ECE 546
Lecture -15
Circuit Synthesis

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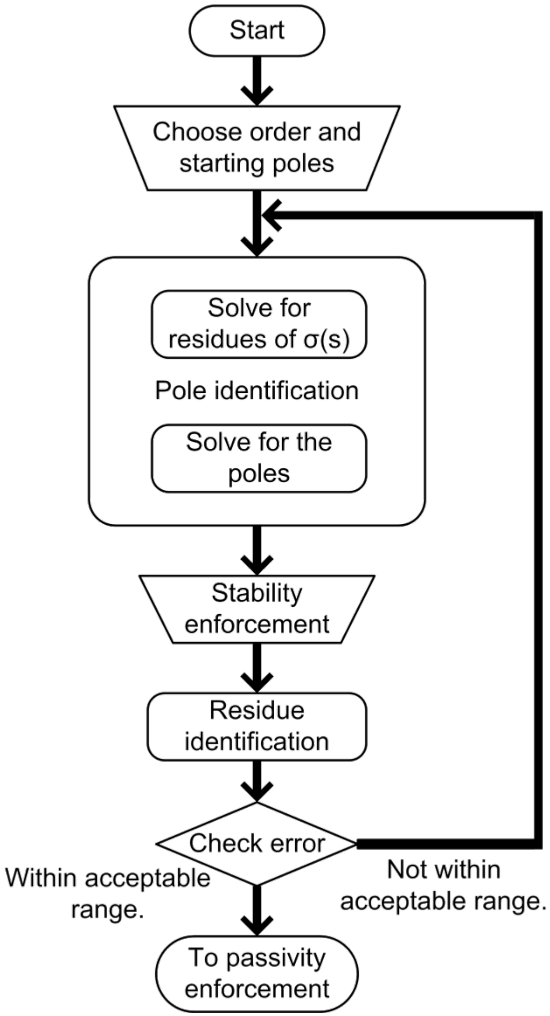
MOR via Vector Fitting

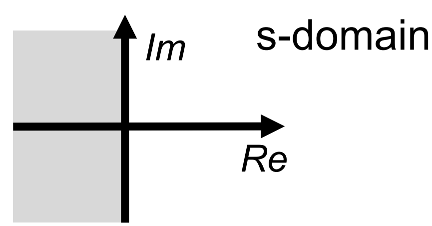


- Rational function approximation: $$f(s) \approx \sum_{n=1}^{N} \frac{c_n}{s-a_n} + d + sh$$

- Introduce an unknown function $\sigma(s)$ that satisfies: $$\begin{bmatrix} \sigma(s) f(s) \\ \sigma(s) \end{bmatrix} \approx \begin{bmatrix} \sum_{n=1}^{N} \frac{c_n}{s-\tilde{a}_n} + d + sh \\ \sum_{n=1}^{N} \frac{\tilde{c}_n}{s-\tilde{a}_n} + 1 \end{bmatrix}$$

- Poles of $f(s)$ = zeros of $\sigma(s)$: $$f(s) \approx \frac{\sum_{n=1}^{N} \frac{c_n}{s-\tilde{a}_n} + d + sh}{\sum_{n=1}^{N} \frac{\tilde{c}_n}{s-\tilde{a}_n} + 1} = \frac{\prod_{n=1}^{N+1} (s - z_n)}{\prod_{n=1}^{N} (s - \tilde{z}_n)}$$

- Flip unstable poles into the left half plane.



Blackbox Formulation

Transfer function is approximated

$$H(\omega) = d + \sum_{k=1}^{L} \frac{c_k}{1 + j\omega / \omega_{ck}}$$

Using curve fitting technique (e.g. vector fitting)

In the time domain, recursive convolution is used

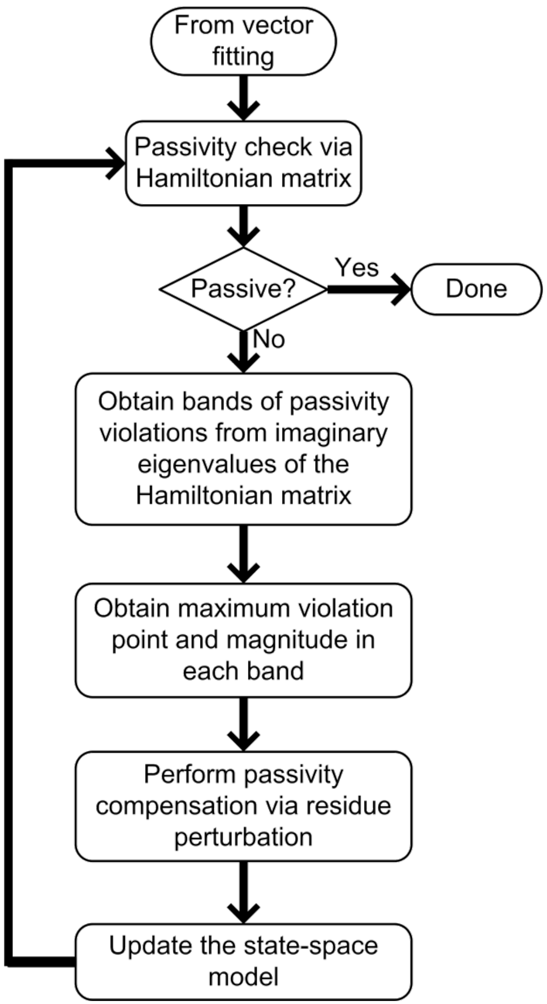
$$y(t) = dx(t-T) + \sum_{k=1}^{L} y_{pk}(t)$$

where

$$y_{pk}(t) = a_k x(t-T)\left(1 - e^{-\omega_{ck}T}\right) + e^{-\omega_{ck}T} y_{pk}(t-T)$$

Recursive convolution is fast

Passivity Enforcement



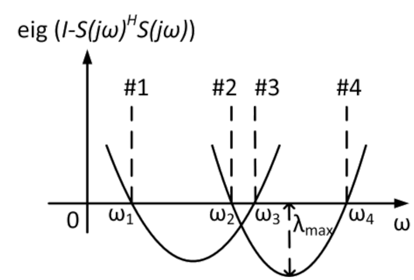
- State-space form: $\dot{x} = Ax + Bu$, $y = Cx + Du$

- Hamiltonian matrix: $M = \begin{bmatrix} A + BKD^T C & BKB^T \\ -C^T LC & -A^T - C^T DKB^T \end{bmatrix}$

$$K = \left(I - D^T D\right)^{-1} \quad L = \left(I - DD^T\right)^{-1}$$

- Passive if *M* has no imaginary eigenvalues.

- Sweep: $eig\left(I - S(j\omega)^H S(j\omega)\right)$



- Quadratic programming:
  - Minimize *(change in response)* subject to *(passivity compensation).*

$$\min\left(vec(\Delta C)^T H\, vec(\Delta C)\right) \quad \text{subject to} \quad \Delta\lambda = G \cdot vec(\Delta C).$$

Macromodel Circuit Synthesis

Use of Macromodel

- Time-Domain simulation using recursive convolution
- Frequency-domain circuit synthesis for SPICE netlist

Macromodel Circuit Synthesis

Objective: Determine equivalent circuit from macromodel representation

Motivation

- Circuit can be used in SPICE

Goal

- Generate a netlist of circuit elements

Circuit Realization

Circuit realization consists of interfacing the reduced model with a general circuit simulator such as SPICE

Model order reduction gives a transfer function that can be presented in matrix form as

$$S(s) = \begin{bmatrix} s_{11}(s) & \cdot & s_{1N}(s) \\ \cdot & \cdot & \cdot \\ s_{N1}(s) & \cdot & s_{NN}(s) \end{bmatrix}$$

or

$$Y(s) = \begin{bmatrix} y_{11}(s) & \cdot & y_{1N}(s) \\ \cdot & \cdot & \cdot \\ y_{N1}(s) & \cdot & y_{NN}(s) \end{bmatrix}$$

Method 1: Y-Parameter/MOR*

Each of the Y-parameters can be represented as

$$y_{ij}(s) = d + \sum_{k=1}^{L} \frac{a_k}{s - p_k}$$

where the a_k's are the residues and the p_k's are the poles. d is a constant

*Giulio Antonini "SPICE Equivalent Circuits of Frequency-Domain Responses", IEEE Transactions on Electromagnetic Compatibility, pp 502-512, Vol. 45, No. 3, August 2003.

Equivalent-Circuit Extraction

Macromodel is curve-fit to take the form

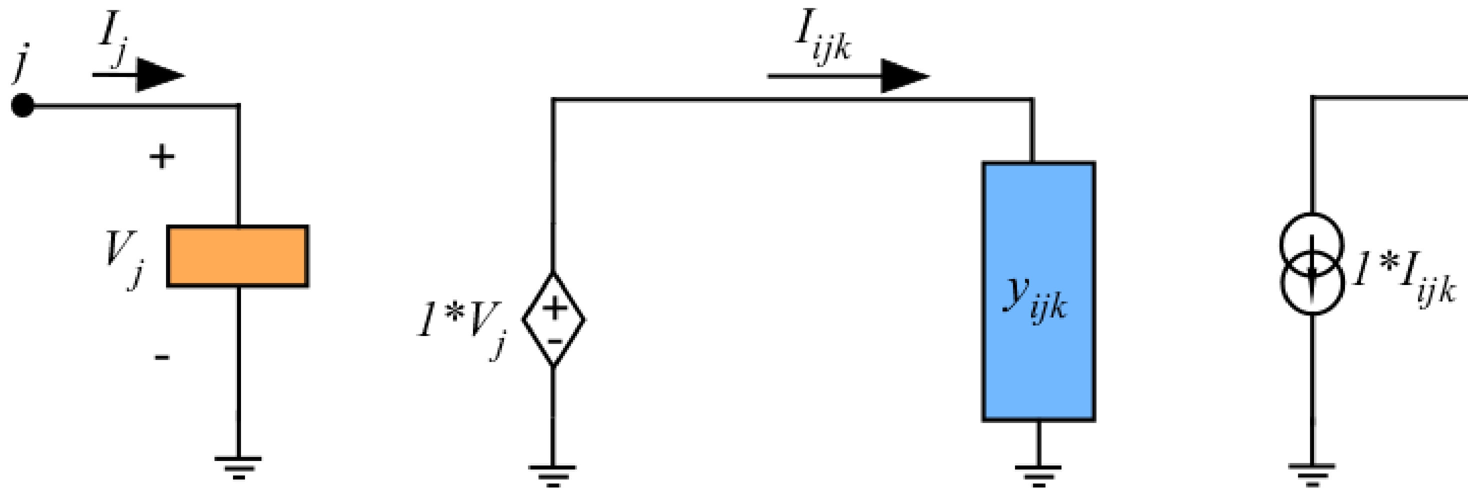
$$Y(s) = d + \sum_{k=1}^{L} \frac{r_k}{s - p_k}$$

Need to find equivalent circuit associated with

- Constant term d
- Real Poles
- Complex Poles

Y-Parameter - Circuit Realization

Each of the I_{ij} can be realized with a circuit having the following topology:



The resulting current sources can then be superposed for the total current I_i leaving port *i*

All Y parameters are treated as if they were <u>one-port</u> Y parameters

Y-Parameter - Circuit Realization

We try to find the circuit associated with each term:

$$y_{ij}(s) = d_{ij} + \sum_{k=1}^{L} \frac{a_{ijk}}{s - p_{ijk}}$$

For a given port i, the total current due to all ports with voltages V_j $(j=1,\ldots,P)$ is given by

$$I_i = \sum_{j=1}^{P} y_{ij} V_j = \sum_{j=1}^{P} d_{ij} V_j + \sum_{j=1}^{P} V_j \sum_{k=1}^{L} \frac{a_{ijk}}{s - p_{ijk}}$$

For each contributing port, with voltage i, the total current due to a voltage V_j at port j is given by

$$I_{ij} = d_{ij} V_j + V_j \sum_{k=1}^{L} \frac{a_{ijk}}{s - p_{ijk}}$$

Y-Parameter - Circuit Realization

We try to find the circuit associated with each term:

$$y_{ij}(s) = d + \sum_{k=1}^{L} \frac{a_k}{s - p_k}$$

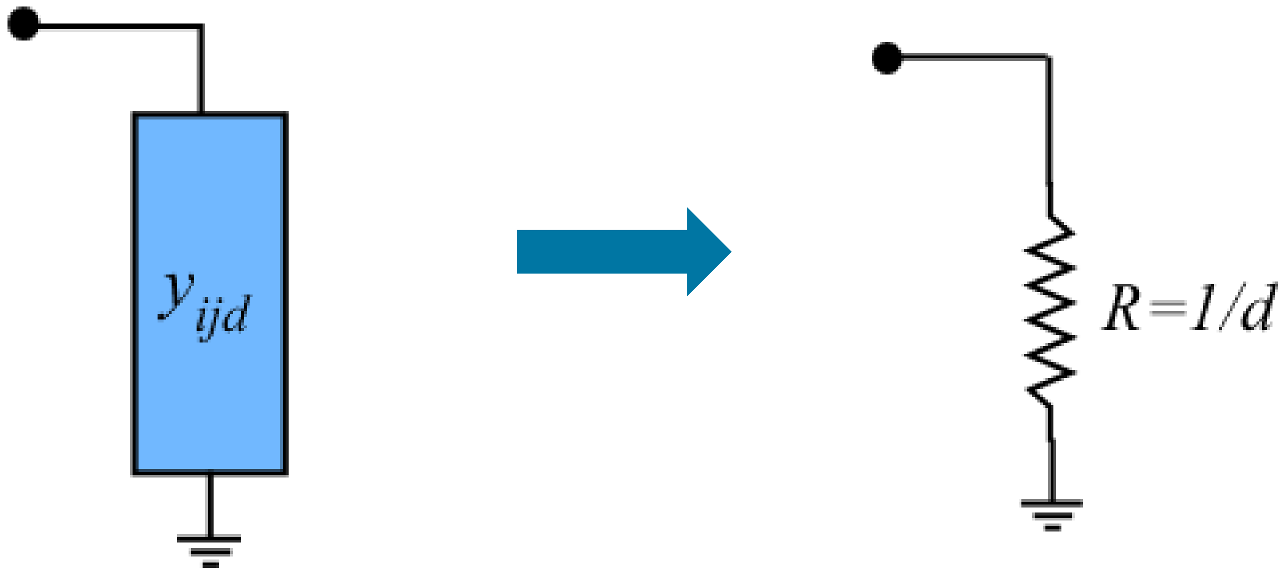
1. Constant term *d*

$$y_{ijd}(s) = d$$

2. Each pole-residue pair

$$y_{ijk}(s) = \frac{a_k}{s - p_k}$$

Circuit Realization – Constant Term



$$R = \frac{1}{d}$$

Circuit Realization – Pole/Residue

In the pole-residue case, we must distinguish two cases

(a) Pole is real

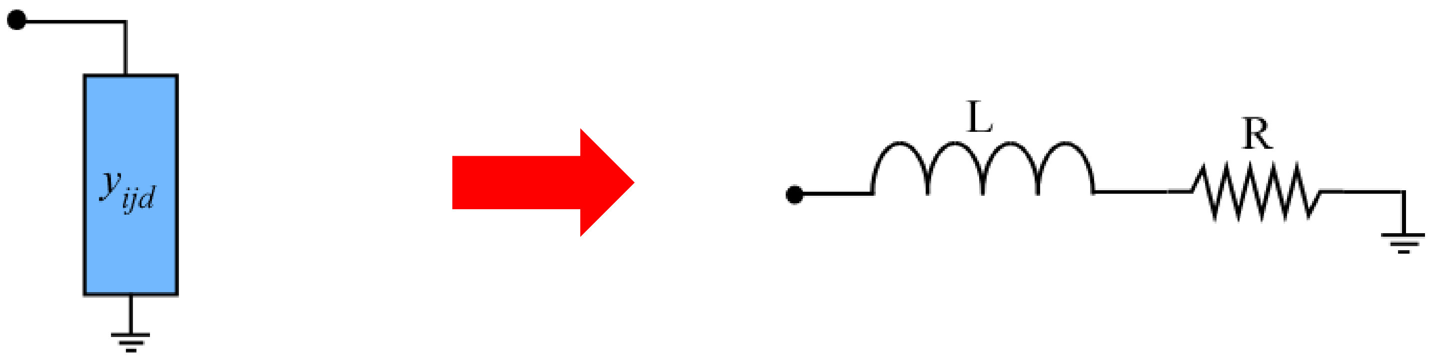
$$y_{ijk}(s) = \frac{a_k}{s - p_k}$$

(b) Complex conjugate pair of poles

$$y_{ijk}(s) = \frac{\alpha_k + j\beta_k}{s - \sigma_k - j\omega_k} + \frac{\alpha_k - j\beta_k}{s - \sigma_k + j\omega_k}$$

In all cases, we must find an equivalent circuit consisting of lumped elements that will exhibit the same behavior

Case (a) - Real Pole



Consider the circuit shown above. The input impedance *Z* as a function of the complex frequency *s* can be expressed as:

from circuit | *from pole and residue*

$$Z = sL + R \qquad Y(s) = \frac{1/L}{s + R/L} \qquad \hat{Y}(s) = \frac{a_k}{s - p_k}$$

Comparing *Y(s)* and $\hat{Y}(s)$ yields the solution

$$L = 1/a_k \qquad R = -p_k/a_k$$

Circuit Realization - Complex Poles

Each term associated with a complex pole pair in the expansion gives:

$$\hat{Y} = \frac{r_1}{s - p_1} + \frac{r_2}{s - p_2}$$

Where r_1, r_2, p_1 **and** p_2 **are the complex residues and poles. They satisfy:** $r_1 = r_2^*$ **and** $p_1 = p_2^*$

It can be re-arranged as:

$$\hat{Y} = \left(r_1 + r_2\right) \frac{\left[s - \left(r_1 p_2 + r_2 p_1\right) / \left(r_1 + r_2\right)\right]}{s^2 - s\left(p_1 + p_2\right) + p_1 p_2}$$

Circuit Realization - Complex Poles

The pole/residue representation of the Y parameters is given by:

$$\hat{Y} = \left(r_1 + r_2\right)\frac{\left[s - \left(r_1 p_2 + r_2 p_1\right)/\left(r_1 + r_2\right)\right]}{s^2 - s\left(p_1 + p_2\right) + p_1 p_2}$$

DEFINE

$p = p_1 p_2$ product of poles

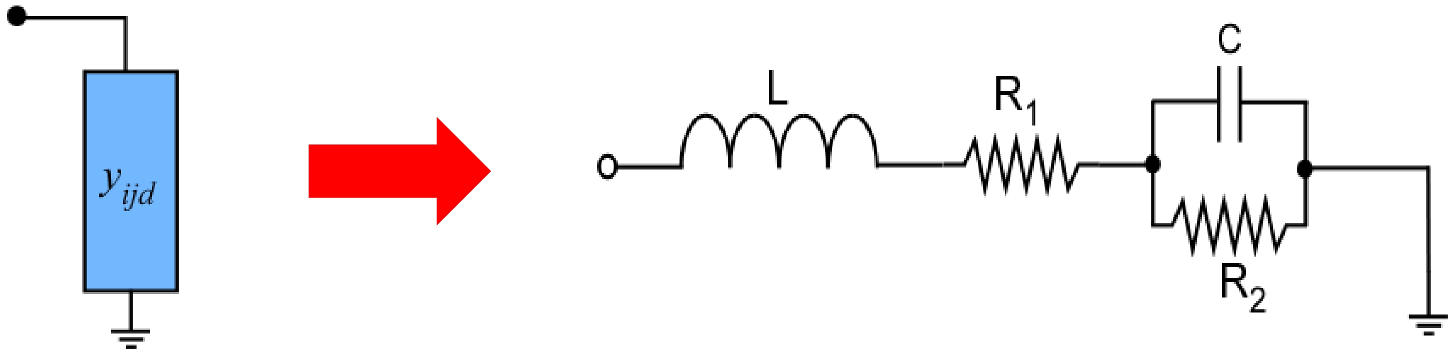
$a = r_1 + r_2$ sum of residues

$g = p_1 + p_2$ sum of poles

$x = r_1 p_2 + r_2 p_1$ cross product

$$\hat{Y} = a\frac{\left[s - x/a\right]}{s^2 - sg + p}$$

Circuit Realization - Complex Poles

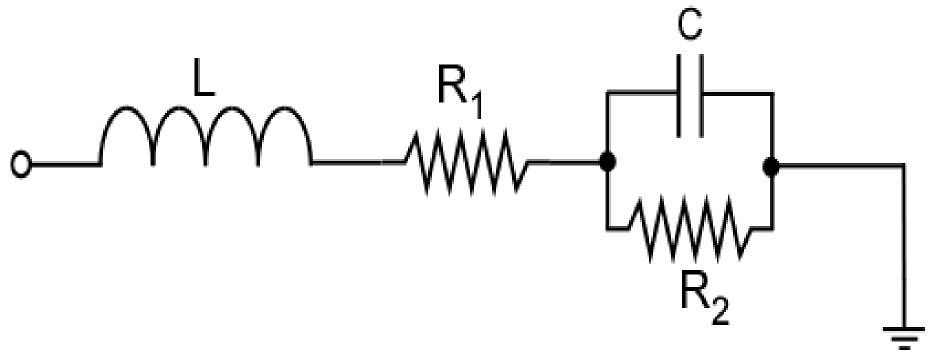


Consider the circuit shown above. The input impedance *Z* as a function of the complex frequency *s* can be expressed as:

$$Z = sL + R_1 + \frac{1}{1/R_2 + sC} = sL + R_1 + \frac{R_2}{1 + sCR_2}$$

$$Z = \frac{(R_1 + sL)(1 + sCR_2) + R_2}{1 + sCR_2}$$

Circuit Realization - Complex Poles



$$Y = \frac{CR_2\left(s + 1/CR_2\right)}{LCR_2\left[s^2 + s\left(\frac{L + CR_1R_2}{LR_2C}\right) + \frac{\left(R_1 + R_2\right)}{LR_2C}\right]}$$

$$Y = \frac{1}{L}\frac{\left(s + 1/CR_2\right)}{\left[s^2 + s\left(\frac{L + CR_1R_2}{LR_2C}\right) + \frac{\left(R_1 + R_2\right)}{LR_2C}\right]}$$

Circuit Realization - Complex Poles

Comparing

$$Y = \frac{1}{L}\frac{\left(s + 1/CR_2\right)}{\left[s^2 + s\left(\frac{L + CR_1R_2}{LR_2C}\right) + \frac{\left(R_1 + R_2\right)}{LR_2C}\right]}$$

with

$$\hat{Y} = a\frac{\left[s - x/a\right]}{s^2 - sg + p}$$

We can identify the circuit elements

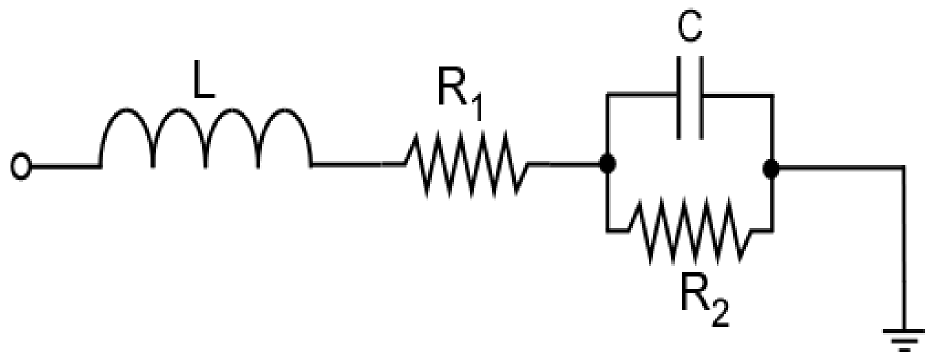
$$L = 1/a$$

$$R_1 = \frac{x}{a^2} - \frac{g}{a}$$

$$R_2 = -\frac{p}{x} - \frac{x}{a^2} + \frac{g}{a}$$

$$C = \frac{pa}{x^2} + \frac{1}{a} - \frac{g}{x}$$

Circuit Realization - Complex Poles



$$L = 1/a$$

$$R_1 = \frac{x}{a^2} - \frac{g}{a}$$

$$R_2 = -\frac{p}{x} - \frac{x}{a^2} + \frac{g}{a}$$

$$C = \frac{pa}{x^2} + \frac{1}{a} - \frac{g}{x}$$

Negative Elements

In the circuit synthesis process, it is possible that some circuit elements come as negative. To prevent this situation, we add a contribution to the real parts of the residues of the system. In the case of a complex residue, for instance, assume that

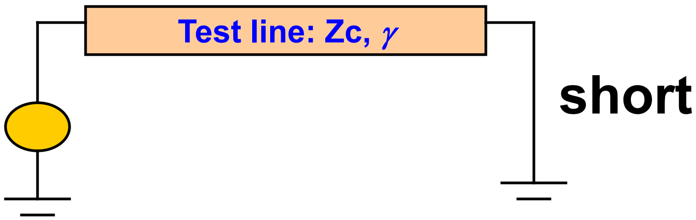
$$\hat{Y} = \frac{r_1}{s - p_1} + \frac{r_2}{s - p_2}$$

$$\hat{Y} = \underbrace{\frac{r_1 + \Delta}{s - p_1} + \frac{r_2 + \Delta}{s - p_2}}_{Augmented\ Circuit} - \underbrace{\left(\frac{\Delta}{s - p_1} + \frac{\Delta}{s - p_2} \right)}_{Compensation\ Circuit}$$

Can show that both augmented and compensation circuits will have positive elements

Why S Parameters?

Y-Parameter

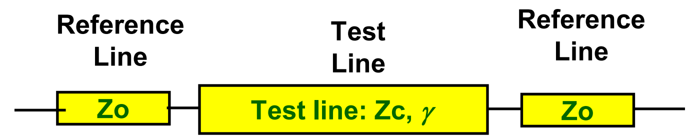


$$Y_{11} = \frac{1 + e^{-2\gamma l}}{Z_c (1 - e^{-2\gamma l})}$$

Z_c : microstrip characteristic impedance
γ : complex propagation constant
l : length of microstrip

Y_{11} can be unstable

S-Parameter



$$S_{11} = \frac{(1 - e^{-2\gamma l})\Gamma}{1 - \Gamma^2 e^{-2\gamma l}}$$

$$\Gamma = \frac{Z_c - Z_o}{Z_c + Z_o}$$

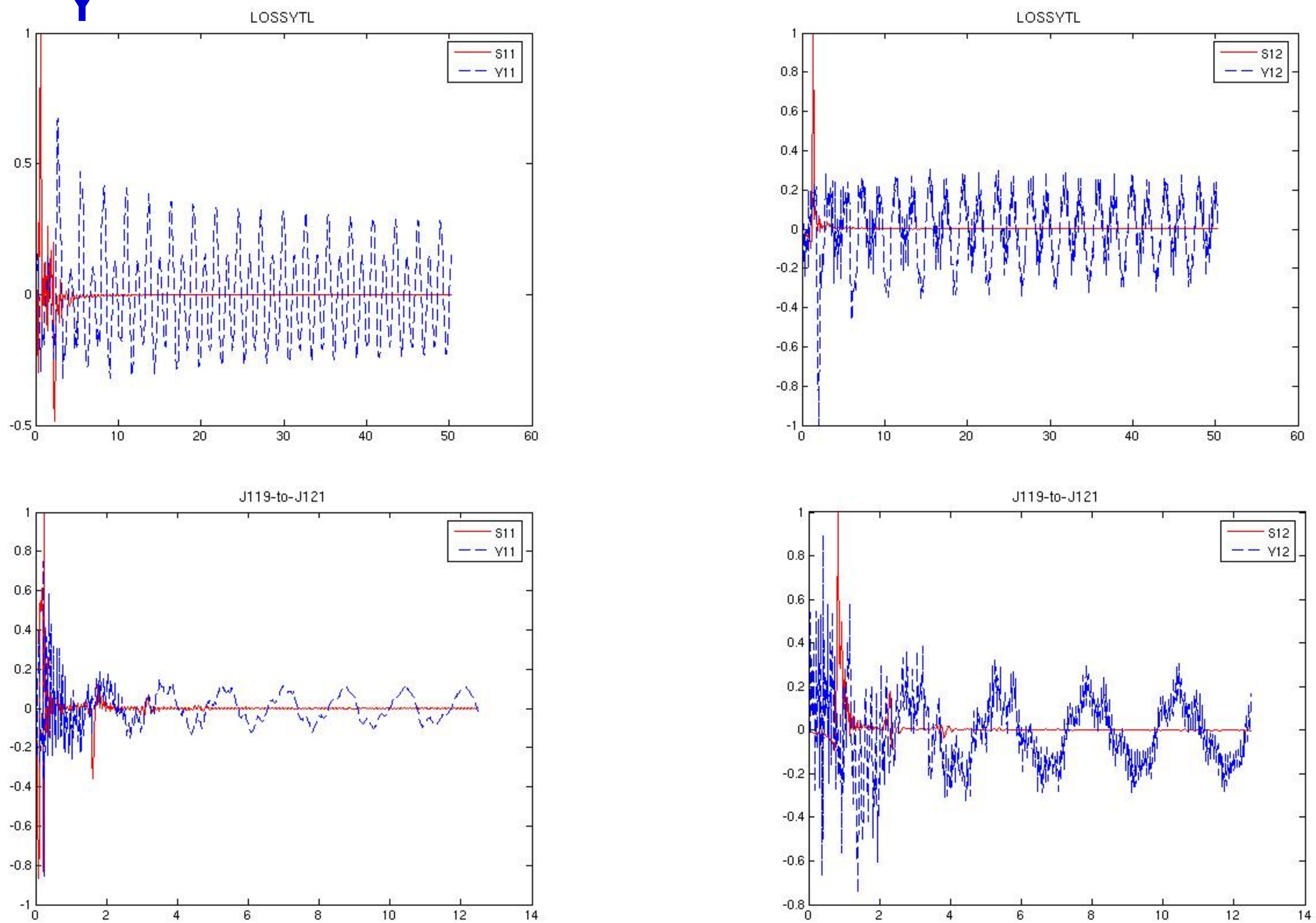
S_{11} is always stable

Y Versus S Parameters

S

Y

Impulse Responses



Observation: S-parameters decay rapidly; Y parameters do not.

Optimizing Reference System

$$S = \left[ZZ_o^{-1} + I \right]^{-1} \left[ZZ_o^{-1} - I \right]$$

$$Z = [I + S][I - S]^{-1} Z_o$$

using

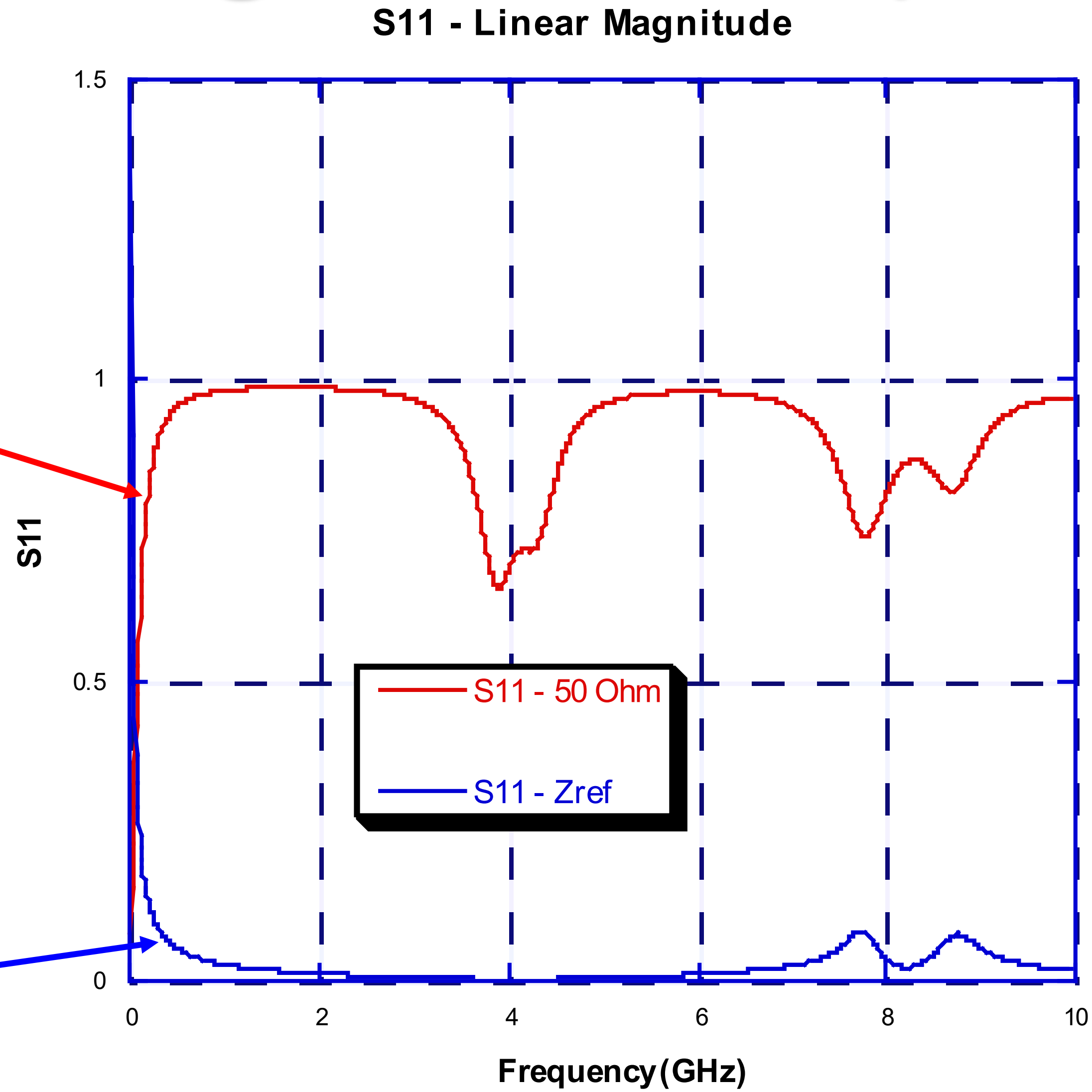
$$Z_o = \begin{matrix} 50.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 50.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 50.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 50.0 \end{matrix}$$

as reference…

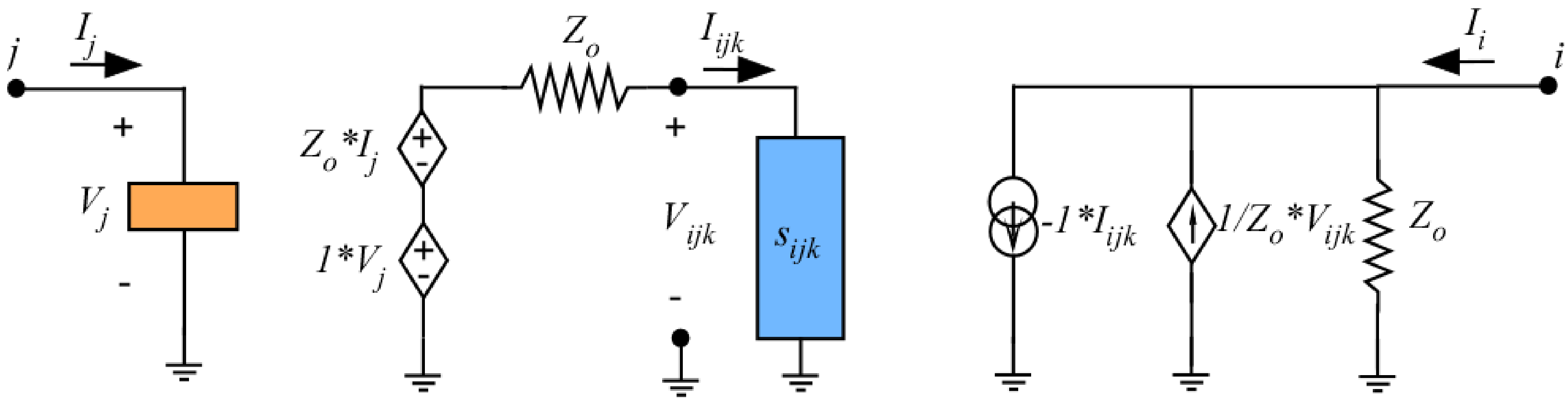
using

$$Z_o = \begin{matrix} 328.0 & 69.6 & 328.9 & 69.6 \\ 69.6 & 328.8 & 69.6 & 328.9 \\ 328.9 & 69.6 & 328.8 & 69.6 \\ 69.6 & 328.9 & 69.6 & 328.8 \end{matrix}$$

as reference…



Method 2: S-Parameter /MOR



$$A_i(\omega) = \frac{1}{2}\left[V_i(\omega) + Z_o I_i(\omega)\right]$$

$$B_i(\omega) = \frac{1}{2}\left[V_i(\omega) - Z_o I_i(\omega)\right]$$

Need equivalent circuit for S_{ijk}

All S parameters are treated as if they were <u>one-port</u> S parameters

Strategy

For a given circuit, a relationship between the input admittance $Y_{ijk}(s)$ of the circuit and the associated one-port S-parameter representation $S_{ijk}(s)$ can be described by

$$S_{ijk}(s) = \frac{Y_o - Y_{ijk}(s)}{Y_o + Y_{ijk}(s)} \qquad Y_{ijk}(s) = Y_o \frac{1 - S_{ijk}(s)}{1 + S_{ijk}(s)}$$

Y_o is the reference admittance

Equivalent-Circuit Extraction

Macromodel is curve-fit to take the form

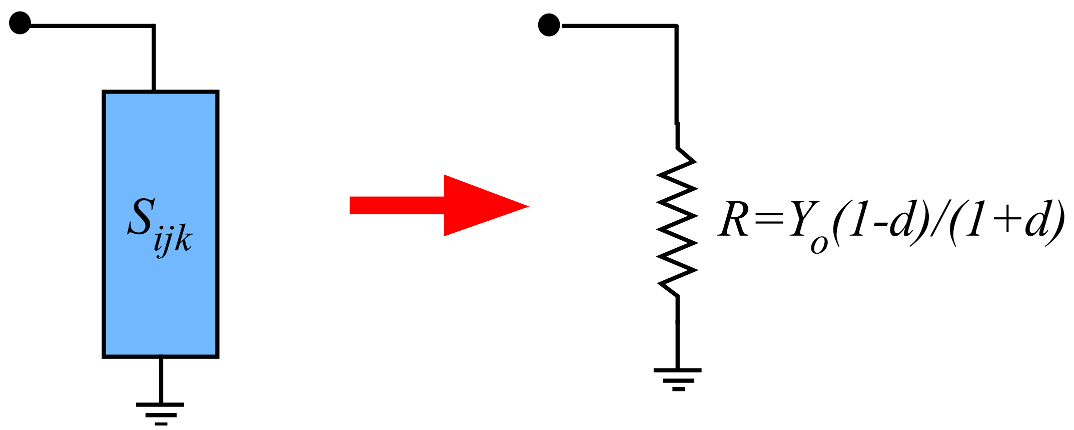
$$S(s) = d + \sum_{k=1}^{L} \frac{r_k}{s - p_k}$$

Need to find equivalent circuit associated with

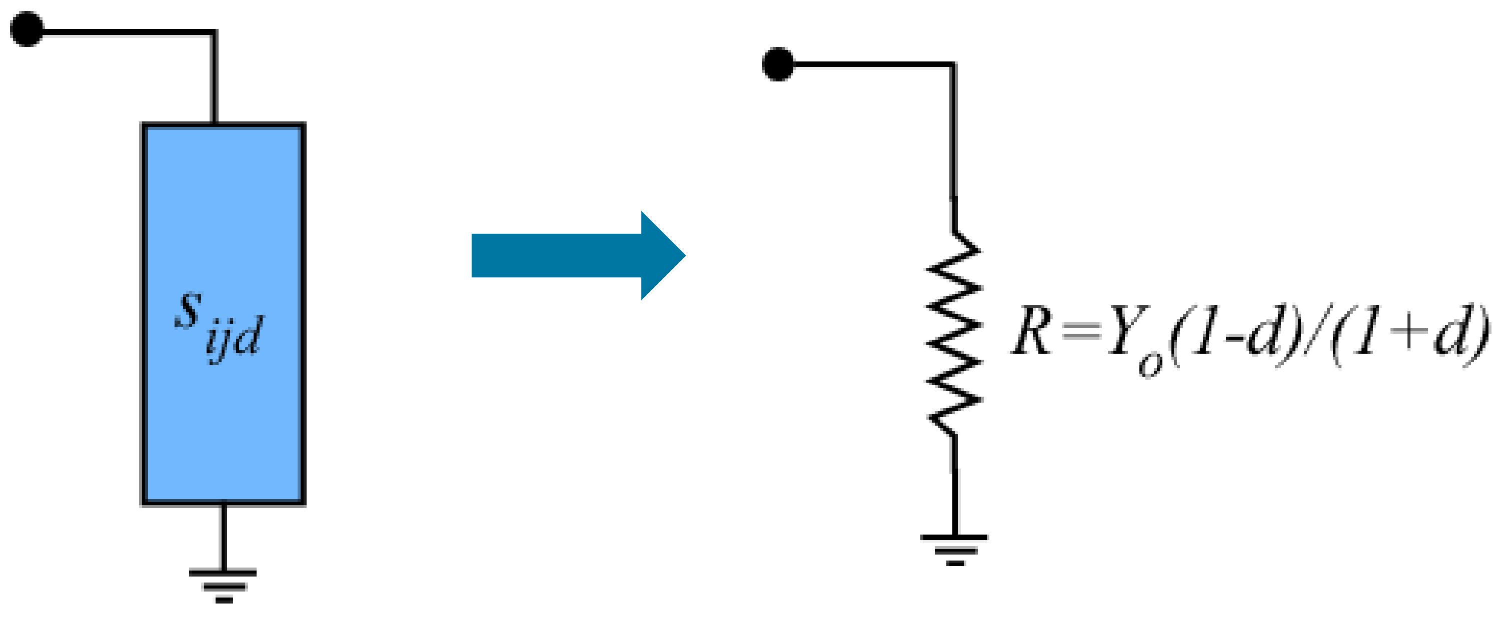
- Constant term d
- Real Poles
- Complex Poles

Constant Term

$$R = \left(\frac{1 - S_{ijk}}{1 + S_{ijk}}\right) Y_o = \left(\frac{1 - d}{1 + d}\right) Y_o$$



S- Circuit Realization – Constant Term



$$R = Y_o\left(\frac{1-d}{1+d}\right)$$

S-Parameters - Poles and Residues

In the pole-residue case, we must distinguish two cases

(a) Pole is real

$$s_{ijk}(s) = \frac{a_k}{s - p_k}$$

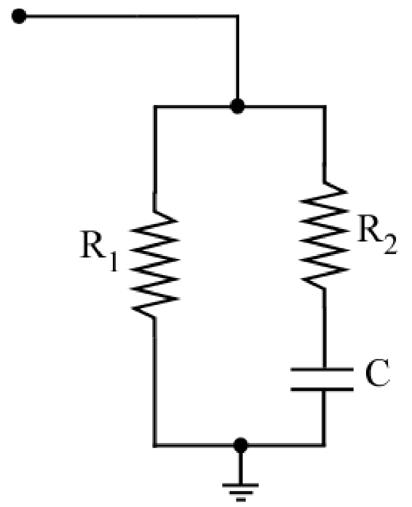
(b) Complex conjugate pair of poles

$$s_{ijk}(s) = \frac{\alpha_k + j\beta_k}{s - \sigma_k - j\omega_k} + \frac{\alpha_k - j\beta_k}{s - \sigma_k + j\omega_k}$$

In all cases, we must find an equivalent circuit consisting of lumped elements that will exhibit the same behavior

S-Realization – Real Poles

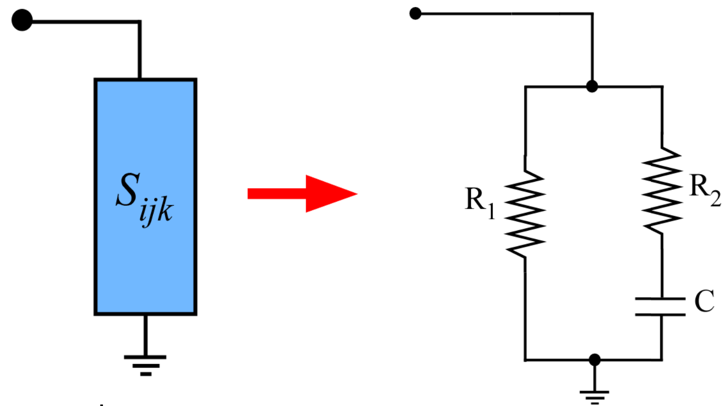
Proposed Circuit Model



Admittance of proposed model is given by:

$$Y = \frac{(R_1 + R_2)}{R_1 R_2} \left[\frac{s + \dfrac{1}{(R_1 + R_2) C}}{s + \dfrac{1}{R_2 C}} \right]$$

Real Poles



from circuit

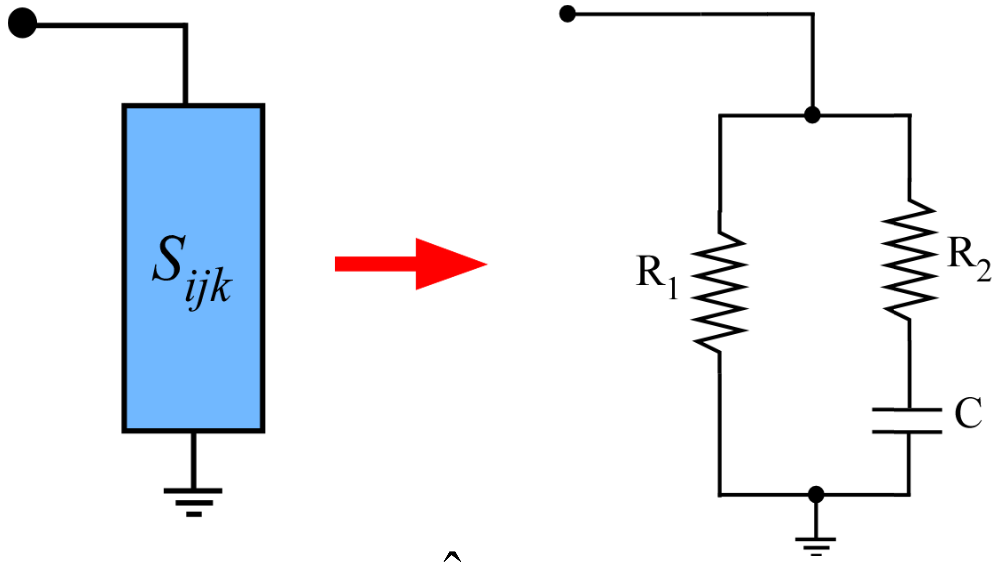
$$Y = \frac{\left(R_1 + R_2\right)}{R_1 R_2}\left[\frac{s + \dfrac{1}{\left(R_1 + R_2\right)C}}{s + \dfrac{1}{R_2 C}}\right]$$

$$S_{ijk}(s) = \frac{a_k}{s - p_k}$$

$$\hat{Y} = Y_o\left(\frac{1 - S_{ijk}}{1 + S_{ijk}}\right) = Y_o\left(\frac{1 - \dfrac{r}{s-p}}{1 + \dfrac{r}{s-p}}\right) = Y_o\left(\frac{s-a}{s+a}\right)$$

from pole and residue

Solution for Real Poles



Comparing $\hat{Y}(s)$ *with* $Y(s)$ *gives*

$$C = -\frac{(b-a)}{b^2 Z_o}$$

$$R_2 = \frac{-1}{bC}$$

$$R_1 = -R_2 - \frac{1}{aC}$$

where $a = p_k + r_k$, and $b = p_k - r_k$

Realization – Complex Poles

From the S-parameter expansion, the complex pole pair gives:

$$\hat{S} = \frac{r_1}{s-p_1} - \frac{r_2}{s-p_2} = \frac{s\left(r_1+r_2\right)-\left(r_1 p_2 + r_2 p_1\right)}{s^2 - s\left(p_1+p_2\right)+p_1 p_2}$$

which corresponds to an admittance of:

$$\hat{Y} = Y_o\left(\frac{1-\hat{S}}{1+\hat{S}}\right) = \left(\frac{1-\dfrac{sa-x}{s^2-sg+p}}{1+\dfrac{sa-x}{s^2-sg+p}}\right)Y_o$$

Realization – Complex Poles

The admittance expression can be re-arranged as

$$\hat{Y} = \left(\frac{s^2 - sg + p - sa + x}{s^2 - sg + p + sa - x} \right) Y_o = \left(\frac{s^2 - s(g+a) + p + x}{s^2 - s(g-a) + p - x} \right) Y_o$$

WE HAD DEFINED

$p = p_1 p_2$ **product of poles**

$a = r_1 + r_2$ **sum of residues**

$g = p_1 + p_2$ **sum of poles**

$x = r_1 p_2 + r_2 p_1$ **cross product**

Realization – Complex Poles

$$\hat{Y} = \left(\frac{s^2 - sg + p - sa + x}{s^2 - sg + p + sa - x} \right) Y_o = \left(\frac{s^2 - s(g+a) + p + x}{s^2 - s(g-a) + p - x} \right) Y_o$$

This can be further rearranged as

$$\hat{Y} = \left(\frac{s^2 + sA + B}{s^2 + sD + F} \right) E$$

in which

$$A = -(g+a),\ \ B = p + x,\ \ D = -(g-a),\ \ F = p - x$$

and $\quad E = Y_o = \dfrac{1}{H}$

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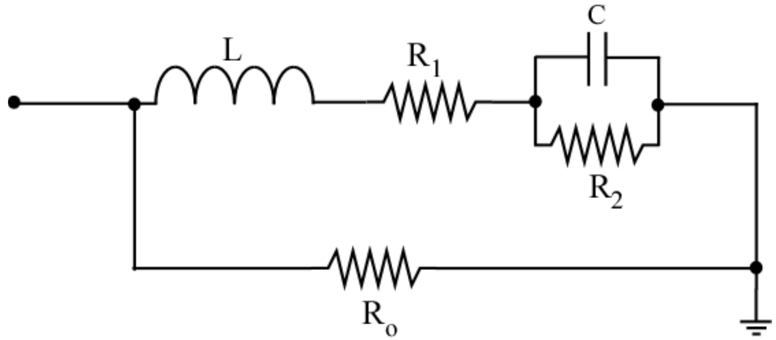
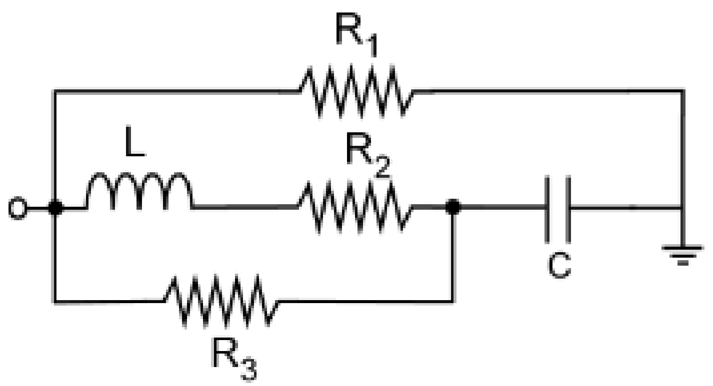
Realization – Complex Poles

There are several circuit topologies that will work

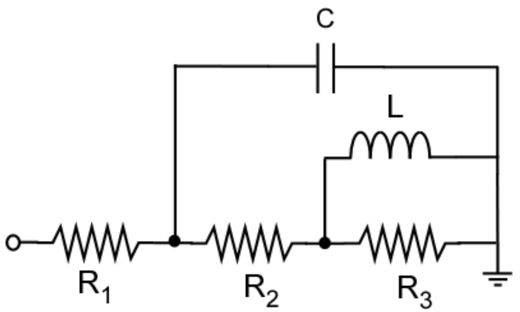
Model 1



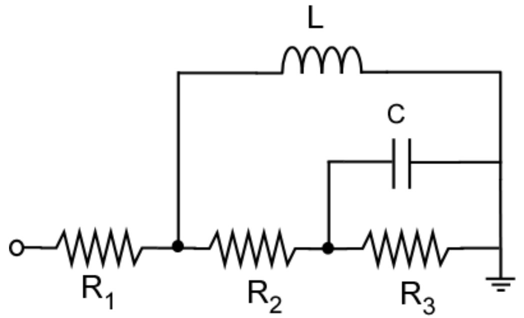
Model 9



Model 13



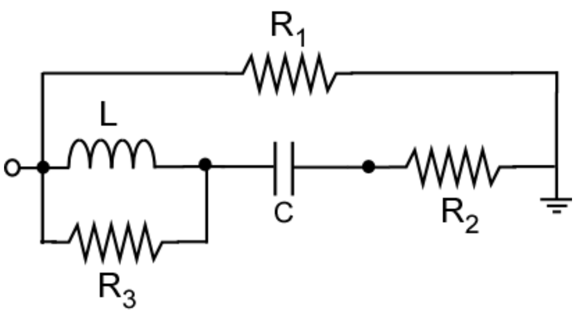
Model 12



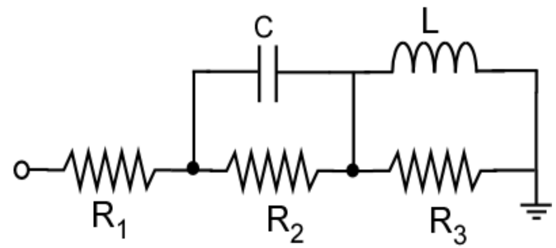
Realization – Complex Poles

More circuit topologies that will work

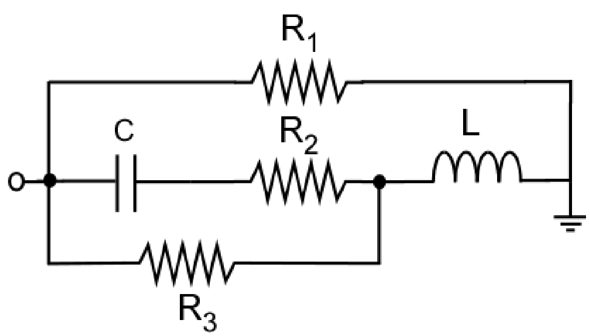
Model 10



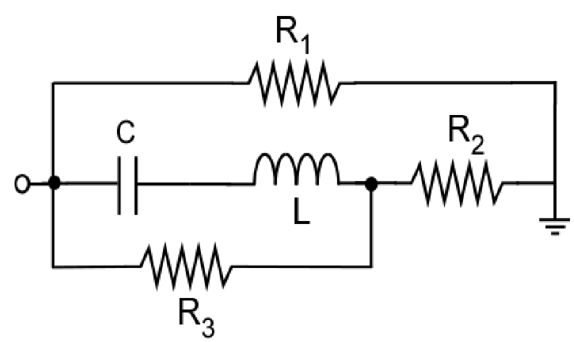
Model 11



Model 8

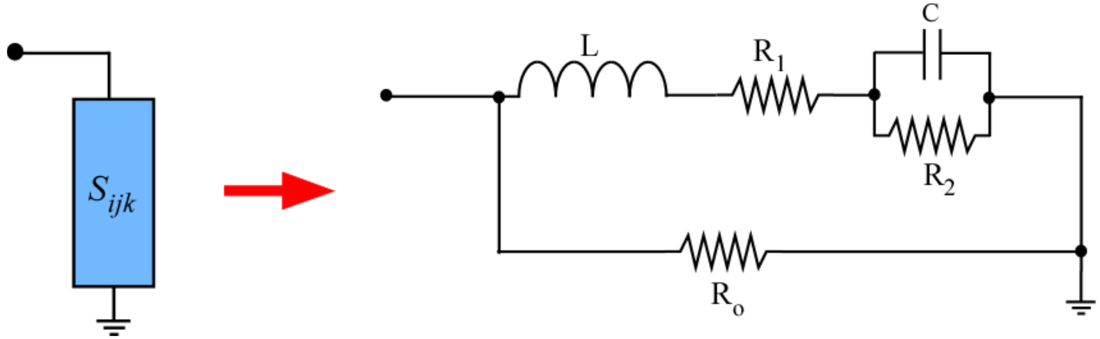


Model 12



Complex Poles – Model 1

In particular, if we choose model 1



$$Y = \frac{1}{R_o}\left[\frac{s^2 + s\left(\dfrac{L + R_1R_2C + R_oR_2C}{LCR_2}\right) + \dfrac{R_o + R_1 + R_2}{LCR_2}}{s^2 + s\left(\dfrac{L + R_1R_2C}{LCR_2}\right)_o + \dfrac{R_1 + R_2}{LCR_2}}\right]$$

Must be matched with

$$\hat{Y} = Y_o\left(\frac{1 - S_{ijk}}{1 + S_{ijk}}\right) = \left(\frac{1 - \dfrac{sa - x}{s^2 - sg + p}}{1 + \dfrac{sa - x}{s^2 - sg + p}}\right)Y_o$$

Complex Poles – Model 1

Matching the terms with like coefficients gives

$$R_o = \frac{1}{Y_o}$$

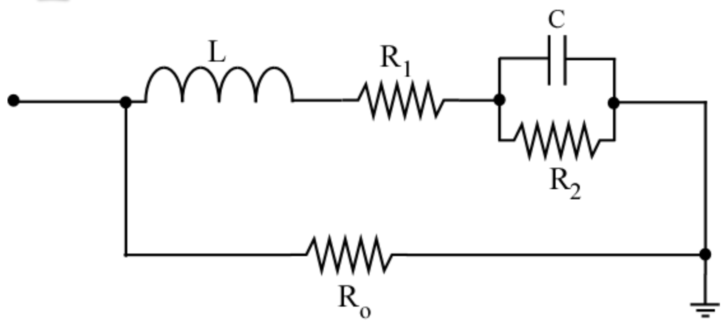
$$p + x = \frac{R_o + R_1 + R_2}{LCR_2}$$

$$p - x = \frac{R_1 + R_2}{LCR_2}$$

$$2p = \frac{R_o + 2R_1 + 2R_2}{LCR_2}$$

$$2x = \frac{R_o}{LCR_2}$$

Complex Poles – Model 1



Solving gives

$$L = -\frac{R_o}{2a}$$

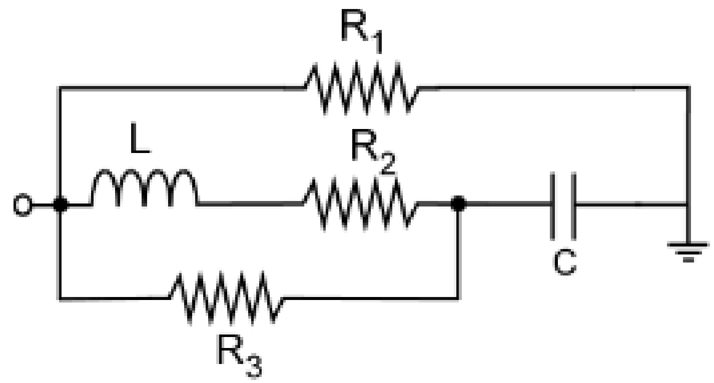
$$R_2 = \frac{R_o\left(\frac{p}{x} - 1\right)}{2} - R_1$$

$$R_1 = \frac{1}{2}\left(\frac{gR_o}{a} + \frac{2Lx}{a} - R_o\right)$$

$$C = \frac{-a}{R_2 x}$$

Complex Poles – Model 9

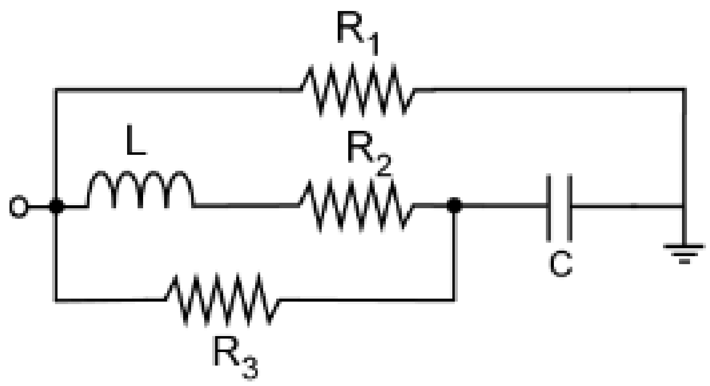
If we choose Model 9



$$Y_9 = \overbrace{\frac{R_1 + R_3}{R_1 R_3}}^{E} \left[\frac{s^2 + s\overbrace{\frac{L + CR_2R_3 + CR_1\left(R_2 + R_3\right)}{LC\left(R_1 + R_3\right)}}^{A} + \overbrace{\frac{R_2 + R_3}{LC\left(R_1 + R_3\right)}}^{B}}{s^2 + s\underbrace{\frac{CR_1R_2R_3 + LR_1}{LCR_1R_3}}_{D} + \underbrace{\frac{R_1\left(R_2 + R_3\right)}{LCR_1R_3}}_{F}} \right]$$

from which the circuit elements can be extracted

Complex Poles – Model 9



$$R_1 = \frac{F}{BH}$$

$$C = \frac{\left(-BD + AF\right)H}{F^2}$$

$$R_2 = \frac{\left(-BD^2 + BF + ADF - F^2\right)}{\left(B^2 - ABD + BD^2 + A^2F - 2BF - ADF + F^2\right)H}$$

$$R_3 = \frac{F}{\left(-B + F\right)H}$$

$$L = \frac{-BD + AF}{\left(B^2 - ABD + BD^2 + A^2F - 2BF - ADF + F^2\right)H}$$

Special Case – Model 4

A special case exists when $x=0$

or $x = r_1 p_2 + r_2 p_1 = 0$

$$\hat{Y} = \left(\frac{s^2 - s(g+a) + p + x}{s^2 - s(g-a) + p - x} \right) Y_o = \left(\frac{s^2 - s(g+a) + p}{s^2 - s(g-a) + p} \right) Y_o$$

$$\hat{Y} = \left(\frac{s^2 + sA + B}{s^2 + sD + B} \right) E$$

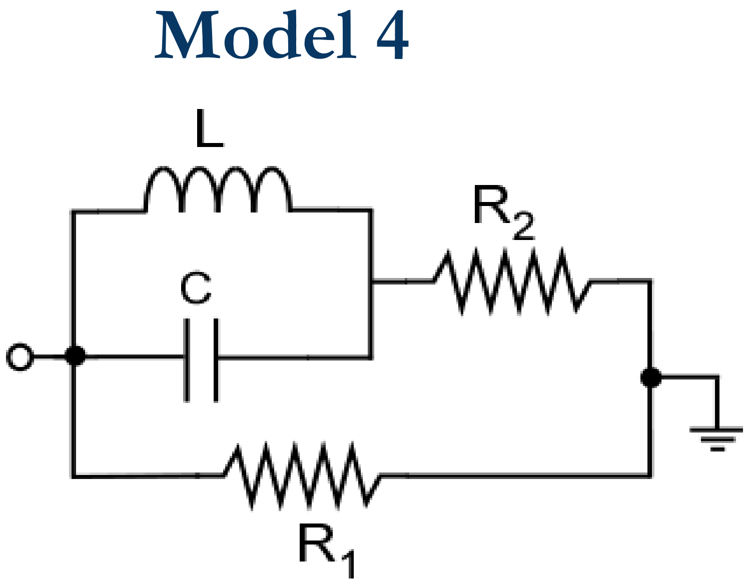
in which

$$A = -(g+a), \;\; B = p + x, \;\; D = -(g-a), \;\; F = p$$

and $E = Y_o = \dfrac{1}{H}$

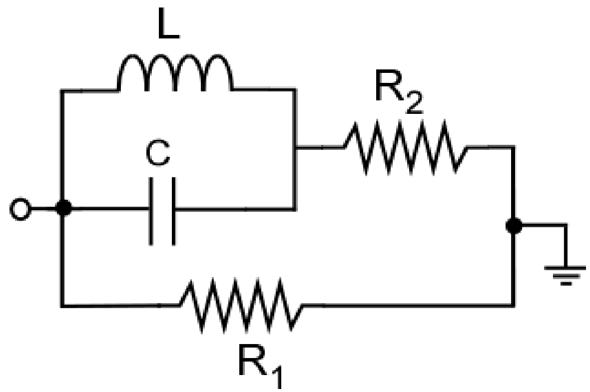
Special Case – Model 4

The proposed circuit model is



This model has one inductor, one capacitor and two resistors.

Special Case – Model 4



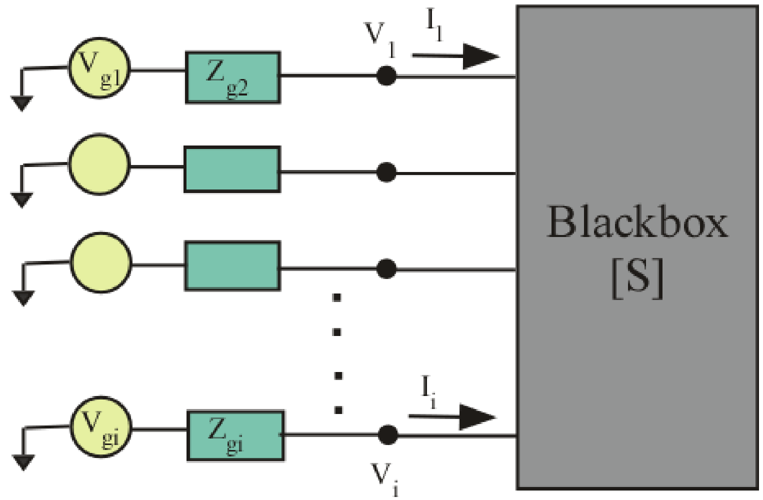
The admittance seen at the input is:

$$Y_4 = \frac{\overbrace{R_1 + R_2}^{E}}{R_1 R_2}\left[\frac{s^2 + s\overbrace{\frac{1}{C\left(R_1 + R_2\right)}}^{A} + \overbrace{\frac{1}{LC}}^{B=F}}{s^2 + s\underbrace{\frac{1}{CR_2}}_{D} + \underbrace{\frac{1}{LC}}_{F=B}}\right]$$

The element are extracted as

$$R_1 = \frac{DE}{A} \qquad R_2 = \frac{R_1 E}{R_1 - E} \qquad C = \frac{1}{DR_2} \qquad L = \frac{1}{FC}$$

Method 3 - Convolution with S Parameters

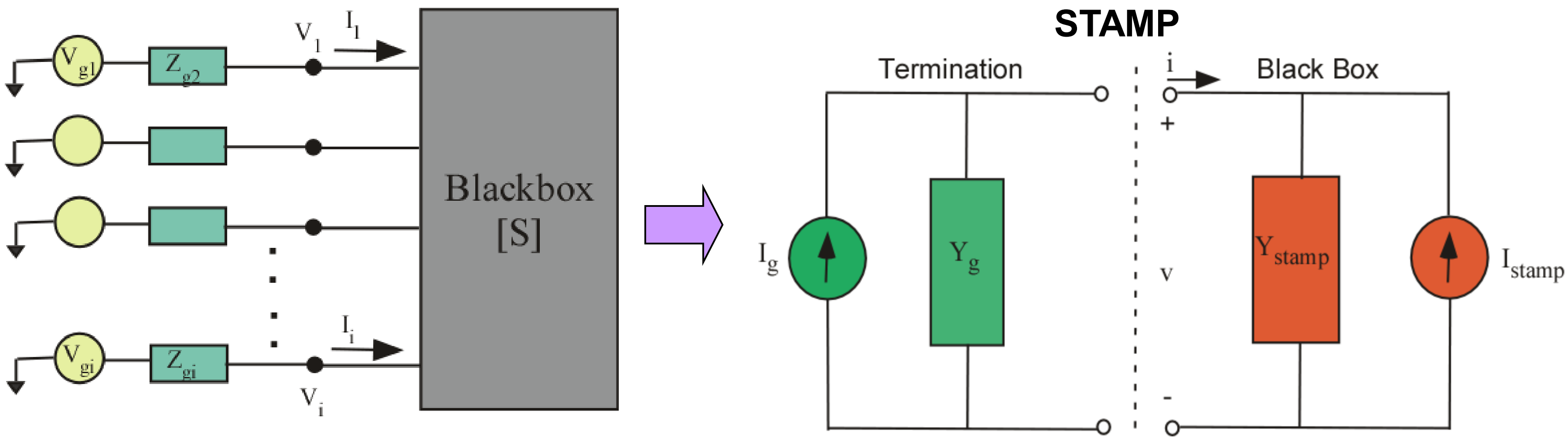


In frequency domain $B=SA$

In time domain $b(t) = s(t)*a(t)$

Convolution: $$s(t)*a(t)=\int_{-\infty}^{\infty} s(t-\tau)a(\tau)d\tau$$

SPICE Macromodel



$$Y_{stamp} = Z_o^{-1}\left[1+s'(0)\right]^{-1}\left[1-s'(0)\right]$$

$$I_{stamp} = 2Z_o^{-1}\left[1+s'(0)\right]^{-1} H(t)$$

Most of the computational effort is in the convolution calculation of the history *H(t)* at each time step.

$$H(t) = \sum_{\tau=1}^{t-1} s(\tau)a(t-\tau)\Delta\tau$$

This operation can be accelerated

We make the convolution faster using a δ-function expansion for *s(t)*

S-Parameter Expansion

In the frequency domain, assume that S parameter can be expanded in the form:

$$S(l) = \sum_{k=1}^{M} c_k e^{j2\pi lk}$$

In the time domain, this corresponds to:

$$s(p) = \sum_{k=1}^{M} c_k \delta(p-k)$$

Which is an impulse train of order *M*. Convolution will then give:

$$y(p) = s(p) * x(p) = \left[\sum_{k=1}^{M} c_k \delta(p-k) \right] * x(p) = \sum_{k=1}^{M} c_k x(p-k)$$

If we truncate the summation to the *Q* largest impulses:

$$y(p) = \left[\sum_{k=1}^{Q} c_k \delta(p-k) \right] * x(p) = \sum_{k=1}^{Q} c_k x(p-k)$$

***If the reference system is optimized*, most of the coefficients c_k will be negligibly small; therefore $\Rightarrow$ *Q* will be a relatively small number**

Fast Convolution is achieved by making Q small

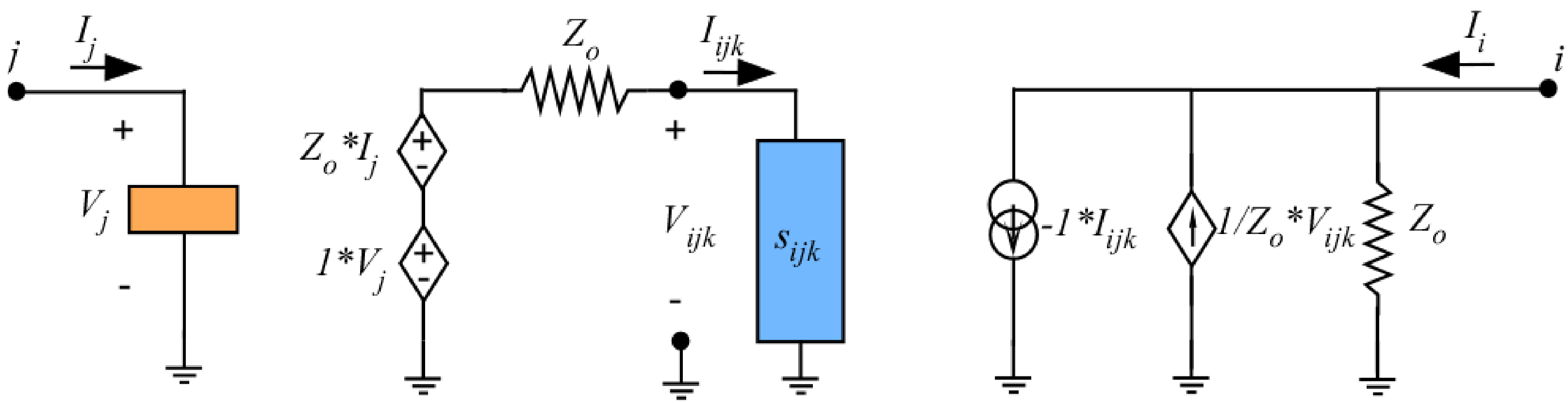
Strategy

$$s_{ijk}(u) = c_{ijk}\delta(u-k)$$

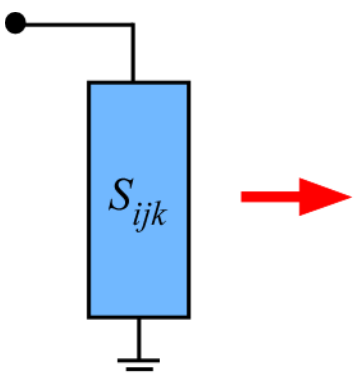
Since the S-parameter is a simple delay, the one-port circuit representation is a transmission line with characteristic impedance Z_o and delay k

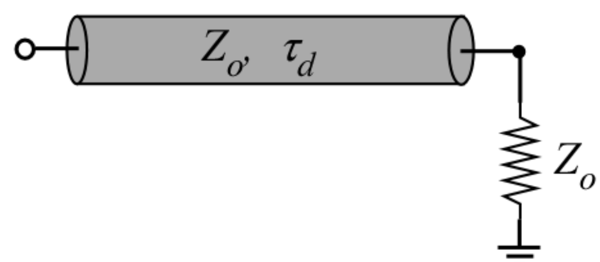
Circuit Interpretation

$$s_{ijk}(u) = c_{ijk}\delta(u-k)$$

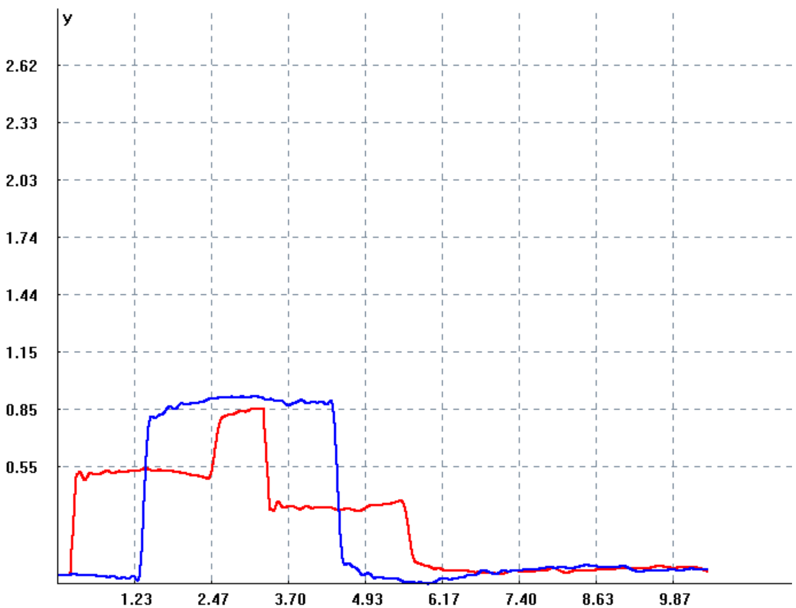


$$\tau_d = k\Delta t \qquad \Delta t = \text{time step}$$

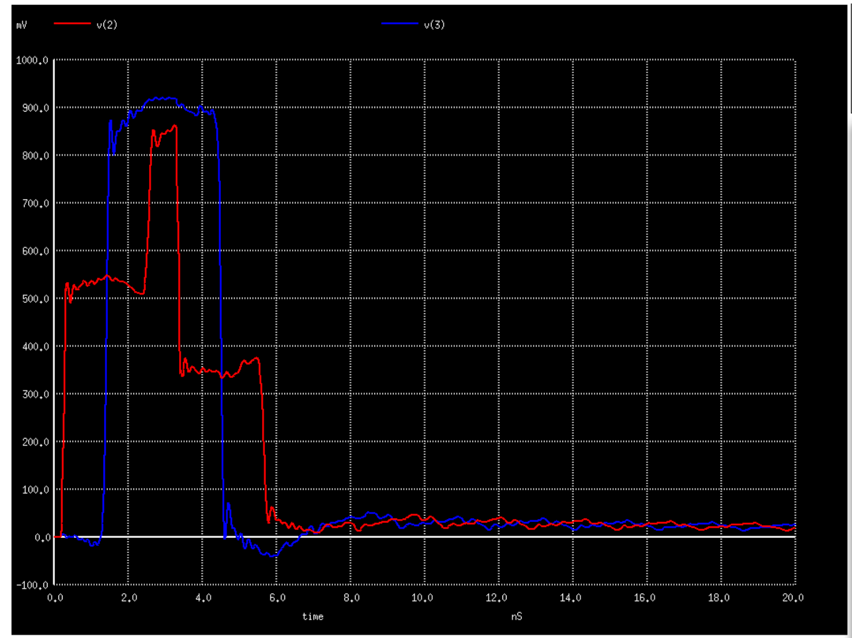




Method 1: Y-Parameters/MOR

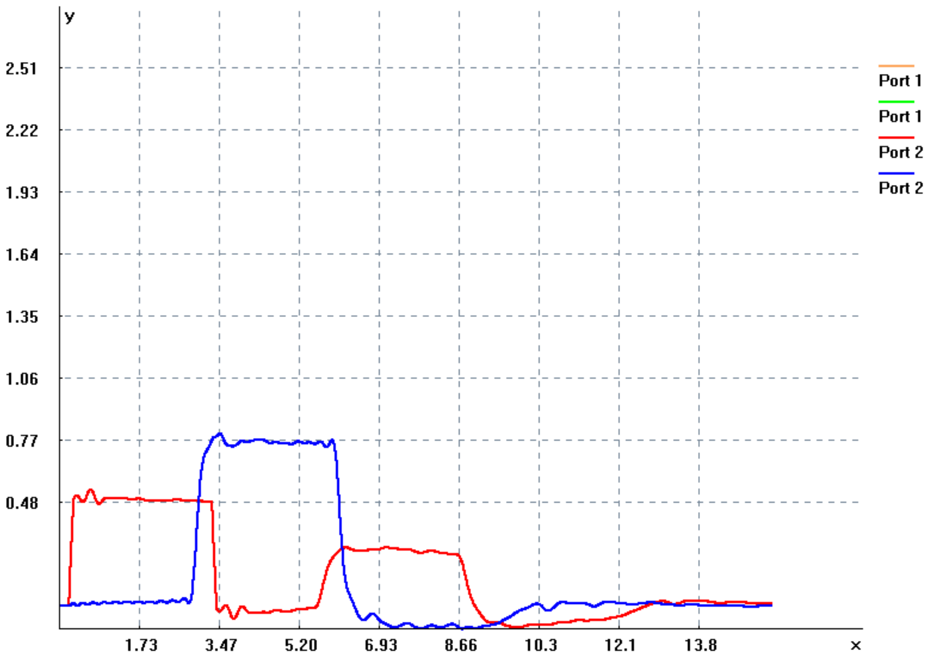


Recursive convolution

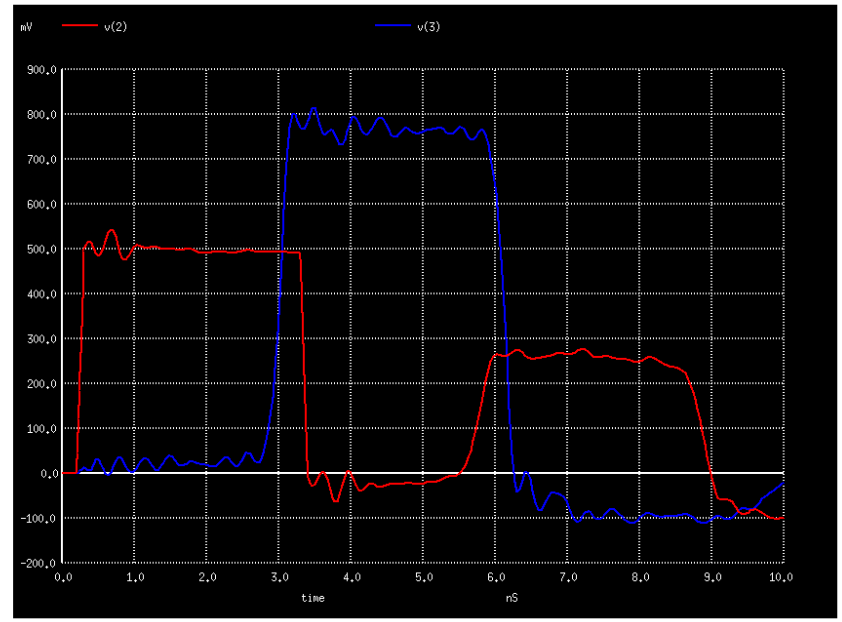


SPICE realization

Method 2: S-Parameters/MOR



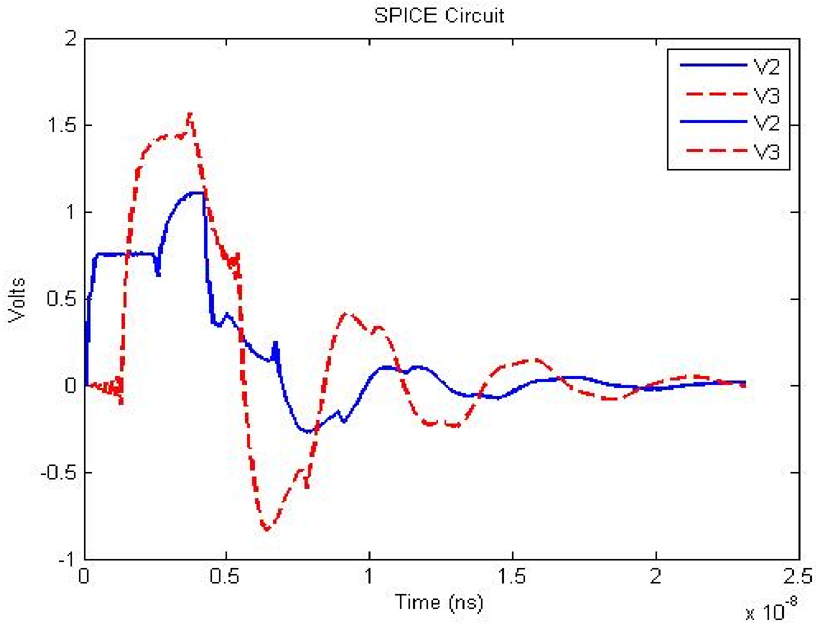
Recursive convolution



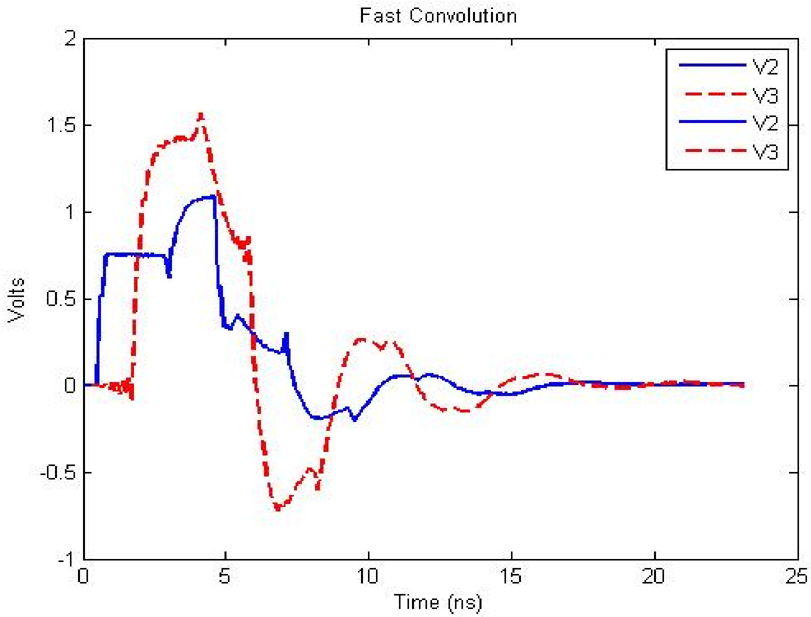
SPICE realization

Comparisons

SPICE simulation from MOR generated Netlist (Method 2)

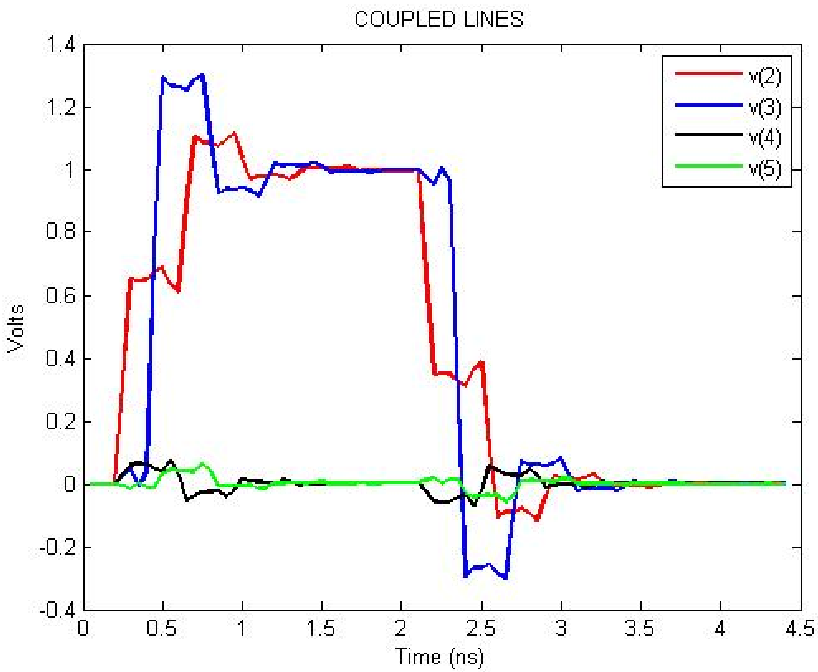


Direct Convolution

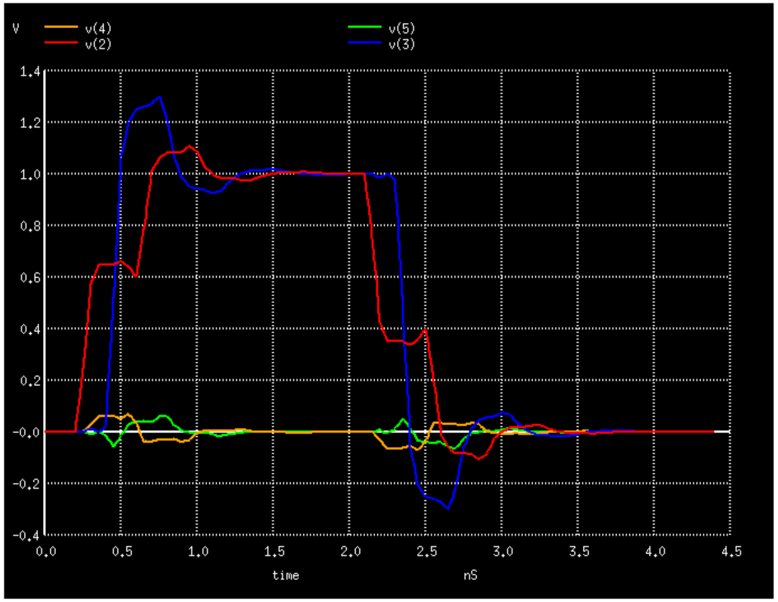


Coupled Lines
(4-port)

Direct

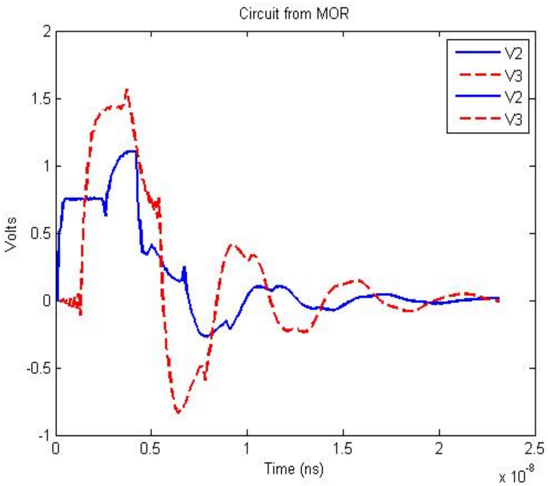


SPICE simulation
Using generated netlist
(Method 2)

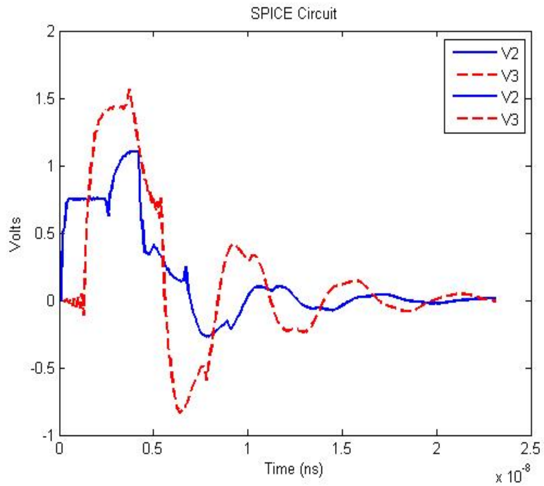


Comparison of S-methods

SPICE Netlist from MOR with S (Method 2)



SPICE Netlist from Fast convolution with S (Method 3)



Convolution (no SPICE netlist)