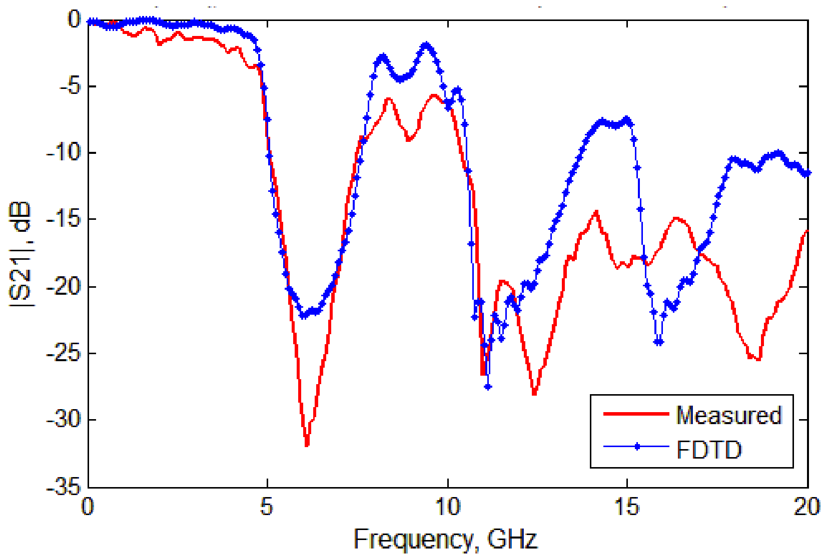


Simulation and Measurements

- Scattering parameters for the m-line #3

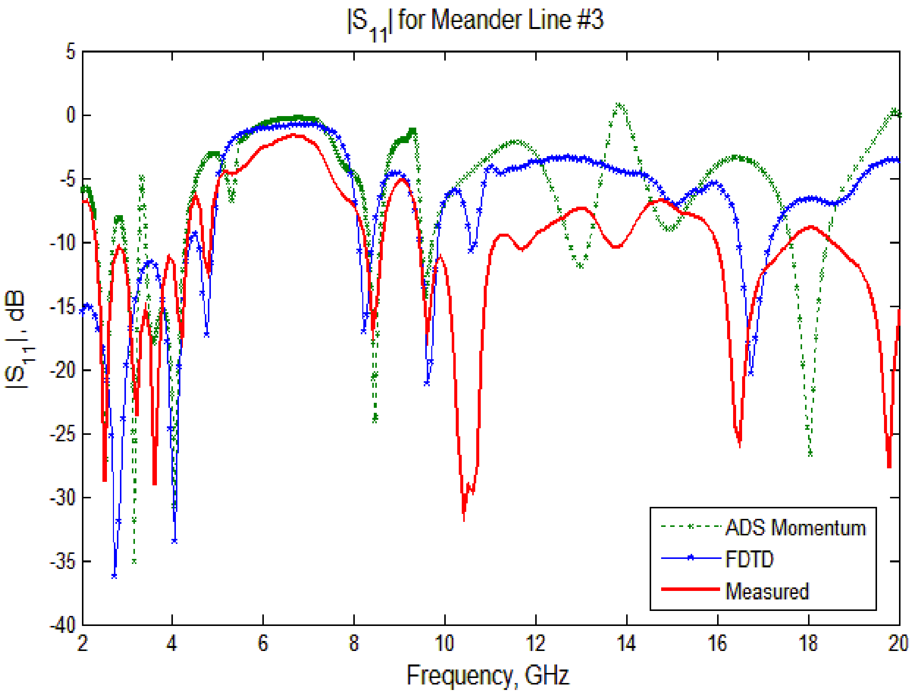


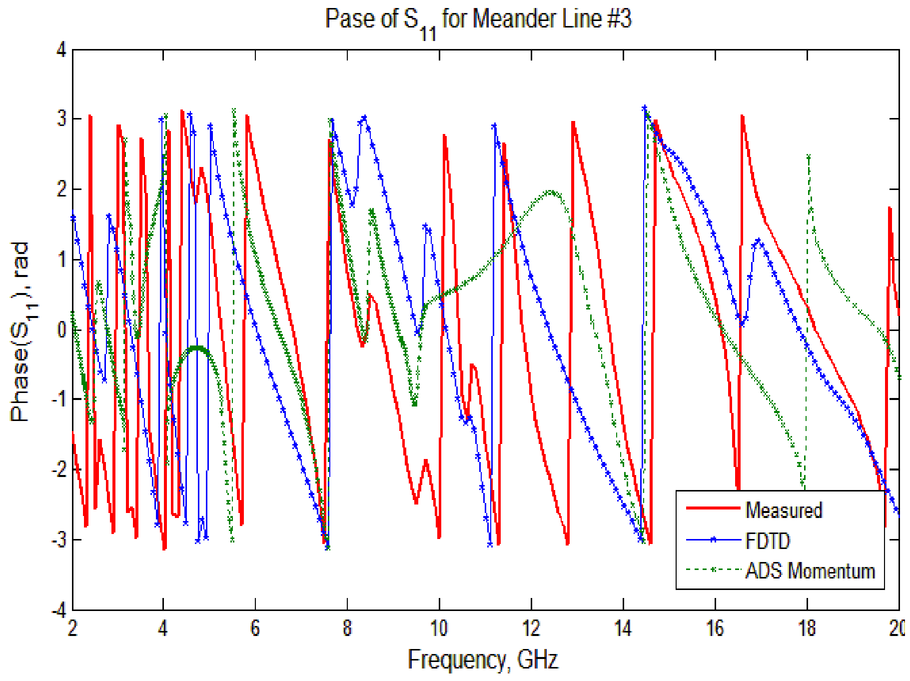




Comparison with ADS Momentum

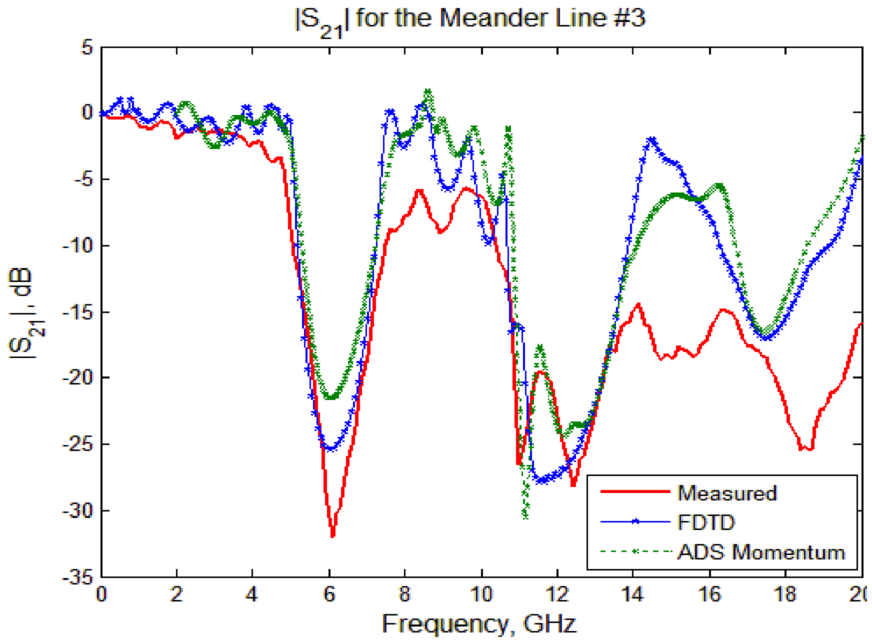
- The line was also simulated with Agilent ADS Momentum EM simulator

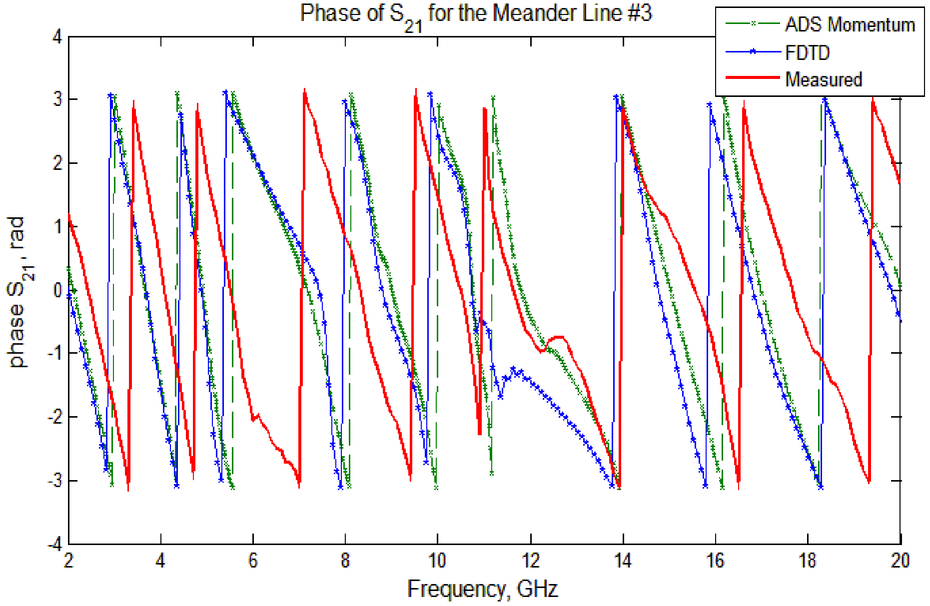




Comparison with ADS Momentum

- S_{21} parameters





FDTD References

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- D. Sullivan, *Electromagnetic simulation using the FDTD method, IEEE Press series on RF and microwave technology*, 2000.
- D.M. Sheen, S.M. Ali, M.D. Abouzahra, J.A. Kong, “Application of the Three-Dimensional Finite-Difference Time-Domain Method to the Analysis of Planar Microstrip Circuits”, *IEEE Trans. Microwave Theory Tech*., vol. 38, no 7, July 1990.