Transfer Function Representation

Use a two-terminal representation of system for input and output
Y-parameter Representation

\[ I_1 = y_{11} V_1 + y_{12} V_2 \]
\[ I_2 = y_{21} V_1 + y_{22} V_2 \]
Y Parameter Calculations

\[
y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} \quad y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0}
\]

To make \( V_2 = 0 \), place a short at port 2
Z Parameters

\[ V_1 = z_{11} I_1 + z_{12} I_2 \]

\[ V_2 = z_{21} I_1 + z_{22} I_2 \]
Z-parameter Calculations

\[ Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} \quad Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} \]

To make \( I_2 = 0 \), place an open at port 2
H Parameters

\[ V_1 = h_{11} I_1 + h_{12} V_2 \]
\[ I_2 = h_{21} I_1 + h_{22} V_2 \]
H Parameter Calculations

To make $V_2 = 0$, place a short at port 2

$$h_{11} = \frac{V_1}{I_1}_{V_2=0}$$

$$h_{21} = \frac{I_2}{I_1}_{V_2=0}$$
\[ I_1 = g_{11} V_1 + g_{12} I_2 \]

\[ V_2 = g_{21} V_1 + g_{22} I_2 \]
G-Parameter Calculations

\[ g_{11} = \frac{I_1}{V_1} \bigg|_{I_2=0} \quad \quad g_{21} = \frac{V_2}{V_1} \bigg|_{I_2=0} \]

To make \( I_2 = 0 \), place an open at port 2
TWO-PORT NETWORK REPRESENTATION

- At microwave frequencies, it is more difficult to measure total voltages and currents.

- Short and open circuits are difficult to achieve at high frequencies.

- Most active devices are not short- or open-circuit stable.

Z Parameters
\[ V_1 = Z_{11} I_1 + Z_{12} I_2 \]
\[ V_2 = Z_{21} I_1 + Z_{22} I_2 \]

Y Parameters
\[ I_1 = Y_{11} V_1 + Y_{12} V_2 \]
\[ I_2 = Y_{21} V_1 + Y_{22} V_2 \]
Wave Approach

Use a travelling wave approach

\[ V_1 = E_{i1} + E_{r1} \quad V_2 = E_{i2} + E_{r2} \]

\[ I_1 = \frac{E_{i1} - E_{r1}}{Z_o} \quad I_2 = \frac{E_{i2} - E_{r2}}{Z_o} \]

- Total voltage and current are made up of sums of forward and backward traveling waves.

- Traveling waves can be determined from standing-wave ratio.
Wave Approach

\[ a_1 = \frac{E_{i1}}{\sqrt{Z_o}} \quad a_2 = \frac{E_{i2}}{\sqrt{Z_o}} \]

\[ b_1 = \frac{E_{r1}}{\sqrt{Z_o}} \quad b_2 = \frac{E_{r2}}{\sqrt{Z_o}} \]

\( Z_o \) is the reference impedance of the system

\[ b_1 = S_{11} \ a_1 + S_{12} \ a_2 \]

\[ b_2 = S_{21} \ a_1 + S_{22} \ a_2 \]
Wave Approach

\[
S_{11} = \frac{b_1}{a_1} \mid a_2=0 \\
S_{12} = \frac{b_1}{a_2} \mid a_1=0 \\
S_{21} = \frac{b_2}{a_1} \mid a_2=0 \\
S_{22} = \frac{b_2}{a_2} \mid a_1=0
\]

To make \( a_i = 0 \)
1) Provide no excitation at port \( i \)
2) Match port \( i \) to the characteristic impedance of the reference lines.

CAUTION : \( a_i \) and \( b_i \) are the traveling waves in the reference lines.
S-Parameters of TL

\[ S_{11} = S_{22} = \frac{(1 - X^2) \Gamma}{1 - X^2 \Gamma^2} \]

\[ S_{12} = S_{21} = \frac{(1 - \Gamma^2) X}{1 - X^2 \Gamma^2} \]

\[ \gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \]

\[ Z_c = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \]

\[ \Gamma = \frac{Z_c - Z_{\text{ref}}}{Z_c + Z_{\text{ref}}} \]

\[ X = e^{-\gamma l} \]
\[ \beta = \omega \sqrt{LC} \]

\[ Z_c = \sqrt{\frac{L}{C}} \]

\[ S_{11} = S_{22} = \frac{(1 - X^2) \Gamma}{1 - X^2 \Gamma^2} \]

\[ S_{12} = S_{21} = \frac{(1 - \Gamma^2) X}{1 - X^2 \Gamma^2} \]

\[ \Gamma = \frac{Z_c - Z_{\text{ref}}}{Z_c + Z_{\text{ref}}} \]

\[ X = e^{-j\beta l} \]

If \( Z_c = Z_{\text{ref}} \)

\[ S_{11} = S_{22} = 0 \]

\[ S_{12} = S_{21} = e^{-j\beta l} \]
N-Port S Parameters

\[
\begin{bmatrix}
  b_1 \\
  b_2 \\
  \vdots \\
  b_n
\end{bmatrix}
= 
\begin{bmatrix}
  S_{11} & S_{12} & \cdots & \\
  S_{21} & S_{22} & \cdots & \\
  \vdots & \vdots & \ddots & \\
  \vdots & \vdots & \vdots & S_{nn}
\end{bmatrix}
\begin{bmatrix}
  a_1 \\
  a_2 \\
  \vdots \\
  a_n
\end{bmatrix}
\]

\[b = Sa\]

If \( b_i = 0 \), then no reflected wave on port \( i \) \( \Rightarrow \) port is matched.

\[a_i = \frac{V_i^+}{\sqrt{Z_{oi}}}\]

\( V_i^+ \): incident voltage wave in port \( i \)

\[b_i = \frac{V_i^-}{\sqrt{Z_{oi}}}\]

\( V_i^- \): reflected voltage wave in port \( i \)

\( Z_{oi} \): impedance in port \( i \)
N-Port S Parameters

\[ v = \sqrt{Z_o} (a + b) \quad (1) \quad i = \frac{1}{\sqrt{Z_o}} (a - b) \quad (2) \quad v = Zi \quad (3) \]

Substitute (1) and (2) into (3)

\[ \sqrt{Z_o} (a + b) = Z \frac{1}{\sqrt{Z_o}} (a - b) \]

Defining S such that \( b = Sa \) and substituting for b

\[ Z_o (U + S)a = Z_o (U - S)a \quad \text{U : unit matrix} \]

\[ S \rightarrow Z \]

\[ Z = Z_o (U + S)(U - S)^{-1} \]

\[ Z \rightarrow S \]

\[ S = (Z + Z_o U)^{-1} (Z - Z_o U) \]
N-Port S Parameters

If the port reference impedances are different, we define \( k \) as

\[
k = \begin{bmatrix}
\sqrt{Z_{o1}} \\
\sqrt{Z_{o2}} \\
\sqrt{Z_{on}}
\end{bmatrix}.
\]

\[
v = k(a + b) \quad \text{and} \quad i = k^{-1}(a - b) \quad \text{and} \quad k(a + b) = Zk^{-1}(a - b)
\]

\[
\begin{align*}
Z & \rightarrow S \\
S & = (Zk^{-1} + k)(Zk^{-1} - k) \\
S & \rightarrow Z \\
Z &= k(U + S)(U - S)^{-1}k
\end{align*}
\]
Normalization

Assume original S parameters as $S_1$ with system $k_1$. Then the representation $S_2$ on system $k_2$ is given by

$$
S_2 = \left[ k_1 (U + S_1)(U - S_1)^{-1} k_1 k_2 + k_2 \right]^{-1} \left[ k_1 (U + S_1)(U - S_1)^{-1} k_1 k_2 - k_2 \right]
$$

Transformation Equation

If $Z$ is symmetric, $S$ is also symmetric
Dissipated Power

\[ P_d = \frac{1}{2} a^T (U - S^T S^*) a^* \]

The dissipation matrix \( D \) is given by:

\[ D = U - S^T S^* \]

Passivity insures that the system will always be stable provided that it is connected to another passive network.

For passivity
- (1) the determinant of \( D \) must be \( \geq 0 \)
- (2) the determinant of the principal minors must be \( \geq 0 \)
Dissipated Power

When the dissipation matrix is 0, we have a lossless network

$$S^T S^* = U$$

The S matrix is unitary.

For a lossless two-port:

$$|S_{11}|^2 + |S_{21}|^2 = 1$$

$$|S_{22}|^2 + |S_{12}|^2 = 1$$

If in addition the network is reciprocal, then

$$S_{12} = S_{21} \quad \text{and} \quad |S_{11}| = |S_{22}| = \sqrt{1 - |S_{12}|^2}$$
Lossy and Dispersive Line

\[ S_{11} = S_{22} = \frac{(1 - \alpha^2)\rho}{1 - \rho^2 \alpha^2} \]

\[ S_{21} = S_{12} = \frac{(1 - \rho^2)\alpha}{1 - \rho^2 \alpha^2} \]

\[ \alpha = e^{-\gamma l} \]

\[ \rho = \frac{Z_c(\omega) - Z_o}{Z_c(\omega) + Z_o} \]
Frequency-Domain Formulation*

Frequency-Domain

\[ B_1(\omega) = S_{11}(\omega) A_1(\omega) + S_{12}(\omega) A_2(\omega) \]

\[ B_2(\omega) = S_{21}(\omega) A_1(\omega) + S_{22}(\omega) A_2(\omega) \]
Time-Domain Formulation
Time-Domain Formulation

\[ b_1(t) = s_{11}(t)^* a_1(t) + s_{12}(t)^* a_2(t) \]

\[ b_2(t) = s_{21}(t)^* a_1(t) + s_{22}(t)^* a_2(t) \]

\[ a_1(t) = \Gamma_1(t)b_1(t) + T_1(t)g_1(t) \]

\[ a_2(t) = \Gamma_2(t)b_2(t) + T_2(t)g_2(t) \]

\[ T_i(t) = \frac{Z_o}{Z_i(t) + Z_o} \]

\[ \Gamma_i(t) = \frac{Z_i(t) - Z_o}{Z_i(t) + Z_o} \]
Time-Domain Solutions

\[ a_1(t) = \frac{\left[ 1 - \Gamma_2(t)s'_{22}(0) \right] \left[ T_1(t)g_1(t) + \Gamma_1(t)M_1(t) \right]}{\Delta(t)} + \frac{\left[ \Gamma_1(t)s'_{12}(0) \right] \left[ T_2(t)g_2(t) + \Gamma_2(t)M_2(t) \right]}{\Delta(t)} \]

\[ a_2(t) = \frac{\left[ 1 - \Gamma_1(t)s'_{11}(0) \right] \left[ T_2(t)g_2(t) + \Gamma_2(t)M_2(t) \right]}{\Delta(t)} + \frac{\left[ \Gamma_2(t)s'_{21}(0) \right] \left[ T_1(t)g_1(t) + \Gamma_1(t)M_1(t) \right]}{\Delta(t)} \]
Time-Domain Solutions

\[ b_1(t) = s_{11}'(0)a_1(t) + s_{12}'(0)a_2(t) + M_1(t) \]

\[ b_2(t) = s_{21}'(0)a_1(t) + s_{22}'(0)a_2(t) + M_2(t) \]

\[ \Delta(t) = [1 - \Gamma_1(t)s_{11}'(0)][1 - \Gamma_2(t)s_{22}'(0)] - \Gamma_1(t)s_{12}'(0)\Gamma_2(t)s_{21}'(0) \]

\[ M_1(t) = H_{11}(t) + H_{12}(t) \]

\[ M_2(t) = H_{21}(t) + H_{22}(t) \]

\[ s_{ij}'(0) = s_{ij}(0)\Delta \tau \]

\[ H_{ij}(t) = \sum_{\tau=1}^{t-1} s_{ij}(t - \tau)a_j(\tau)\Delta \tau \]
Special Case – Lossless Line

\[ s_{11}(t) = s_{22}(t) = 0 \]

\[ s_{12}(t) = s_{21}(t) = \delta \left( t - \frac{l}{v} \right) \]

\[ M_1(t) = a_2 \left( t - \frac{l}{v} \right) \]

\[ M_2(t) = a_1 \left( t - \frac{l}{v} \right) \]

\[ a_1(t) = T_1(t) g_1(t) + \Gamma_1(t) a_2 \left( t - \frac{l}{v} \right) \]

\[ a_2(t) = T_2(t) g_2(t) + \Gamma_2(t) a_1 \left( t - \frac{l}{v} \right) \]

\[ b_1(t) = a_2 \left( t - \frac{l}{v} \right) \]

\[ b_2(t) = a_1 \left( t - \frac{l}{v} \right) \]

Wave Shifting Solution
Time-Domain Solutions

\[ v_1(t) = a_1(t) + b_1(t) \]

\[ v_2(t) = a_2(t) + b_2(t) \]

\[ i_1(t) = \frac{a_1(t)}{Z_o} - \frac{b_1(t)}{Z_o} \]

\[ i_2(t) = \frac{a_2(t)}{Z_o} - \frac{b_2(t)}{Z_o} \]
Simulations

Line length = 1.27m

$Z_0 = 73 \, \Omega$

$v = 0.142 \, \text{m/ns}$
Simulations
Simulations

Line length = 25 in
L = 539 nH/m

C = 39 pF/m
R_o = 1 kΩ (GHz)^{1/2}

Pulse magnitude = 4V
Pulse width = 20 ns
Rise and fall times = 1 ns
N-Line S-Parameters*

\[
B_1 = S_{11} A_1 + S_{12} A_2 \quad \quad B_2 = S_{21} A_1 + S_{22} A_2
\]

Scattering Parameters for N-Line

\[ S_{21} = S_{12} = 2E_0E^{-1}[1 - \Gamma]\Psi[1 - \Gamma\Psi\Gamma\Psi]^{-1}T \]

\[ S_{11} = S_{22} = T^{-1}[\Gamma - \Psi\Gamma\Psi][1 - \Gamma\Psi\Gamma\Psi]^{-1}T \]

\[ \Gamma = \left[1 + EE_0^{-1}Z_0H_oH^{-1}Z_m^{-1}\right]^{-1}\left[1 - EE_0^{-1}Z_0H_oH^{-1}Z_m^{-1}\right] \]

\[ T = \left[1 + EE_0^{-1}Z_0H_oH^{-1}Z_m^{-1}\right]^{-1}EE_0^{-1} \]

\[ \Psi = W(-l) \]
Scattering Parameter Matrices

\( E_o \): Reference system voltage eigenvector matrix

\( E \): Test system voltage eigenvector matrix

\( H_o \): Reference system current eigenvector matrix

\( H \): Test system current eigenvector matrix

\( Z_o \): Reference system modal impedance matrix

\( Z_m \): Test system modal impedance matrix
Eigen Analysis

* Diagonalize $Z Y$ and $Y Z$ and find eigenvalues.
* Eigenvalues are complex: $\lambda_i = \alpha_i + j\beta_i$

\[
W(u) = \begin{bmatrix}
  e^{\alpha_1 u + j\beta_1 u} \\
  \cdot \\
  e^{\alpha_n u + j\beta_n u}
\end{bmatrix}
\]
Solution

\[ V_m = EV \]

\[ I_m = HI \]

\[ V_m(x) = [W(-x)A + W(x)B] \]

\[ I_m(x) = Z_m^{-1} [W(-x)A + W(x)B] \]

\[ Z_m = \Lambda_m^{-1}EZH^{-1} \]

\[ Z_c = E^{-1}Z_mH = E^{-1}\Lambda_m^{-1}EZ \]
Solutions

\[ a_1(t) = \Delta_1^{-1} \left[ 1 - \Gamma_1(t)s'_{11}(0) \right]^{-1} \left[ T_1(t)g_1(t) + \Gamma_1(t)M_1(t) \right] \]

\[ - \Delta_1^{-1} \left[ 1 - \Gamma_1(t)s'_{11}(0) \right]^{-1} \left[ 1 - \Gamma_2(t)s'_2(0) \right]^{-1} \times \]

\[ \left[ \Gamma_1(t)s'_{21}(0) \right] \left[ T_2(t)g_2(t) + \Gamma_2(t)M_2(t) \right] \]

\[ a_2(t) = \Delta_2^{-1} \left[ 1 - \Gamma_2(t)s'_2(0) \right]^{-1} \left[ T_2(t)g_2(t) + \Gamma_2(t)M_2(t) \right] \]

\[ - \Delta_2^{-1} \left[ 1 - \Gamma_2(t)s'_2(0) \right]^{-1} \left[ 1 - \Gamma_1(t)s'_{11}(0) \right]^{-1} \times \]

\[ \left[ \Gamma_1(t)s'_{12}(0) \right] \left[ T_1(t)g_1(t) + \Gamma_1(t)M_1(t) \right] \]
Solutions

\[ \Delta_1(t) = 1 - \left[ 1 - \Gamma_1(t)s'_{11}(0) \right]^{-1} \left[ 1 - \Gamma_2(t)s'_{22}(0) \right]^{-1} \Gamma_1(t)s'_{21}(0)\Gamma_2(t)s'_{12}(0) \]

\[ \Delta_2(t) = 1 - \left[ 1 - \Gamma_2(t)s'_{22}(0) \right]^{-1} \left[ 1 - \Gamma_1(t)s'_{11}(0) \right]^{-1} \Gamma_2(t)s'_{12}(0)\Gamma_1(t)s'_{21}(0) \]

\[ b_1(t) = s'_{11}(0)a_1(t) + s'_{12}(0)a_2(t) + M_1(t) \]

\[ b_2(t) = s'_{21}(0)a_1(t) + s'_{22}(0)a_2(t) + M_2(t) \]
Solutions

\[ v_{m1}(t) = a_1(t) + b_1(t) \Rightarrow v_1(t) = E_o^{-1} \left[ a_1(t) + b_1(t) \right] \]

\[ v_{m2}(t) = a_2(t) + b_2(t) \Rightarrow v_2(t) = E_o^{-1} \left[ a_2(t) + b_2(t) \right] \]
Lossless Case – Wave Shifting

\[ s_{21}(t) = s_{12}(t) = \delta(t - \tau_m) \]

\[ M_1(t) = a_2(t - \tau_m) \]

\[ M_2(t) = a_1(t - \tau_m) \]

\[ a_1(t) = T_1(t) g_1(t) + \Gamma_1(t) a_2(t - \tau_m) \]

\[ a_2(t) = T_2(t) g_3(t) + \Gamma_3(t) a_1(t - \tau_m) \]

\[ b_1(t) = a_2(t - \tau_m) \]

\[ b_2(t) = a_2(t - \tau_m) \]
Solution for Lossless Lines

\[
\delta(t - \tau_m) = \begin{cases} 
\delta(t - \tau_{m1}) \\
\delta(t - \tau_{m2}) \\
\vdots \\
\delta(t - \tau_{mn}) 
\end{cases}
\]

\[
a_i(t - \tau_m) = \begin{bmatrix}
a_1(t - \tau_{m1}) \\
a_2(t - \tau_{m2}) \\
\vdots \\
a_n(t - \tau_{mn})
\end{bmatrix}
\]
Why Use S Parameters?

Y-Parameter

\[
Y_{11} = \frac{1 + e^{-2\gamma l}}{Z_c (1 - e^{-2\gamma l})}
\]

Z_c : microstrip characteristic impedance
\(\gamma\) : complex propagation constant
l : length of microstrip

Y_{11} can be unstable

S-Parameter

\[
S_{11} = \frac{(1 - e^{-2\gamma l})\Gamma}{1 - \Gamma^2 e^{-2\gamma l}}
\]

\[
\Gamma = \frac{Z_c - Z_o}{Z_c + Z_o}
\]

S_{11} is always stable
Choice of Reference

\[ \Gamma = \frac{Z_c - Z_{ref}}{Z_c + Z_{ref}} \]

\[ Z_c = \sqrt[4]{\frac{R + j\omega L}{G + j\omega C}} \]

\( Z_{ref} \) is arbitrary

**What is the best choice for** \( Z_{ref} \)?

At high frequencies

\[ Z_c \rightarrow \sqrt{\frac{L}{C}} \]

Thus, if we choose

\[ Z_{ref} = \sqrt{\frac{L}{C}} \]

\[ S_{12} \rightarrow e^{-j\omega \sqrt{LCd}} = X_o \]

\[ S_{11} \rightarrow 0 \]
Choice of Reference

S-Parameter measurements (or simulations) are made using a 50-ohm system. For a 4-port, the reference impedance is given by:

\[
Z_0 = \begin{bmatrix}
50.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 50.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 50.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 50.0
\end{bmatrix}
\]

- **Z**: Impedance matrix (of blackbox)
- **S**: S-parameter matrix
- **Z_0**: Reference impedance
- **I**: Unit matrix

\[
S = \left( ZZ_0^{-1} + I \right)^{-1} \left( ZZ_0^{-1} - I \right)
\]

\[
Z = \left( I + S \right) \left( I - S \right)^{-1} Z_0
\]
Reference Transformation

Method: Change reference impedance from uncoupled to coupled system to get new S-parameter representation

\[
Z_o = \begin{bmatrix}
50.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 50.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 50.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 50.0 \\
\end{bmatrix} \quad \text{Uncoupled system}
\]

\[
Z_o = \begin{bmatrix}
328.0 & 69.6 & 328.9 & 69.6 \\
69.6 & 328.8 & 69.6 & 328.9 \\
328.9 & 69.6 & 328.8 & 69.6 \\
69.6 & 328.9 & 69.6 & 328.8 \\
\end{bmatrix} \quad \text{Coupled system}
\]

as an example…
Choice of Reference

\[ Z_0 = \]

using

\[
\begin{array}{cccc}
50.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 50.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 50.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 50.0 \\
\end{array}
\]
as reference...

using

\[
\begin{array}{cccc}
328.0 & 69.6 & 328.9 & 69.6 \\
69.6 & 328.8 & 69.6 & 328.9 \\
328.9 & 69.6 & 328.8 & 69.6 \\
69.6 & 328.9 & 69.6 & 328.8 \\
\end{array}
\]
as reference...

Easier to approximate (up to 6 GHz)

Harder to approximate
Choice of Reference

Using
\[ Z_0 = \begin{bmatrix} 50.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 50.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 50.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 50.0 \end{bmatrix} \]
as reference...

Using
\[ Z_0 = \begin{bmatrix} 328.0 & 69.6 & 328.9 & 69.6 \\ 69.6 & 328.8 & 69.6 & 328.9 \\ 328.9 & 69.6 & 328.8 & 69.6 \\ 69.6 & 328.9 & 69.6 & 328.8 \end{bmatrix} \]
as reference...
Choice of Reference

\[ Z_0 = \begin{bmatrix} 50.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 50.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 50.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 50.0 \end{bmatrix} \]

using \( Z_0 \) as reference...

\[ Z_0 = \begin{bmatrix} 328.0 & 69.6 & 328.9 & 69.6 \\ 69.6 & 328.8 & 69.6 & 328.9 \\ 328.9 & 69.6 & 328.8 & 69.6 \\ 69.6 & 328.9 & 69.6 & 328.8 \end{bmatrix} \]

using \( Z_0 \) as reference...

Easier to approximate (up to 6 GHz)

Harder to approximate
Choice of Reference

\[ Z_0 = \begin{align*}
328.0 & \quad 69.6 & \quad 328.9 & \quad 69.6 \\
69.6 & \quad 328.8 & \quad 69.6 & \quad 328.9 \\
328.9 & \quad 69.6 & \quad 328.8 & \quad 69.6 \\
69.6 & \quad 328.9 & \quad 69.6 & \quad 328.8
\end{align*} \]

as reference…

\[ Z_0 = \begin{align*}
50.0 & \quad 0.0 & \quad 0.0 & \quad 0.0 \\
0.0 & \quad 50.0 & \quad 0.0 & \quad 0.0 \\
0.0 & \quad 0.0 & \quad 50.0 & \quad 0.0 \\
0.0 & \quad 0.0 & \quad 0.0 & \quad 50.0
\end{align*} \]

as reference…

Using \( Z_0 = 50 \, \text{Ohm} \) as reference...

Using \( Z_0 = Z_{\text{ref}} \) as reference...

Easier to approximate

Harder to approximate
Choice of Reference

![Graph showing S11 Magnitude for different reference impedances](image)

- **S11 Magnitude**
  - **Zref=Zo**
  - **Zref=80 ohms**
  - **Zref=100 ohms**

Frequency (GHz) vs. S11 Magnitude plot.
Choice of Reference

S21 Magnitude

- Red line: \( Z_{\text{ref}} = Z_0 \)
- Blue line: \( Z_{\text{ref}} = 80 \text{ ohms} \)
- Black line: \( Z_{\text{ref}} = 100 \text{ ohms} \)

- Frequency (GHz)
- S21 Magnitude
Modeling of Discontinuities

1. Tapered Lines

2. Capacitive Discontinuities
Tapered Microstrip

General topology of tapered microstrip with $d_w$: width at wide end, $d_n$: width at narrow end, $l_w$: length of wide section, $l_n$: length of narrow section, $l_t$: length of tapered section.
Tapered Line Analysis Using S Parameters*

\[ u_j(t) = s_{21}^{(j)}(t) * u_{j-1}(t) + s_{22}^{(j)}(t) * w_j(t) \]

\[ w_j(t) = s_{11}^{(j+1)}(t) * u_j(t) + s_{12}^{(j+1)}(t) * w_{j+1}(t) \]

Tapered Transmission Line

Small End
Excitation at small end

Wide End
Excitation at small end

Small End
Excitation at wide end

Wide End
Excitation at wide end
Tapered Transmission Line

Varying tapering rate

Near End

Far End

Varying tapering rate

Time (ns)

Near End

Far End

Varying tapering rate
Capacitive Load

![Diagram of a capacitive load circuit]

- Near End -- C=4 pF
- Far end -- C=4 pF
Capacitive Load

Near end -- C=40 pF

Volts

Time (ns)

Far end -- C=40 pF

Volts

Time (ns)
Multidrop Buses

- Stubs of TL with nonlinear loads
- Reduce speed and bandwidth
- Limit driving capabilities
Transmission Lines with Capacitive Discontinuities
Capacitive Discontinuity

\[ V_i + V_r = V_t \]

\[ \frac{V_i - V_r}{Z_0} + \frac{E}{R} - \frac{V_i - V_r}{R} = \frac{V_t}{Z_0} \]

\[ V_r = T_c E + \Gamma_c V_i \]
Scattering Parameter Analysis

\[ u_j(t) = s_{21}^{(j)}(t) * u_{j-1}(t) + s_{22}^{(j)}(t) * w_j(t) \]

\[ u_j'(t) = u_j(t) + u_j''(t) \]

\[ w_j(t) = w_j'(t) + u_j''(t) \]
Capacitive Loading
Computer-simulated near end responses for capacitively loaded transmission line with $l = 3.6$ in, $w = 8$ mils, $h = 5$ mils. Pulse parameters are $V_{\text{max}} = 4$ V, $t_r = t_f = 0.5$ ns, $t_w = 4$ ns. Left: Varying $P$ with $C = 2$ pF. Right: Varying $C$ with $P = 300$ mils.