

# ECE 546

## Lecture - 06

# Coupled Lines

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# Crosstalk Noise

## Signal Integrity

**Crosstalk**

**Dispersion**

**Attenuation**

**Reflection**

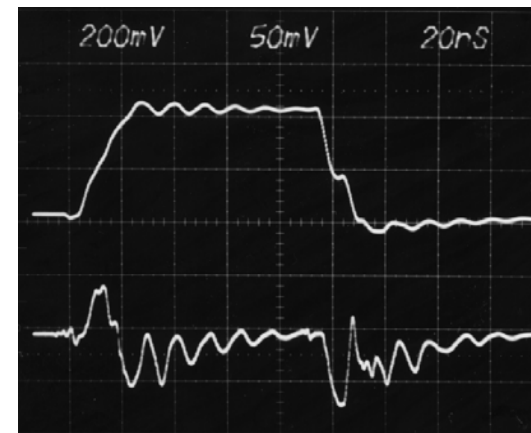
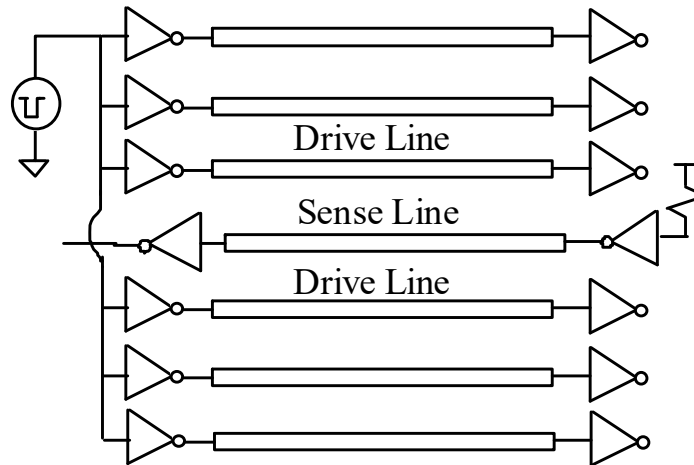
**Distortion**

**Loss**

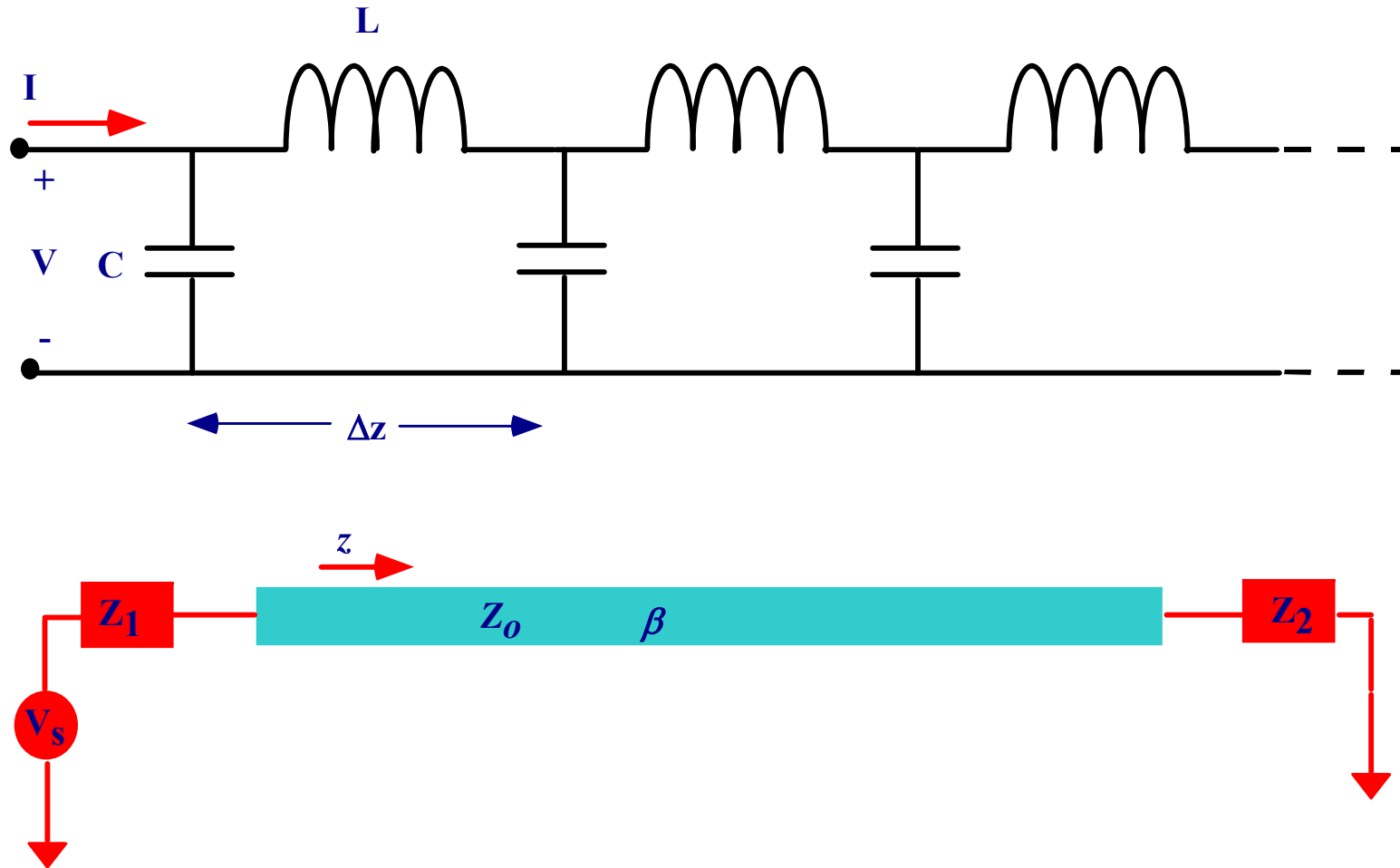
**Delta I Noise**

**Ground Bounce**

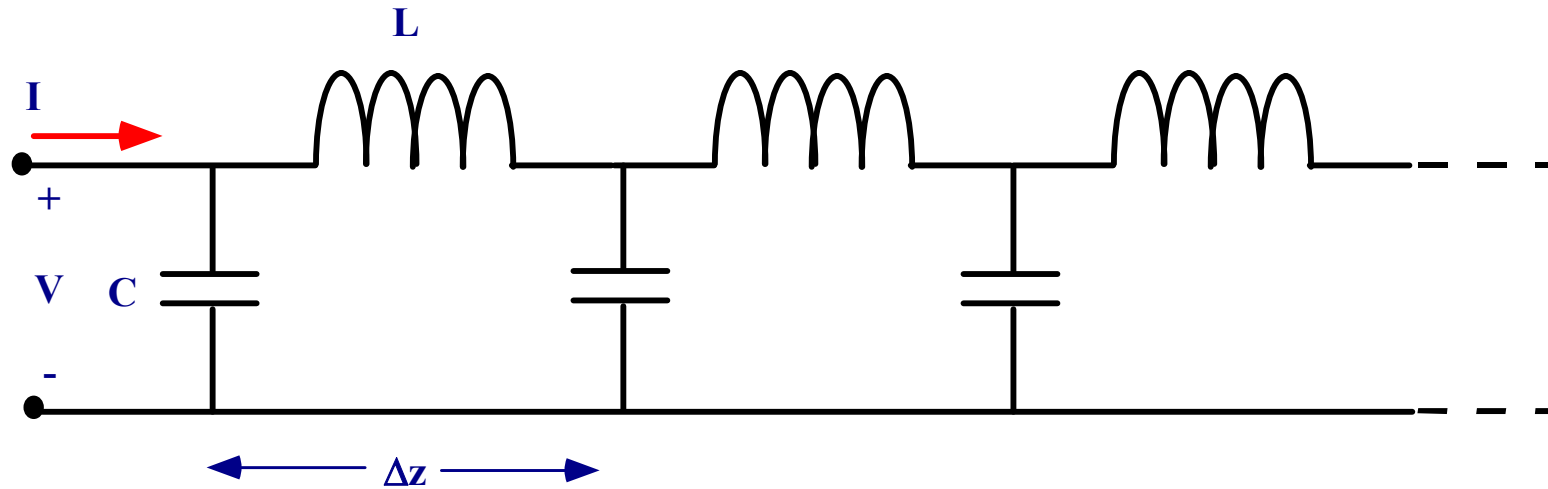
**Radiation**



# TEM PROPAGATION



# Telegrapher's Equations



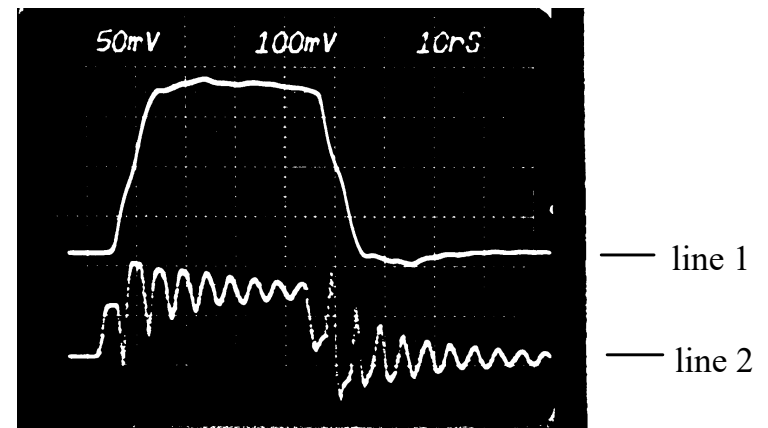
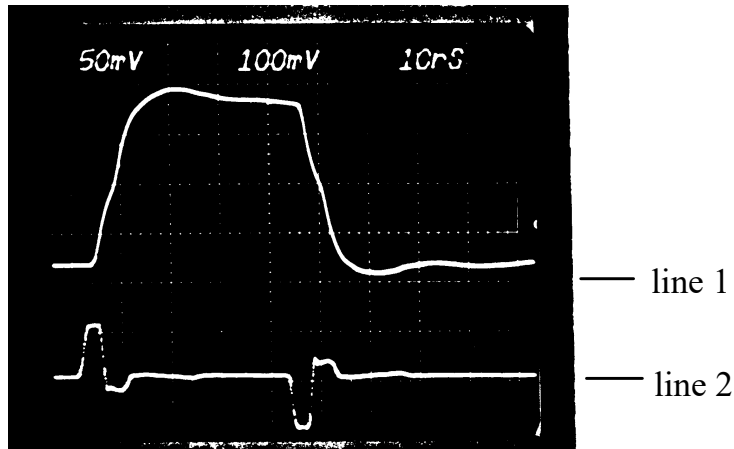
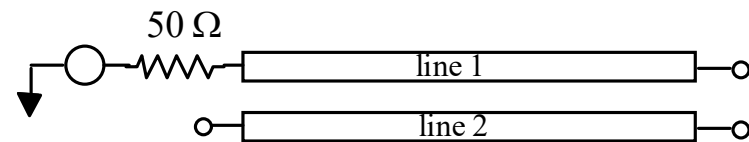
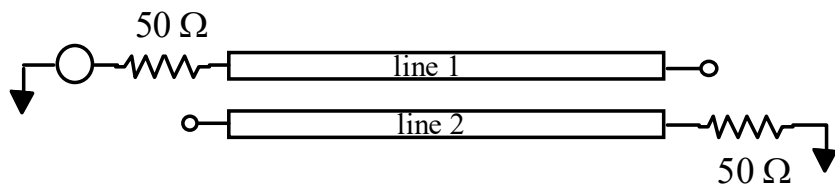
$$-\frac{\partial V}{\partial z} = L \frac{\partial I}{\partial t}$$

$$-\frac{\partial I}{\partial z} = C \frac{\partial V}{\partial t}$$

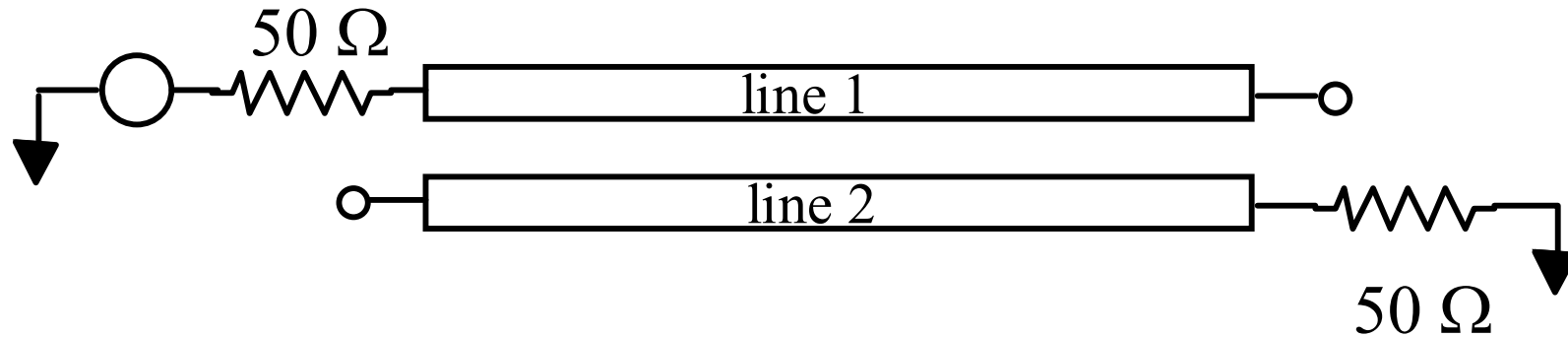
**$L$ : Inductance per unit length.**

**$C$ : Capacitance per unit length.**

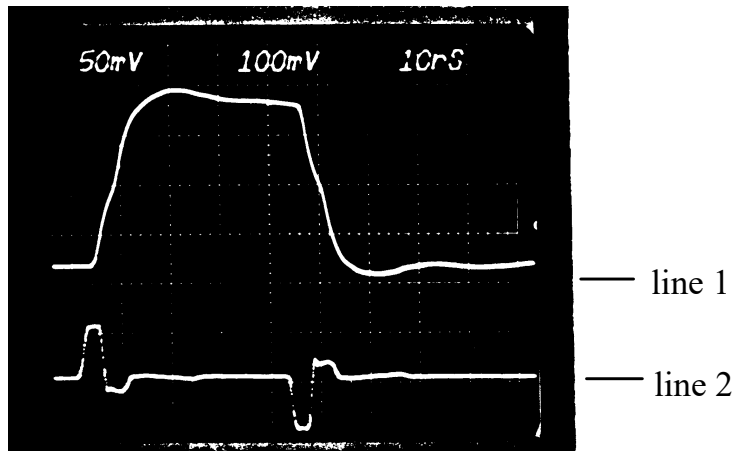
# Crosstalk noise depends on termination



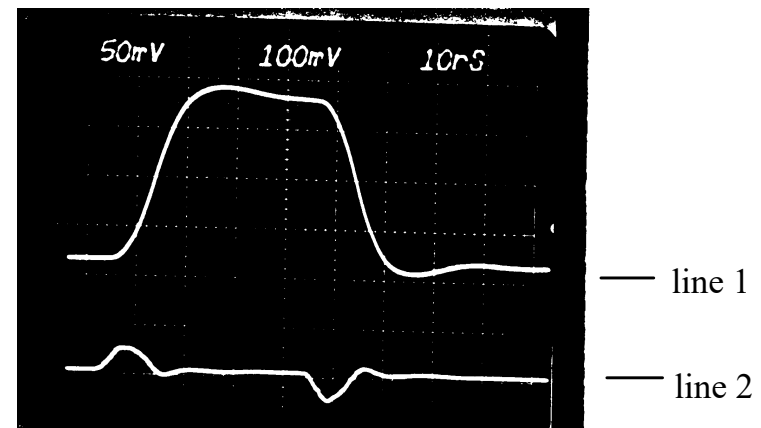
# Crosstalk depends on signal rise time



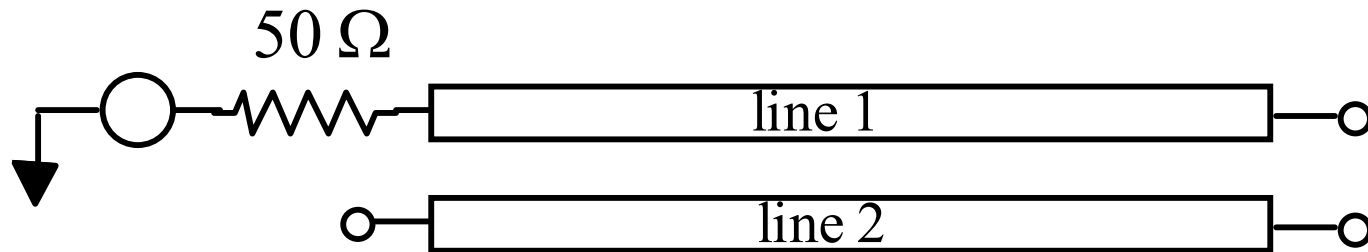
$t_r = 1\ \text{ns}$



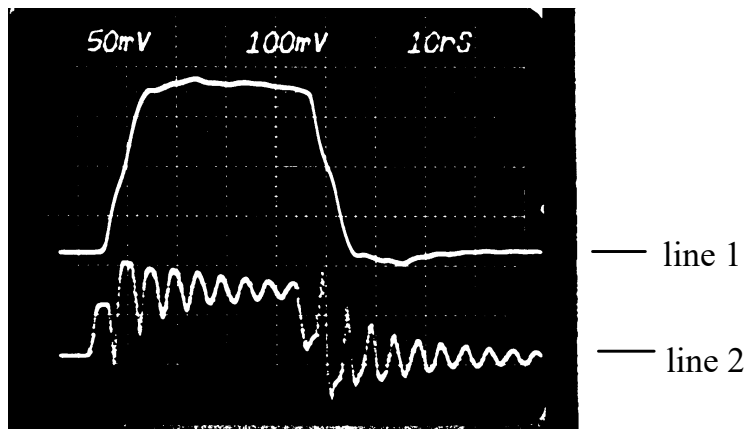
$t_r = 7\ \text{ns}$



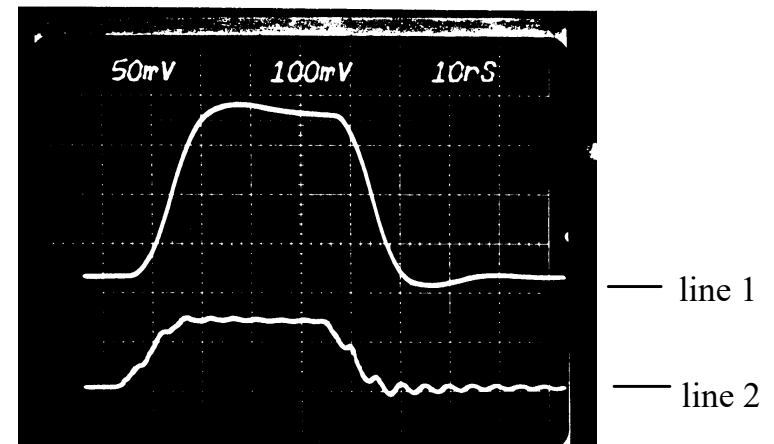
# Crosstalk depends on signal rise time



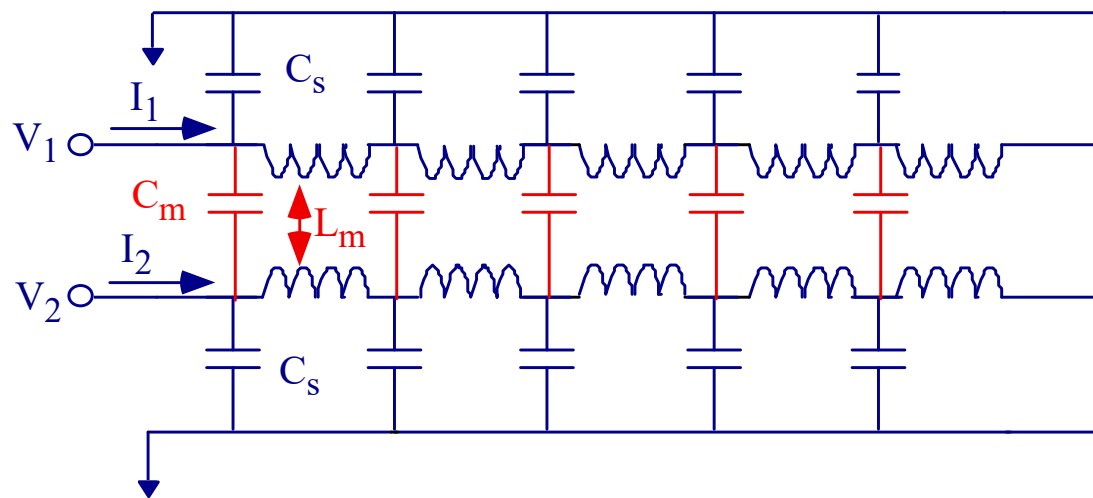
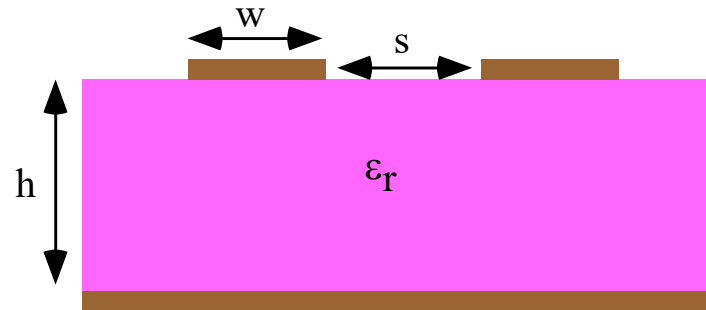
$t_r = 1\ \text{ns}$



$t_r = 7\ \text{ns}$



# Coupled Transmission Lines





# Telegraphers Equations for Coupled Transmission Lines

## Maxwellian Form

$$-\frac{\partial V_1}{\partial z} = L_{11} \frac{\partial I_1}{\partial t} + L_{12} \frac{\partial I_2}{\partial t}$$

$$-\frac{\partial V_2}{\partial z} = L_{21} \frac{\partial I_1}{\partial t} + L_{22} \frac{\partial I_2}{\partial t}$$

$$-\frac{\partial I_1}{\partial z} = C_{11} \frac{\partial V_1}{\partial t} + C_{12} \frac{\partial V_2}{\partial t}$$

$$-\frac{\partial I_2}{\partial z} = C_{21} \frac{\partial V_1}{\partial t} + C_{22} \frac{\partial V_2}{\partial t}$$

# Telegraphers Equations for Coupled Transmission Lines

Physical form

$$-\frac{\partial V_1}{\partial z} = L_s \frac{\partial I_1}{\partial t} + L_m \frac{\partial I_2}{\partial t}$$

$$-\frac{\partial V_2}{\partial z} = L_m \frac{\partial I_1}{\partial t} + L_s \frac{\partial I_2}{\partial t}$$

$$-\frac{\partial I_1}{\partial z} = C_s \frac{\partial V_1}{\partial t} + C_m \frac{\partial V_1}{\partial t} - C_m \frac{\partial V_2}{\partial t}$$

$$-\frac{\partial I_2}{\partial z} = -C_m \frac{\partial V_1}{\partial t} + C_m \frac{\partial V_2}{\partial t} + C_s \frac{\partial V_2}{\partial t}$$

# Relations Between Physical and Maxwellian Parameters (symmetric lines)

$$L_{11} = L_{22} = L_s$$

$$L_{12} = L_{21} = L_m$$

$$C_{11} = C_{22} = C_s + C_m$$

$$C_{12} = C_{21} = - C_m$$

# Even Mode

$$-\frac{\partial V_e}{\partial z} = (L_{11} + L_{12}) \frac{\partial I_e}{\partial t}$$

$$-\frac{\partial I_e}{\partial z} = (C_{11} + C_{12}) \frac{\partial V_e}{\partial t}$$

**Add voltage  
and current  
equations**

**$V_e$  : Even mode voltage**  $V_e = \frac{1}{2}(V_1 + V_2)$

**$I_e$  : Even mode current**  $I_e = \frac{1}{2}(I_1 + I_2)$

$$Z_e = \sqrt{\frac{L_{11} + L_{12}}{C_{11} + C_{12}}} = \sqrt{\frac{L_s + L_m}{C_s}}$$

**Impedance**

$$v_e = \frac{1}{\sqrt{(L_{11} + L_{12})(C_{11} + C_{12})}} = \frac{1}{\sqrt{(L_s + L_m)C_s}}$$

**velocity**

# Odd Mode

$$-\frac{\partial V_d}{\partial z} = (L_{11} - L_{12}) \frac{\partial I_d}{\partial t}$$

$$-\frac{\partial I_d}{\partial z} = (C_{11} - C_{12}) \frac{\partial V_d}{\partial t}$$

**Subtract voltage  
and current  
equations**

**$V_d$  : Odd mode voltage**

$$V_d = \frac{1}{2}(V_1 - V_2)$$

**$I_d$  : Odd mode current**

$$I_d = \frac{1}{2}(I_1 - I_2)$$

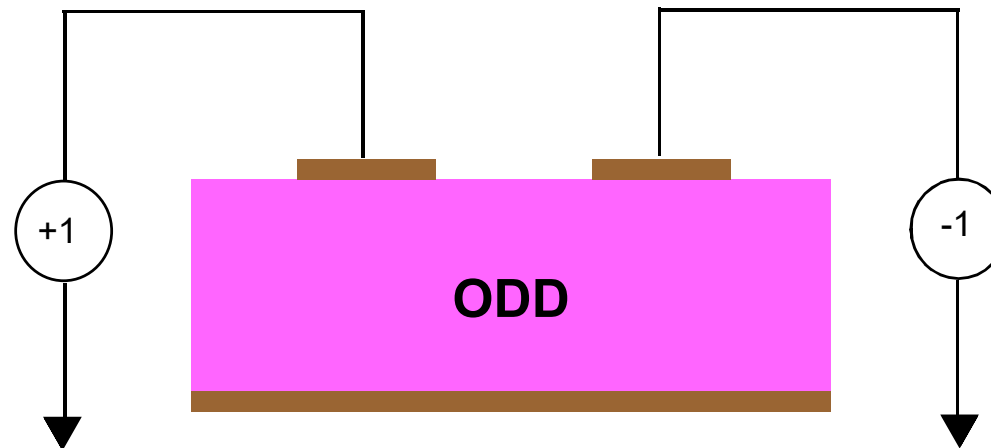
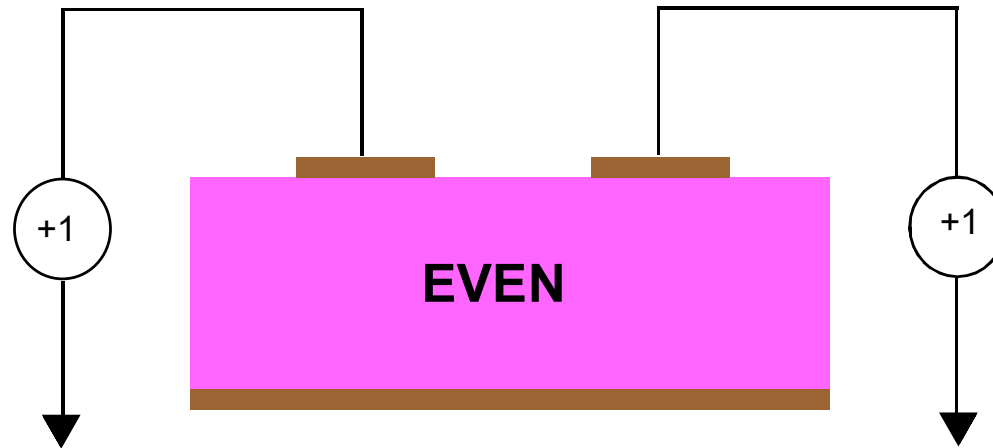
$$Z_d = \sqrt{\frac{L_{11} - L_{12}}{C_{11} - C_{12}}} = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}}$$

**Impedance**

$$V_d = \frac{1}{\sqrt{(L_{11} - L_{12})(C_{11} - C_{12})}} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}}$$

**velocity**

# Mode Excitation



# PHYSICAL SIGNIFICANCE OF EVEN- AND ODD-MODE IMPEDANCES

- \*  $Z_e$  and  $Z_d$  are the wave resistance seen by the even and odd mode travelling signals respectively.
- \* The impedance of each line is no longer described by a single characteristic impedance; instead, we have

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

# Definitions

## Even-Mode Impedance: $Z_e$

Impedance seen by wave propagating through the coupled-line system when excitation is symmetric (1, 1).

## Odd-Mode Impedance: $Z_d$

Impedance seen by wave propagating through the coupled-line system when excitation is anti-symmetric (1, -1).

## Common-Mode Impedance: $Z_c = 0.5Z_e$

Impedance seen by a pair of line and a common return by a common signal.

## Differential Impedance: $Z_{\text{diff}} = 2Z_d$

Impedance seen across a pair of lines by differential mode signal.



# EVEN AND ODD-MODE IMPEDANCES

**$Z_{11}, Z_{22}$  : Self Impedances**

**$Z_{12}, Z_{21}$  : Mutual Impedances**

**For symmetrical lines,**

$$\mathbf{Z_{11} = Z_{22} \text{ and } Z_{12} = Z_{21}}$$

# Coupled Lines

## Line Space

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

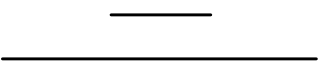
## Modal Space

$$V_e = Z_e I_e$$

$$V_d = Z_d I_d$$

$$\begin{bmatrix} V_e \\ V_d \end{bmatrix} = \begin{bmatrix} Z_e & 0 \\ 0 & Z_d \end{bmatrix} \begin{bmatrix} I_e \\ I_d \end{bmatrix}$$

## EXAMPLE (Microstrip)


$$\epsilon_r = 4.3$$
$$Z_s = 56.4 \Omega$$

### Single Line

Dielectric height = 6 mils


Width = 8 mils

### Coupled Lines

Height = 6 mils

Width = 8 mils

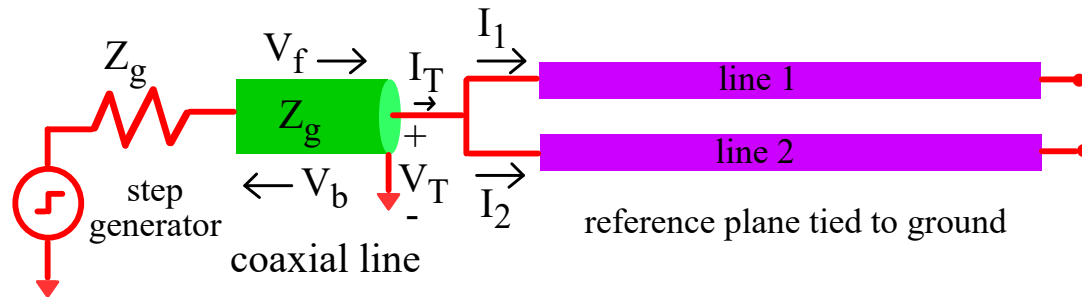
Spacing = 12 mils


$$\epsilon_r = 4.3$$

$$Z_e = 68.1 \Omega \quad Z_d = 40.8 \Omega$$

$$Z_{11} = 54.4 \Omega \quad Z_{12} = 13.6 \Omega$$

# Even Mode

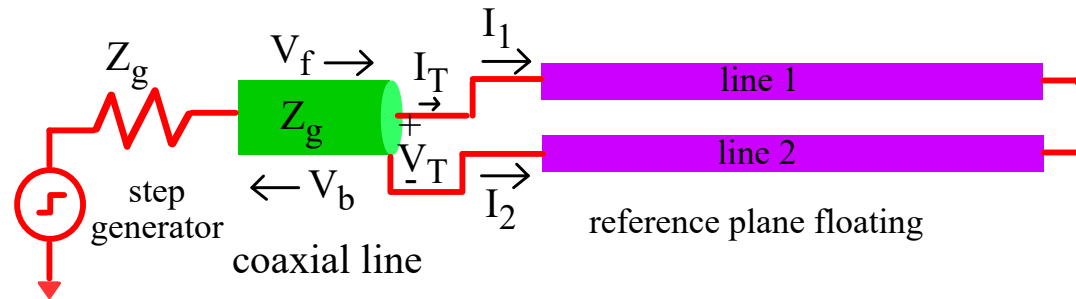


$$I_{tdr} = \left[ \frac{a_e(t,0)}{Z_e} + \frac{a_d(t,0)}{Z_d} \right] + \left[ \frac{a_e(t,0)}{Z_e} - \frac{a_d(t,0)}{Z_d} \right]$$

$$V_{tdr} = a_e(t,0) - a_d(t,0) \quad a_d(t,0) = 0$$

$$\frac{V_{tdr}}{I_{tdr}} = \frac{Z_e}{2} \quad Z_e = 2 \left( \frac{1 + \rho_e}{1 - \rho_e} \right) Z_g \quad v_e = \frac{2l}{\tau_e}$$

# Odd Mode



$$V_{tdr} = a_e(t,0) + a_d(t,0) - [a_e(t,0) - a_d(t,0)] = V_f + V_b$$

$$I_{tdr} = \left[ \frac{a_e(t,0)}{Z_e} + \frac{a_d(t,0)}{Z_d} \right]$$

$$I_{tdr} = - \left[ \frac{a_e(t,0)}{Z_e} - \frac{a_d(t,0)}{Z_d} \right]$$

$$a_e(t,0) = 0,$$

$$\frac{V_{tdr}}{I_{tdr}} = 2Z_d$$

$$Z_d = \frac{1}{2} \left( \frac{1 + \rho_d}{1 - \rho_d} \right) Z_g, \quad V_d = \frac{2V}{\tau_d}$$

# EXTRACT INDUCTANCE AND CAPACITANCE COEFFICIENTS

$$L_s = \frac{1}{2} \left[ \frac{Z_e}{v_e} + \frac{Z_d}{v_d} \right]$$

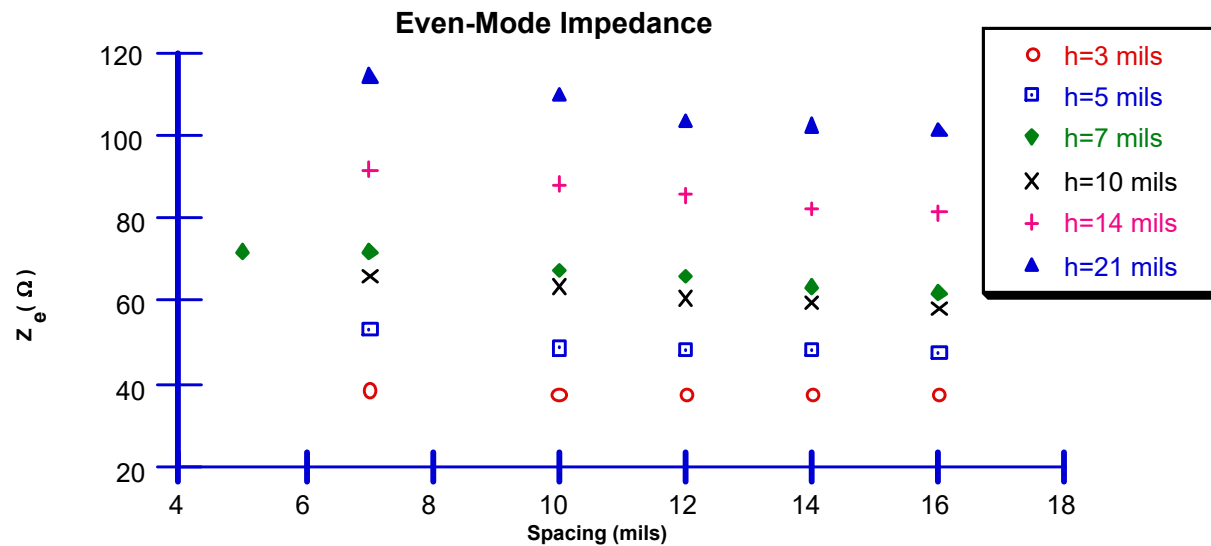
$$C_s = \frac{1}{Z_e v_e}$$

$$L_m = \frac{1}{2} \left[ \frac{Z_e}{v_e} - \frac{Z_d}{v_d} \right]$$

$$C_m = \frac{1}{2} \left[ \frac{1}{Z_e v_e} - \frac{1}{Z_d v_d} \right]$$

$$Z_d < Z_s < Z_e$$

# Measured even-mode impedance

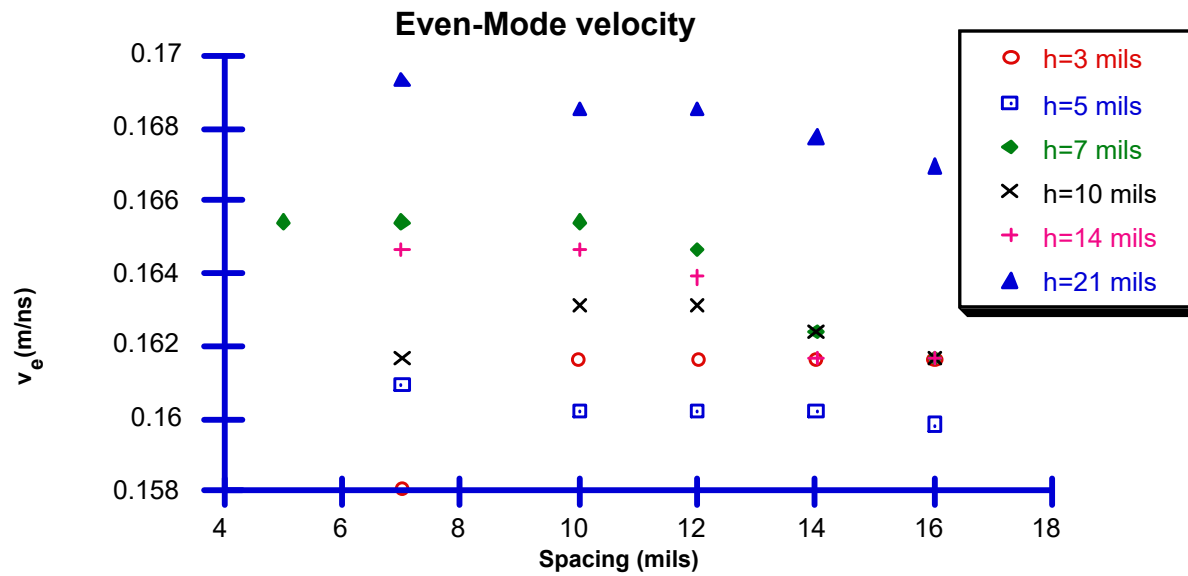


# Measured odd-mode impedance

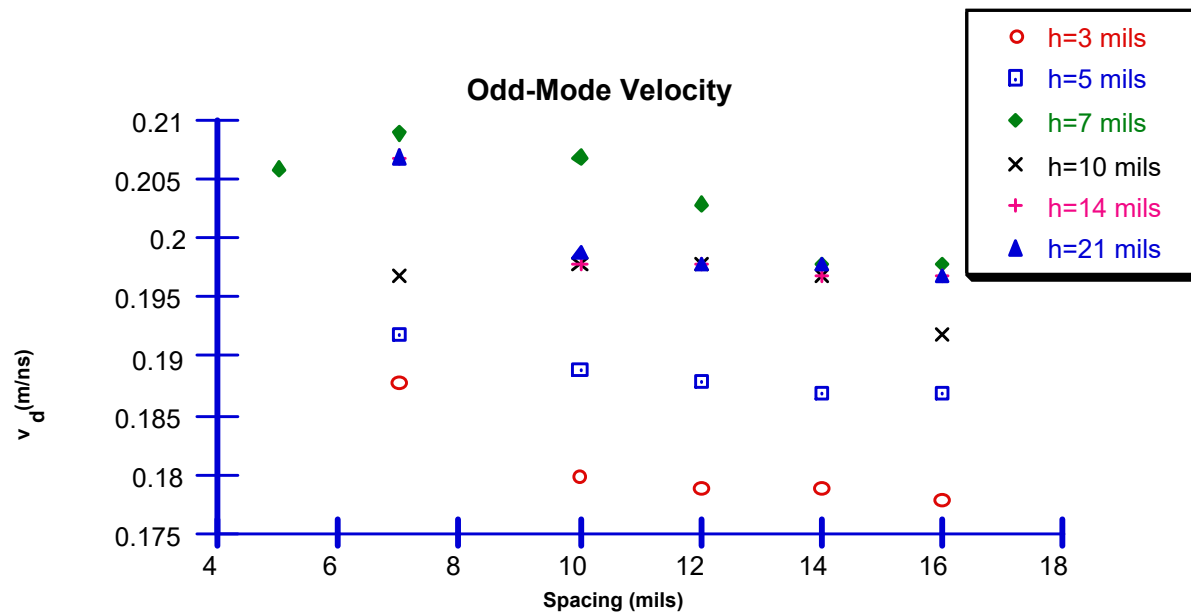




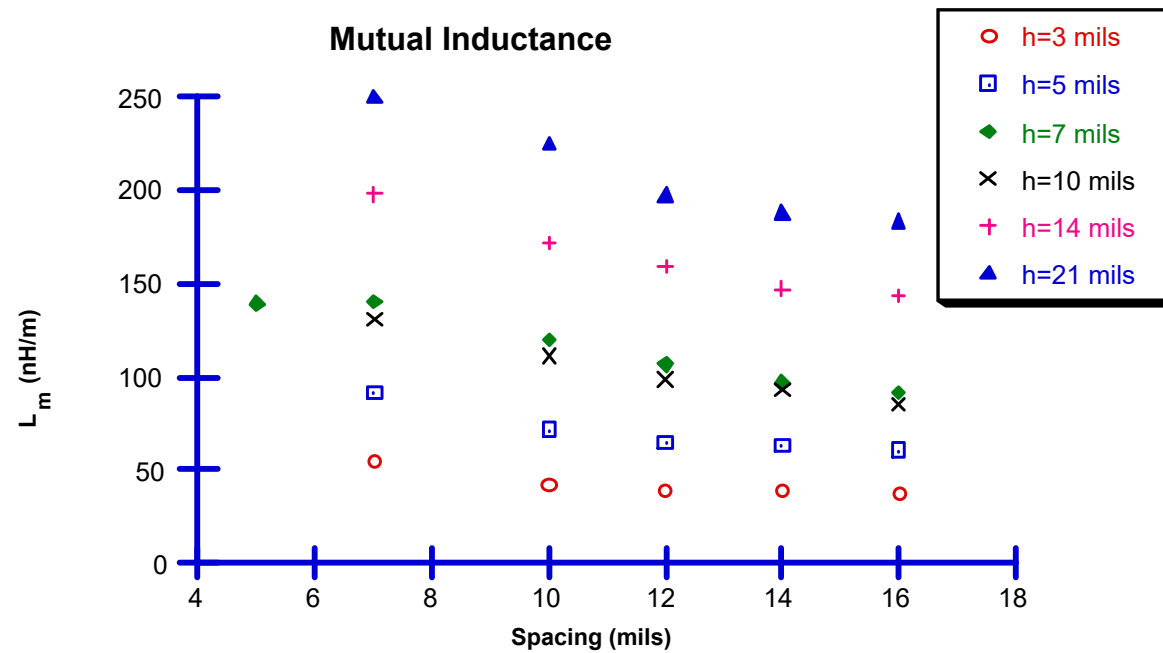
# Measured even-mode velocity



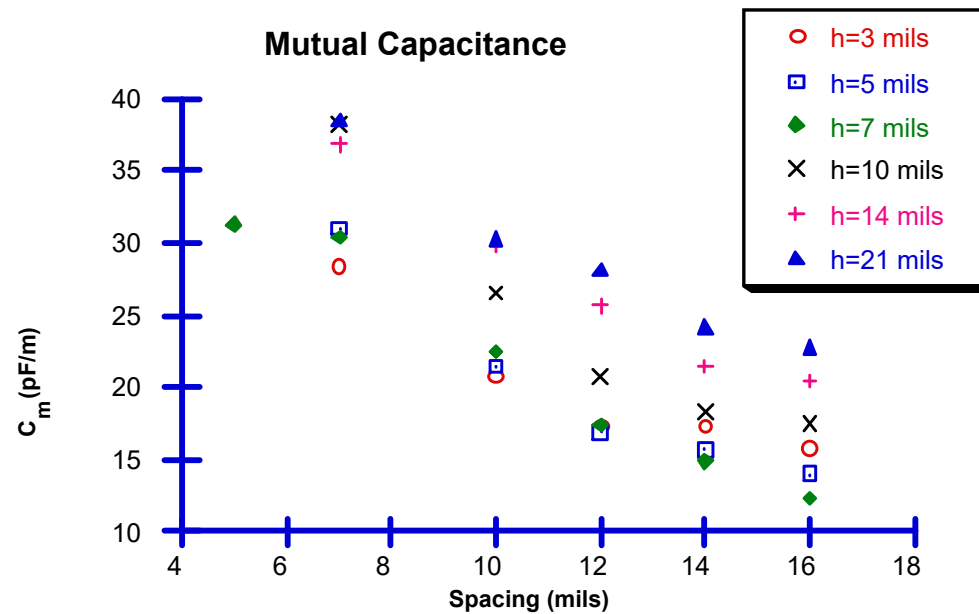
# Measured odd-mode velocity



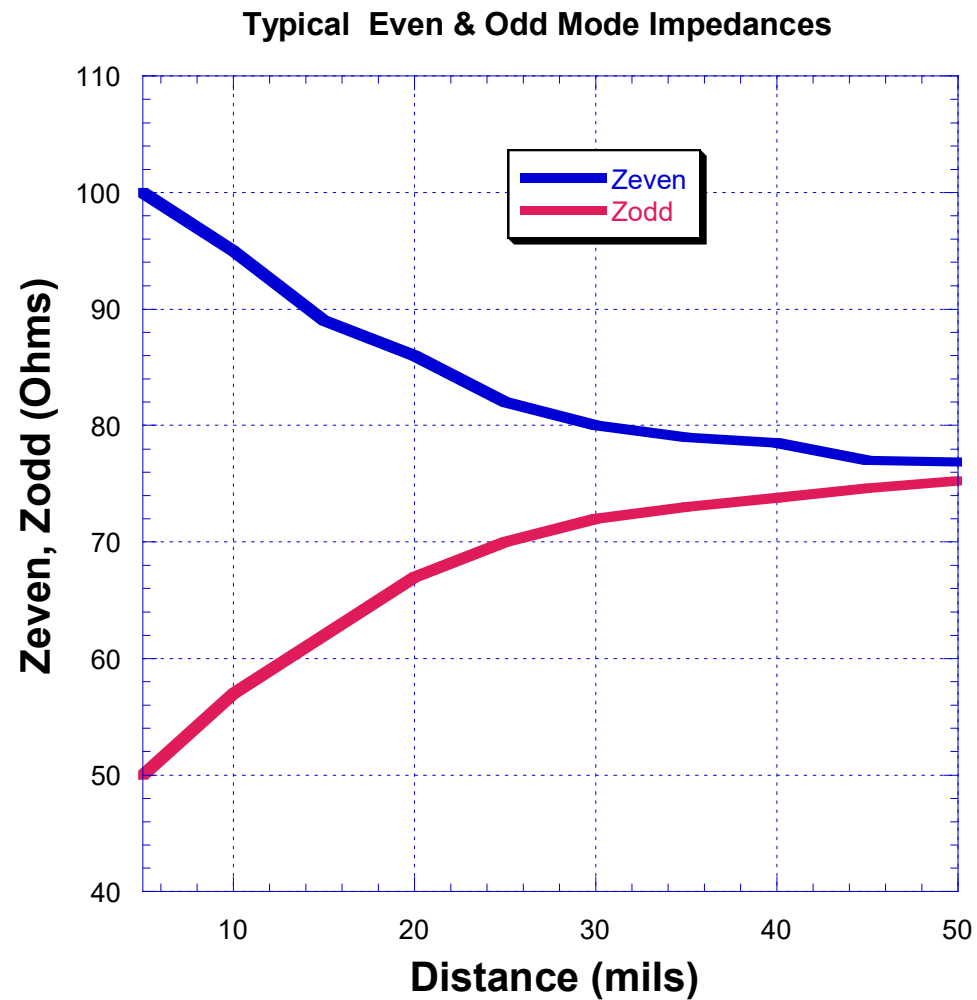
# Measured mutual inductance



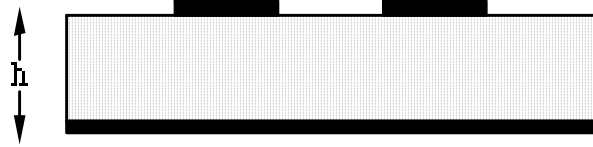
# Measured mutual capacitance



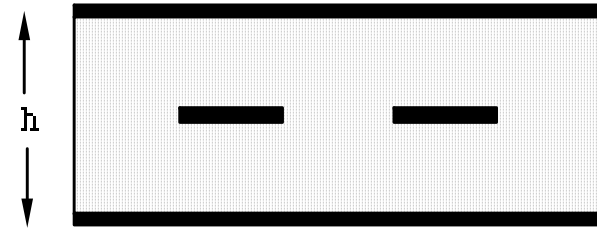
# Even & Odd Mode Impedances



# Modal Velocities in Stripline and Microstrip

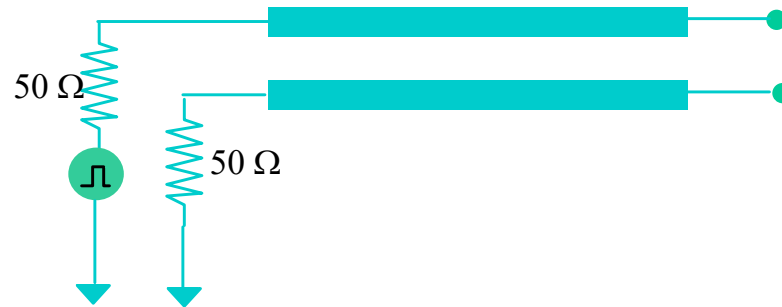


**Microstrip : Inhomogeneous structure, odd and even-mode velocities must have different values.**



**Stripline : Homogeneous configuration, odd and even-mode velocities have approximately the same values.**

# Microstrip vs Stripline



## Microstrip (h = 8 mils)

$$w = 8 \text{ mils}$$

$$\epsilon_r = 4.32$$

$$L_s = 377 \text{ nH/m}$$

$$C_s = 82 \text{ pF/m}$$

$$L_m = 131 \text{ nH/m}$$

$$C_m = 23 \text{ pF/m}$$

$$v_e = 0.155 \text{ m/ns}$$

$$v_d = 0.178 \text{ m/ns}$$

## Stripline (h = 16 mils)

$$w = 8 \text{ mils}$$

$$\epsilon_r = 4.32$$

$$L_s = 466 \text{ nH/m}$$

$$C_s = 86 \text{ pF/m}$$

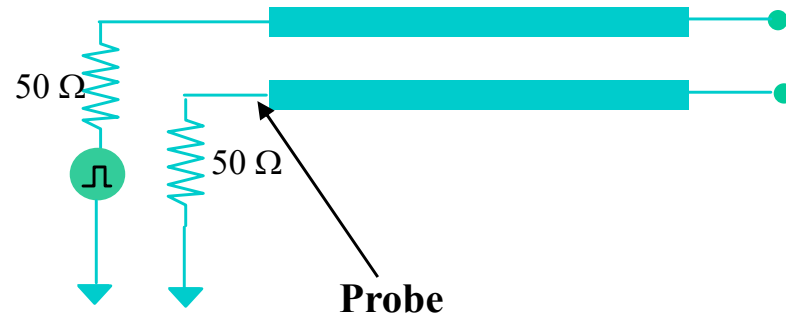
$$L_m = 109 \text{ nH/m}$$

$$C_m = 26 \text{ pF/m}$$

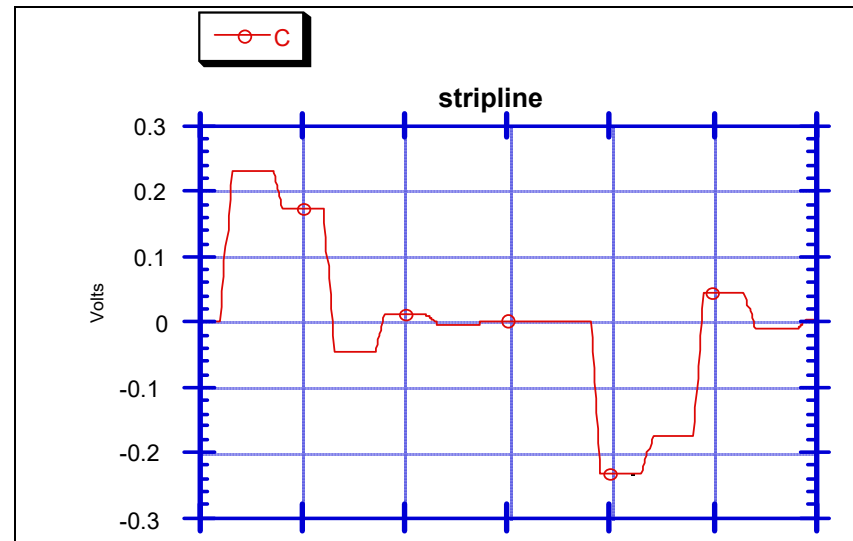
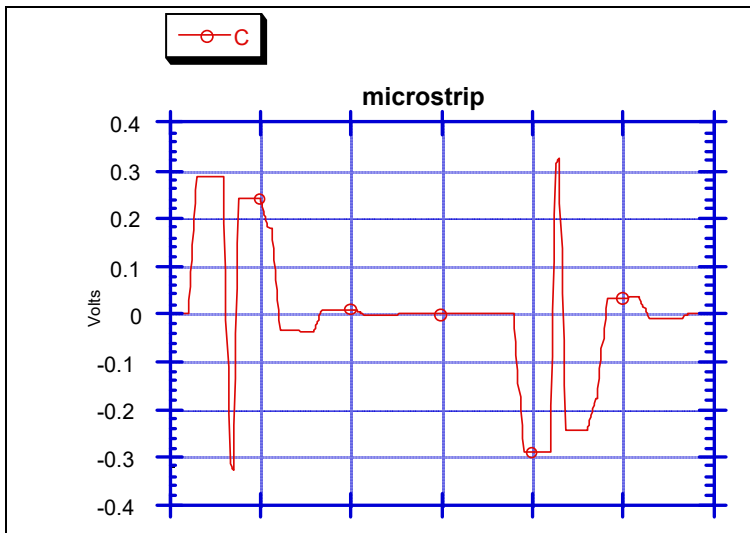
$$v_e = 0.142 \text{ m/ns}$$

$$v_d = 0.142 \text{ m/ns}$$

# Microstrip vs Stripline

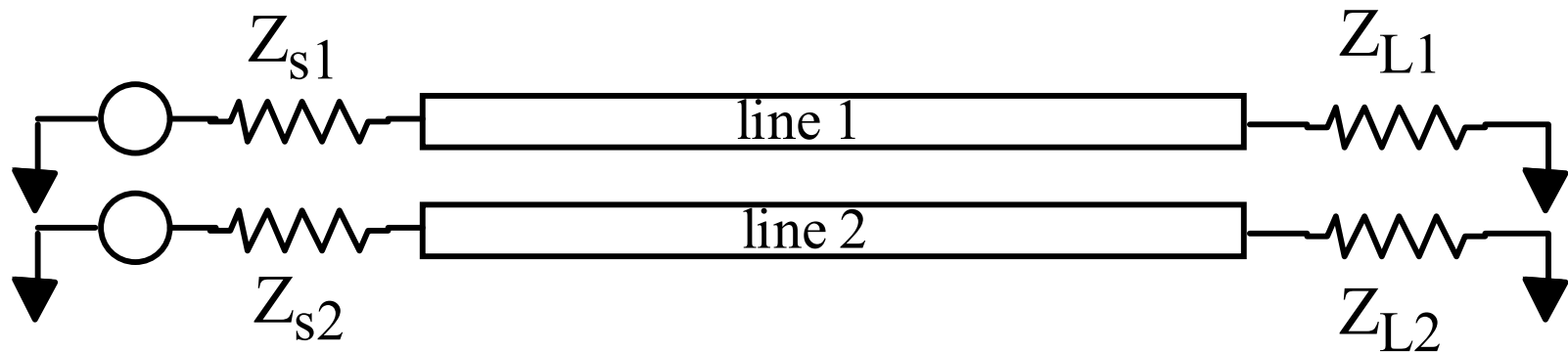


## Sense line response at near end





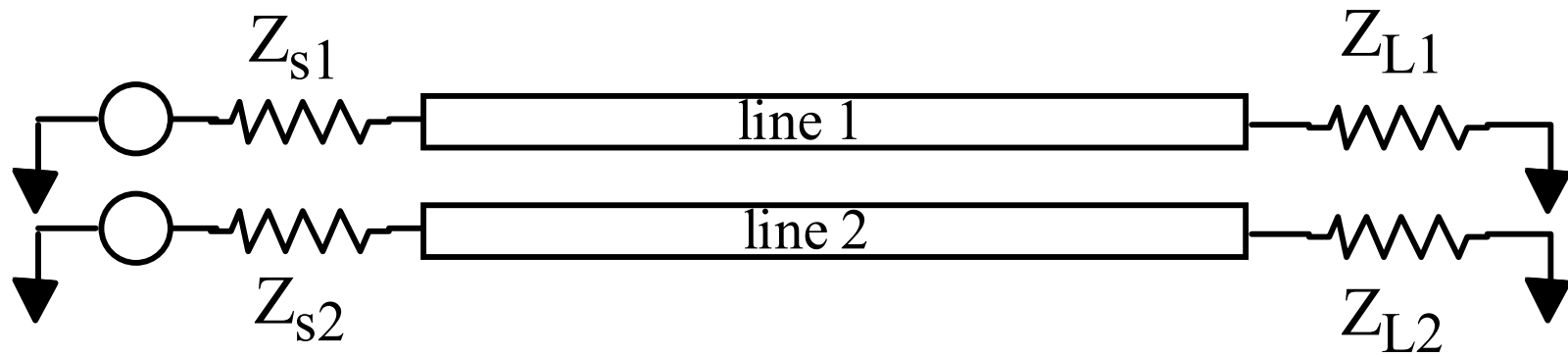
# General Solution for Voltages



$$V_1(z) = \underbrace{A_e e^{-\frac{j\omega z}{v_e}} + B_e e^{+\frac{j\omega z}{v_e}}}_{\text{even}} + \underbrace{A_d e^{-\frac{j\omega z}{v_d}} + B_d e^{+\frac{j\omega z}{v_d}}}_{\text{odd}}$$

$$V_2(z) = \underbrace{A_e e^{-\frac{j\omega z}{v_e}} + B_e e^{+\frac{j\omega z}{v_e}}}_{\text{even}} - \underbrace{A_d e^{-\frac{j\omega z}{v_d}} + B_d e^{+\frac{j\omega z}{v_d}}}_{\text{odd}}$$

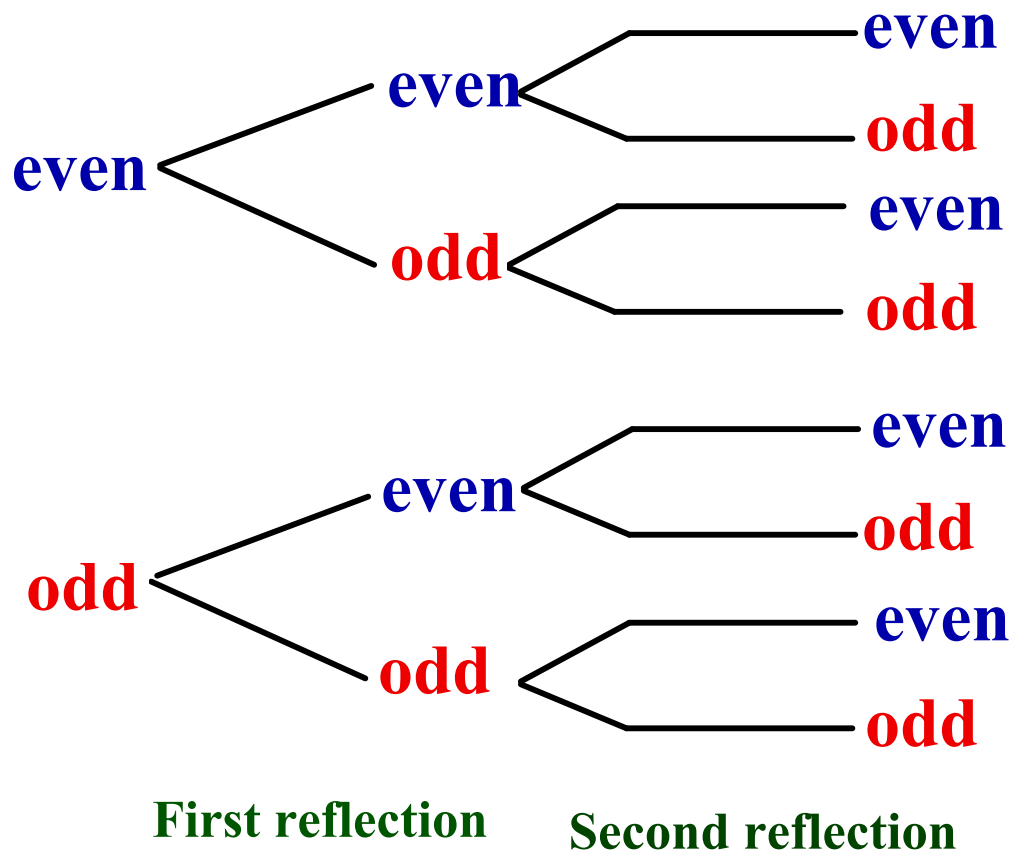
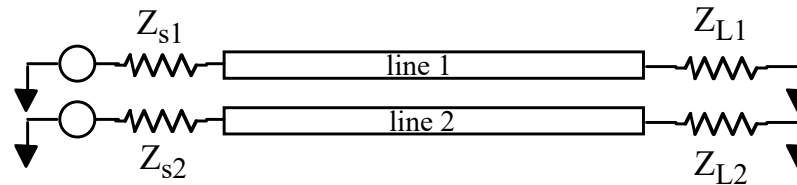
# General Solution for Currents



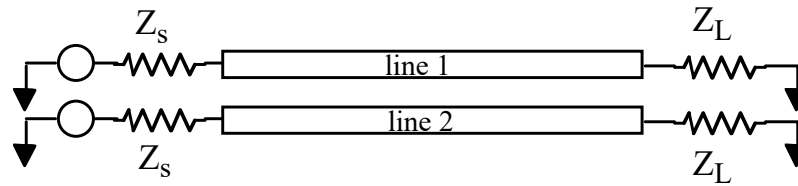
$$I_1(z) = \underbrace{\frac{1}{Z_e} \left[ A_e e^{-\frac{j\omega z}{v_e}} - B_e e^{+\frac{j\omega z}{v_e}} \right]}_{\text{even}} + \underbrace{\frac{1}{Z_d} \left[ A_d e^{-\frac{j\omega z}{v_d}} - B_d e^{+\frac{j\omega z}{v_d}} \right]}_{\text{odd}}$$

$$I_2(z) = \underbrace{\frac{1}{Z_e} \left[ A_e e^{-\frac{j\omega z}{v_e}} - B_e e^{+\frac{j\omega z}{v_e}} \right]}_{\text{even}} - \underbrace{\frac{1}{Z_d} \left[ A_d e^{-\frac{j\omega z}{v_d}} - B_d e^{+\frac{j\omega z}{v_d}} \right]}_{\text{odd}}$$

# Coupling of Modes (asymmetric load)



# Coupling of Modes (symmetric load)



**even** ————— **even** ————— **even**

**odd** ————— **odd** ————— **odd**

**First reflection**

**Second reflection**

# Capacitive Terminations

