ECE 546
Lecture -17
Latency Insertion Method
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Jose E. Schutt-Aine
Electrical & Computer Engineering
University of Illinois
jesa@illinois.edu
Challenges in Integration

Packaging Complexity
- Up to 16 layers
- Hundreds of vias
- Thousands of TLs
- High density
- Nonuniformity

Chip Complexity
Vertical parallel-plate capacitance 0.05 fF/μm²
Vertical parallel-plate capacitance (min width) 0.03 fF/μm
Vertical fringing capacitance (each side) 0.01 fF/μm
Horizontal coupling capacitance (each side) 0.03

TSV Density: 10/cm² - 10⁸/cm²


Typical workstation simulation time for a 1200-cell network is 2 h 40 min.

Too time consuming!
Why LIM?

• MNA has super-linear numerical complexity
• LIM has linear numerical complexity
• LIM has no matrix ill-conditioning problems
• Accuracy and stability in LIM are easily controlled
• LIM is much faster than MNA for large circuits
Latency Insertion Method
Latency Insertion Method

Each branch must have an inductor*
Each node must have a shunt capacitor*
Express branch current in terms of history of adjacent node voltages
Express node voltage in terms of history of adjacent branch currents

* If branch or node has no inductor or capacitor, insert one with very small value
LIM Algorithm

• Represents network as a grid of nodes and branches

\[
\begin{align*}
V_{i}^{n+1/2} & = \frac{C_i V_i^{n-1/2}}{\Delta t} + H_i^n - \sum_{k=1}^{N_a} I_{ik}^n \\
I_{ij}^{n+1} & = I_{ij}^n + \frac{\Delta t}{L_{ij}} \left( V_{i}^{n+1/2} - V_{j}^{n+1/2} - R_{ij} I_{ij}^n \right)
\end{align*}
\]

• Discretizes Kirchhoff's current and voltage equations

• Uses "leapfrog" scheme to solve for node voltages and branch currents

• Presence of reactive elements is required to generate latency
LIM: Leapfrog Method

\[ V_{i}^{n+1/2} = \frac{C_{i}V_{i}^{n-1/2}}{\Delta t} + H_{i}^{n} - \sum_{k=1}^{N_{a}} I_{ik}^{n} \]

\[ I_{ij}^{n+1} = I_{ij}^{n} + \frac{\Delta t}{L_{ij}} \left( V_{i}^{n+1/2} - V_{j}^{n+1/2} - R_{ij}I_{ij}^{n} \right) \]

Leapfrog method achieves second-order accuracy, i.e., error is proportional to \( \Delta t^2 \)
For time=1, $N_t$

For branch =1, $N_b$
Update current as per Equation:

$$I_{ij}^{n+1} = I_{ij}^n + \frac{\Delta t}{L_{ij}} \left( V_{ij}^{n+1/2} - V_{ij}^{n+1/2} - R_{ij} I_{ij}^n + E_{ij}^{n+1/2} \right)$$

Next branch;

For node=1, $N_n$
Update voltage as per Equation

$$V_{ij}^{n+1/2} = \frac{C_i V_{ij}^{n-1/2} + H_{ij}^n - \sum_{k=1}^{M_i} I_{ik}^n - \sum_{q=1}^{N} J_{iq}^n}{\Delta t} + \frac{C_i}{\Delta t} + G_i$$

Next node;
Next time;
## Standalone LIM vs SPICE

### Simulation Times

<table>
<thead>
<tr>
<th>No of Nodes</th>
<th>20,000</th>
<th>30,000</th>
<th>40,000</th>
<th>50,000</th>
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<tr>
<td><strong>SPICE</strong></td>
<td>1224</td>
<td>2935</td>
<td>4741</td>
<td>7358</td>
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<tr>
<td>(sec)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>LIM</strong></td>
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<td>13</td>
<td>17</td>
<td>21</td>
</tr>
<tr>
<td>(sec)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Speedup</strong></td>
<td>136</td>
<td>225</td>
<td>278</td>
<td>350</td>
</tr>
</tbody>
</table>
LIM vs SPICE

Log Size (normalized to 2,000 nodes)

Speedup Factor

LIM v.s. SPICE
LIM: Problem Elements

- Mutual inductance ➔ use reluctance (1/L)
- Branch capacitors ➔ special formulation
- Shunt inductors ➔ special formulation
- Dependent sources ➔ account for dependencies
- Frequency dependence ➔ use macromodels
We wish to determine the current update equations for that branch.

In order to handle the branch capacitor with LIM, we introduce a series inductor $L$. 
LIM Branch capacitor

In the augmented circuit, we have:

\[ I_{ij}^n = C \left( \frac{V_c^{n+1/2} - V_c^{n-1/2}}{\Delta t} \right) \]

where \( V_c \) is the voltage drop across the capacitor; \( V_c = V_o - V_j \). Solving for \( V_c^{n+1/2} \)

\[ V_c^{n+1/2} = V_c^{n-1/2} + \frac{\Delta t}{C} I_{ij}^n \]
LIM Branch capacitor

The voltage drop across the inductor is given by:

\[ V_{L}^{n+1/2} = L \left( \frac{I_{ij}^{n+1} - I_{ij}^{n}}{\Delta t} \right) \]

so that

\[ V_{L}^{n+1/2} = V_{i}^{n+1/2} - V_{j}^{n+1/2} - V_{c}^{n+1/2} = L \left( \frac{I_{ij}^{n+1} - I_{ij}^{n}}{\Delta t} \right) \]

which leads to

\[ I_{ij}^{n+1} = I_{ij}^{n} + \frac{\Delta t}{L} \left( V_{i}^{n+1/2} - V_{j}^{n+1/2} - V_{c}^{n+1/2} \right) \]
LIM Branch capacitor

After substitution and rearrangement, we get:

\[ I_{ij}^{n+1} = I_{ij}^n \left(1 + \frac{\Delta t^2}{LC}\right) + \frac{\Delta t}{L} \left(V_i^{n+1/2} - V_j^{n+1/2} - V_c^{n-1/2}\right) \]

In order to minimize the effect of L while maintaining stability, we see that

- L must be very small

- L must be \( \gg \frac{\Delta t^2}{C} \)

A large branch capacitor helps as well as small time steps.
LIM Branch capacitor

Apply the difference equation directly to branch

\[
\begin{align*}
I_{ij} &= C \frac{dV_c}{dt} \\
V_L &= L_p \frac{dI_{ij}}{dt} \\
V_{c}^{n+1/2} &= \sum_{k=0}^{n} \left( \frac{I_{ij}^k C}{\Delta t} \right) \equiv T_{ij}^n \\
V_{L}^{n+1/2} &= L_p \frac{I_{ij}^{n+1} - I_{ij}^n}{\Delta t} \\
T_{ij}^n &= T_{ij}^{n-1} + \frac{I_{ij}^n C}{\Delta t}
\end{align*}
\]

\[
\begin{align*}
I_{ij}^{n+1} &= I_{ij}^n + \frac{\Delta t}{L_p} V_{ij}^{n+1/2} - \frac{\Delta t}{L_p} T_{ij}^n
\end{align*}
\]
LIM Node Formulations

\[ C_i \left( \frac{V_i^{n+1/2} - V_i^{n-1/2}}{\Delta t} \right) + G_i V_i^{n-1/2} - H_{ij}^n = -\sum_{k=1}^{M_i} I_{ik}^n \]  
Explicit

\[ C_i \left( \frac{V_i^{n+1/2} - V_i^{n-1/2}}{\Delta t} \right) + G_i V_i^{n+1/2} - H_{ij}^n = -\sum_{k=1}^{M_i} I_{ik}^n \]  
Implicit

\[ C_i \left( \frac{V_i^{n+1/2} - V_i^{n-1/2}}{\Delta t} \right) + \frac{G_i}{2} \left( V_i^{n+1/2} + V_i^{n-1/2} \right) - H_{ij}^n = -\sum_{k=1}^{M_i} I_{ik}^n \]  
Semi-Implicit
LIM Branch Formulations

\[ V_{i}^{n+1/2} - V_{j}^{n+1/2} = L_{ij} \left( \frac{I_{ij}^{n+1} - I_{ij}^{n}}{\Delta t} \right) + R_{ij} I_{ij}^{n} - E_{ij}^{n+1/2} \]  \hspace{2cm} \text{Explicit}\\

\[ V_{i}^{n+1/2} - V_{j}^{n+1/2} = L_{ij} \left( \frac{I_{ij}^{n+1} - I_{ij}^{n}}{\Delta t} \right) + R_{ij} I_{ij}^{n+1} - E_{ij}^{n+1/2} \]  \hspace{2cm} \text{Implicit}\\

\[ V_{i}^{n+1/2} - V_{j}^{n+1/2} = L_{ij} \left( \frac{I_{ij}^{n+1} - I_{ij}^{n}}{\Delta t} \right) + \frac{R_{ij}}{2} \left( I_{ij}^{n+1} + I_{ij}^{n} \right) - E_{ij}^{n+1/2} \]  \hspace{2cm} \text{Semi-Implicit}
FDTD & LIM

Field Solution

Yee Algorithm

\[
E_x^n(i,j,k) = E_x^{n-1} + \frac{c \Delta t}{\varepsilon} \left( H_{z}^{n-1/2}(i,j,k) - H_{z}^{n-1/2}(i,j-1,k) \right)
\]

\[
-\frac{c \Delta t}{\varepsilon} \left( H_{y}^{n-1/2}(i,j,k) - H_{y}^{n-1/2}(i,j,k-1) \right)
\]

\[
H_x^{n+1/2}(i,j,k) = H_x^{n-1/2} - \frac{c \Delta t}{\mu} \left( E_z^n(i,j,k+1) - E_z^n(i,j,k) \right)
\]

\[
-\frac{c \Delta t}{\mu} \left( H_y^n(i,j,k+1) - H_y^n(i,j,k) \right)
\]

Circuit Solution

Latency Insertion Method (LIM)

\[
V_{i}^{n+1/2} = \frac{C_i V_{i}^{n-1/2}}{\Delta t} + \frac{H_i^n}{\Delta t} - \sum_{k=1}^{M_i} I_{ik}^n
\]

\[
I_{ij}^{n+1} = I_{ij}^n + \frac{\Delta t}{L_{ij}} \left( V_{i}^{n+1/2} - V_{j}^{n+1/2} - R_{ij} I_{ij}^n + E_{ij}^{n+1/2} \right)
\]

Courant-Friedrichs-Lewy criteria

\[
\Delta t < \sqrt{LC}
\]

stability condition
LIM: Stability Analysis

\[ \Delta t_{\text{max}} = 0.20099505 \text{ ns} \]

\[ 0.99\Delta t_{\text{max}} \]

\[ 1.01\Delta t_{\text{max}} \]
LIM and Dependent Sources

\[ C_i \left( \frac{V_i^{n+1/2} - V_i^{n-1/2}}{\Delta t} \right) + \frac{G_i}{2} \left( V_i^{n+1/2} + V_i^{n-1/2} \right) - H_i^n + I_{Li}^n \]

\[ -\frac{B_{ik}}{2} \left( V_k^{n+1/2} + V_k^{n-1/2} \right) - S_{ip} I_p^n = -\sum_{j=1}^{M_i} I_{ij}^n \]
LIM and Dependent Sources

\[
C \left( \frac{v^{n+1/2} - v^{n-1/2}}{\Delta t} \right) + \frac{1}{2} G \left( v^{n+1/2} + v^{n-1/2} \right) - h^n + i_L^n
\]

\[
-\frac{1}{2} B \left( v^{n+1/2} + v^{n-1/2} \right) - S_i^n = -M_i^n
\]
LIM and Dependent Sources

where $M$ is the incidence matrix. $M$ is defined as follows

$$M_{qp} = 1 \text{ if branch } p \text{ is incident at node } q \text{ and the current flows away from node } q.$$  

$$M_{qp} = -1 \text{ if branch } p \text{ is incident at node } q \text{ and the current flows into node } q.$$  

$$M_{qp} = 0 \text{ if branch } p \text{ is not incident at node } q.$$
Incidence Matrix

\[
\begin{pmatrix}
\text{node 1} & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
\text{node 2} & -1 & 0 & 0 & 1 & 0 & 1 \\
\text{node 3} & 0 & 0 & -1 & -1 & 1 & 0 \\
\text{node 4} & 0 & -1 & 0 & 0 & -1 & -1 \\
\end{pmatrix}
\]
LIM and Dependent Sources

\[ i_{L}^{n+1} = i_{L}^{n} + \Delta t L_n^{-1} v^{n+1/2} \]

\[
C \left( \frac{v^{n+1/2} - v^{n-1/2}}{\Delta t} \right) + \frac{1}{2} G' \left( v^{n+1/2} + v^{n-1/2} \right) - h^n + i_{L}^{n} = -M'i^n
\]

\[ G' = G + B \quad \quad M' = M + S \]

\[
v^{n+1/2} = \left( \frac{C}{\Delta t} + \frac{G'}{2} \right)^{-1} \left[ \left( \frac{C}{\Delta t} - \frac{G'}{2} \right) v^{n-1/2} + h^n - i_{L}^{n} - M'i^n \right]
\]
LIM and Dependent Sources

\[ V_{i}^{n+1/2} - V_{j}^{n+1/2} = L_{ij} \left( \frac{I_{ij}^{n+1} - I_{ij}^{n}}{\Delta t} \right) + \frac{R_{ij}}{2} \left( I_{ij}^{n+1} + I_{ij}^{n} \right) - E_{ij}^{n+1/2} + V_{cij}^{n+1/2} \]

\[-T_{ijk} V_{k}^{n+1/2} - \frac{Z_{ijpq}}{2} \left( I_{pq}^{n+1} + I_{pq}^{n} \right)\]

\[ M^{T} v^{n+1/2} = \frac{L}{\Delta t} \left( i^{n+1} - i^{n} \right) + \frac{R}{2} \left( i^{n+1} + i^{n} \right) - e^{n+1/2} + v_{c}^{n+1/2} \]

\[-Tv^{n+1/2} - \frac{Z}{2} \left( i^{n+1} + i^{n} \right)\]
LIM and Dependent Sources

\[ v^{n+1/2}_c = v^{n-1/2}_c + \Delta t C_b^{-1} i^n \]

\[ i^{n+1}_L = i^n_L + \Delta t L_n^{-1} v^{n+1/2} \]

\[ i^{n+1} = \left( \frac{L}{\Delta t} + \frac{R'}{2} \right)^{-1} \left[ \left( \frac{L}{\Delta t} - \frac{R'}{2} \right) i^n + e^{n+1/2} - v^{n+1/2}_c + M^T v^{n+1/2} \right] \]

\[ v^{n+1/2} = \left( \frac{C}{\Delta t} + \frac{G'}{2} \right)^{-1} \left[ \left( \frac{C}{\Delta t} - \frac{G'}{2} \right) v^{n-1/2} + h^n - i^n_L - M' i^n \right] \]
LIM and Dependent Sources

\[ i^{n+1} = \left( \frac{L}{\Delta t} + \frac{R'}{2} \right)^{-1} \left( \frac{L}{\Delta t} - \frac{R'}{2} \right) i^n + \left( \frac{L}{\Delta t} + \frac{R'}{2} \right)^{-1} M'^T v^{n+1/2} \]

\[ -\left( \frac{L}{\Delta t} + \frac{R'}{2} \right)^{-1} v_c^{n+1/2} \]

\[ v^{n+1/2} = \left( \frac{C}{\Delta t} + \frac{G'}{2} \right)^{-1} \left( \frac{C}{\Delta t} - \frac{G'}{2} \right) v^{n-1/2} - \left( \frac{C}{\Delta t} + \frac{G'}{2} \right)^{-1} M' i^n \]

\[ -\left( \frac{C}{\Delta t} + \frac{G'}{2} \right)^{-1} i_L^n \]
Stability Analysis

**DEFINE:**

\[
P_+ = \left( \frac{C + G'}{\Delta t} \right)^{-1}
\]

\[
Q_+ = \left( \frac{L + R'}{\Delta t} \right)^{-1}
\]

\[
P_- = \left( \frac{C - G}{\Delta t} \right)
\]

\[
Q_- = \left( \frac{L - R}{\Delta t} \right)
\]

so that

\[
i^{n+1} = Q_+ Q_i^n + Q_+ M^T v^{n+1/2}
\]

\[
v^{n+1/2} = P_+ P_- v^{n-1/2} - P_+ M' i^n
\]

\[
i^{n+1} = Q_+ Q_i^n + Q_+ M^T P_+ P_- v^{n-1/2} - Q_+ M^T P_+ M' i^n
\]
Stability Analysis

\[
v^{n+1/2} = P_+P_- v^{n-1/2} - P_+M'i^n
\]

\[
i^{n+1} = Q_+M^TP_+P_- v^{n-1/2} + \left[ Q_+Q_- - Q_+M^TP_+M' \right]i^n
\]

The 2 matrix equations can be combined in a single matrix equation that reads

\[
\begin{bmatrix}
  v^{n+1/2} \\
  i^{n+1}
\end{bmatrix} =
\begin{bmatrix}
P_+P_- & -P_+M' \\
Q_+M^TP_+P_- & Q_+Q_- - Q_+M^TP_+M'
\end{bmatrix}
\begin{bmatrix}
v^{n-1/2} \\
i^n
\end{bmatrix}
\]

or

\[
\begin{bmatrix}
v^{n+1/2} \\
 i^{n+1}
\end{bmatrix} = A
\begin{bmatrix}
v^{n-1/2} \\
 i^n
\end{bmatrix}
\]
Amplification Matrix

$$A = \begin{bmatrix}
P_+P_- & -P_+M' \\
Q_+M^{T'}P_+P_- & Q_+Q_- - Q_+M^{T'}P_+M'
\end{bmatrix}$$

$$A_{11} = P_+P_- \quad A_{12} = -P_+M'$$
$$A_{21} = Q_+M^{T'}P_+P_- \quad A_{22} = Q_+Q_- - Q_+M^{T'}P_+M'$$

$A$ is the amplification matrix. All the eigenvalues of $A$ must be less than 1 in order to guarantee stability. Therefore to insure stability, we must choose $\Delta t$ such that all the eigenvalues of $A$ are less than 1.
Stability Methods for LIM

• LIM is conditionally stable $\rightarrow$ upper bound on the time step $\Delta t$.

• For uniform 1-D LC circuits $[2]$:

$$\Delta t < \sqrt{LC}$$

• For RLC/GLC circuits $[3]$:

$$\Delta t \leq \sqrt{2 \min_{i=1}^{N_b}} \left( \frac{C_i}{N_b} \min_{p=1}^{N_p} (L_{i,p}) \right)$$

• For general circuits $[4]$:

$$A = \begin{bmatrix} P_+ P_- & -P_+ M \\ Q_+ M^T P_+ & Q_+ Q_- - Q_+ M^T P_+ M \end{bmatrix}$$

Amplification matrix.

Example – RLGC Grid

* Current excitation with amplitude of 6A, rise and fall times of 10 ps and pulse width of 100 ps.

* SPECTRE – A SPICE-like simulator from Cadence Design Systems Inc.
Example – RLGC Grid

• Comparison of runtime for LIM and SPECTRE*.

<table>
<thead>
<tr>
<th>Circuit Size</th>
<th># nodes (SPECTRE)</th>
<th>SPECTRE (s)</th>
<th>LIM (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 × 20</td>
<td>1160</td>
<td>0.15</td>
<td>1.53</td>
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<tr>
<td>40 × 40</td>
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<td>60 × 60</td>
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<td>80 × 80</td>
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<td>120 × 120</td>
<td>42960</td>
<td>945.00</td>
<td>62.89</td>
</tr>
</tbody>
</table>

• LIM exhibits linear numerical complexity!
  – Outperforms conventional SPICE-like simulators.
• Expand on LIM as a multi-purpose circuit simulator.

* Intel 3.16 GHz processor, 32 GB RAM.
Computer simulations of distributed model (top) and experimental waveforms of coupled microstrip lines (bottom) for the transmission-line circuit shown at the near end (x=0) for line 1 (left) and line 2 (right). The pulse characteristics are magnitude = 4 V; width = 12 ns; rise and fall times = 1 ns. The photograph probe attenuation factors are 40 (left) and 10 (right).
LIM Simulation Examples

Twisted-pair cable

Simulation setup

Generalized model [3]

Ideal line

lossy line

Simulation results

Measured response