ECE 546
Lecture - 09
Lossy Transmission Lines

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Loss in Transmission Lines

Signal amplitude decreases with distance from the source.
Skin Effect in Lines

Low Frequency

High Frequency

Very High Frequency
Skin Effect in Microstrip

Skin Effect in Microstrip

Current density varies as

\[ J = J_0 e^{-y/\delta} e^{-jy/\delta} \]

Note that the phase of the current density varies as a function of \( y \)

\[ I = \int_{0}^{\infty} J_0 w e^{-y/\delta} e^{-jy/\delta} dy = \frac{J_0 \omega \delta}{1 + j} \]

\[ \sigma E_0 = J_0 \Rightarrow E_0 = \frac{J_0}{\sigma} \]

The voltage measured over a section of conductor of length \( D \) is:

\[ V = E_0 D = \frac{J_0 D}{\sigma} \]
Skin Effect in Microstrip

The skin effect impedance is

\[ Z_{\text{skin}} = \frac{V}{I} = \frac{J_o D}{\sigma J_o w\delta} = \frac{D}{w} (1 + j) \sqrt{\pi f \mu \rho} \]

where \( \rho = \frac{1}{\sigma} \) is the bulk resistivity of the conductor

\[ Z_{\text{skin}} = R_{\text{skin}} + jX_{\text{skin}} \]

with

\[ R_{\text{skin}} = X_{\text{skin}} = \frac{D}{w} \sqrt{\pi f \mu \sigma} \]

⇒ Skin effect has reactive (inductive) component
Lossy Transmission Line

Telegraphers Equation: Time Domain

\[-\frac{\partial V}{\partial z} = RI + L \frac{\partial I}{\partial t}\]

\[-\frac{\partial I}{\partial z} = GV + C \frac{\partial V}{\partial t}\]
Lossy Transmission Line

Telegraphers Equation: Frequency Domain

\[-\frac{\partial V}{\partial z} = (R + j\omega L)I = ZI\]

\[-\frac{\partial I}{\partial z} = (G + j\omega C)V = YV\]
Lossy Transmission Line

R, L, G, C,

forward wave

backward wave
Lossy Transmission Line

\[ V(z) = Ae^{-\alpha z} e^{-j\beta z} + Be^{+\alpha z} e^{+j\beta z} \]

\[ I(z) = \frac{1}{Z_o} \left[ Ae^{-\alpha z} e^{-j\beta z} - Be^{+\alpha z} e^{+j\beta z} \right] \]

\[ Z_o = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} \]
\[ \gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \]
Effects of Losses

- Signal attenuation

- Dispersion \( \gamma = \alpha(\omega) + j\beta(\omega) = \sqrt{(R + j\omega L)(G + j\omega C)} \)

- Rise time degradation
RC Transmission Line

\[ Z_{\text{in}} = \frac{Rl \ \coth \left( \frac{Rl}{\sqrt{2}} \sqrt{\frac{C\omega}{R}} \right)}{\left(1 + j\right)} \]

For very high \( \omega \), \( \arg(Z_{\text{in}}) \approx 45^\circ \)

R : series resistance per unit length
C : shunt capacitance per unit length
RC Transmission Line

If $\omega \ll \frac{2}{RCl^2}$ then

$$Z_{in} = \frac{Rl}{2} + \frac{1}{jCl\omega} = \frac{R_T}{2} + \frac{1}{jC_T\omega}$$

$R_T = Rl$ : total resistance
$C_T = Cl$ : total capacitance
## RC Transmission Line

### Pulse Characteristics:
- rise time: 100 ps
- fall time: 100 ps
- pulse width: 4ns

### Line Characteristics
- length: 3 mm
- near end termination: 50 Ω
- far end termination: 65 Ω
Long Cable

100m Category-5 Cable
Short Cable

1m Category-5 Cable
Category 5 Cable

Resistance and velocity

Category 5/100-meter

Resistance (Ohms/m) vs. Frequency (GHz)

Category 5/100-meter

Velocity Ratio vs. Frequency (GHz)
Cable Loss Model

\[ R(f) = R_s \cdot f^p \]

\[ v_r = v_{ro} + v_{rs} \cdot f \]

\[ Z = R(f) + j\omega L = R_{skin} + j(R_{skin} + \omega L) \]

<table>
<thead>
<tr>
<th></th>
<th>( Z_0 )</th>
<th>( v_{ro} )</th>
<th>( v_{rs} )</th>
<th>( R_s )</th>
<th>( p )</th>
<th>( f_{\text{max}} )</th>
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<td>Category 5</td>
<td>100</td>
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<td>0.113</td>
<td>7.94</td>
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</table>
Lossy TL Simulation

• To simulate lossy TL with resistive loads
  ➢ No closed form solution
  ➢ Simplest method is to use IFFT

\[
v(t, z) = \text{IFFT}\left\{ A e^{-\alpha z} e^{-j\beta z} + B e^{+\alpha z} e^{+j\beta z} \right\}
\]

\[
i(t, z) = \text{IFFT}\left\{ \frac{1}{Z_o}\left[ A e^{-\alpha z} e^{-j\beta z} + A e^{+\alpha z} e^{+j\beta z} \right] \right\}
\]

\[
Z_o = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}
\]

\[
\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}
\]

\[
T = \frac{Z_o}{Z_1 + Z_o}
\]

\[
A = \frac{TV_s(\omega)}{1 - \Gamma_1 \Gamma_2 e^{-2\gamma l}} \quad B = \Gamma_2 e^{-2\gamma l} A \quad \Gamma_2 = \frac{Z_2 - Z_o}{Z_2 + Z_o}
\]

\[
\Gamma_1 = \frac{Z_1 - Z_o}{Z_1 + Z_o}
\]

Time-Domain Simulations

\[ Z_s = 50 \, \Omega \]

\[ V_s \]

cable

near end

far end

open
Pulse Propagation (CAT-5)
Pulse Propagation (MP/CM)

MP/CM Shielded Near End

MP/CM Shielded Far End
Pulse Propagation (RG174)