ECE 546
Lecture 11
MOS Amplifiers

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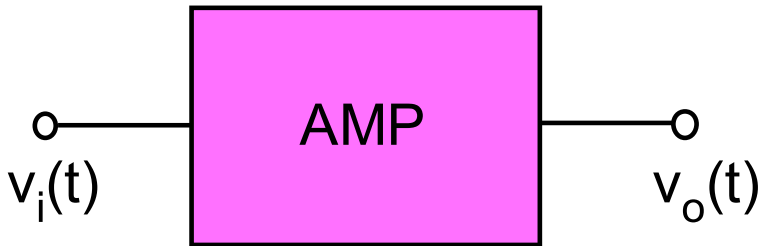
Amplifiers

- **Definitions**
  - Used to increase the amplitude of an input signal to a desired level
  - This is a fundamental signal processing function
  - Must be linear (free of distortion) – Shape of signal preserved



$$v_o(t) = Av_i(t),\ \ where\ A\ is\ the\ voltage\ gain$$

$$Voltage\ Gain: A_v = \frac{v_o}{v_i}$$

$$Power\ \ Gain: A_p = \frac{Load\ \ Power\ (P_L)}{Input\ \ Power\ (P_I)}$$

Amplifiers

$$A_p = \frac{v_o i_o}{v_I i_I}$$

$$Current\ Gain: A_i = \frac{i_o}{i_i}$$

$$Note: A_p = A_v A_i$$

Expressing gain in dB (decibels)

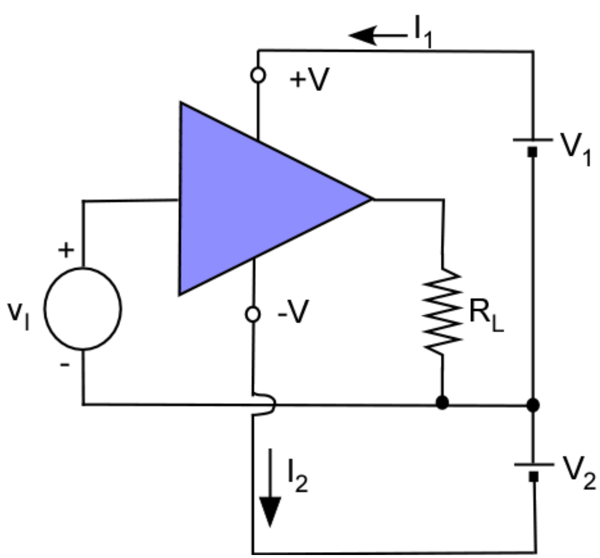
$$Voltage\ gain\ in\ dB = 20\log\left|A_V\right|$$

$$Current\ gain\ in\ dB = 20\log\left|A_I\right|$$

$$Power\ gain\ in\ dB = 10\log\left|A_P\right|$$

Amplifiers

Since output associated with the signal is larger than the input signal, power must come from DC supply



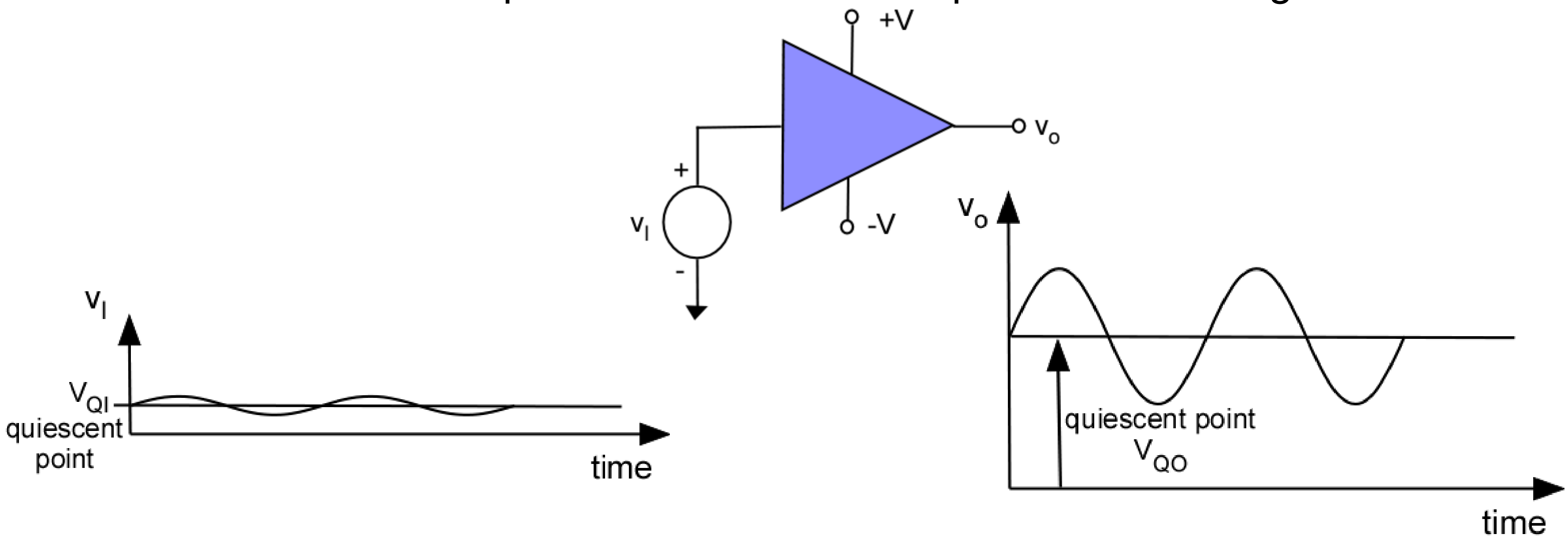
$$P_{DC} = V_1 I_1 + V_2 I_2$$

$$P_{DC} + P_I = P_L + P_{dissipated}$$

$$\eta = \frac{P_L}{P_{DC}} \times 100 = Power\ Efficiency$$

Biasing of Amp

Bias will provide quiescent points for input and output about which variations will take place. Bias maintain amplifier in active region.



$$V_I(t) = V_{QI} + v_I(t)$$

$$V_o(t) = V_{QO} + v_o(t)$$

$$v_o(t) = A_v v_I(t)$$

$$A_v = \left.\frac{dv_o}{dv_I}\right|_{at\ Q}$$

Amplifier characteristics are determined by bias point

Small-Signal Model

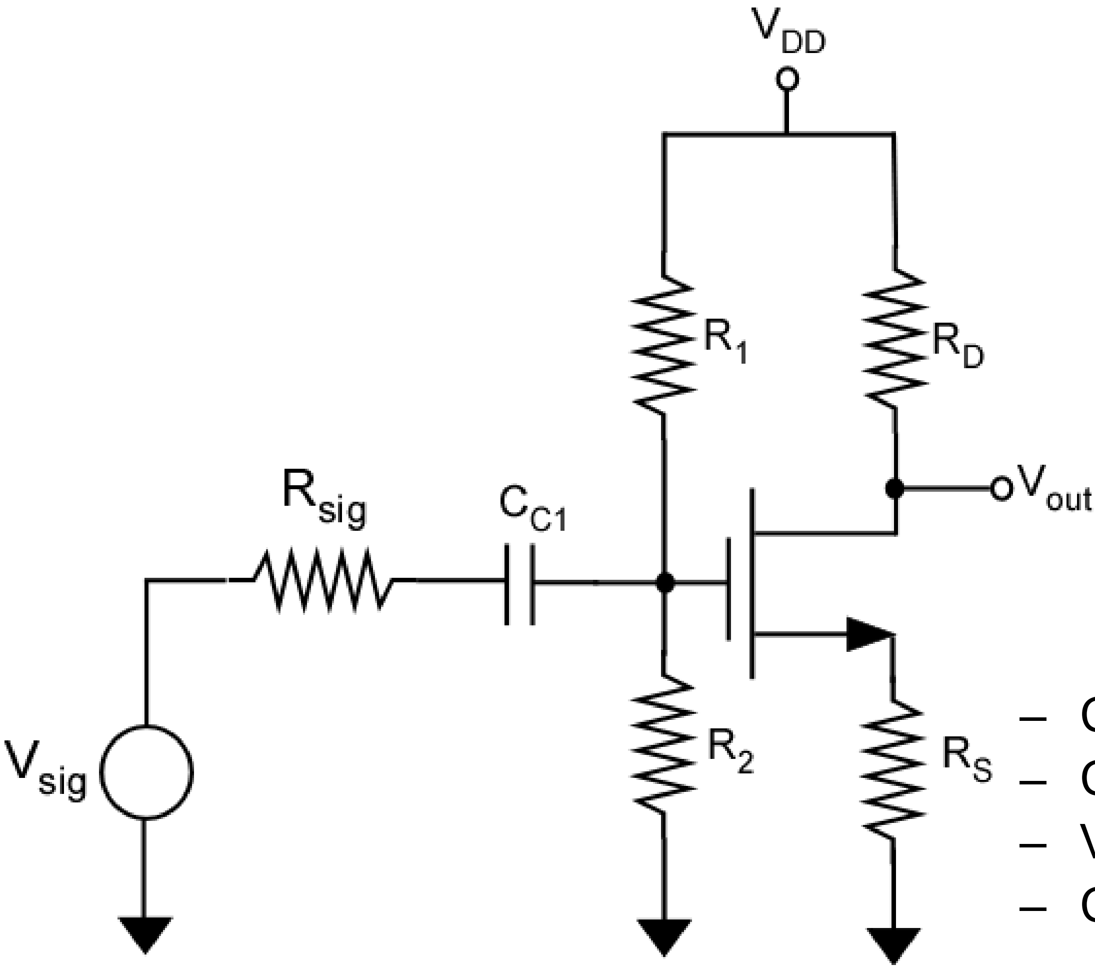
- **What is a small-signal incremental model?**
  - Equivalent circuit that only accounts for signal level fluctuations about the DC bias operating points
  - Fluctuations are assumed to be small enough so as not to drive the devices out of the proper range of operation
  - Assumed to be linear
  - Derives from superposition principle

Biasing of MOS Transistors

- **Bias Characteristics**
  - Operation in saturation region
  - Stable and predictable drain current

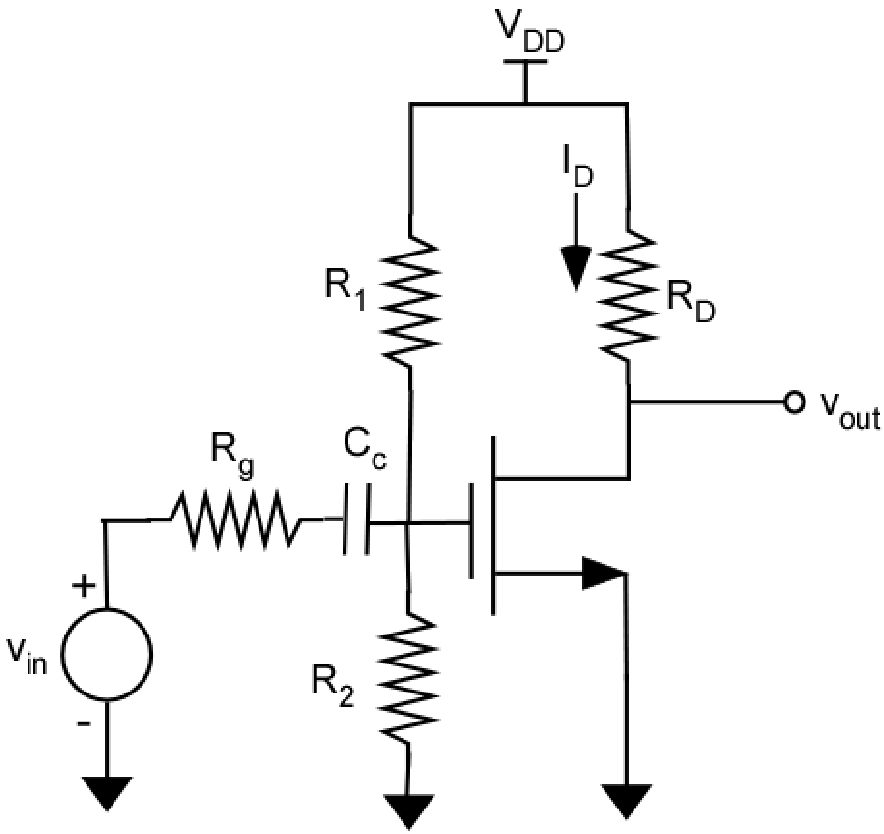
$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_T\right)^2$$

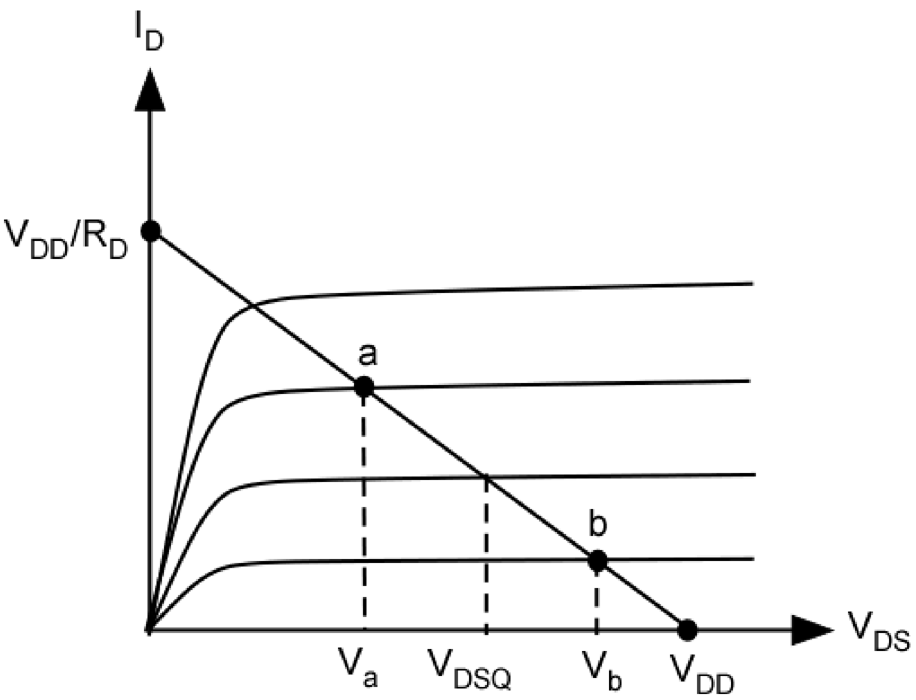
Single-Supply MOS Bias



- Choose R_1 and R_2 to fix V_G
- Choose R_S and R_2 to fix V_S
- V_{GS} determines I_D
- Choose R_D to fix V_D

Common Source MOSFET Amplifier

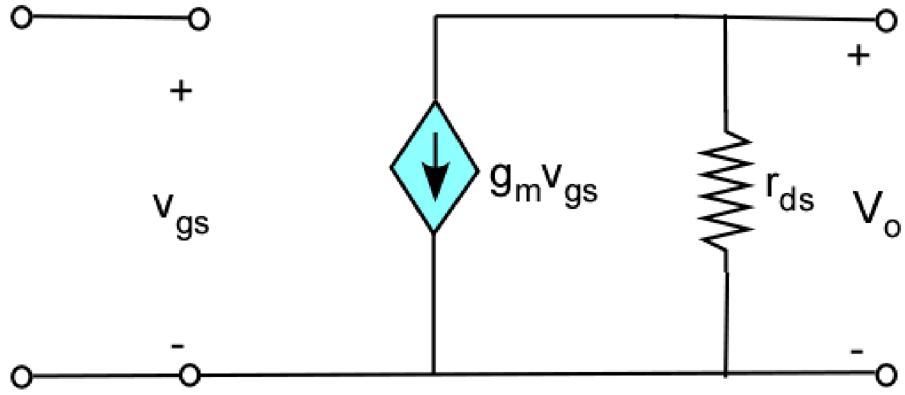




Bias is to keep MOS in saturation region

Common Source MOSFET Amplifier

Small-Signal Equivalent Circuit for MOS (device only)



$$I_D = \frac{1}{2} k_n' \frac{W}{L} \left(V_{GS} - V_T\right)^2$$

$$g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{GS}=V_{GSQ}} = \frac{2I_D}{V_{eff}}$$

$$where\ V_{GS} - V_T = V_{eff}$$

Which leads to

$$g_m = \sqrt{2k_n'}\sqrt{W/L}\sqrt{I_D}$$

$$g_m\ is\ proportional\ to = \sqrt{W/L}$$

MOSFET Output Impedance

To calculate r_{ds}, account for λ

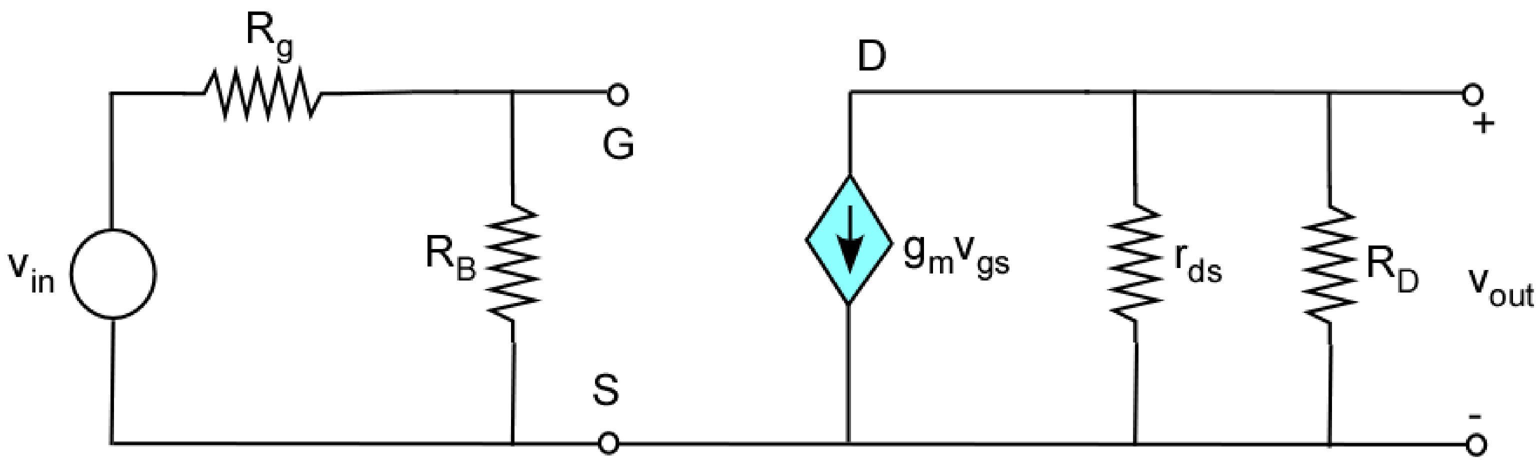
$$r_{ds} = \left.\frac{\partial V_{DS}}{\partial I_D}\right|_{V_{GS}=V_{GSQ}} = \frac{1}{\lambda\mu\frac{W}{2L}C_{ox}\left[V_{GS}-V_T\right]^2} = \frac{1}{\lambda I_{DP}}$$

$$I_{DP} = \frac{1}{2}k_n'\frac{W}{L}\left(V_{GS}-V_T\right)^2$$

r_{ds}, accounts for channel width modulation resistance.

Midband Frequency Gain

Incremental model for complete amplifier



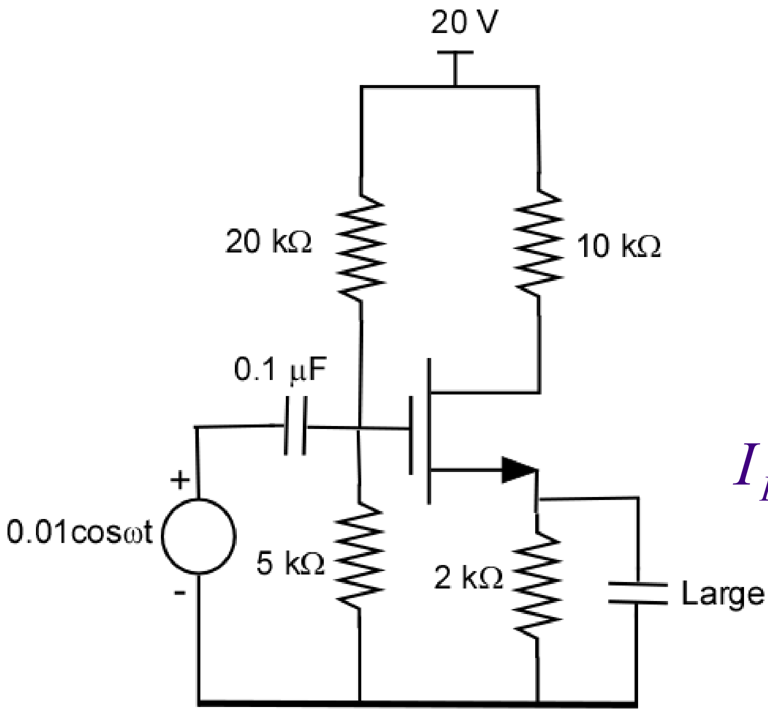
$$A_{MB} = \frac{v_{out}}{v_{in}} = -\frac{R_B}{R_B + R_g} g_m \frac{r_{ds} R_D}{r_{ds} + R_D}$$

Example

For the circuit shown, k=75 μA/V², V_T=1 V, λ=0

(a) Find V_{DQ}, V_{SQ}

(b) Find the midband gain



$$V_{GQ} = V_{DD}\frac{R_2}{R_1+R_2} = \frac{20\times 5}{25} = 4\,V$$

$$V_{GSQ} = V_{GQ} - V_{SQ} = 4 - 2I_{DQ}$$

$$I_{DQ} = K\left[V_{GSQ} - V_T\right]^2 = 0.075\left[4 - 2I_{DQ} - 1\right]^2$$

$$I_{DQ} = 0.075(9 - 12I_{DQ} + 4I_{DQ}^2)$$

$$4I_{DQ}^2 - 12I_{DQ} + 9 = 13.3I_{DQ} \Rightarrow I_{DQ}^2 - 6.33I_{DQ} + 2.25 = 0$$

Example (Cont')

$$I_{DQ} = 3.167 \pm \frac{\sqrt{6.33^2 - 9}}{2} = 0.378\,mA \text{ or } 5.953\,mA$$

reject since voltage drop across R_D will be too large

$$I_{DQ} = 0.378\,mA$$

$$V_{DQ} = V_{DD} - R_D I_{DQ} = 20 - 10 \times 0.378 = 16.22\,V$$

$$V_{SQ} = R_S I_{DQ} = 2 \times 0.378 = 0.756\,V$$

$$V_{DQ} = 16.22\,V$$

$$V_{SQ} = 0.756\,V$$

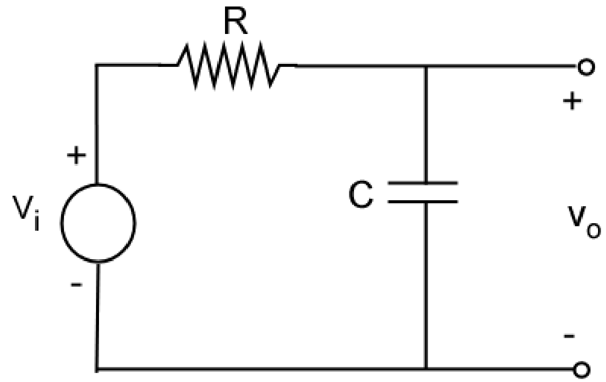
Example (Cont')

$$g_m = \sqrt{2k_n^{'} \frac{W}{L} I_{DQ}} = \sqrt{4 \times 0.075 \times 0.378} = 0.337$$

$$A_{MB} = -g_m R_D = -0.337 \times 10 = -3.37$$

$$A_{MB} = -3.37$$

Low-Pass Circuit



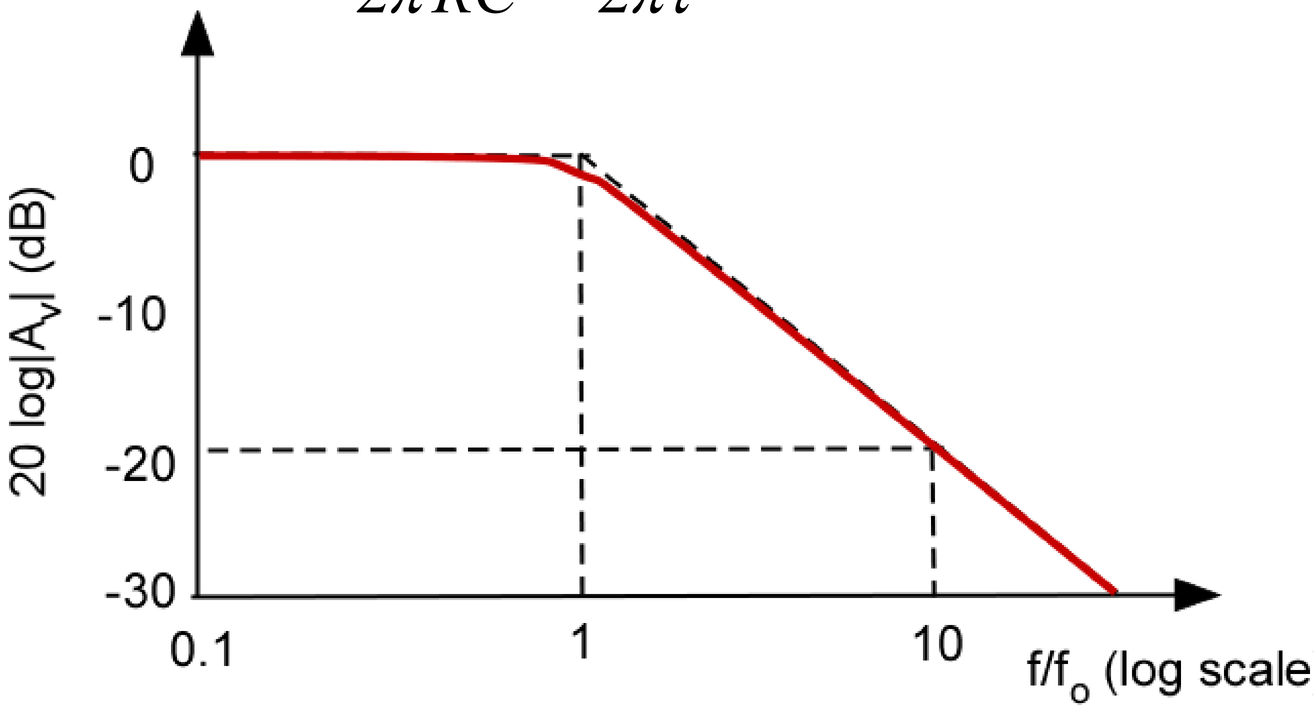
In frequency domain: $$V_o = \frac{V_i}{R + \dfrac{1}{j\omega C}} \cdot \frac{1}{j\omega C}$$

$$V_o = \frac{V_i}{1 + j\omega RC} \Rightarrow A_v = \frac{V_o}{V_i} = \frac{1}{1 + j\omega RC}$$

$$A_v = \frac{1}{1 + j\omega RC} = \frac{1}{1 + jf / f_2}$$

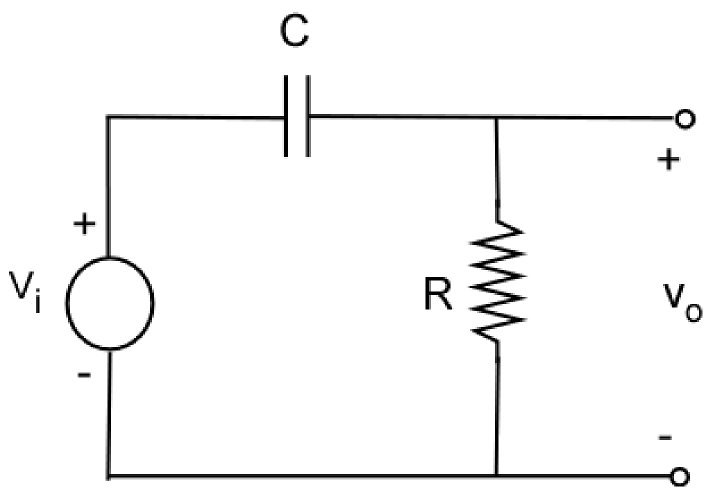
Low-Pass Circuit

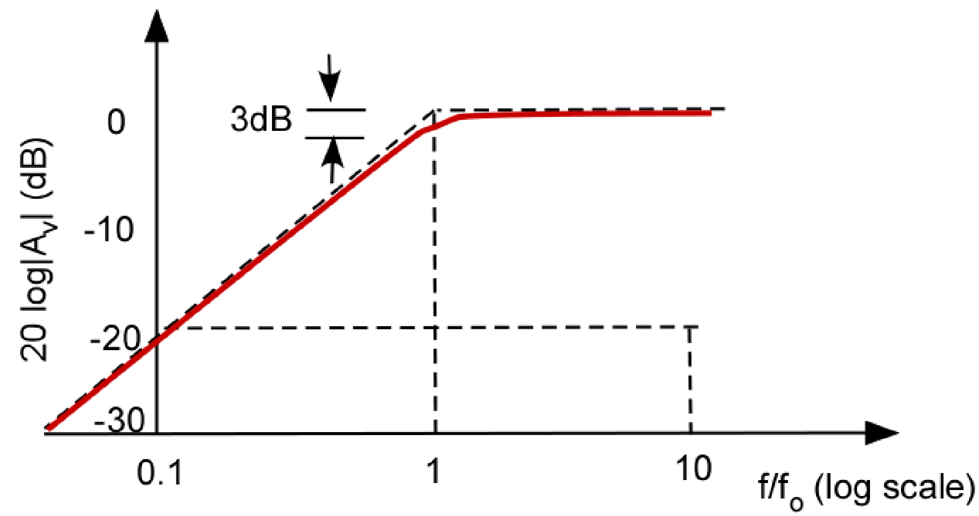
$$f_2 = \frac{1}{2\pi RC} = \frac{1}{2\pi\tau}$$



$$\tau = 2\pi RC = \text{time constant}$$

High-Pass Circuit





$$V_o = \frac{V_i R}{R + \frac{1}{j\omega C}} = \frac{V_i}{1 + \frac{1}{j\omega RC}}$$

$$A_v = \frac{V_o}{V_i} = \frac{1}{1 - j\frac{1}{2\pi fRC}} = \frac{1}{1 - jf_2 / f}$$

$$f_2 = \frac{1}{2\pi RC}$$

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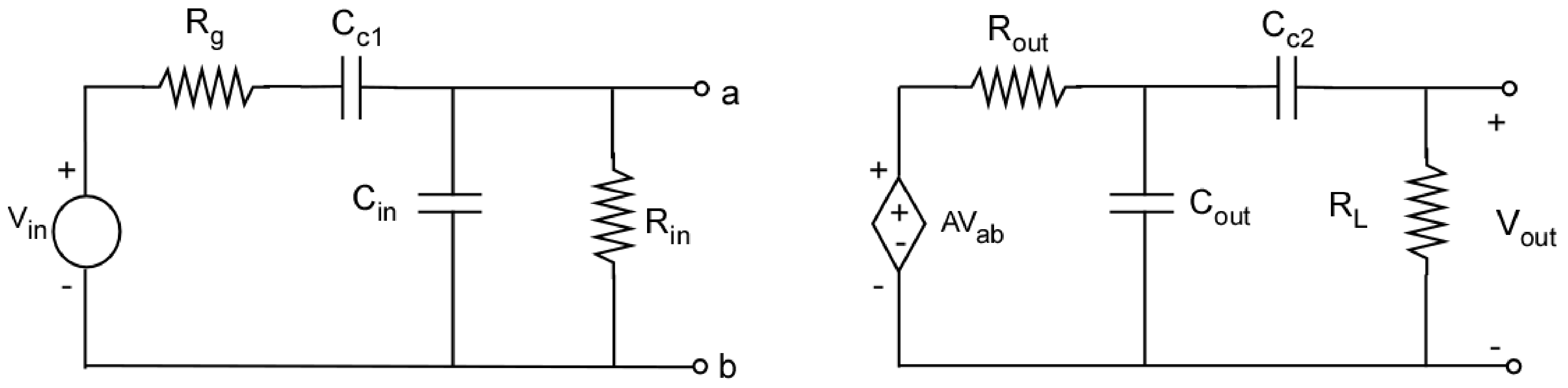
Model for general Amplifying Element

C_{c1} and C_{c2} are coupling capacitors (large) ➔ μF

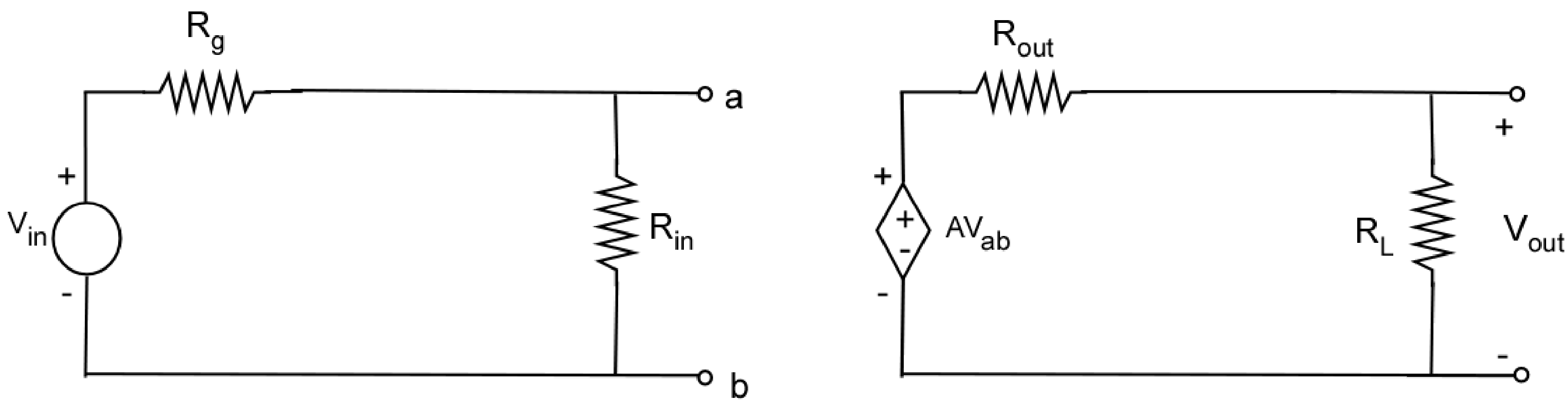
C_{in} and C_{out} are parasitic capacitors (small) ➔ pF



Midband Frequencies

- Coupling capacitors are short circuits

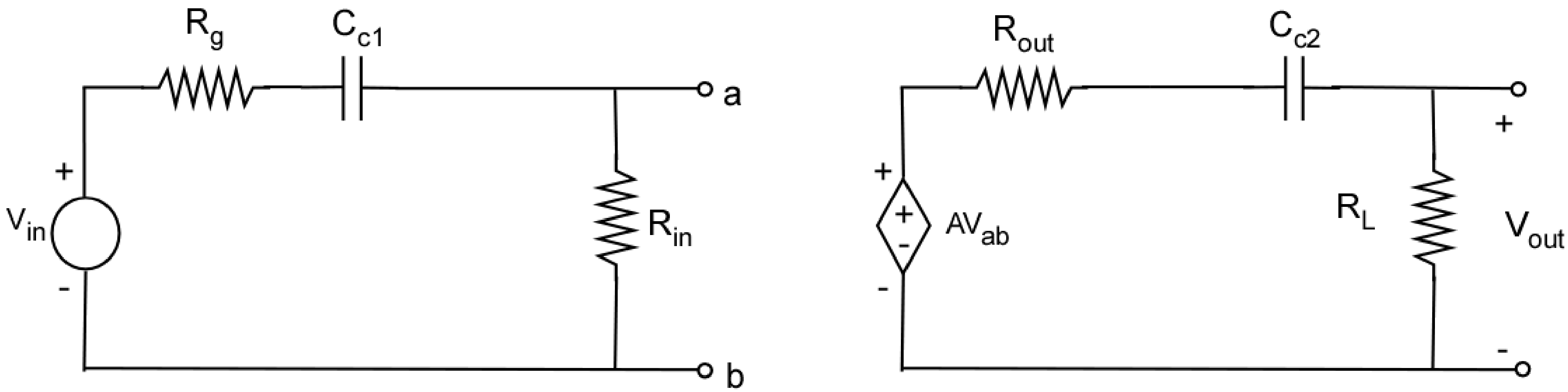
- Parasitic capacitors are open circuits



$$A_{MB} = \frac{v_{out}}{v_{in}} = \frac{R_{in}}{R_g + R_{in}} A \frac{R_L}{R_{out} + R_L}$$

Low Frequency Model

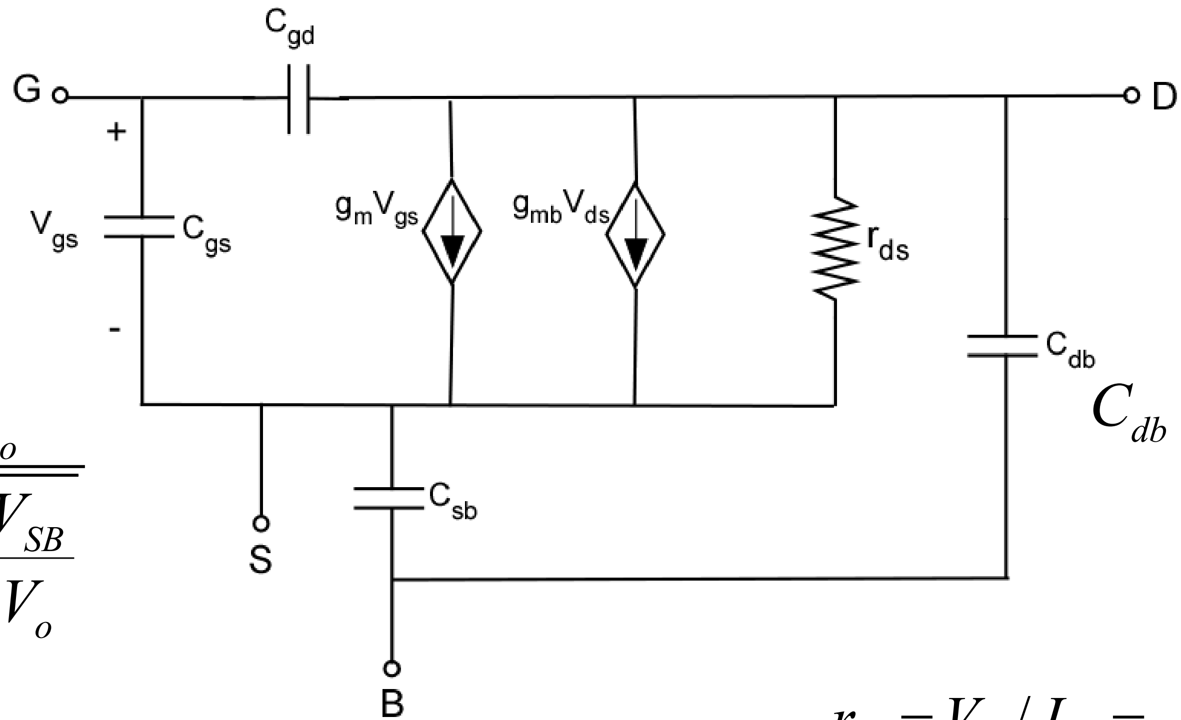
- Coupling capacitors are present
- Parasitic capacitors are open circuits



$$v_{ab} = \frac{v_{in} R_{in}}{R_g + R_{in} + \dfrac{1}{j\omega C_{c1}}} = \frac{v_{in} j\omega C_{c1} R_{in}}{1 + j\omega C_{c1}(R_g + R_{in})}$$

$$v_{ab} = v_{in} \frac{R_{in}}{R_g + R_{in}} \cdot \frac{j\omega C_{c1}(R_g + R_{in})}{\left[1 + j\omega C_{c1}(R_g + R_{in})\right]}$$

MOSFET High-Frequency Model



$$C_{sb} = \frac{C_{sbo}}{\sqrt{1 + \frac{V_{SB}}{V_o}}}$$

$$C_{db} = \frac{C_{dbo}}{\sqrt{1 + \frac{V_{DB}}{V_o}}}$$

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{eff} = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \frac{2I_D}{V_{eff}}$$

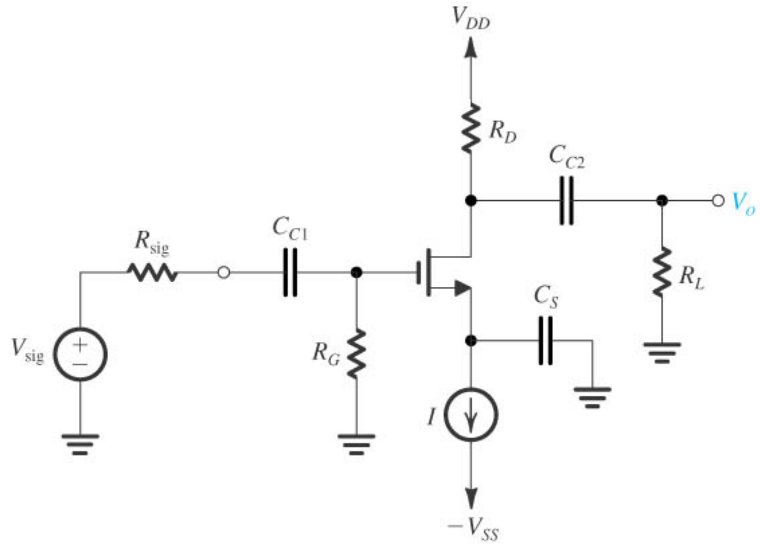
$$g_{mb} = \chi g_m = \frac{\gamma}{2\sqrt{2\phi_F + V_{sb}}} g_m$$

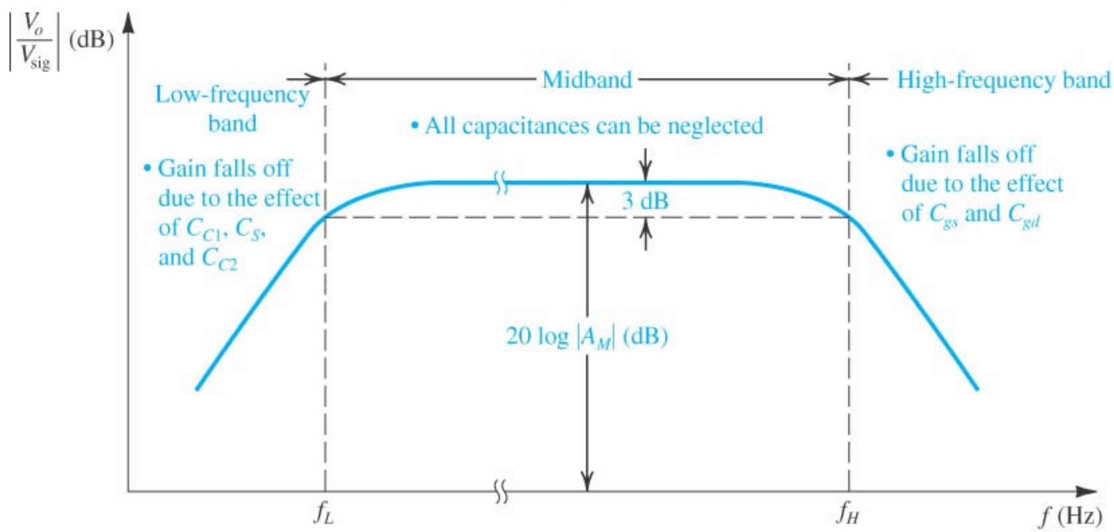
$$r_{ds} = V_A / I_D = \frac{1}{\lambda I_D}$$

$$C_{gs} = \frac{2}{3} WLC_{ox} + WL_{ov} C_{ox}$$

$$C_{gd} = WL_{ov} C_{ox}$$

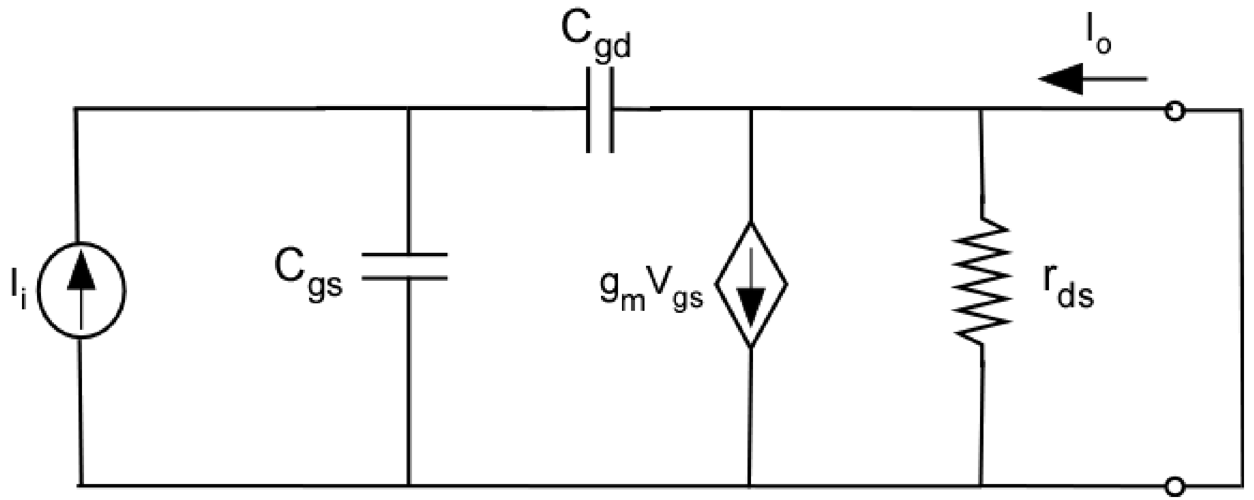
CS - Three Frequency Bands





Unity-Gain Frequency f_T

f_T is defined as the frequency at which the short-circuit current gain of the common source configuration becomes unity



Define:

$$s = j\omega$$

(neglect $sC_{gd}V_{gs}$ since C_{gd} is small)

$$I_o = g_m V_{gs} - sC_{gd}V_{gs}$$

$$I_o \simeq g_m V_{gs}$$

$$V_{gs} = \frac{I_i}{s\left(C_{gs} + C_{gd}\right)}$$

$$\frac{I_o}{I_i} = \frac{g_m}{s\left(C_{gs} + C_{gd}\right)}$$

Calculating f_T

For $s=j\omega$, magnitude of current gain becomes unity at

$$\omega_T = \frac{g_m}{C_{gs} + C_{gd}} \Rightarrow f_T = \frac{g_m}{2\pi\left(C_{gs} + C_{gd}\right)}$$

f_T ~ 100 MHz for 5-μm CMOS, f_T ~ several GHz for 0.13μm CMOS