

ECE 546

Lecture 11

Amplifiers

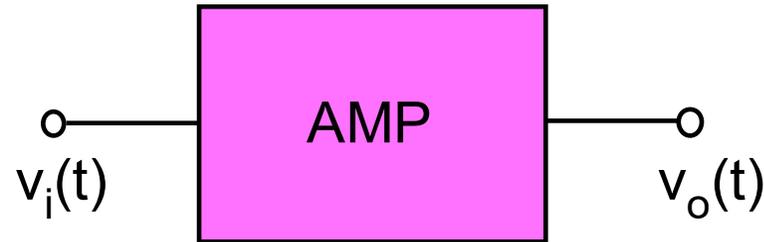
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Amplifiers

- **Definitions**

- Used to increase the amplitude of an input signal to a desired level
- This is a fundamental signal processing function
- Must be linear (free of distortion) – Shape of signal preserved



$v_o(t) = Av_i(t)$, where A is the voltage gain

$$\text{Voltage Gain: } A_v = \frac{v_o}{v_i}$$

$$\text{Power Gain: } A_p = \frac{\text{Load Power } (P_L)}{\text{Input Power } (P_I)}$$

Amplifiers

$$A_p = \frac{v_o i_o}{v_I i_I}$$

$$\text{Current Gain: } A_i = \frac{i_o}{i_i}$$

$$\text{Note: } A_p = A_v A_i$$

Expressing gain in dB (decibels)

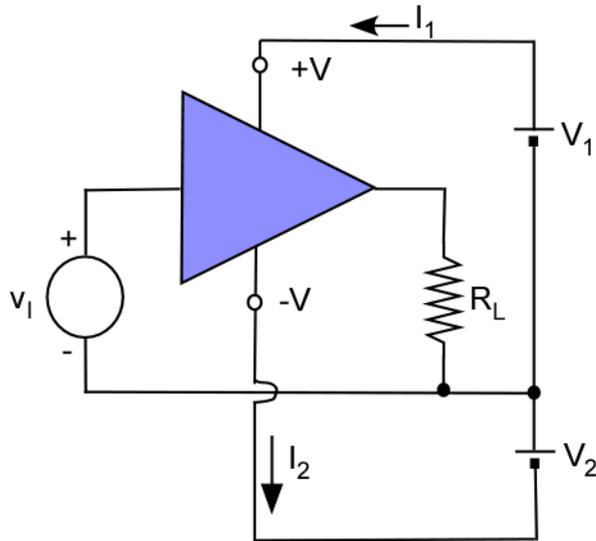
$$\text{Voltage gain in dB} = 20 \log |A_v|$$

$$\text{Current gain in dB} = 20 \log |A_i|$$

$$\text{Power gain in dB} = 10 \log |A_p|$$

Amplifiers

Since output associated with the signal is larger than the input signal, power must come from DC supply



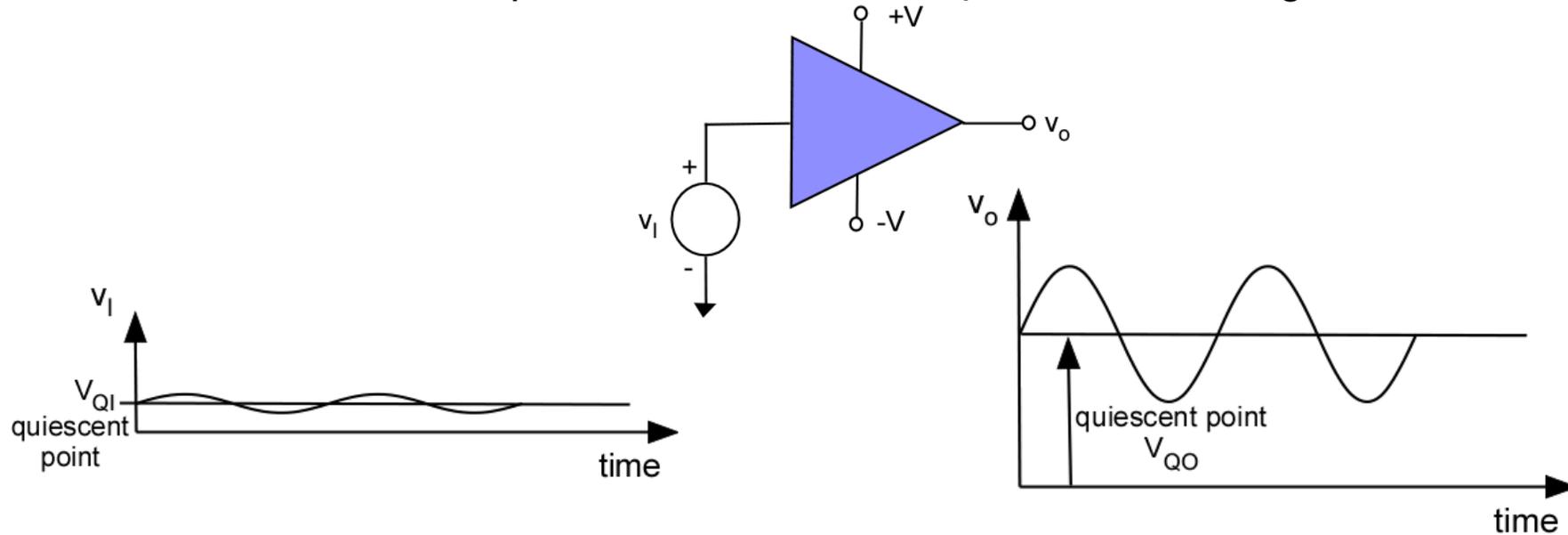
$$P_{DC} = V_1 I_1 + V_2 I_2$$

$$P_{DC} + P_I = P_L + P_{dissipated}$$

$$\eta = \frac{P_L}{P_{DC}} \times 100 = \text{Power Efficiency}$$

Biassing of Amp

Bias will provide quiescent points for input and output about which variations will take place. Bias maintain amplifier in active region.



$$V_I(t) = V_{QI} + v_I(t)$$

$$V_o(t) = V_{QO} + v_o(t)$$

$$v_o(t) = A_v v_I(t)$$

$$A_v = \left. \frac{dv_o}{dv_I} \right|_{at Q}$$

Amplifier characteristics are determined by bias point

Small-Signal Model

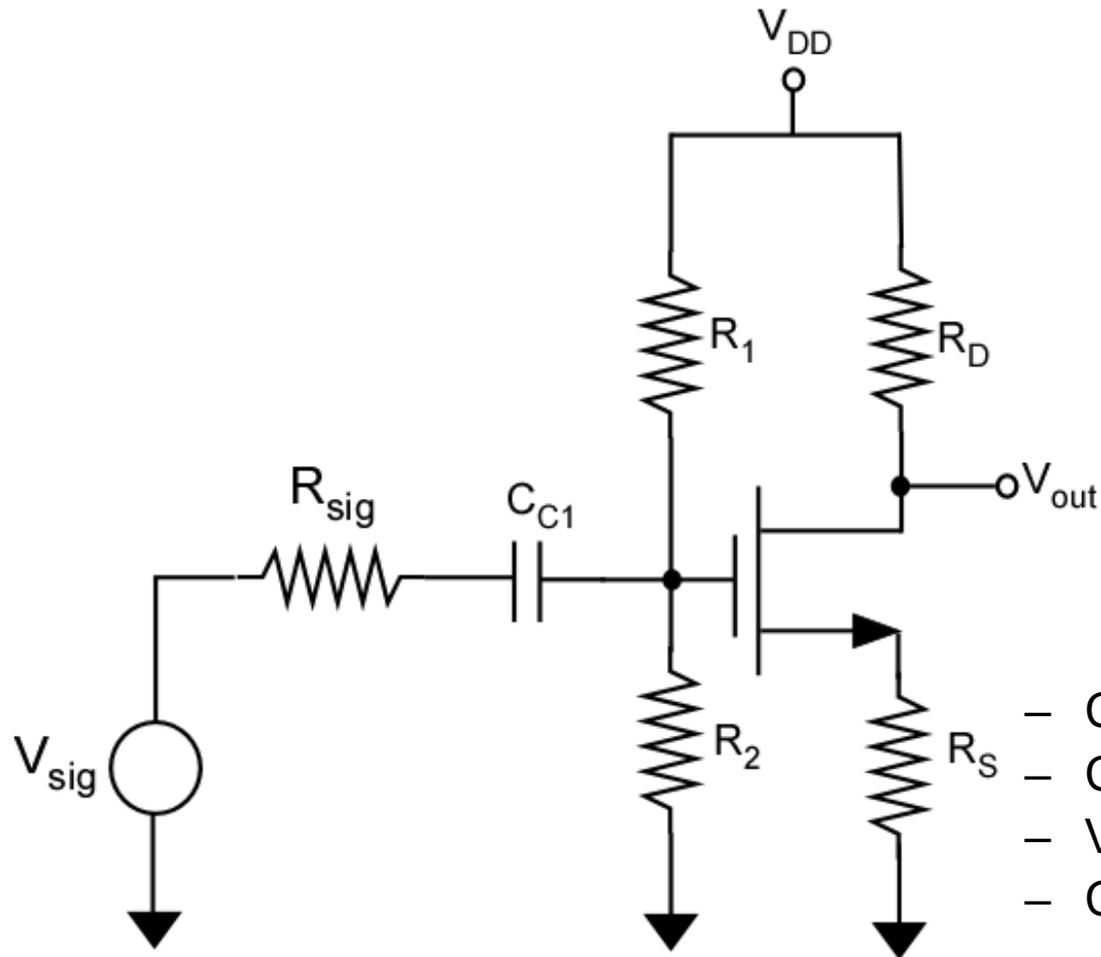
- **What is a small-signal incremental model?**
 - Equivalent circuit that only accounts for signal level fluctuations about the DC bias operating points
 - Fluctuations are assumed to be small enough so as not to drive the devices out of the proper range of operation
 - Assumed to be linear
 - Derives from superposition principle

Biasing of MOS Transistors

- **Bias Characteristics**
 - Operation in saturation region
 - Stable and predictable drain current

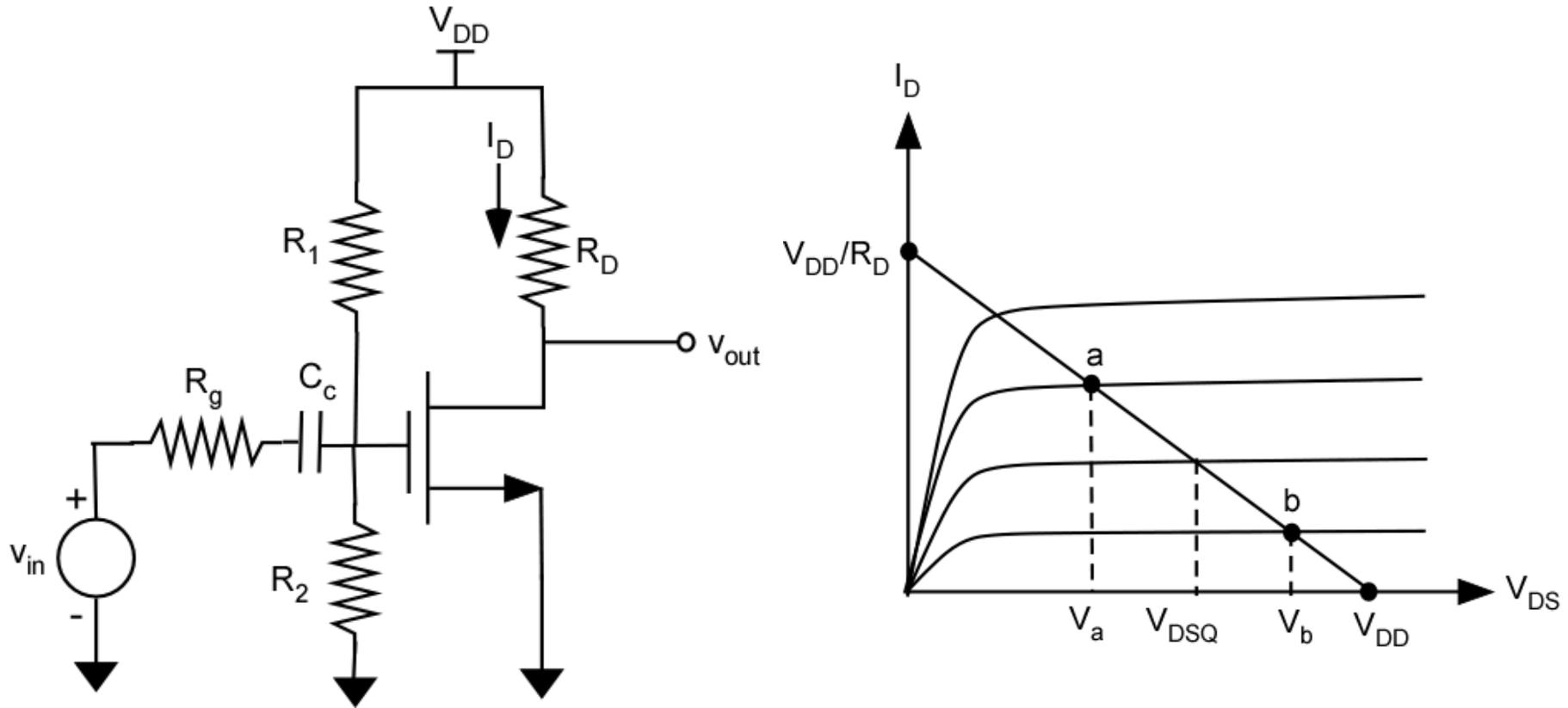
$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)^2$$

Single-Supply MOS Bias



- Choose R_1 and R_2 to fix V_G
- Choose R_S and R_2 to fix V_S
- V_{GS} determines I_D
- Choose R_D to fix V_D

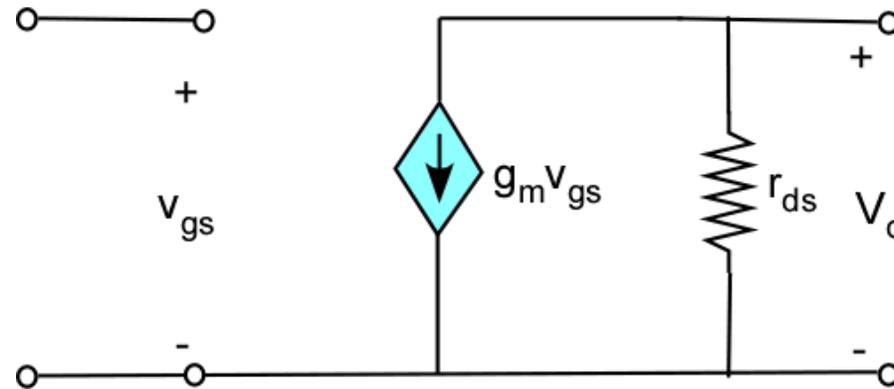
Common Source MOSFET Amplifier



Bias is to keep MOS in saturation region

Common Source MOSFET Amplifier

Small-Signal Equivalent Circuit for MOS (device only)



$$I_D = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_T)^2$$

$$g_m = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V_{GS} = V_{GSQ}} = \frac{2I_D}{V_{eff}}$$

where $V_{GS} - V_T = V_{eff}$

Which leads to

$$g_m = \sqrt{2k'_n} \sqrt{W/L} \sqrt{I_D}$$

g_m is proportional to $= \sqrt{W/L}$

MOSFET Output Impedance

To calculate r_{ds} , account for λ

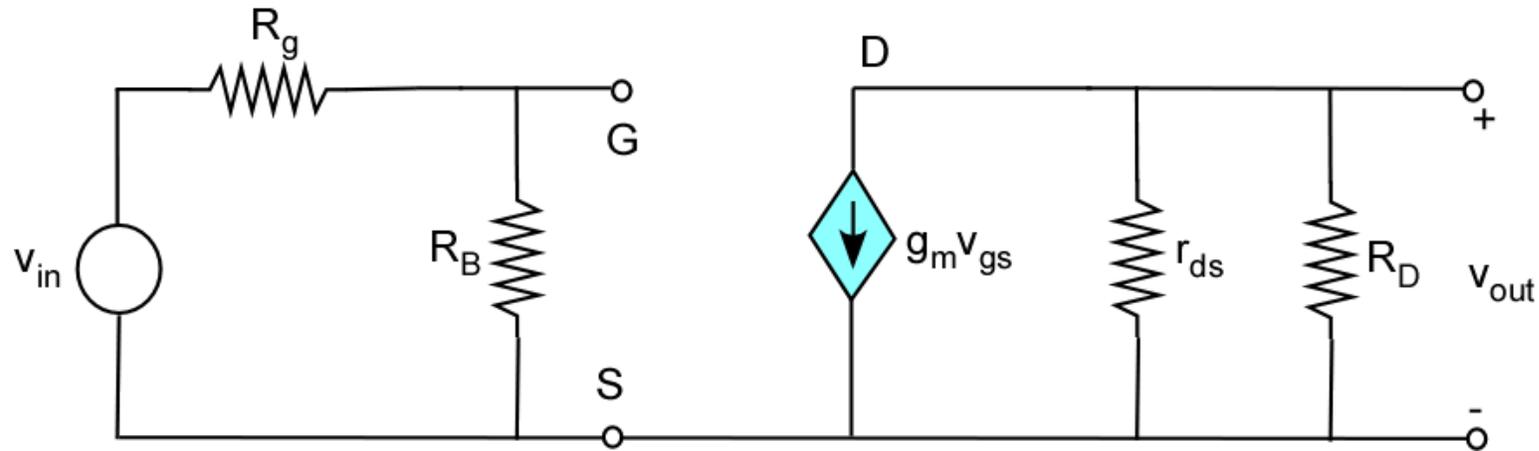
$$r_{ds} = \left. \frac{\partial V_{DS}}{\partial I_D} \right|_{V_{GS}=V_{GSQ}} = \frac{1}{\lambda \mu \frac{W}{2L} C_{ox} [V_{GS} - V_T]^2} = \frac{1}{\lambda I_{DP}}$$

$$I_{DP} = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_T)^2$$

r_{ds} , accounts for channel width modulation resistance.

Midband Frequency Gain

Incremental model for complete amplifier

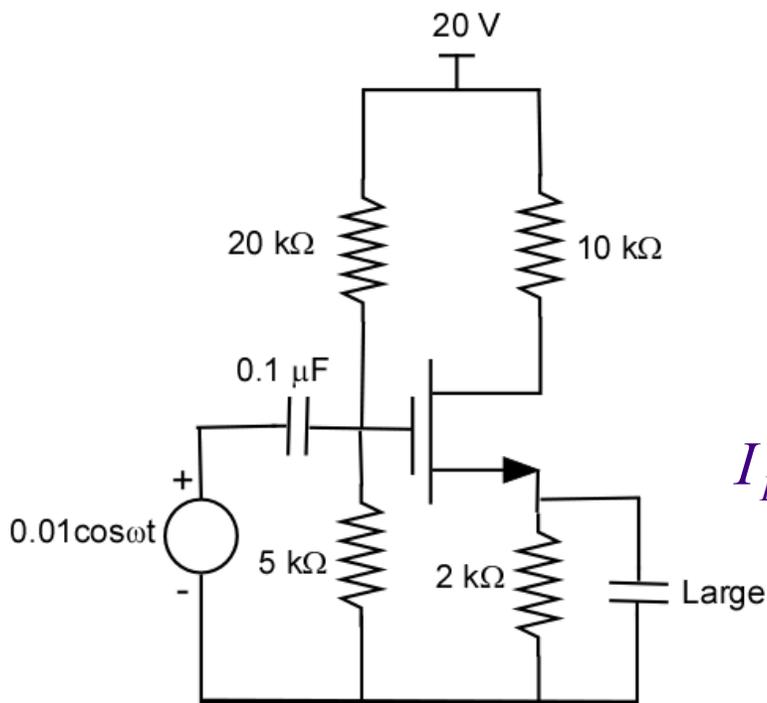


$$A_{MB} = \frac{v_{out}}{v_{in}} = -\frac{R_B}{R_B + R_g} g_m \frac{r_{ds} R_D}{r_{ds} + R_D}$$

Example

For the circuit shown, $k=75 \mu\text{A}/\text{V}^2$, $V_T=1 \text{ V}$, $\lambda=0$

- (a) Find V_{DQ} , V_{SQ}
- (b) Find the midband gain



$$V_{GQ} = V_{DD} \frac{R_2}{R_1 + R_2} = \frac{20 \times 5}{25} = 4 \text{ V}$$

$$V_{GSQ} = V_{GQ} - V_{SQ} = 4 - 2I_{DQ}$$

$$I_{DQ} = K [V_{GSQ} - V_T]^2 = 0.075 [4 - 2I_{DQ} - 1]^2$$

$$I_{DQ} = 0.075(9 - 12I_{DQ} + 4I_{DQ}^2)$$

$$4I_{DQ}^2 - 12I_{DQ} + 9 = 13.3I_{DQ} \Rightarrow I_{DQ}^2 - 6.33I_{DQ} + 2.25 = 0$$

Example (Cont')

$$I_{DQ} = 3.167 \pm \frac{\sqrt{6.33^2 - 9}}{2} = 0.378 \text{ mA or } 5.953 \text{ mA}$$

reject since voltage drop across R_D will be too large

$$I_{DQ} = 0.378 \text{ mA}$$

$$V_{DQ} = V_{DD} - R_D I_{DQ} = 20 - 10 \times 0.378 = 16.22 \text{ V}$$

$$V_{SQ} = R_S I_{DQ} = 2 \times 0.378 = 0.756 \text{ V}$$

$$V_{DQ} = 16.22 \text{ V}$$

$$V_{SQ} = 0.756 \text{ V}$$

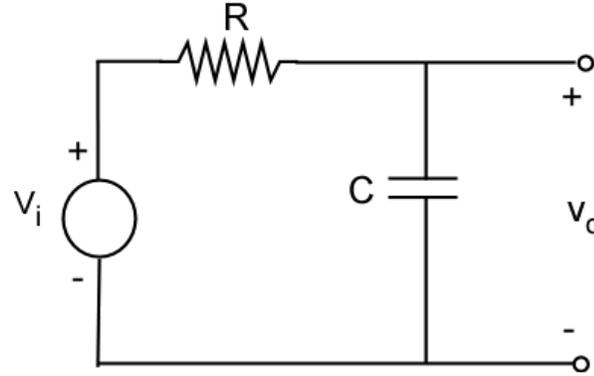
Example (Cont')

$$g_m = \sqrt{2k'_n \frac{W}{L} I_{DQ}} = \sqrt{4 \times 0.075 \times 0.378} = 0.337$$

$$A_{MB} = -g_m R_D = -0.337 \times 10 = -3.37$$

$$A_{MB} = -3.37$$

Low-Pass Circuit



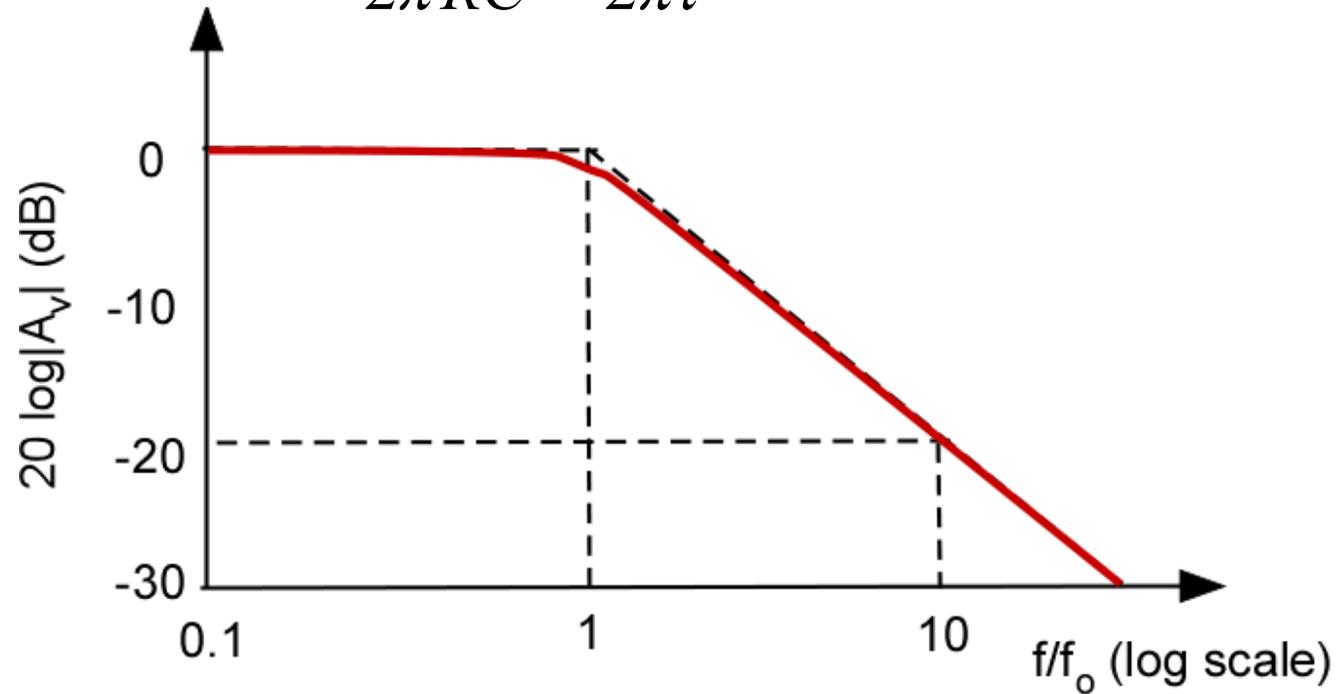
In frequency domain:
$$V_o = \frac{V_i}{R + \frac{1}{j\omega C}} \cdot \frac{1}{j\omega C}$$

$$V_o = \frac{V_i}{1 + j\omega RC} \Rightarrow A_v = \frac{V_o}{V_i} = \frac{1}{1 + j\omega RC}$$

$$A_v = \frac{1}{1 + j\omega RC} = \frac{1}{1 + jf / f_2}$$

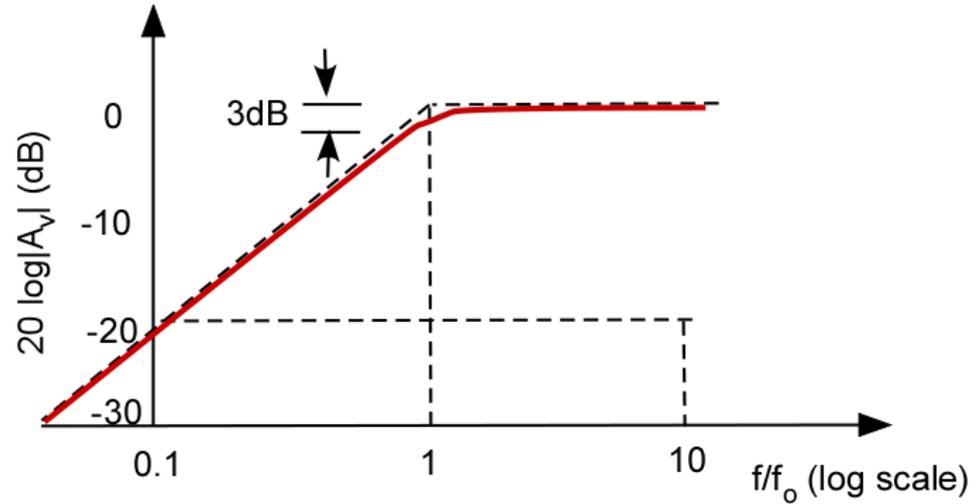
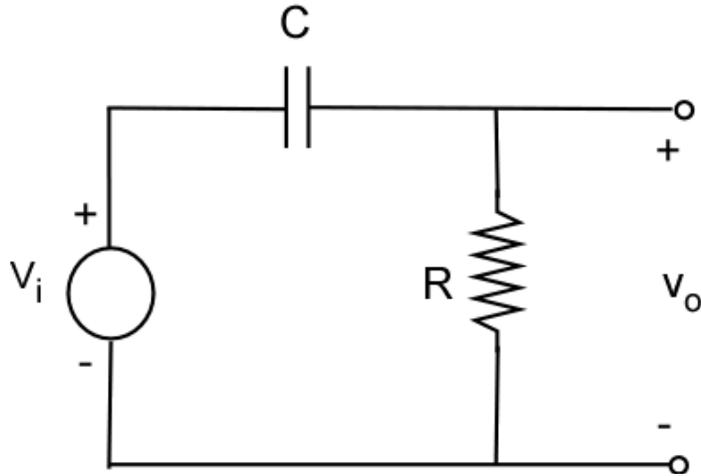
Low-Pass Circuit

$$f_2 = \frac{1}{2\pi RC} = \frac{1}{2\pi\tau}$$



$$\tau = 2\pi RC = \text{time constant}$$

High-Pass Circuit



$$V_o = \frac{V_i R}{R + \frac{1}{j\omega C}} = \frac{V_i}{1 + \frac{1}{j\omega RC}}$$

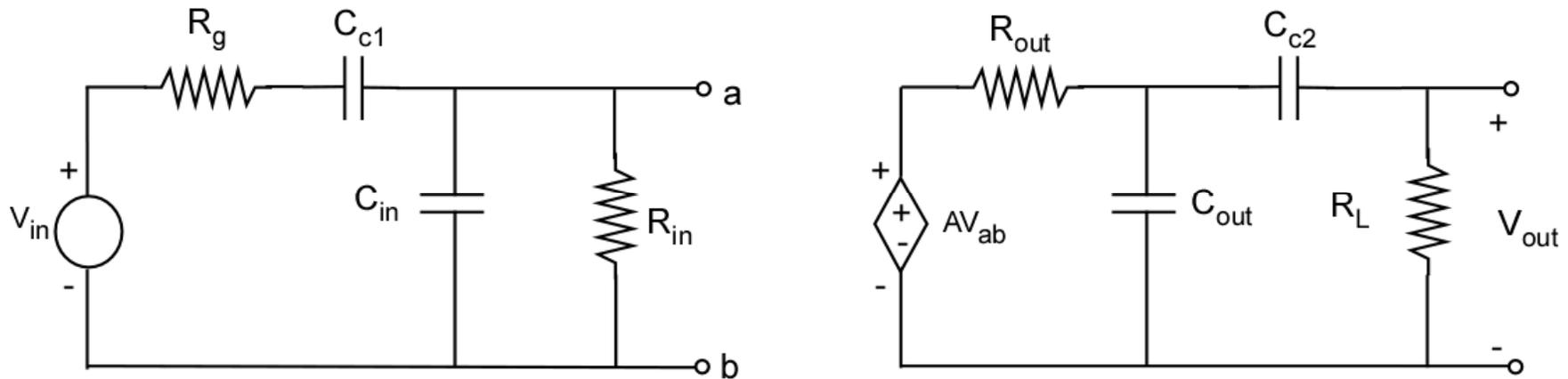
$$A_v = \frac{V_o}{V_i} = \frac{1}{1 - j \frac{1}{2\pi f RC}} = \frac{1}{1 - j f_2 / f}$$

$$f_2 = \frac{1}{2\pi RC}$$

Model for general Amplifying Element

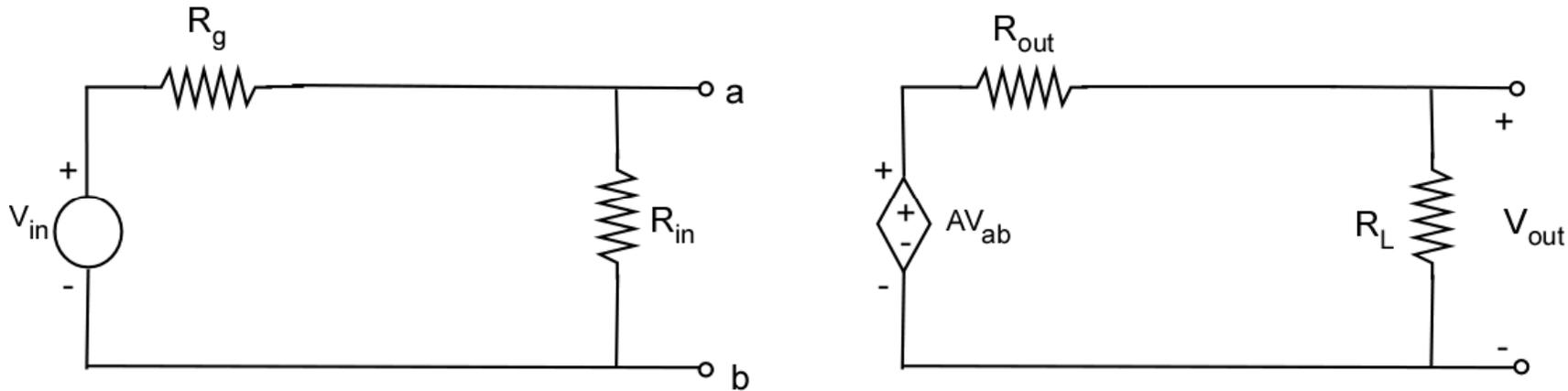
C_{c1} and C_{c2} are coupling capacitors (large) $\rightarrow \mu\text{F}$

C_{in} and C_{out} are parasitic capacitors (small) $\rightarrow \text{pF}$



Midband Frequencies

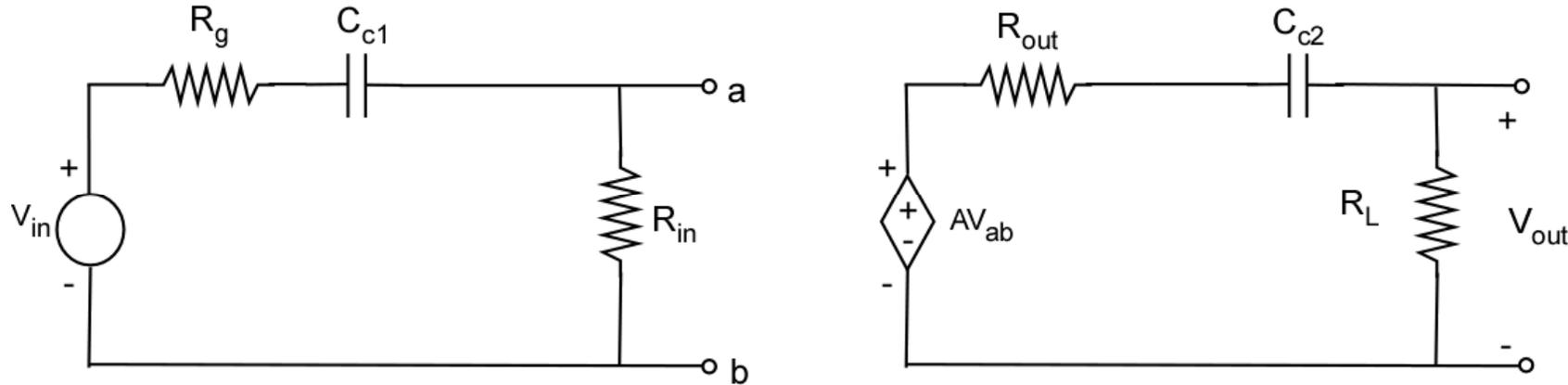
- Coupling capacitors are short circuits
- Parasitic capacitors are open circuits



$$A_{MB} = \frac{v_{out}}{v_{in}} = \frac{R_{in}}{R_g + R_{in}} A \frac{R_L}{R_{out} + R_L}$$

Low Frequency Model

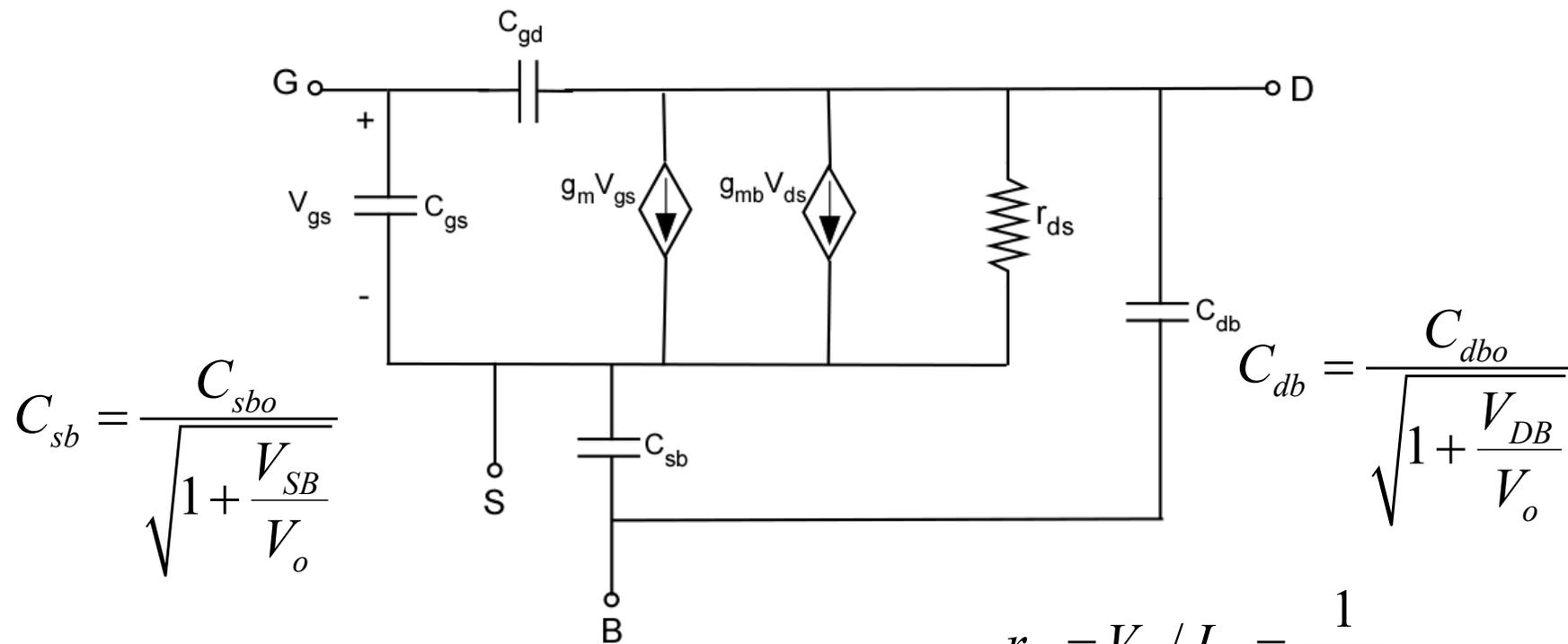
- Coupling capacitors are present
- Parasitic capacitors are open circuits



$$v_{ab} = \frac{v_{in} R_{in}}{R_g + R_{in} + \frac{1}{j\omega C_{c1}}} = \frac{v_{in} j\omega C_{c1} R_{in}}{1 + j\omega C_{c1} (R_g + R_{in})}$$

$$v_{ab} = v_{in} \frac{R_{in}}{R_g + R_{in}} \cdot \frac{j\omega C_{c1} (R_g + R_{in})}{\left[1 + j\omega C_{c1} (R_g + R_{in})\right]}$$

MOSFET High-Frequency Model



$$C_{sb} = \frac{C_{sbo}}{\sqrt{1 + \frac{V_{SB}}{V_o}}}$$

$$C_{db} = \frac{C_{dbo}}{\sqrt{1 + \frac{V_{DB}}{V_o}}}$$

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{eff} = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} = \frac{2I_D}{V_{eff}}$$

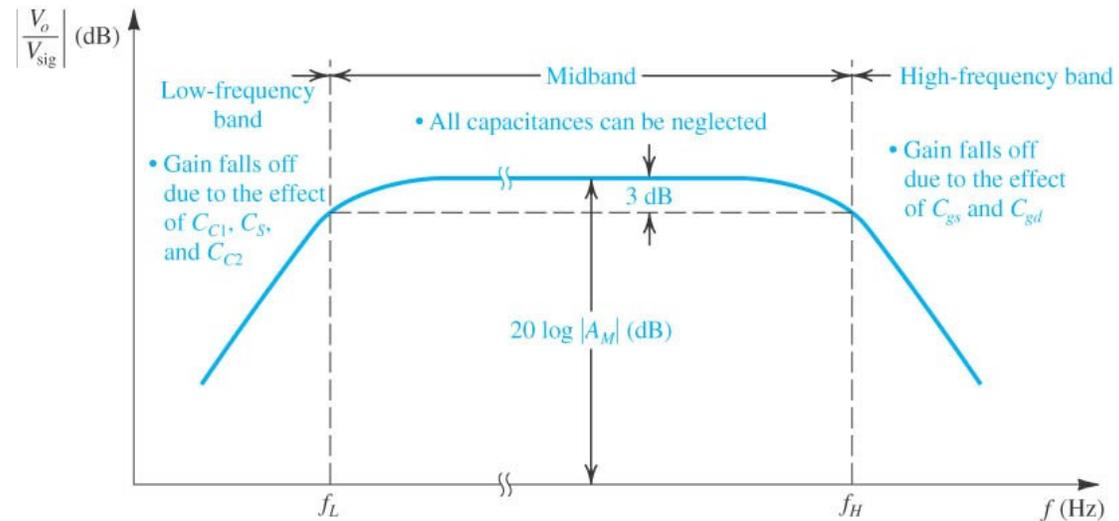
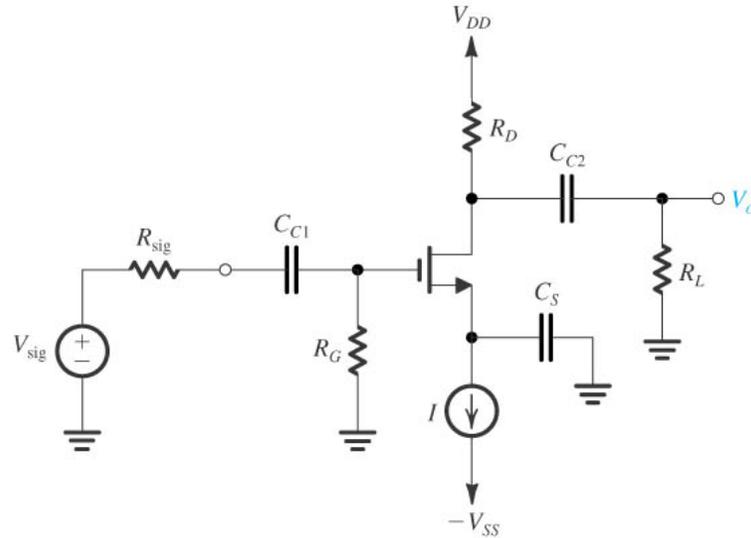
$$r_{ds} = V_A / I_D = \frac{1}{\lambda I_D}$$

$$C_{gs} = \frac{2}{3} W L C_{ox} + W L_{ov} C_{ox}$$

$$g_{mb} = \chi g_m = \frac{\gamma}{2\sqrt{2\phi_F + V_{sb}}} g_m$$

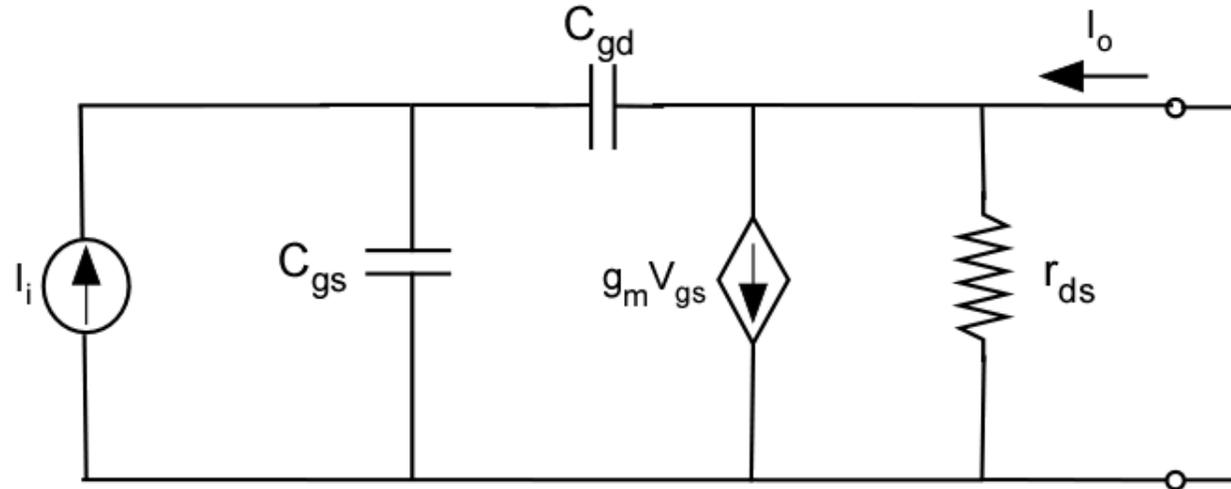
$$C_{gd} = W L_{ov} C_{ox}$$

CS - Three Frequency Bands



Unity-Gain Frequency f_T

f_T is defined as the frequency at which the short-circuit current gain of the common source configuration becomes unity



Define:
 $s = j\omega$

(neglect $sC_{gd}V_{gs}$ since C_{gd} is small)

$$I_o = g_m V_{gs} - sC_{gd} V_{gs} \qquad \frac{I_o}{I_i} = \frac{g_m}{s(C_{gs} + C_{gd})}$$

$$I_o \approx g_m V_{gs} \qquad V_{gs} = \frac{I_i}{s(C_{gs} + C_{gd})}$$

Calculating f_T

For $s=j\omega$, magnitude of current gain becomes unity at

$$\omega_T = \frac{g_m}{C_{gs} + C_{gd}} \Rightarrow f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

$f_T \sim 100$ MHz for $5\text{-}\mu\text{m}$ CMOS, $f_T \sim$ several GHz for $0.13\mu\text{m}$ CMOS