ECE 546
Lecture -19
X-Parameters

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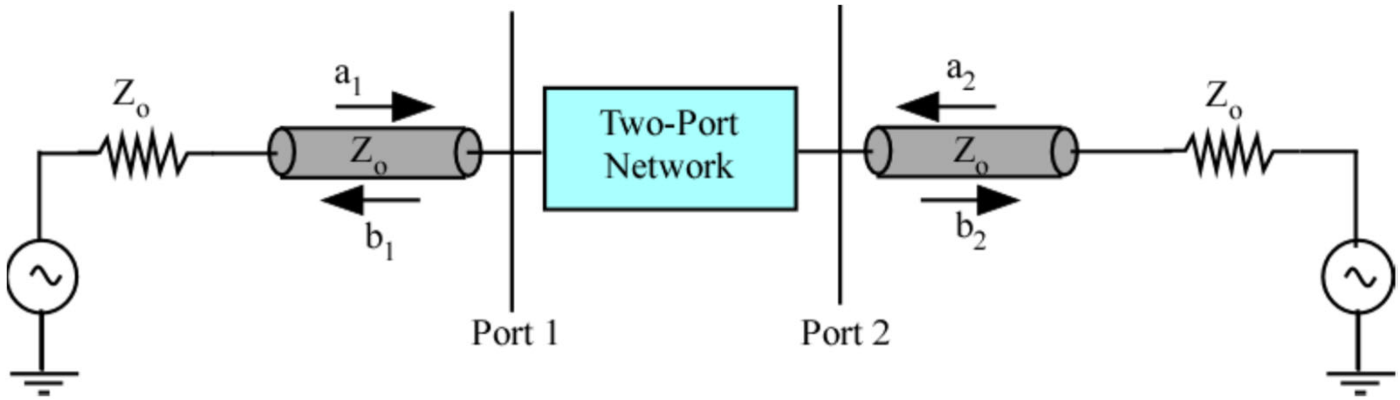
References

[1] J J. Verspecht and D. E. Root, "Polyharmonic Distortion Modeling," IEEE Magazine, June 2006, pp. 44-57.

[2] D.E. Root, J. Verspecht, D. Sharrit, J. Wood, and A. Cognata, "Broad-band poly-harmonic distortion (PHD) behavioral models from fast automated simulations and large-signal vectorial network measurements,"IEEE Trans. Microwave Theory Tech., vol. 53, no. 11, pp. 3656–3664, Nov. 2005.

[3] D.E. Root, J. Verspecht, J. Horn, J. Wood, and M. Marcu, "X Parameters", Cambridge University Press, 2013.

Scattering Parameters



For a two-port

$$V_1 = a_1 + b_1$$

$$I_1 = \frac{a_1 - b_1}{Z_o}$$

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

$$V_2 = a_2 + b_2$$

$$I_2 = \frac{a_2 - b_2}{Z_o}$$

For a general N-port

$$B = SA$$

$$B_i = \sum_{j=1}^{N} S_{ij} A_j$$

$$S_{ij} = \left.\frac{B_i}{A_j}\right|_{\substack{A_k=0 \\ k\neq j \\ k=1,\ldots,N}}$$

" ...most successful behavioral models..."

X Parameters: Motivation

S Parameters are a very powerful tool for signal integrity analysis.

Today, X parameters are primarily used to characterize power amplifiers and nonlinear devices. Not yet applied to signal integrity.

X Parameters

Purpose

Characterize nonlinear behavior of devices and systems

Advantages

- **Mathematically robust framework**
- **Can handle nonlinearities**
- **Instrument exists (NVNA)**
- **Blackbox format → vendor IP protection**
- **Matrix format → easy incorporation in CAD tools**
- **X Parameters are a *superset* of S parameters**

Challenges in HS Links

High speed Serial channels are pushing the current limits of simulation. Models/Simulator need to handle current challenges

- **Need to accurately handle very high data rates**
- **Simulate large number of bits to achieve low BER**
- **Non-linear blocks with time variant systems**
- **Model TX/RX equalization**
- **All types of jitter: (random, deterministic, etc.)**
- **Crosstalk, loss, dispersion, attenuation, etc…**
- **Handle and manage vendor specific device settings**
- **Clock data recovery (CDR) circuits**

These cannot be accurately modeled with S parameters

Motivation

Limitation: S Parameters only work for linear systems. Many networks and systems are nonlinear

- **Applications**
  - High-speed links, power amplifiers, mixed-signal circuits
- **Existing Methods**
  - Load pull techniques
  - IBIS models
  - Models are flawed and incomplete

PHD Modeling

- Polyharmonic distortion (PHD) modeling is a frequency-domain modeling technique
- PHD model defines X parameters which form a superset of S parameters
- To construct PHD model, DUT is stimulated by a set of harmonically related discrete tones
- In stimulus, fundamental tone is dominant and higher-order harmonics are smaller

PHD Framework

- Signal is represented by a fundamental with harmonics
- Signals are periodic or narrowband modulated versions of a fundamental with harmonics
- Harmonic index: 0 for dc contribution, 1 for fundamental and 2 for second harmonic
- Power level, fundamental frequency can be varied to generate complete data for DUT

PHD Framework

- **Stimulus**
  - A-waves are incident and B-waves are scattered
- **Reference System Z_C**
  - Default value is 50 ohm

For a given port with voltage V and current I

$$A = \frac{V + Z_C I}{2}$$

$$B = \frac{V - Z_C I}{2}$$

PHD Framework

F_{pm} describes a time-invariant system → delay in time domain corresponds to phase shift in frequency domain

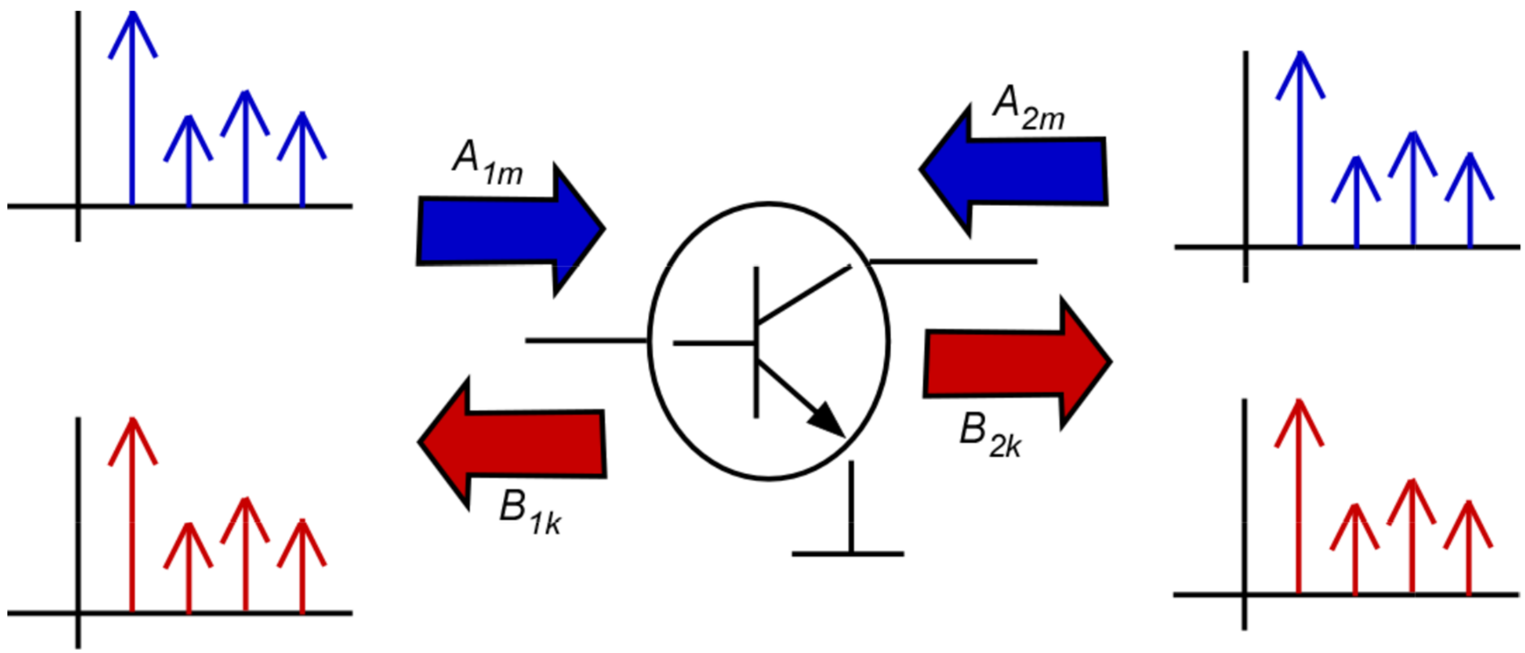
$$B_{pm}e^{jm\theta} = F_{pm}\left(A_{11}e^{j\theta}, A_{12}e^{j2\theta}, \ldots, A_{21}e^{j\theta}, A_{22}e^{j2\theta}, \ldots\right)$$

For phase normalization, define

$$P = e^{+j\varphi(A_{11})}$$

$$B_{pm} = F_{pm}\left(\left|A_{11}\right|, A_{12}P^{-2}, A_{13}P^{-3}, \ldots, A_{21}P^{-1}, A_{22}P^{-2}, \ldots\right)P^{+m}$$

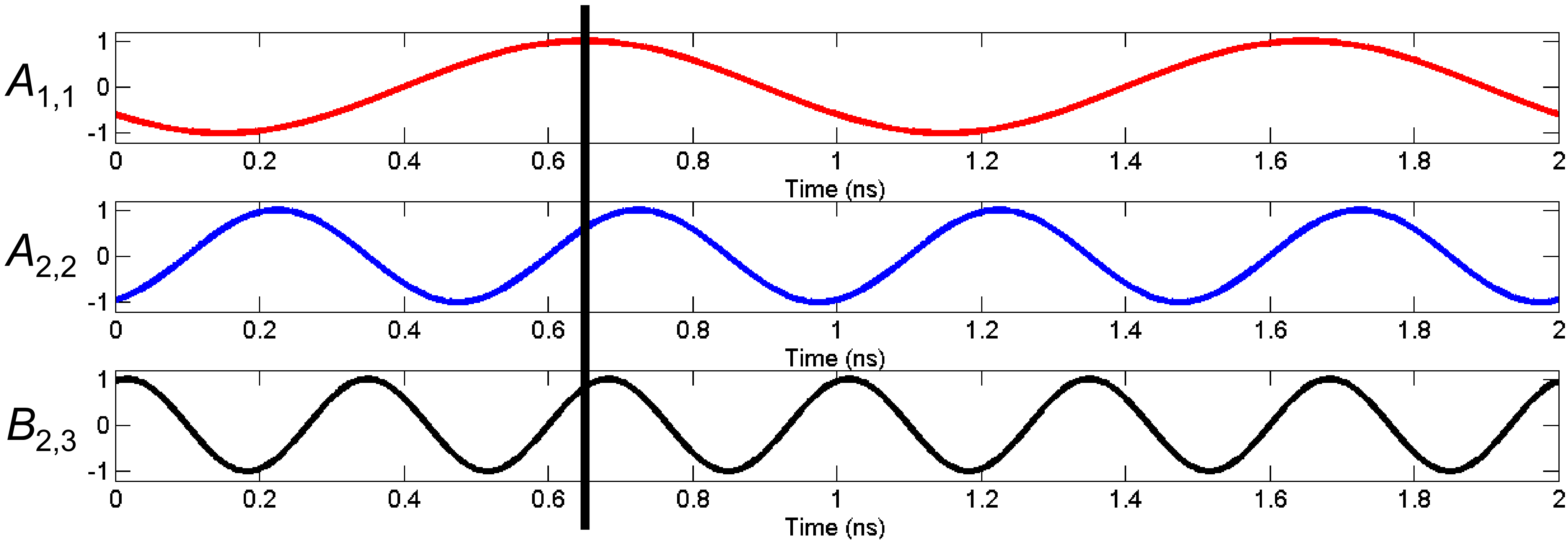
PHD Framework



$$B_{1k}=F_{1k}(A_{11}, A_{12}, \ldots, A_{21}, A_{22},\ldots)$$
$$B_{2k}=F_{2k}(A_{11}, A_{12}, \ldots, A_{21}, A_{22},\ldots)$$

Cross-Frequency Phase for Commensurate Tones

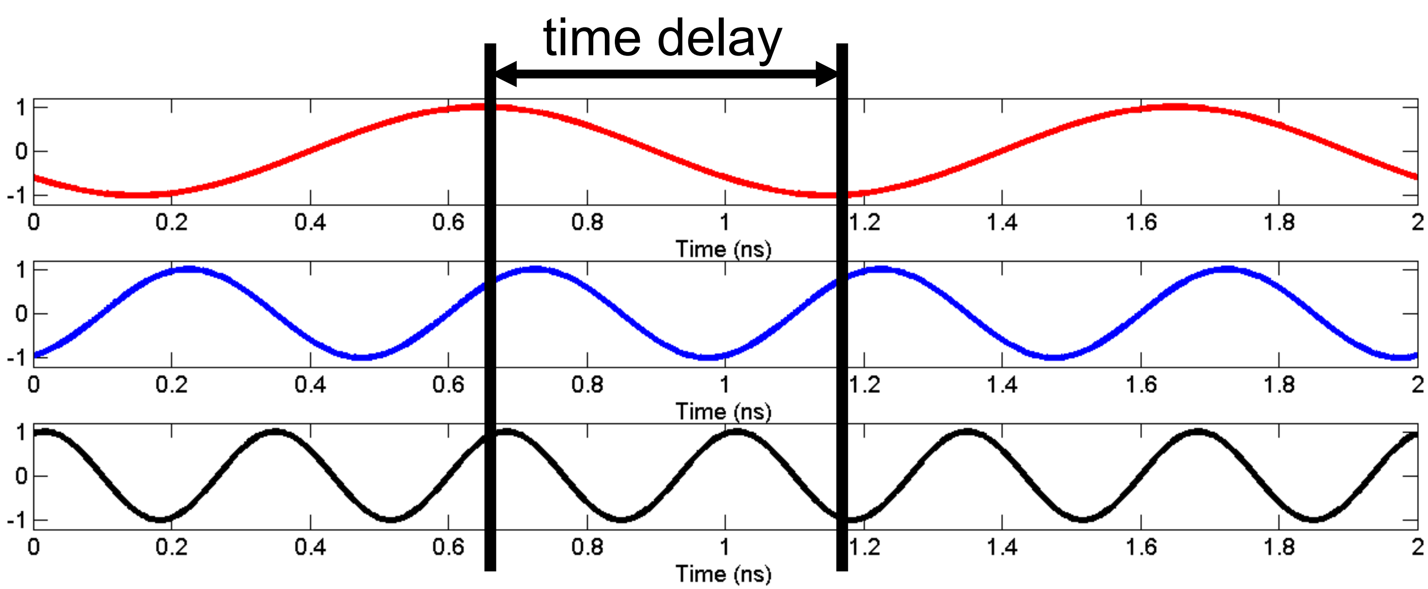
- Defined as the phase of each pseudowave when the fundamental, $A_{1,1}$, has zero phase.
- $B_{2,3}$ can be related to $A_{2,2}$ in magnitude and phase.



Time-Invariance Property of Nonlinear Scattering Function

$$F_{p,k}(A_{1,1}e^{j\theta}, A_{1,2}(e^{j\theta})^2, A_{1,3}(e^{j\theta})^3, \ldots)$$
$$= F_{p,k}(A_{1,1}, A_{1,2}, A_{1,3}, \ldots)(e^{j\theta})^k$$

- Shifting all of the inputs by the same time means that different harmonic components are shifted by different phases.



f_0 180º phase shift

$2f_0$ 360º phase shift

$3f_0$ 540º phase shift

Defining Phase Reference

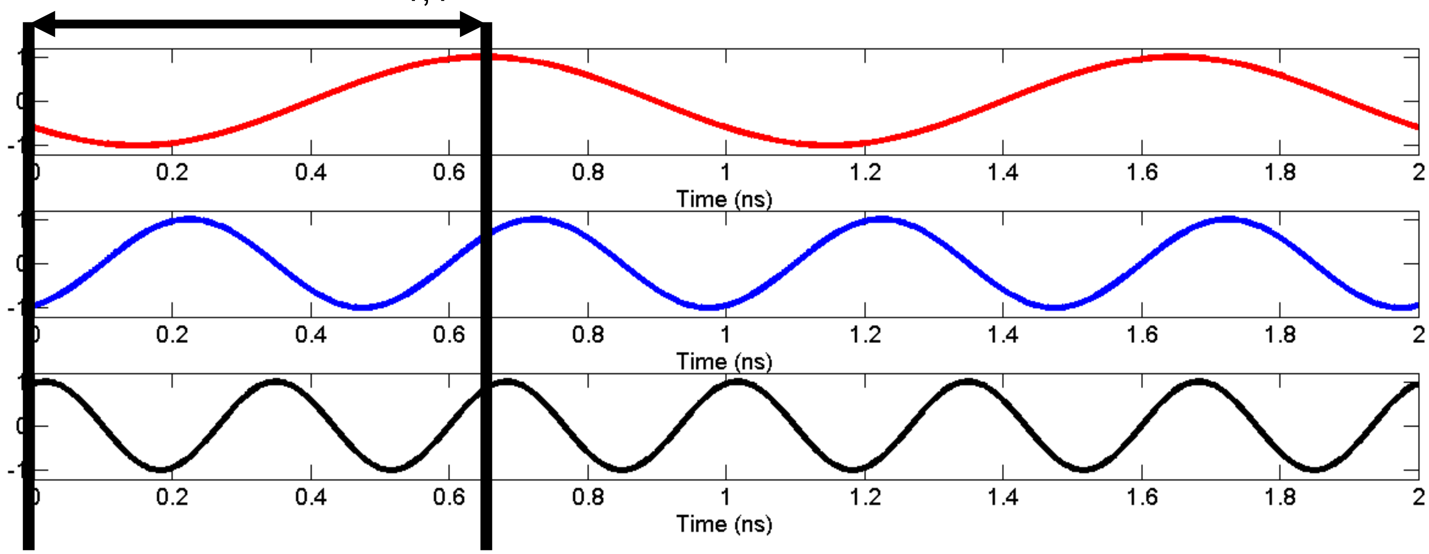
- Can use time-invariance to separate magnitude and phase dependence of one incident pseudowave.

$$B_{p,k} = F_{p,k}(A_{1,1}, A_{1,2}, A_{1,3}, \ldots)$$
$$= F_{p,k}(|A_{1,1}|, A_{1,2}P^{-2}, A_{1,3}P^{-3}, \ldots)P^k$$

using

$$P = \frac{A_{1,1}}{|A_{1,1}|} = e^{j\arg(A_{1,1})}$$

Shifting reference to zero phase of $A_{1,1}$.



Commensurate Tones X-Parameter Formalism*

- Define $$X_{p,k}^{(FB)}\left(\left|A_{1,1}\right|, A_{1,2}P^{-2}, A_{1,3}P^{-3}, \ldots\right)$$
$$= F_{p,k}\left(A_{1,1}, A_{1,2}, A_{1,3}, \ldots\right)P^{-k}$$

⇩

$$B_{p,k} = X_{p,k}^{(FB)}\left(\left|A_{1,1}\right|, A_{1,2}P^{-2}, A_{1,3}P^{-3}, \ldots\right)P^{k}$$

- Still difficult to characterize this nonlinear term.
- If only one incident pseudowave, $A_{1,1}$, is large then the other smaller inputs can be linearized about the large-signal response of $F_{p,k}$ to only $A_{1,1}$.

*D. E. Root, *et al.*, *X-Parameters*, 2013.

PHD Framework

Define variables

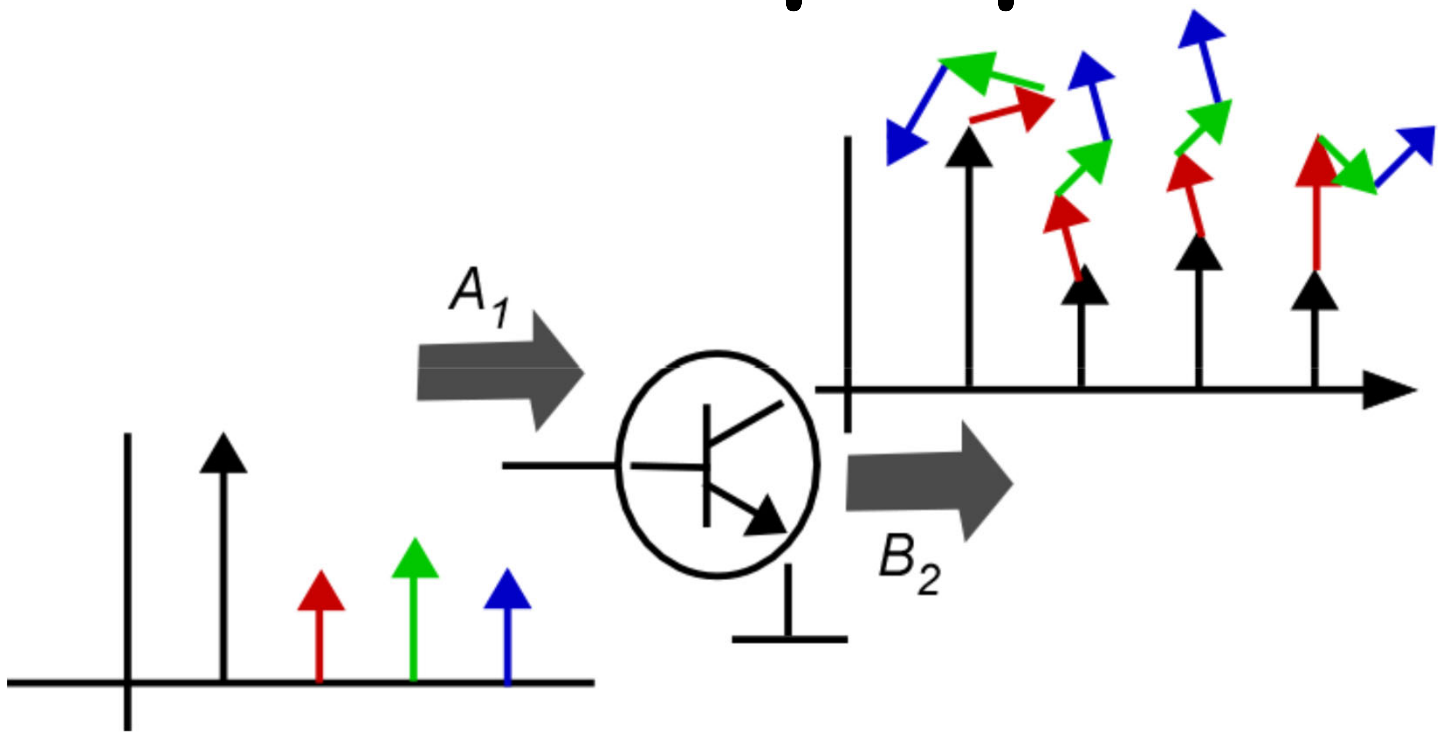
A_{pm} (p = port, m = harmonic)

B_{pm} (p = port, m = harmonic)

- Introduce multivariate complex function F_{pm} such that

$$B_{pm} = F_{pm}\left(A_{11}, A_{12}, \ldots, A_{21}, A_{22}, \ldots\right)$$

Harmonic Superposition



In many situations, there is only one dominant large-signal input component present. The harmonic frequency components are relatively small→ harmonic components can be superposed

Harmonic superposition principle is key to PHD model

Nonanalytical Mapping*

A nonlinearity described by:

$$f(x) = \alpha x + \gamma x^3$$

Signal is sum of main signal and additional perturbation term which is assumed to be small

$$x(t) = x_o(t) + \Delta x(t)$$

* see: J. Verspecht and D. E. Root, "Polyharmonic Distortion Modeling," IEEE Magazine, June 2006, pp. 44-57.

Case 1

Consider the signal x(t), given by the sum of a real dc component and a small tone at frequency f

$$x_o(t) = A$$

A is real

$$\Delta x(t) = \frac{\delta e^{j\omega t} + \delta^* e^{-j\omega t}}{2}$$

δ is a small complex number

The linear response in Δx(t) can be computed by

$$\Delta\left(y(t)\right) = f\left(x_o(t) + \Delta x(t)\right) - f\left(x_o(t)\right)$$

Case 1

$$\Delta\left(y(t)\right) \approx f'\left(x_o(t)\right)\Delta x(t)$$

For case 1, we evaluate the conductance nonlinearity *f'(xo)*

at the fixed value $x_o = A$

$$f'\left(A\right) = \alpha + 3\gamma A^2$$

After substitution, we get

$$\Delta\left(y(t)\right) = \left[\alpha + 3\gamma A^2\right]\left(\frac{\delta e^{j\omega t} + \delta^* e^{-j\omega t}}{2}\right)$$

The complex coefficient of term proportional to $e^{j\omega t}$ is

$$\left[\frac{\alpha + 3\gamma A^2}{2}\right]\delta$$

➔ **Linear input-output relationship**

Case 2

Now, $x_o(t)$ is a periodically time-varying signal:

$$x_o(t) = A\cos\left(\omega t\right)$$

$$\Delta x(t) = \frac{\delta e^{j\omega t} + \delta^* e^{-j\omega t}}{2}$$

Evaluating the conductance nonlinearity at $x_o(t)$ gives

$$f'(A\cos(\omega t)) = \alpha + 3\gamma A^2 \cos(\omega t)$$

$$= \left(\alpha + \frac{3\gamma A^2}{2}\right) + \frac{3\gamma A^2}{2}\cos(2\omega t)$$

Case 2

We can evaluate Δ(y(t)) to get:

$$\Delta\left(y(t)\right)=\left[\left(\alpha+\frac{3\gamma A^2}{2}\right)+\frac{3\gamma A^2}{2}\left(\frac{e^{2j\omega t}+e^{-2j\omega t}}{2}\right)\right]$$

$$\times\left(\frac{\delta e^{j\omega t}+\delta^* e^{-j\omega t}}{2}\right)$$

Now, we have terms proportional to $e^{j\omega t}$ and $e^{j3\omega t}$ and their complex conjugates. Restrict attention to complex term proportional to $e^{j\omega t}$

Case 2

The complex coefficient of term proportional to $e^{j\omega t}$ is

$$\left(\frac{\alpha}{2}+\frac{3\gamma A^2}{4}\right)\delta+\frac{3\gamma A^2}{4}\delta^*$$

We observe that the output phasor at frequency ω is not just proportional to the input phasor δ at frequency ω but has distinct contributions to both δ and δ*

➔ Linearization is not analytic

In Fourier domain, we have:

$$\frac{\Delta\hat{Y}(\omega)}{\Delta\hat{X}(\omega)}\left(\frac{\alpha}{2}+\frac{3\gamma A^2}{4}\right)\delta+\frac{3\gamma A^2}{4}e^{-2\,jPhase(\delta)}$$

PHD Derivation

$$B_{pm} = K_{pm}\left(\left|A_{11}\right|\right)P^{+m} + \sum_{qn} G_{pq,mn}\left(\left|A_{11}\right|\right)P^{+m}\,\mathrm{Re}(A_{qn}P^{-n}) + \sum_{qn} H_{pq,mn}\left(\left|A_{11}\right|\right)P^{+m}\,\mathrm{Im}(A_{qn}P^{-n})$$

in which

$$K_{pm}\left(\left|A_{11}\right|\right) = F_{pm}\left(\left|A_{11}\right|,0,\ldots,0\right)$$

$$G_{pq,mn}\left(\left|A_{11}\right|\right) = \left.\frac{\partial F_{pm}}{\partial\,\mathrm{Re}\left(A_{qn}P^{-n}\right)}\right|_{\left|A_{11}\right|,0,\ldots,0}$$

$$H_{pq,mn}\left(\left|A_{11}\right|\right) = \left.\frac{\partial F_{pm}}{\partial\,\mathrm{Im}\left(A_{qn}P^{-n}\right)}\right|_{\left|A_{11}\right|,0,\ldots,0}$$

Spectral mapping is nonanalytic

PHD Derivation

Since

$$\mathrm{Re}(A_{qn}P^{-n}) = \frac{A_{qn}P^{-n} + conj\left(A_{qn}P^{-n}\right)}{2}$$

$$\mathrm{Im}(A_{qn}P^{-n}) = \frac{A_{qn}P^{-n} - conj\left(A_{qn}P^{-n}\right)}{2j}$$

we get

$$B_{pm} = K_{pm}\left(\left|A_{11}\right|\right)P^{+m}$$

$$+\sum_{qn} G_{pq,mn}\left(\left|A_{11}\right|\right)P^{+m}\left(\frac{A_{qn}P^{-n} + conj\left(A_{qn}P^{-n}\right)}{2}\right)$$

$$+\sum_{qn} H_{pq,mn}\left(\left|A_{11}\right|\right)P^{+m}\left(\frac{A_{qn}P^{-n} - conj\left(A_{qn}P^{-n}\right)}{2j}\right)$$

PHD Model

$$B_{pm} = X_{pm}^{(FB)}\left(|A_{11}|\right)P^{+m} + \sum_{qn} X_{pq,mn}^{(S)}\left(|A_{11}|\right)P^{+m-n}A_{qn}$$
$$+\sum_{qn} X_{pq,mn}^{(T)}\left(|A_{11}|\right)P^{+m+n}conj\left(A_{qn}\right)$$

PHD Model Equation

$$X_{p1,m1}^{(S)}\left(|A_{11}|\right) = \frac{K_{pm}\left(|A_{11}|\right)}{|A_{11}|} \qquad X_{p1,m1}^{(T)}\left(|A_{11}|\right) = 0$$

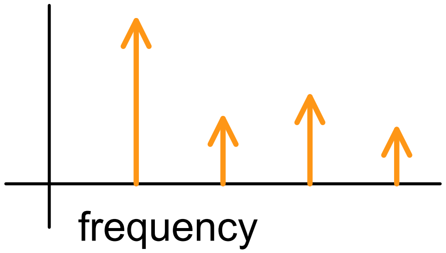
$$\forall\{q,n\} \neq \{1,1\} : X_{pq,mn}^{(S)}\left(|A_{11}|\right) = \frac{G_{pq,mn}\left(|A_{11}|\right) - jH_{pq,mn}\left(|A_{11}|\right)}{2}$$

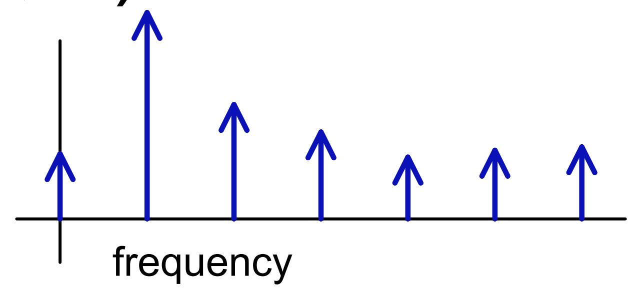
$$\forall\{q,n\} \neq \{1,1\} : X_{pq,mn}^{(T)}\left(|A_{11}|\right) = \frac{G_{pq,mn}\left(|A_{11}|\right) + jH_{pq,mn}\left(|A_{11}|\right)}{2}$$

1-Tone X-Parameter Formalism*

Incident Waves — Approximates — Scattered Waves

$$B_{p,k} = X_{p,k}^{(FB)}\left(\left|A_{1,1}\right|, A_{1,2}P^{-2}, A_{1,3}P^{-3}, \ldots\right)$$

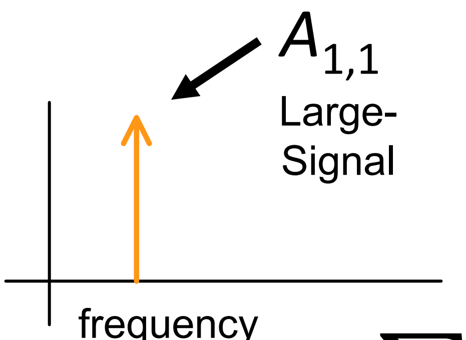


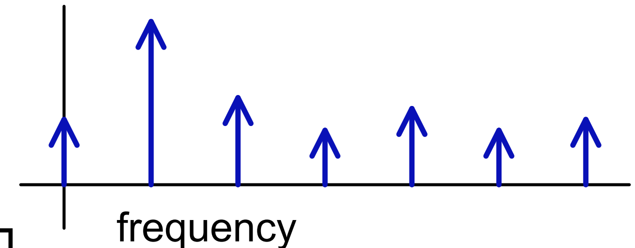


Nonlinear Mapping

$$\approx$$

$$X_{p,k}^{(FB)}\left(\left|A_{1,1}\right|, 0, 0, \ldots\right)$$

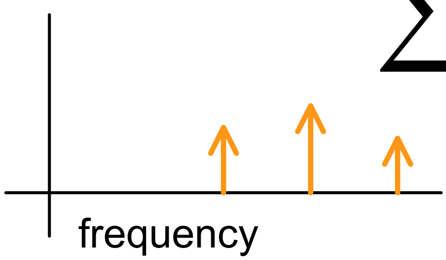


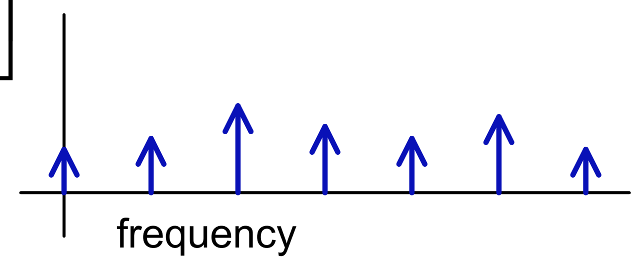


Simple Nonlinear Mapping

$$+$$

$$\sum\left[X_{p,k;q,l}^{(S)} \cdot A_{q,l} + X_{p,k;q,l}^{(T)} \cdot A_{q,l}^{*}\right]$$





Nonanalytic Harmonic Superposition

*J. Verspecht, *et al.*, "Linearization…," 2005.

1-Tone X-Parameter Formalism*

$$B_{p,k} \approx \underbrace{X^{(FB)}_{p,k} \cdot P^{k}}_{\text{Simple nonlinear map}} + \underbrace{\sum_{\substack{q=1,\, l=1 \\ (q,l)\neq(1,1)}}^{q=N,\, l=K} X^{(S)}_{p,k;q,l} \cdot A_{q,l} \cdot P^{k-l}}_{\text{Linear harmonic map function of incident wave}} + \underbrace{\sum_{\substack{q=1,\, l=1 \\ (q,l)\neq(1,1)}}^{q=N,\, l=K} X^{(T)}_{p,k;q,l} \cdot A^{*}_{q,l} \cdot P^{k+l}}_{\text{Linear harmonic map function of conjugate of incident wave}}$$

Simple nonlinear map

Linear harmonic map function of incident wave

Linear harmonic map function of conjugate of incident wave

- X-parameters of type FB, S, and T fully characterize the nonlinear function.
- Depend on
  - frequency
  - large signal magnitude, $|A_{1,1}|$
  - DC bias

$$X^{(S)}_{p,k;q,l}$$

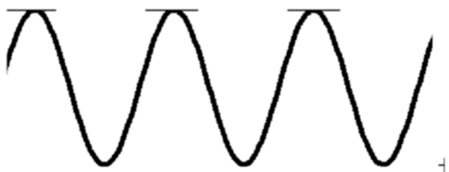
p: output port; k: output harmonic; q: input port; l: input harmonic

$$P = \frac{A_{1,1}}{|A_{1,1}|}$$

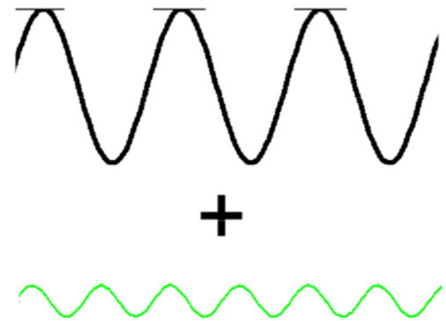
*D. E. Root, *et al.*, *X-Parameters*, 2013.

Excitation Design

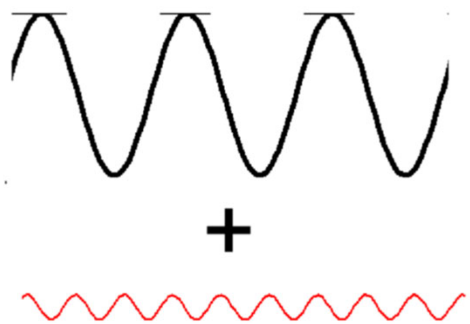
Excitation 1



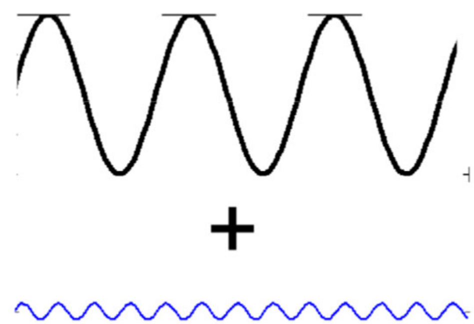
Excitation 2



Excitation 3



Excitation 4



Each excitation will generate response with fundamental and all harmonics

X-Parameter Data File

TOP: FILE DESCRIPTION

! Created Fri Jul 30 07:44:48 2010

! Version = 2.0
! HB_MaxOrder = 25
! XParamMaxOrder = 12
! NumExtractedPorts = 3

! IDC_1=0 NumPts=1
! IDC_2=0 NumPts=1
! VDC_3=12 NumPts=1
! ZM_2_1=50 NumPts=1
! ZP_2_1=0 NumPts=1
! AN_1_1=100e-03(20.000000dBm) NumPts=1
! fund_1=[100 Hz->1 GHz] NumPts=4

X-Parameter Data File

MIDDLE: FORMAT DESCRIPTION

```
BEGIN XParamData
% fund_1(real)  FV_1(real)  FV_2(real)  FI_3(real)  FB_1_1(complex)
% FB_1_2(complex)  FB_1_3(complex)  FB_1_4(complex)
% FB_1_7(complex)  FB_1_8(complex)  FB_1_9(complex)
% FB_1_12(complex)  FB_2_1(complex)  FB_2_2(complex)
% FB_2_5(complex)  FB_2_6(complex)  FB_2_7(complex)
% FB_2_10(complex)  FB_2_11(complex)  FB_2_12(complex)
% T_1_1_1_1(complex)  S_1_2_1_1(complex)  T_1_2_1_1(complex)
% S_1_4_1_1(complex)  T_1_4_1_1(complex)  S_1_5_1_1(complex)
% T_1_6_1_1(complex)  S_1_7_1_1(complex)  T_1_7_1_1(complex)
% S_1_9_1_1(complex)  T_1_9_1_1(complex)  S_1_10_1_1(complex))
% T_1_11_1_1(complex)  S_1_12_1_1(complex)  T_1_12_1_1(complex)
% T_2_1_1_1(complex)  S_2_2_1_1(complex)  T_2_2_1_1(complex)
% S_2_4_1_1(complex)  T_2_4_1_1(complex)  S_2_5_1_1(complex
% T_2_6_1_1(complex)  S_2_7_1_1(complex)  T_2_7_1_1(complex)
% S_2_9_1_1(complex)  T_2_9_1_1(complex)  S_2_10_1_1(complex)
```

X-Parameter Data File

BOTTOM: DATA LISTING

100	0	0.903921	0.0263984	0.316228	-5.41159e-09
-5.8503e-16	-4.19864e-10	-6.37642e-16	-1.6748e-10	-4.62314e-16	
-1.25093e-15	-3.79264e-10	-7.91128e-16	-1.51261e-10	1.93535e-17	
-1.38032e-16	-2.09262e-10	0.107122	-5.52212e-08	0.0739648	
-0.0081633	-2.40901e-08	-0.00739395	-1.21199e-08	-0.000530768	
0.000921039	-4.82427e-09	-0.00230559	1.07836e-08	-0.00288533	
-1.20792e-15	-5.09916e-10	-6.95799e-15	-2.56672e-09	-3.25033e-15	
-1.2948e-14	3.97284e-10	-7.08201e-15	-2.17127e-09	-1.43757e-14	
3.39598e-15	3.66098e-10	-1.08395e-14	-4.05911e-09	1.67366e-14	
2.76565e-14	5.60242e-09	2.69755e-14	-6.60802e-10	3.99868e-14	

Remarks

- Data is measured or generated from a harmonic balance simulator
- Data file can be very large

X-Parameter Relationship

$$b_{ik} = D_{ik}\left(\left|a_{11}\right|\right)P^{k} + \sum_{(j,l)\neq(1,1)}\left[S_{ik,jl}\left(\left|a_{11}\right|\right)P^{k-l}a_{jl} + T_{ik,jl}\left(\left|a_{11}\right|\right)P^{k+l}a_{jl}^{*}\right]$$

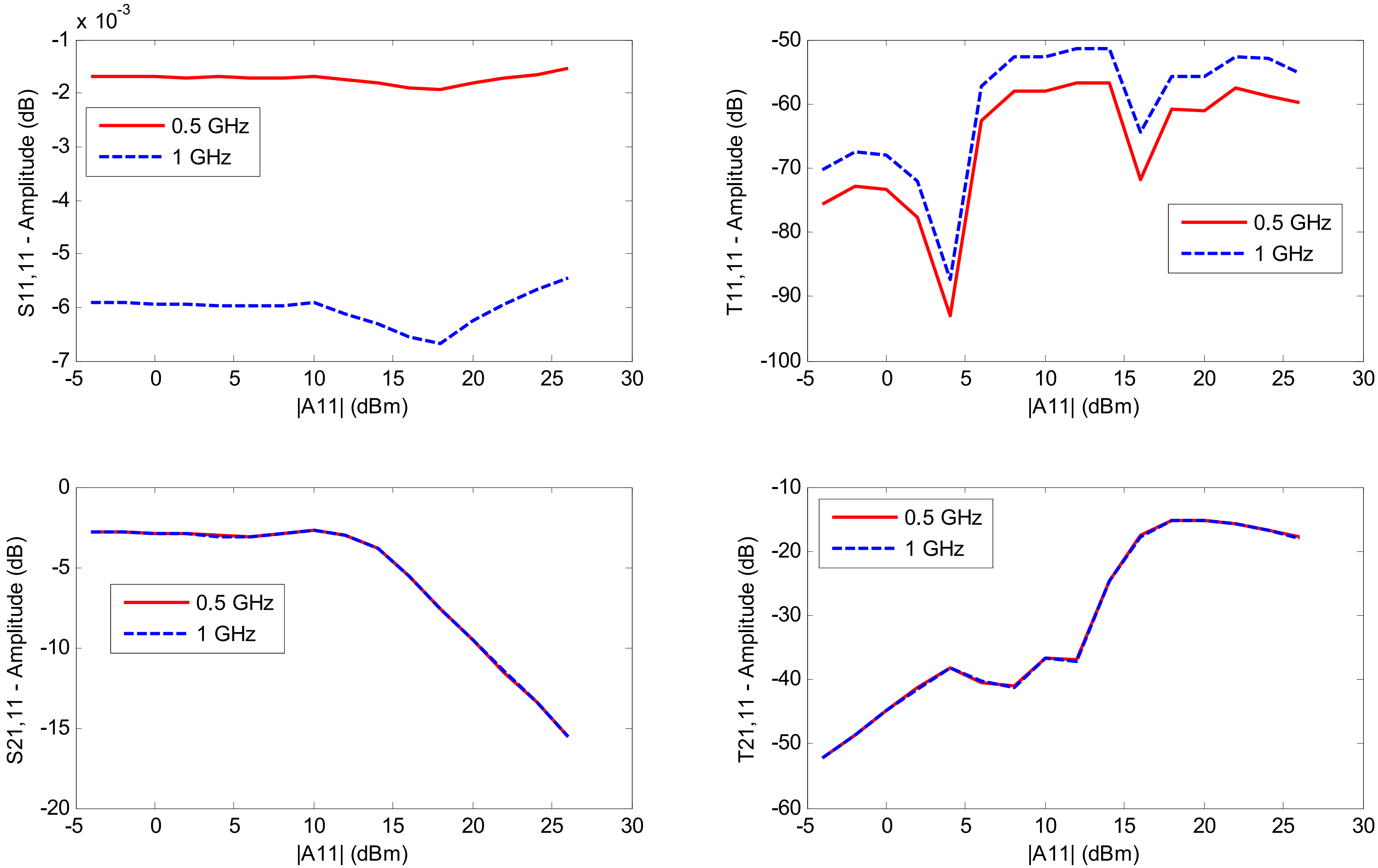
$$P : Phase\ of\ \ a_{11}$$

$$D_{ik} : B-type\ X\ \ parameter$$

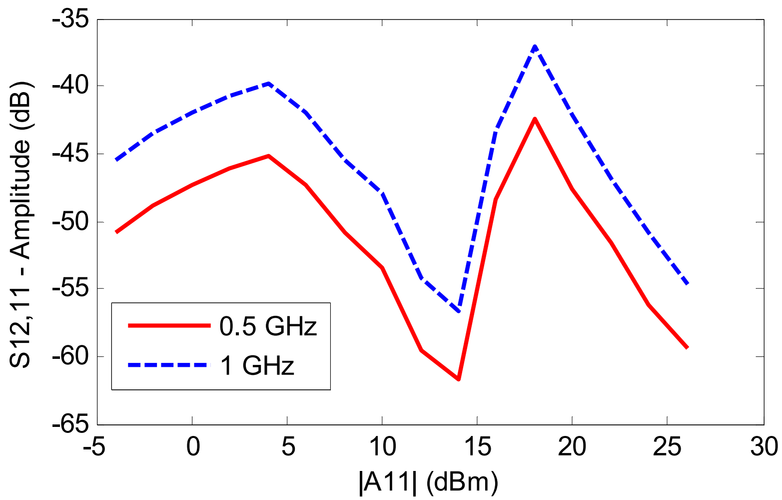
$$S_{ik,jl} : S-type\ X\ \ parameter$$

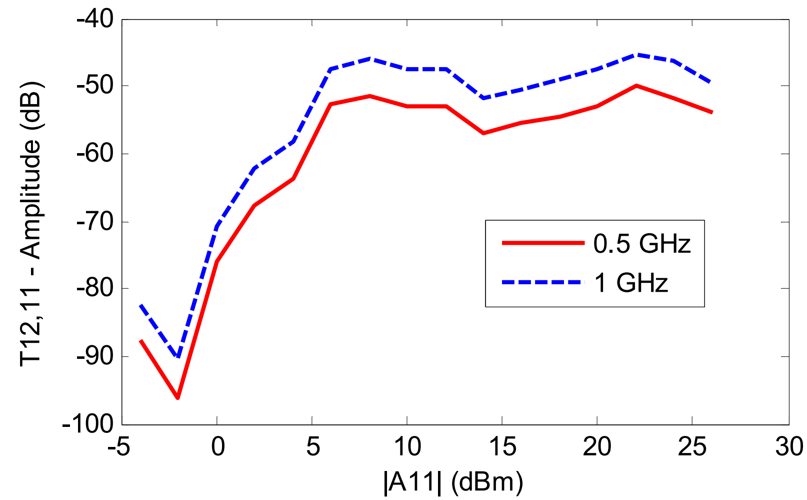
$$T_{ik,jl} : T-type\ X\ \ parameter$$

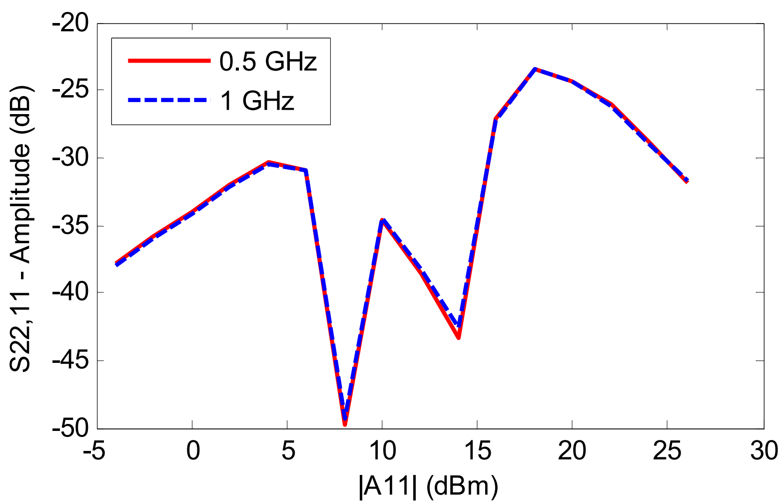
X Parameters of CMOS

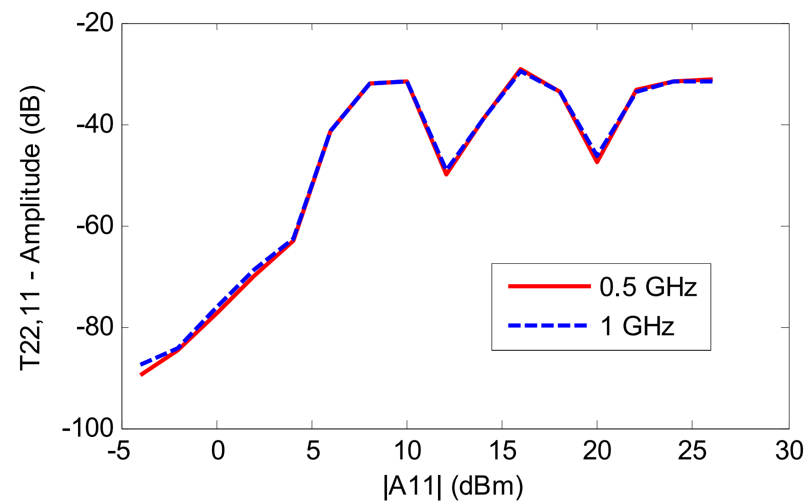


X Parameters of CMOS

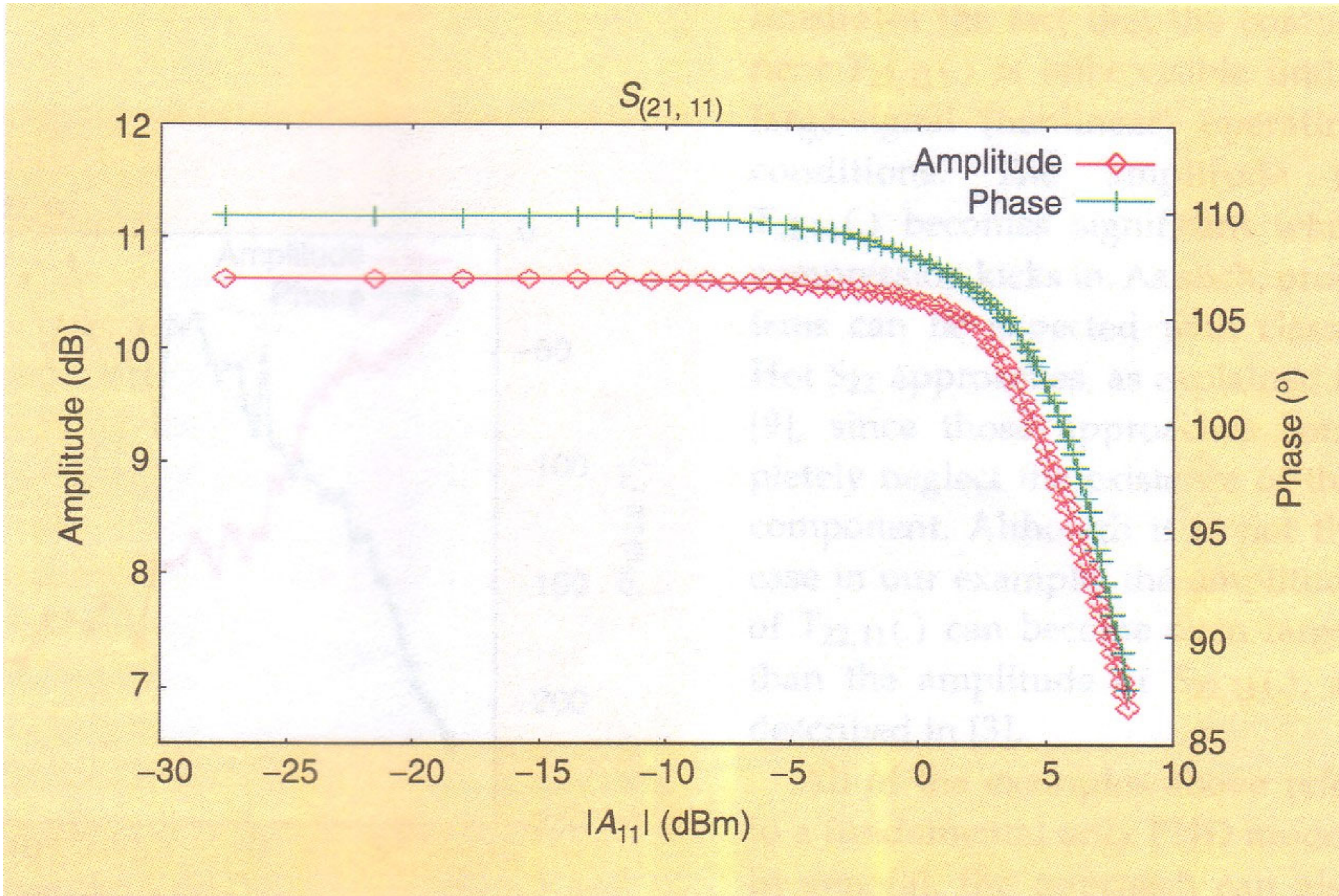






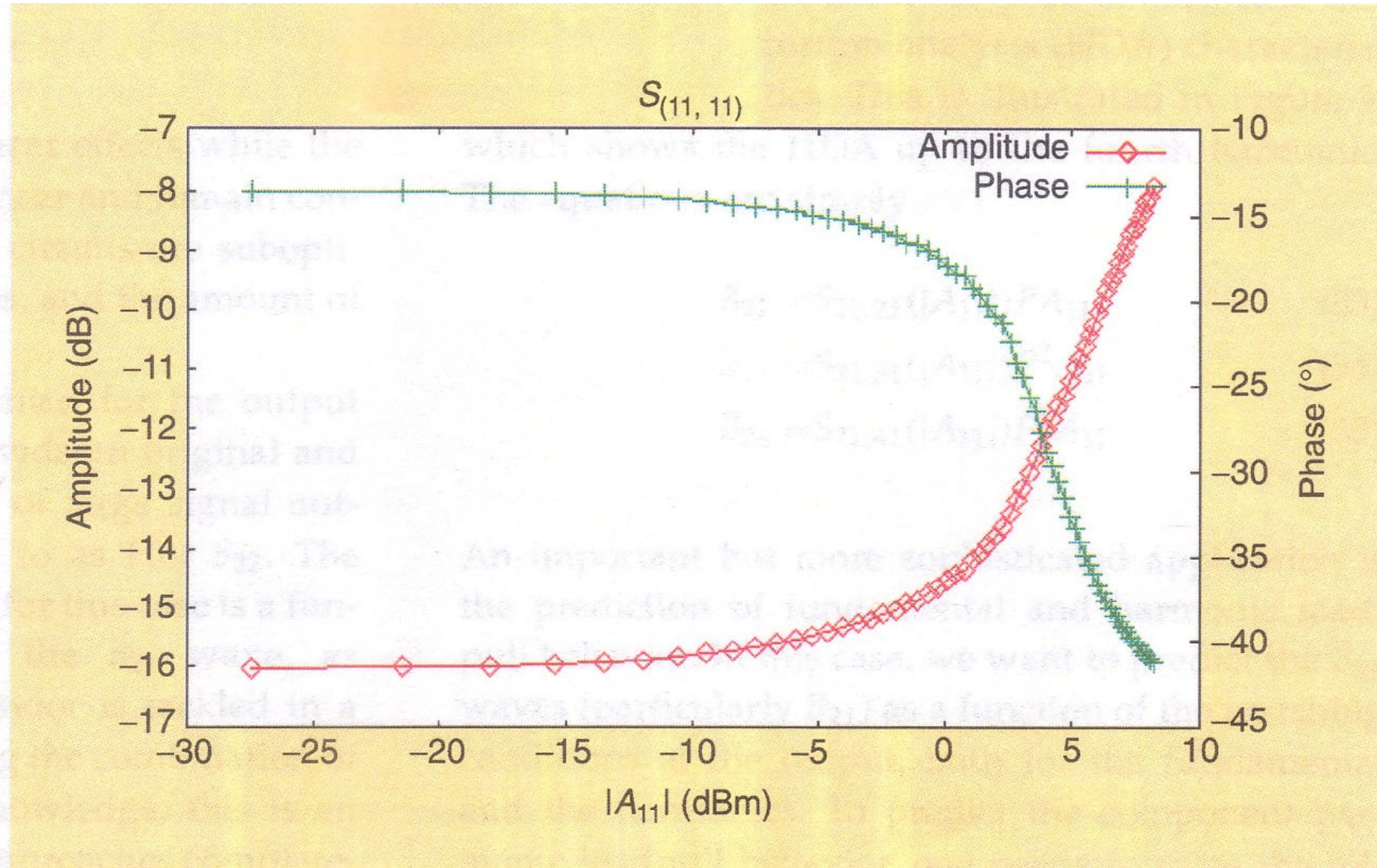


Large-Signal Reflection



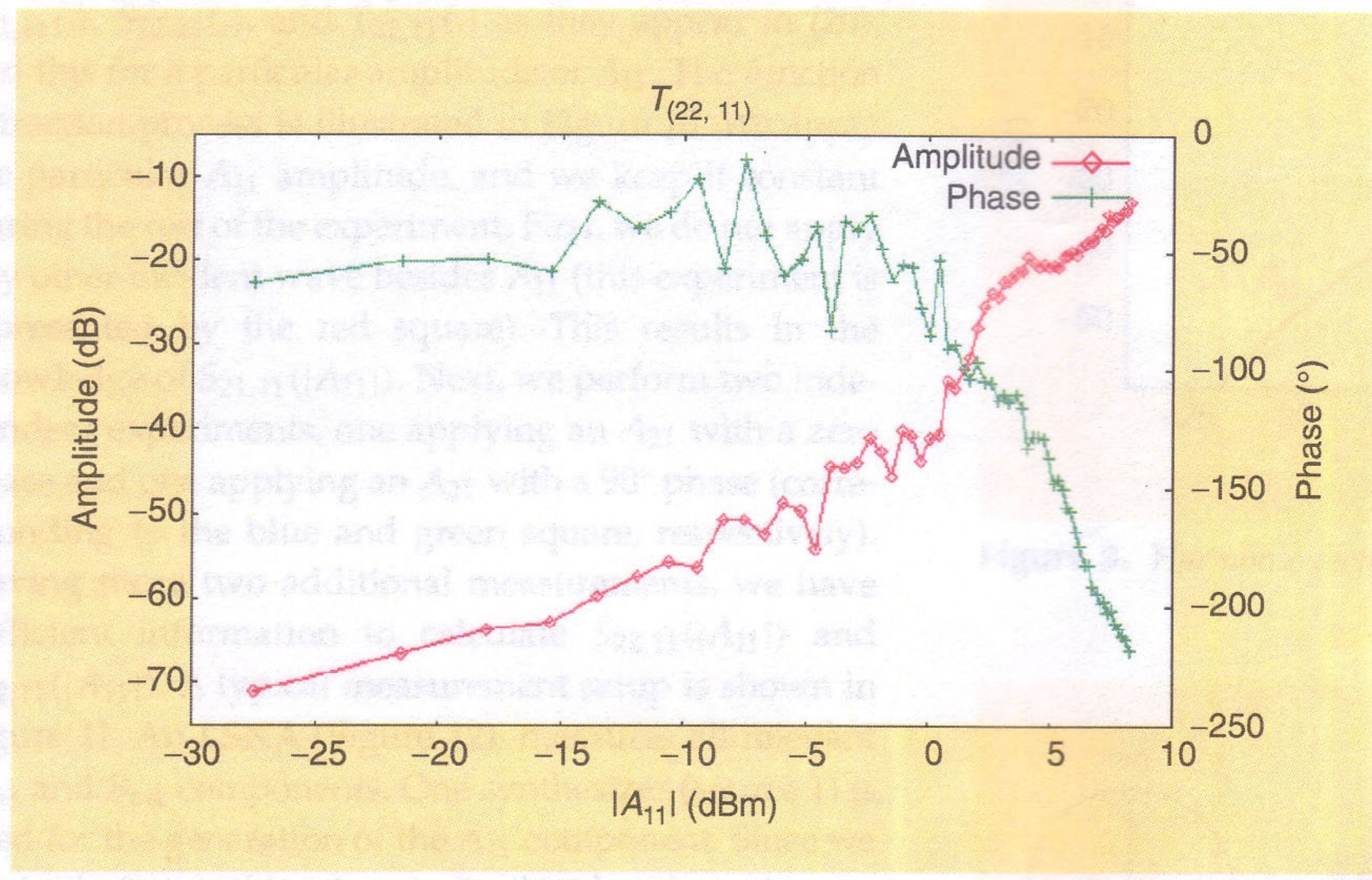
Microwave amplifier with fundamental frequency at 9.9 GHz

Compression and AM-PM



Microwave amplifier with fundamental frequency at 9.9 GHz

$T_{22,11}$



Microwave amplifier with fundamental frequency at 9.9 GHz

Special Terms

- **T-Type X Parameter**
  - ➢ Spectral mapping is non-analytical
  - ➢ Real and imaginary parts in FD are treated differently
  - ➢ Even and odd parts in TD are treated differently
  - ➢ *T* involves non-causal component of signal

- **Phase Term *P***
  - ➢ *P* is phase of large-signal excitation (a_{11})
  - ➢ Contributions to B waves will depend on *P*
  - ➢ In measurements, system must be calibrated for phase

Notation Change

Define

$$S_{ik,jl}\left(\left|a_{11}\right|\right) = X_{ik,jl}^{(S)}(|\ A_{11}\ |)$$

$$T_{ik,jl}\left(\left|a_{11}\right|\right) = X_{ik,jl}^{(T)}(|\ A_{11}\ |)$$

$$D_{ik}\left(\left|a_{11}\right|\right) = X_{ik}^{(FB)}\left(\left|a_{11}\right|\right)$$

Handling Phase Term

$$b_{ik} = D_{ik}\left(|a_{11}|\right)P^{k} + \sum_{(j,l)\neq(1,1)}\left[S_{ik,jl}\left(|a_{11}|\right)P^{k-l}a_{jl} + T_{ik,jl}\left(|a_{11}|\right)P^{k+l}a_{jl}^{*}\right]$$

Multiply through by P^{-k}

$$b_{ik}P^{-k} = D_{ik}\left(|a_{11}|\right) + \sum_{(j,l)\neq(1,1)}\left[S_{ik,jl}\left(|a_{11}|\right)P^{-l}a_{jl} + T_{ik,jl}\left(|a_{11}|\right)P^{+l}a_{jl}^{*}\right]$$

$P = e^{j\phi_{11}}$ where ϕ_{11} is the phase of a_{11}

we can always express the relationship in terms of modified power wave variables

$$\overline{b}_{ik} = D_{ik}\left(|a_{11}|\right) + \sum_{(j,l)\neq(1,1)}\left[S_{ik,jl}\left(|a_{11}|\right)\overline{a}_{jl} + T_{ik,jl}\left(|a_{11}|\right)\overline{a}_{jl}^{*}\right]$$

where $\overline{b}_{ik} = b_{ik}P^{-k}$ and $\overline{a}_{ik} = a_{ik}P^{-k}$

Handling R&I Components

Because of non-analytical nature of spectral mapping, real and imaginary component interactions must be accounted for separately.

we have

$$\begin{pmatrix} b_r \\ b_i \end{pmatrix} = \begin{pmatrix} X_{rr} & X_{ri} \\ X_{ir} & X_{ii} \end{pmatrix} \begin{pmatrix} a_r \\ a_i \end{pmatrix}$$

where

$$X_{rr} = \left(S_r + T_r\right), \;\; X_{ri} = -\left(S_i - T_i\right)$$

$$X_{ir} = \left(S_i + T_i\right), \;\; X_{ii} = \left(S_r - T_r\right)$$

Handling Phase Term

Phase term can be accounted for by applying following transformations

$$\begin{pmatrix} b_r \\ b_i \end{pmatrix} = \begin{pmatrix} X_{rr} & X_{ri} \\ X_{ir} & X_{ii} \end{pmatrix} \begin{pmatrix} a_r \\ a_i \end{pmatrix}$$

$$\begin{pmatrix} \cos\theta_b & -\sin\theta_b \\ -\sin\theta_b & \cos\theta_b \end{pmatrix} \begin{pmatrix} b_r' \\ b_i' \end{pmatrix} = \begin{pmatrix} X_{rr} & X_{ri} \\ X_{ir} & X_{ii} \end{pmatrix} \begin{pmatrix} \cos\theta_a & -\sin\theta_a \\ -\sin\theta_a & \cos\theta_a \end{pmatrix} \begin{pmatrix} a_r' \\ a_i' \end{pmatrix}$$

in which

$$\begin{pmatrix} b_r \\ b_i \end{pmatrix} = \begin{pmatrix} \cos\theta_b & -\sin\theta_b \\ -\sin\theta_b & \cos\theta_b \end{pmatrix} \begin{pmatrix} b_r' \\ b_i' \end{pmatrix}$$

$$\begin{pmatrix} a_r \\ a_i \end{pmatrix} = \begin{pmatrix} \cos\theta_a & -\sin\theta_a \\ -\sin\theta_a & \cos\theta_a \end{pmatrix} \begin{pmatrix} a_r' \\ a_i' \end{pmatrix}$$

X Matrix Construction

- Separate real and imaginary components
- Account for real-imaginary interactions
- Account for harmonic-to-harmonic contributions
- Account for harmonic-to-DC contributions

Matrix size is $2mn \times 2mn$

m: number of harmonics

n: number of ports

Matrix Formulation*

We wish to use:

$$\mathbf{b} = \mathbf{X}\mathbf{a}$$

size:2*mn*

$$\mathbf{a} = \begin{pmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_p \\ \mathbf{a}_n \end{pmatrix}$$

size:2 *mn*

$$\mathbf{b} = \begin{pmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \mathbf{b}_p \\ \mathbf{b}_n \end{pmatrix}$$

$$\mathbf{a}_p = \begin{pmatrix} a_{pr}^{(1)} \\ a_{pi}^{(1)} \\ a_{pr}^{(2)} \\ a_{pi}^{(2)} \\ . \\ . \\ a_{pr}^{(m)} \\ a_{pi}^{(m)} \end{pmatrix} \qquad \mathbf{b}_p = \begin{pmatrix} b_{pr}^{(1)} \\ b_{pi}^{(1)} \\ b_{pr}^{(2)} \\ b_{pi}^{(2)} \\ . \\ . \\ b_{pr}^{(m)} \\ b_{pi}^{(m)} \end{pmatrix}$$

vector size is 2*m*

***m*: number of harmonics**

***n*: number of ports**

(real vectors)

*DC term not included

Matrix Formulation*

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_{11} & \mathbf{X}_{12} & \cdot & \mathbf{X}_{1n} \\ \mathbf{X}_{21} & \mathbf{X}_{22} & \cdot & \cdot \\ \cdot & \cdot & \mathbf{X}_{pq} & \cdot \\ \mathbf{X}_{n1} & \cdot & \cdot & \mathbf{X}_{nn} \end{pmatrix}$$

← **matrix size is 2*mn* × 2*mn***

***m*: number of harmonics**

***n*: number of ports**

size: 2*m* × 2*m*

$$\mathbf{X}_{pq} = \begin{pmatrix} X_{pqrr}^{(11)} & X_{pqri}^{(11)} & X_{pqrr}^{(12)} & X_{pqri}^{(12)} & \cdot & \cdot & X_{pqrr}^{(1m)} & X_{pqri}^{(1m)} \\ X_{pqir}^{(11)} & X_{pqii}^{(11)} & X_{pqir}^{(12)} & X_{pqii}^{(12)} & \cdot & \cdot & \cdot & \cdot \\ X_{pqrr}^{(21)} & X_{pqri}^{(21)} & X_{pqrr}^{(22)} & X_{pqri}^{(22)} & \cdot & \cdot & \cdot & \cdot \\ X_{pqir}^{(21)} & X_{pqii}^{(21)} & X_{pqir}^{(22)} & X_{pqii}^{(22)} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ X_{pqir}^{(m1)} & X_{pqii}^{(m1)} & \cdot & \cdot & \cdot & \cdot & X_{pqir}^{(mm)} & X_{pqii}^{(mm)} \end{pmatrix}$$

(real matrix)

*DC term not included

X Matrix for 2-Port System*

(2 harmonics)

$$\mathbf{X}=\begin{pmatrix}
X_{11rr}^{(11)} & X_{11ri}^{(11)} & X_{11rr}^{(12)} & X_{11ri}^{(12)} & X_{12rr}^{(11)} & X_{12ri}^{(11)} & X_{12rr}^{(12)} & X_{12ri}^{(12)} \\
X_{11ir}^{(11)} & X_{11ii}^{(11)} & X_{11ir}^{(12)} & X_{11ii}^{(12)} & X_{12ir}^{(11)} & X_{12ii}^{(11)} & X_{12ir}^{(12)} & X_{12ii}^{(12)} \\
X_{11rr}^{(21)} & X_{11ri}^{(21)} & X_{11rr}^{(22)} & X_{11ri}^{(22)} & X_{12rr}^{(21)} & X_{12ri}^{(21)} & X_{12rr}^{(21)} & X_{12ri}^{(21)} \\
X_{11ir}^{(21)} & X_{11ii}^{(21)} & X_{11ir}^{(22)} & X_{11ii}^{(22)} & X_{12ir}^{(21)} & X_{12ii}^{(21)} & X_{12ir}^{(22)} & X_{12ii}^{(22)} \\
X_{21rr}^{(11)} & X_{21ri}^{(11)} & X_{21rr}^{(12)} & X_{21ri}^{(12)} & X_{22rr}^{(11)} & X_{22ri}^{(11)} & X_{22rr}^{(12)} & X_{22ri}^{(12)} \\
X_{21ir}^{(11)} & X_{21ii}^{(11)} & X_{21ir}^{(12)} & X_{21ii}^{(12)} & X_{22ir}^{(11)} & X_{22ii}^{(11)} & X_{22ir}^{(12)} & X_{22ii}^{(12)} \\
X_{21rr}^{(21)} & X_{21ri}^{(21)} & X_{21rr}^{(22)} & X_{21ri}^{(22)} & X_{22rr}^{(21)} & X_{22ri}^{(21)} & X_{22rr}^{(22)} & X_{22ri}^{(22)} \\
X_{21ir}^{(21)} & X_{21ii}^{(21)} & X_{21ir}^{(22)} & X_{21ii}^{(22)} & X_{22ir}^{(21)} & X_{22ii}^{(21)} & X_{22ir}^{(22)} & X_{22ii}^{(22)}
\end{pmatrix}$$

(real matrix)

For instance, $X^{(12)}_{21ri}$ is the contribution to the real part of the *1st* harmonic of the wave scattered at port 2 due to the imaginary part of the *2nd* harmonic of the wave incident port in port 1.

***DC term not included**

Polyharmonic Impedance

Linear Impedance	Polyharmonic Impedance	Nonlinear Impedance
- **Time invariant** - **Linear** - **Scalar**	- **Time invariant** - **Linear** - **Matrix**	- **Time variant** - **Nonlinear** - **Function**
$V = ZI$ FD & TD	$[V(f)]=[Z(f)][I(f)]$ FD rop	$V(t) = Z(I(t))$

Model assumes that nonlinear effects are mild and are captured via harmonic superposition.

Polyharmonic Impedance

4-harmonic system

in frequency domain:

$$\begin{bmatrix} V^{(1)} \\ V^{(2)} \\ V^{(3)} \\ V^{(4)} \end{bmatrix} = \begin{bmatrix} Z^{(11)} & Z^{(12)} & Z^{(13)} & Z^{(14)} \\ Z^{(21)} & Z^{(22)} & Z^{(23)} & Z^{(24)} \\ Z^{(31)} & Z^{(32)} & Z^{(33)} & Z^{(34)} \\ Z^{(41)} & Z^{(42)} & Z^{(43)} & Z^{(44)} \end{bmatrix} \begin{bmatrix} I^{(1)} \\ I^{(2)} \\ I^{(3)} \\ I^{(4)} \end{bmatrix}$$

in time domain:

$$v(t) = v^{(1)}(t) + v^{(2)}(t) + v^{(3)}(t) + v^{(4)}(t)$$

$$i(t) = i^{(1)}(t) + i^{(2)}(t) + i^{(3)}(t) + i^{(4)}(t)$$

Polyharmonic Impedance

$\mathbf{Z_o}$: Reference impedance matrix

$\mathbf{Z}$: Polyharmonic impedance matrix

$\mathbf{V}$: Voltage vector

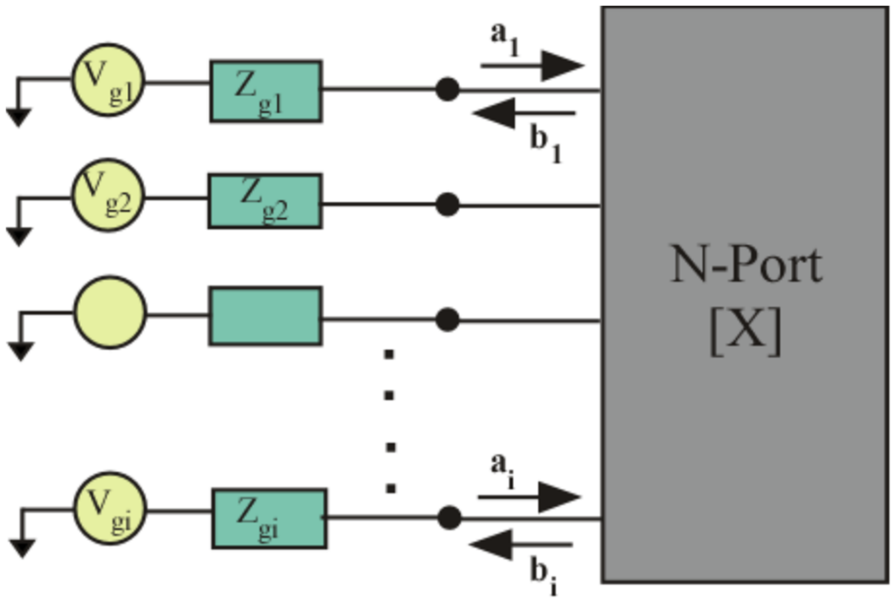
$\mathbf{I}$: Current vector

$$\mathbf{Z} = (\mathbf{1}+\mathbf{X})(\mathbf{1}-\mathbf{X})^{-1}\mathbf{Z_o}$$

$$\mathbf{V} = \mathbf{ZI}$$

Describes interactions between harmonic components of voltage and current.

Network Formulation



Scattered waves

$$\mathbf{b} = \mathbf{X}\mathbf{a}$$

Termination equations

$$\mathbf{a} = \mathbf{D}\mathbf{v}_g + \Gamma\mathbf{b}$$

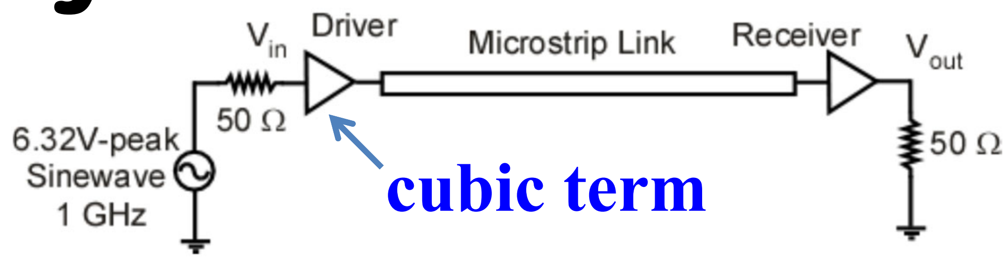
Wave Solution

$$\mathbf{a} = \left[1 - \Gamma\mathbf{X}\right]^{-1}\mathbf{D}\mathbf{v}_g$$

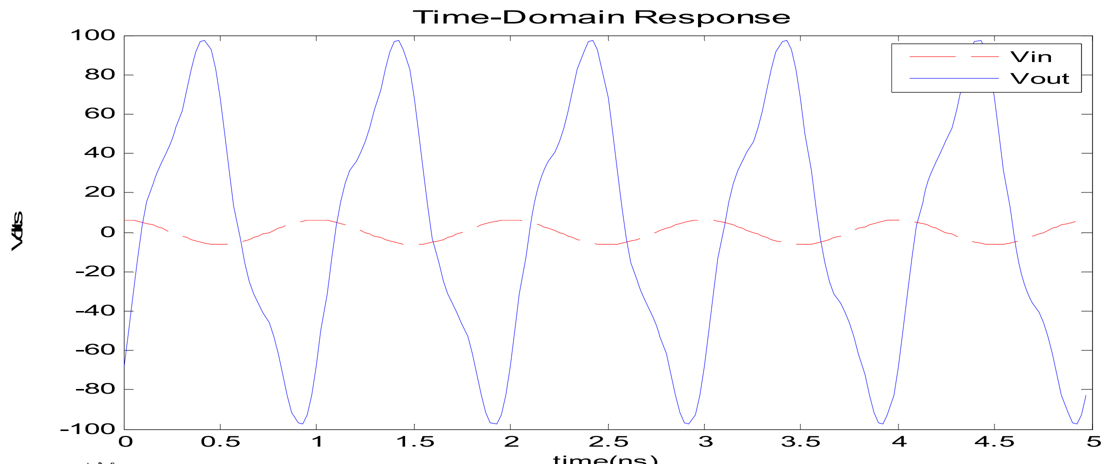
Voltage Solution

$$\mathbf{v} = \left(1 + \mathbf{X}\right)\mathbf{a}$$

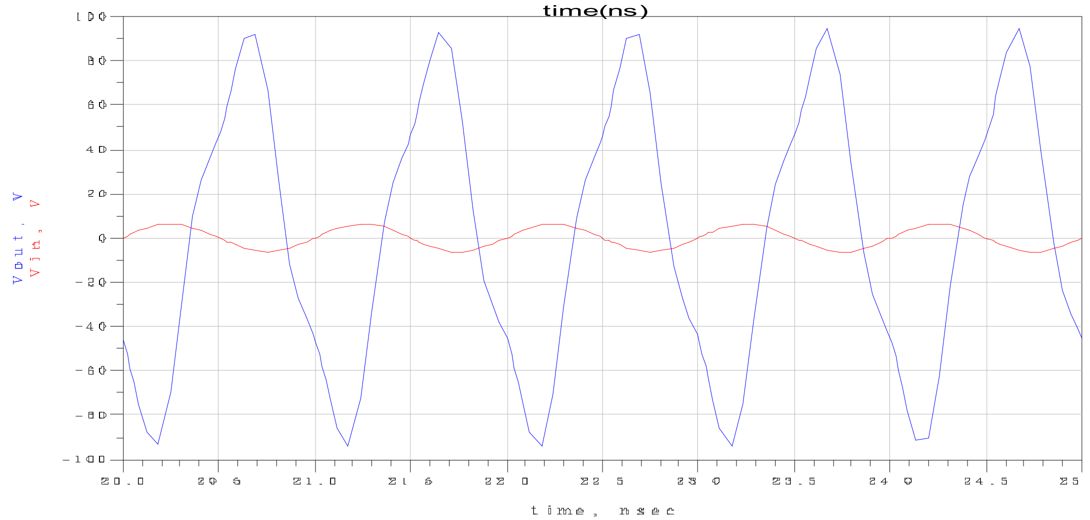
Steady-State Simulations



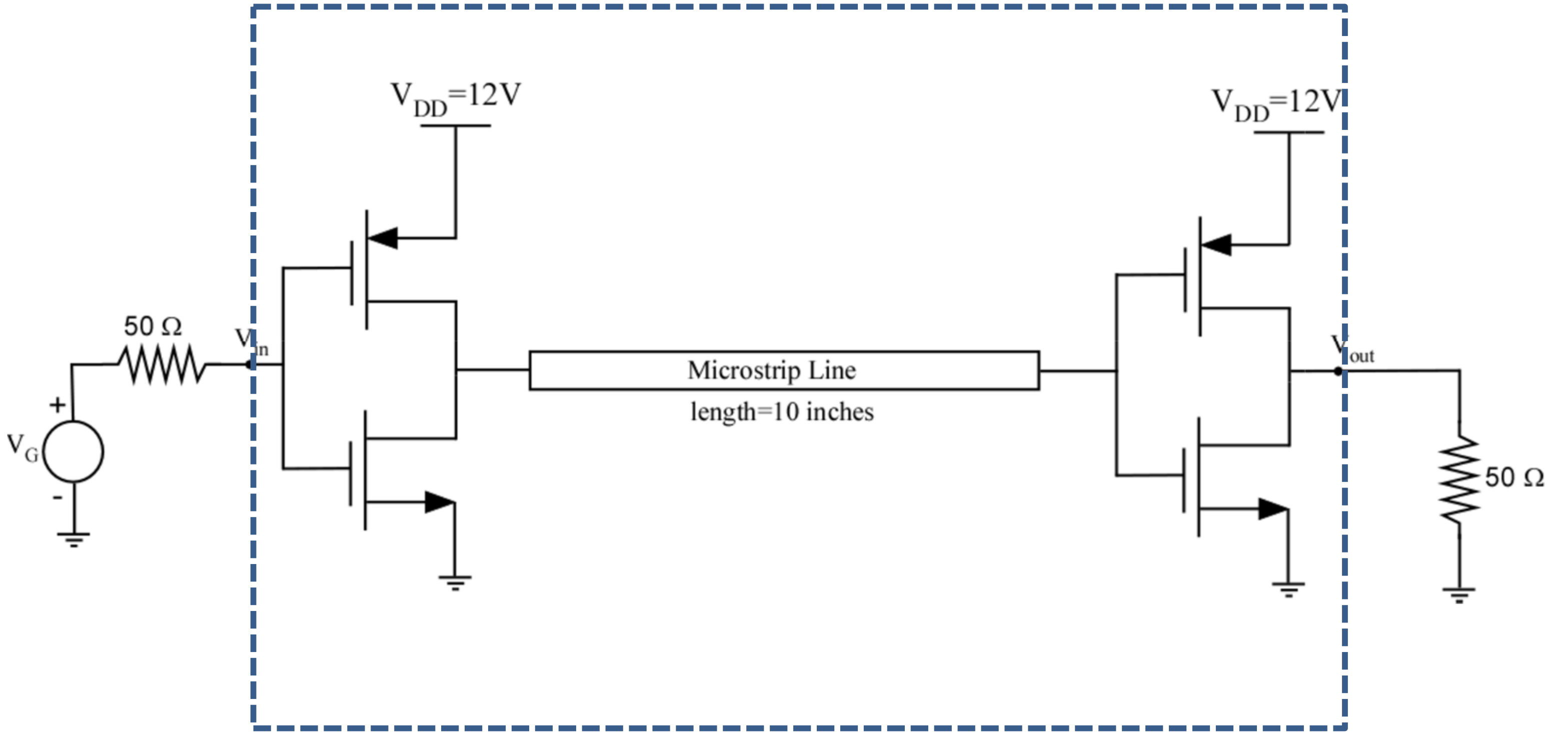
X Parameter



ADS

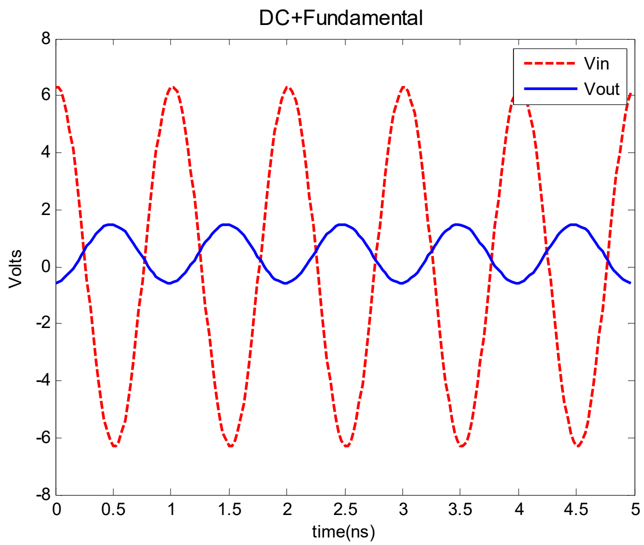


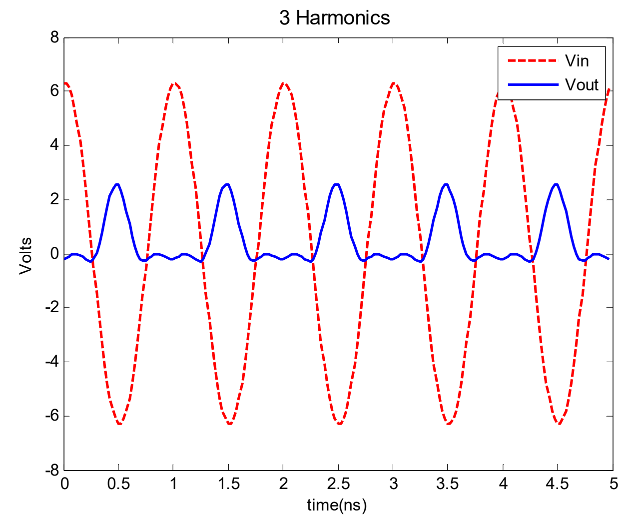
CMOS Driver/Receiver Channel

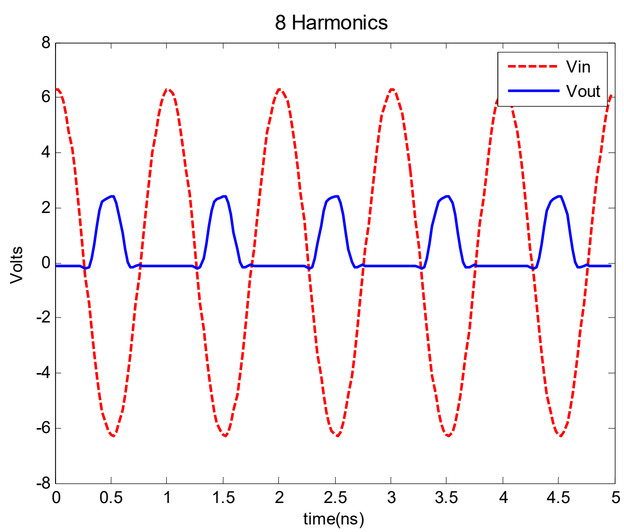


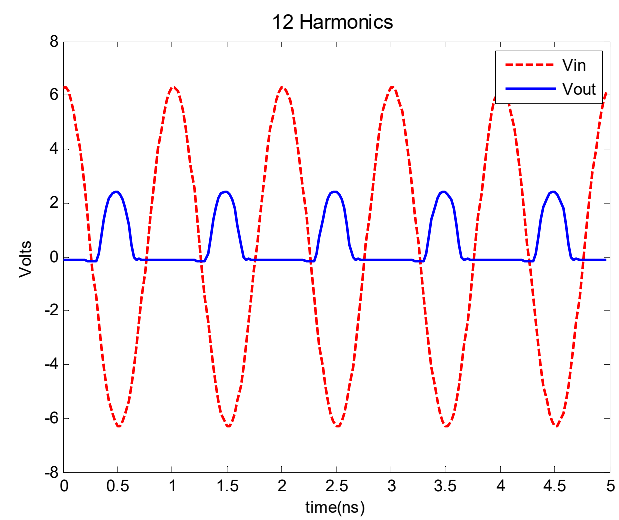
- Generate X parameters for composite system
- Power level: 20 dBm, frequency: 1 GHz
- Construct X matrix
- Combine with terminations for simulation

CMOS Driver/Receiver - Harmonics

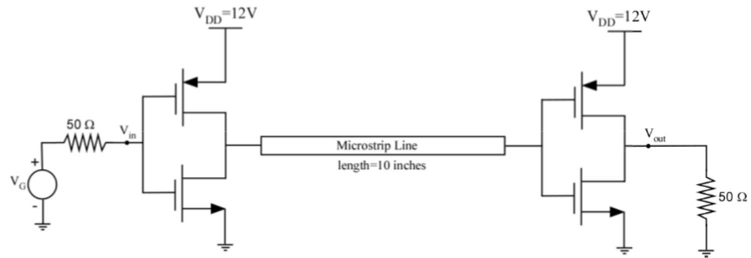




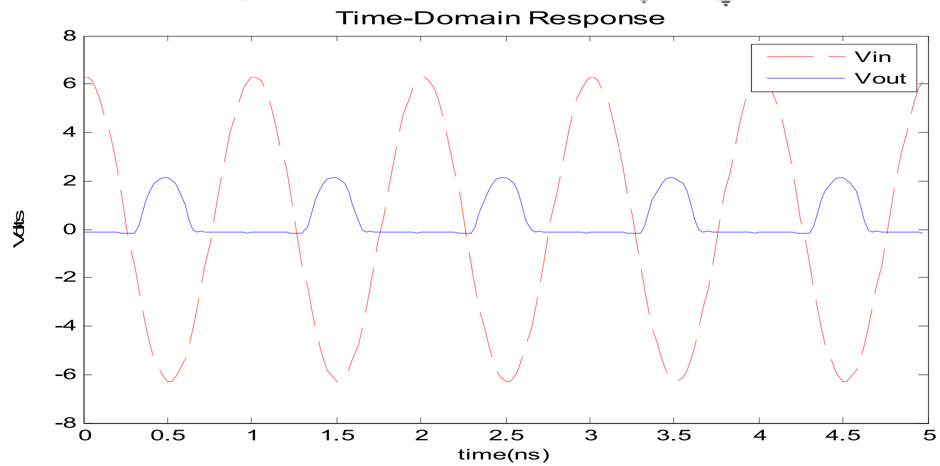




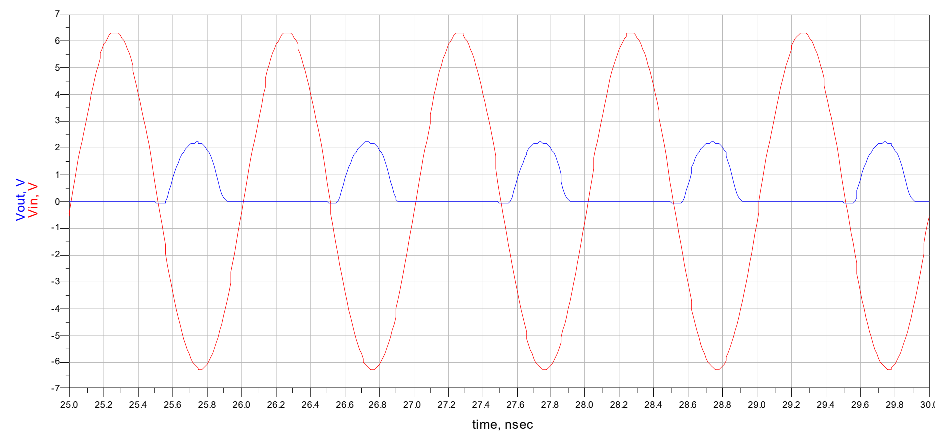
Validation



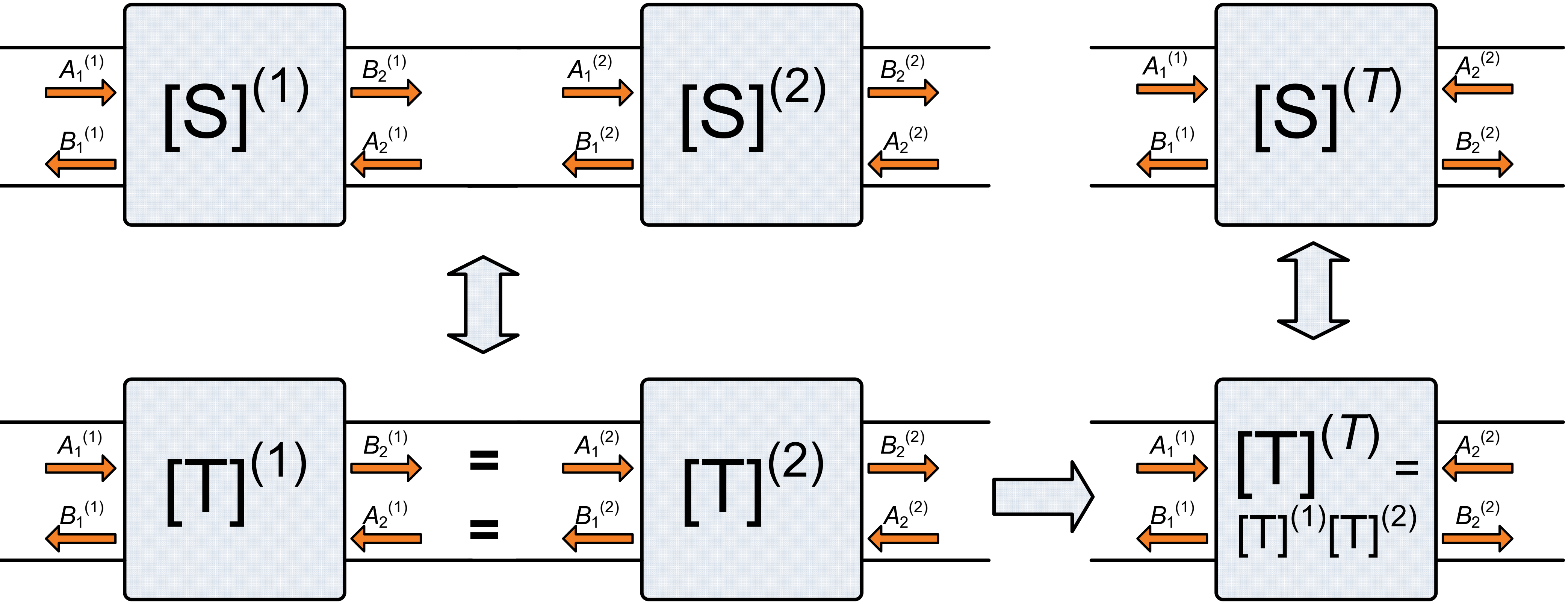
X Parameter



ADS



Cascading S-Parameter Blocks*

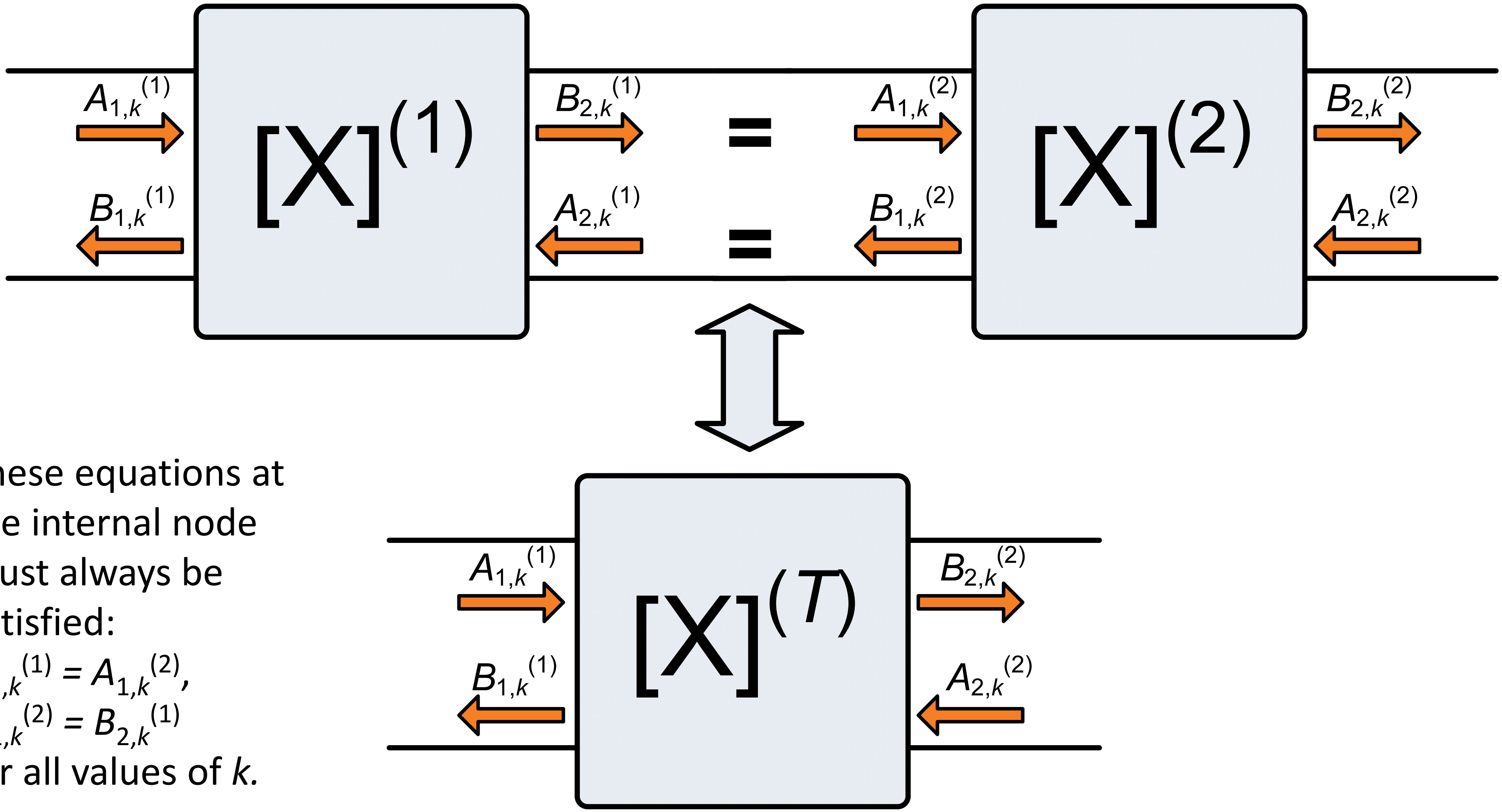


[T] = transfer scattering parameters.

Can disregard circuit behavior at internal node.

*G. Gonzalez, *Microwave Transistor Amplifiers: Analysis and Design*, 2nd ed. Prentice-Hall, 1997.

Cascading X-Parameter Blocks*

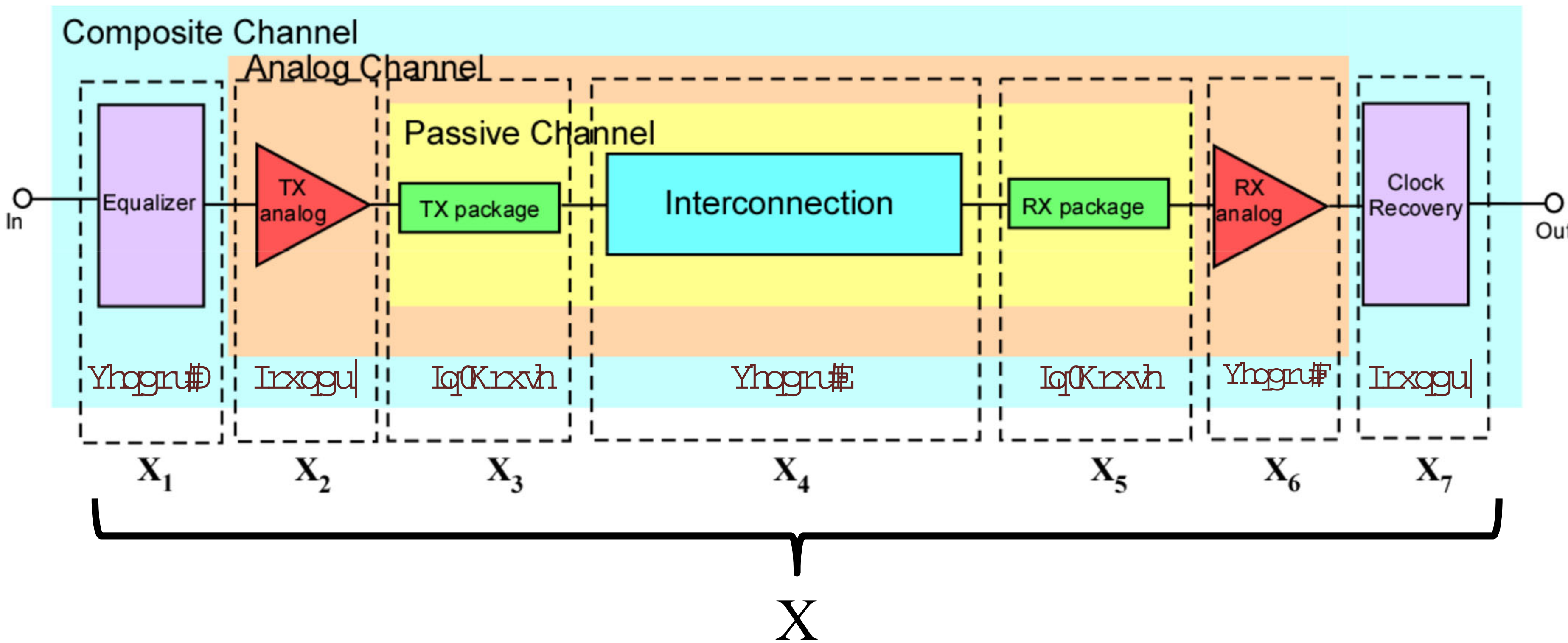


These equations at the internal node must always be satisfied:
$B_{1,k}^{(1)} = A_{1,k}^{(2)}$,
$A_{1,k}^{(2)} = B_{2,k}^{(1)}$
for all values of k.

*D. E. Root, *et al.*, *X-Parameters*, 2013.

Cascading X Parameters

GOAL: Simulate complete channel by combining X-parameter blocks from different sources into a single composite X matrix.



X-parameters of individual devices can be accurately cascaded within a harmonic balance simulator environment.

Volterra Series

A linear causal system with memory can be described by the convolution representation

$$y(t)=\int_{-\infty}^{+\infty}h(\sigma)x(t-\sigma)\,d\sigma$$

where *x(t)* is the input, *y(t)* is the output, and *h(t)* the impulse response of the system.

A nonlinear system without memory can be described with a Taylor series as:

$$y(t)=\sum_{n=1}^{\infty}a_n\left[x(t)\right]^n$$

where *x(t)* is the input and *y(t)* is the output. The an are Taylor series coefficients.

Volterra Series

A Volterra series combines the above two representations to describe a nonlinear system with memory

$$y(t)=\sum_{n=1}^{\infty}\frac{1}{n!}\int_{-\infty}^{\infty}du_1\ldots\int_{-\infty}^{\infty}du_n g_n\left(u_1,\ldots,u_n\right)\prod_{r=1}^{n}x(t-u_r)$$

$$y(t)=\frac{1}{1!}\int_{-\infty}^{\infty}du_1 g_1(u_1)x(t-u_1)$$

← **impulse response**

$$+\frac{1}{2!}\int_{-\infty}^{\infty}du_1\int_{-\infty}^{\infty}du_2 g_2(u_1,u_2)x(t-u_1)x(t-u_2)$$

← **higher-order impulse responses**

$$+\frac{1}{2!}\int_{-\infty}^{\infty}du_1\int_{-\infty}^{\infty}du_2\int_{-\infty}^{\infty}du_3 g_3(u_1,u_2,u_3)x(t-u_1)x(t-u_2)g_2(u_1,u_2)x(t-u_1)x(t-u_2)x(t-u_3)$$

$$+\ldots$$

where *x(t)* is the input and *y(t)* is the output and the $g_n(u_1,\ldots,u_n)$ are called the *Volterra kernels*

Volterra Series

Application to X parameters:

Take order = 2

$$y(t)=\frac{1}{1!}\int_{-\infty}^{+\infty}du_1 g_1(u_1)x(t-u_1)+\frac{1}{2!}\int_{-\infty}^{+\infty}du_1\int_{-\infty}^{+\infty}du_2 g_2(u_1,u_2)x(t-u_1)x(t-u_2)$$

where the input *x(t)* is given by

$$x(t)=\exp(j\omega_1 t)+\exp(j\omega_2 t)$$

This can be written as

$$y(t)=\frac{1}{1!}\int_{-\infty}^{+\infty}du_1 g_1(u_1)\left[e^{j\omega_1(t-u_1)}+e^{j\omega_2(t-u_1)}\right]$$

$$+\frac{1}{2!}\int_{-\infty}^{+\infty}du_1\int_{-\infty}^{+\infty}du_2 g_2(u_1,u_2)\left[e^{j\omega_1(t-u_1)}+e^{j\omega_2(t-u_1)}\right]\left[e^{j\omega_1(t-u_2)}+e^{j\omega_2(t-u_2)}\right]$$

Volterra Series

Define $T_1 = e^{j\omega_1 t}, \quad T_2 = e^{j\omega_2 t}$

$$U_{11} = e^{-j\omega_1 u_1} \qquad U_{12} = e^{-j\omega_1 u_2}$$

$$U_{22} = e^{-j\omega_2 u_2} \qquad U_{21} = e^{-j\omega_2 u_1}$$

This gives

$$y(t) = T_1 \int_{-\infty}^{+\infty} du_1 g_1(u_1) U_{11} + T_2 \int_{-\infty}^{+\infty} du_1 g_1(u_1) U_{21}$$

$$+ \frac{1}{2!} \int_{-\infty}^{+\infty} du_1 \int_{-\infty}^{+\infty} du_2 g_2(u_1, u_2) [T_1 U_{11} + T_2 U_{21}][T_1 U_{12} + T_2 U_{22}]$$

or

$$y(t) = I_1 + I_2 + I_3 + I_4 + I_5 + I_6$$

Volterra Series

in which

$$I_1 = T_1 \int_{-\infty}^{+\infty} du_1 g_1(u_1) U_{11} \quad \rightarrow \quad I_1 = T_1 G_1(f_1)$$

$$I_2 = T_2 \int_{-\infty}^{+\infty} du_1 g_1(u_1) U_{21} \quad \rightarrow \quad I_2 = T_2 G_1(f_2)$$

$G_1(f)$ is the Fourier transform of $g_1(u)$ evaluated at f

Volterra Series

$$I_3 = T_1^2 \frac{1}{2!} \int_{-\infty}^{+\infty} du_1 \int_{-\infty}^{+\infty} du_2 g_2(u_1,u_2) U_{11} U_{12} = \frac{1}{2} T_1^2 G_2(f_1,f_1)$$

$$I_4 = \frac{1}{2!} T_1 T_2 \int_{-\infty}^{+\infty} du_1 \int_{-\infty}^{+\infty} du_2 g_2(u_1,u_2) U_{11} U_{22} = \frac{1}{2} T_1 T_2 G_2(f_1,f_2)$$

$$I_5 = \frac{1}{2!} T_2 T_1 \int_{-\infty}^{+\infty} du_1 \int_{-\infty}^{+\infty} du_2 g_2(u_1,u_2) U_{21} U_{12} = \frac{1}{2} T_2 T_1 G_2(f_2,f_1)$$

$$I_6 = \frac{1}{2!} T_2^2 \int_{-\infty}^{+\infty} du_1 \int_{-\infty}^{+\infty} du_2 g_2(u_1,u_2) U_{21} U_{22} = \frac{1}{2} T_2^2 G_2(f_2,f_2)$$

$G_2(f_1, f_2)$ is the double Fourier transform of $g(u,v)$ evaluated at (f_1, f_2)

Volterra Series

So, $G_1(f)$ is the Fourier transform of $g_1(u)$ evaluated at f and $G_2(f_1, f_2)$ is the double Fourier transform of $g(u,v)$ evaluated at (f_1, f_2)

We can also express y(t) as:

$$y(t) = y_1(t) + y_2(t)$$

in which

$$y_1(t) = T_1 G_1(f_1) + T_2 G_1(f_2)$$

and

$$y_2(t) = \frac{1}{2!}\left[T_1^2 G_2(f_1, f_1) + T_1 T_2 G_2(f_1, f_2) + T_1 T_2 G_2(f_2, f_1) + T_2^2 G_2(f_2, f_2)\right]$$

Volterra Series

If we take into account the respective amplitudes of the tone, we have

$$x(t) = A_1 \exp\left(j\omega_1 t\right) + A_2 \exp\left(j\omega_2 t\right)$$

We can make the transformation

$$T_1 \rightarrow A_1 T_1 \qquad \text{and} \qquad T_2 \rightarrow A_2 T_2$$

$$y(t) = \int_{-\infty}^{+\infty} du_1 g_1\left(u_1\right)\left[A_1 e^{j\omega_1(t-u_1)} + A_2 e^{j\omega_2(t-u_1)}\right]$$

$$+\frac{1}{2!}\int_{-\infty}^{+\infty} du_1 \int_{-\infty}^{+\infty} du_2 g_2\left(u_1,u_2\right)\left[A_1 e^{j\omega_1(t-u_1)} + A_2 e^{j\omega_2(t-u_1)}\right]$$

$$\times\left[A_1 e^{j\omega_1(t-u_2)} + A_2 e^{j\omega_2(t-u_2)}\right]$$

$$y_1(t) = \underbrace{A_1 T_1 G_1\left(f_1\right)}_{A-Term=H_A} + \underbrace{A_2 T_2 G_1\left(f_2\right)}_{B-Term=H_B}$$

Volterra Series

$$y_2(t) = \frac{1}{2!}\left[\underbrace{A_1^2 T_1^2 G_2(f_1, f_1)}_{C-Term=H_C} + \underbrace{A_1 T_1 A_2 T_2 G_2(f_1, f_2) + A_1 T_1 A_2 T_2 G_2(f_2, f_1)}_{D-Term=H_D} + \underbrace{A_2^2 T_2^2 G_2(f_2, f_2)}_{E-Term=H_E}\right]$$

If we choose $f_2 = kf_1$, then $T_1 \rightarrow f_1$ and $T_2 \rightarrow kf_1$.

In general, $\omega_2 = k\omega_1$ so that if

$$T_1 \rightarrow f_1 \, , \; T_2 \rightarrow kf_1$$

H_A=*A-Term* contains terms in f_1

H_B=*B-Term* contains terms in kf_1

H_C=*C-Term* contans terms in $2f_1$

H_D=*D-Term* contains terms in $2kf_1$

H_E=*E-Term* contains terms in $(k+1)f_1$

Volterra Series

- First determine the X parameters of the system

- Next, provide excitation *a(t)*

$$a(t) = A_1 \exp\left(j\omega_1 t\right) + A_2 \exp\left(j\omega_2 t\right)$$

- Next, calculate b in phasor domain using X parameters

b=Xa

- For each port, the scattered wave will include contributions from all harmonics

$$b_p = H_A + H_B + H_C + H_D + H_E$$

Volterra Series

Finally, a relationship can be obtained to extract Volterra kernel Fourier transforms

A-Term:→ $G_1(f_1) = \frac{H_A}{A_1}$

B-Term:→ $G_1(f_2) = \frac{H_B}{A_2}$

C-Term:→ $G_2(f_1, f_1) = \frac{2H_C}{A_1^2}$

D-Term:→ $G_2(f_2, f_2) = \frac{2H_D}{A_2^2}$

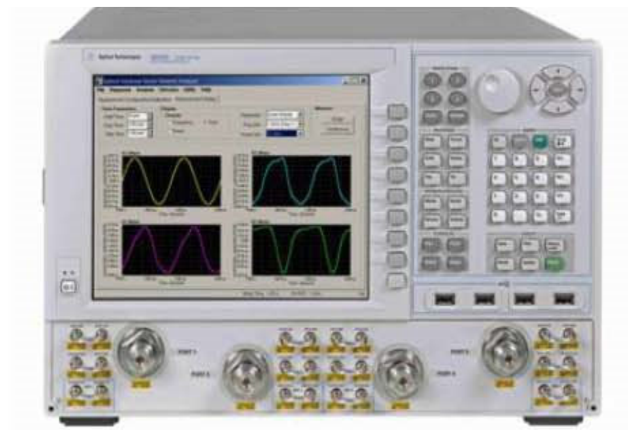
E-Term:→ $G_2(f_1, f_2) = \frac{H_E}{A_1 A_2}$

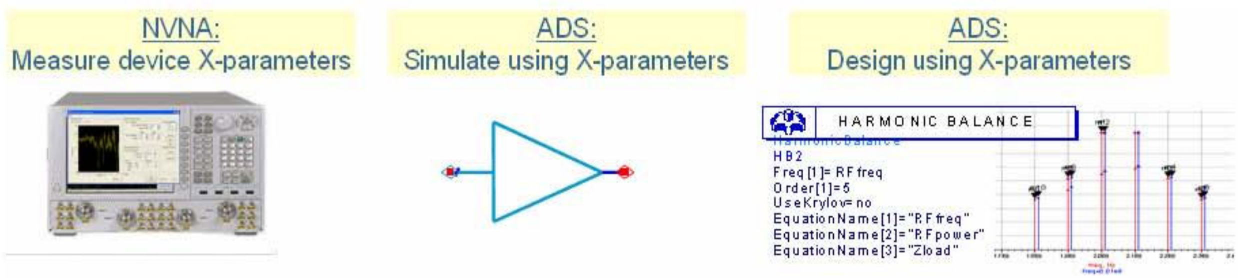
Volterra Series

TABLE OF VOLTERRA KERNEL TRANSFORMS

Term	Wave	Coefficient	Constant	index	k=1	k = 2	k = 3	k = 3
A-Term	T_1	$G_1(f_1)$	A_1	f_1	f_1	f_1	f_1	f_1
B-Term	T_2	$G_1(f_2)$	A_2	$k\ f_1$	f_1	$2\ f_1$	$3\ f_1$	$4\ f_1$
C-Term	T_1^2	$G_2(f_1,f_1)$	$A_1^2/2!$	$2\ f_1$	$2\ f_1$	$2\ f_1$	$2\ f_1$	$2\ f_1$
D-Term	T_2^2	$G_2(f_2,f_2)$	$A_2^2/2!$	$2k\ f_1$	$2\ f_1$	$4\ f_1$	$6\ f_1$	$8\ f_1$
E-Term	T_1T_2	$G_2(f_1,f_2)$	A_1A_2	$(k+1)\ f_1$	$2\ f_1$	$3\ f_1$	$4\ f_1$	$5\ f_1$

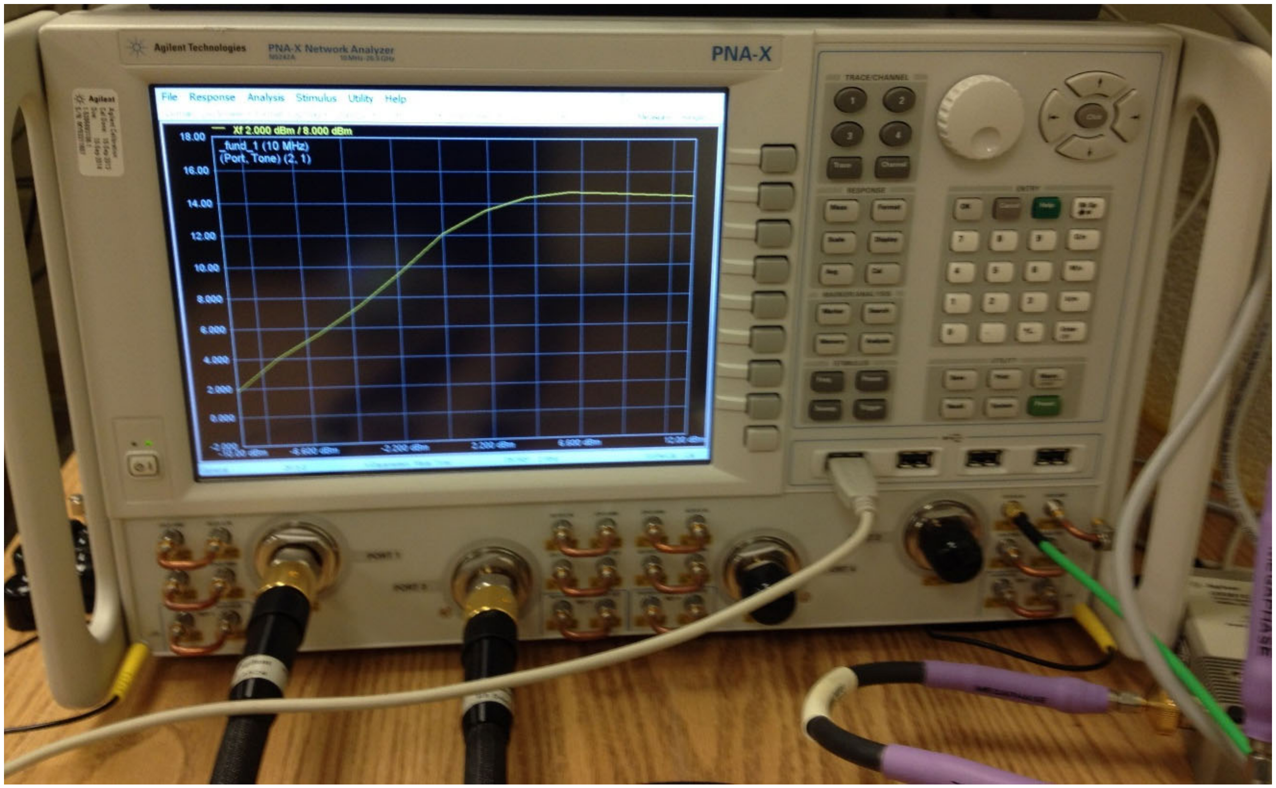
Nonlinear Vector Network Analyzer (NVNA)





NVNA instruments will gradually replace all VNAs

Nonlinear Vector Network Analyzer (NVNA)*



*L. Betts, “X-Parameters and NVNA…,” May 9, 2009.