ECE 546
Lecture - 21
Jitter

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Probe Further

- D. Derickson and M. Muller, “Digital Communications Test and Measurement”, Prentice Hall, 2007.
- Kyung Suk (Dan) Oh and Xingchao (Chuck) Yuan, High-Speed Signaling: Jitter Modeling, Analysis, and Budgeting, Prentice Hall, 2012
- Mike Peng Li, Jitter, Noise and Signal Integrity at High-Speed, Prentice Hall, 2008

Jitter Overview

- Jitter is one of the most critical impairments in:
  - SerDes Links
  - DDR memory interfaces
  - High-speed clocks (PLL, CDRs)
  - RF and mixed-signal systems

*At multi-Gbps data rates, **timing margins shrink**, making jitter a dominant failure mechanism*

Jitter Definition

Jitter is difference *in time* between when an event was ideally to occur and when it actually did occur.

- Timing uncertainties in digital transmission systems
- Utmost importance because timing uncertainties cause bit errors
- There are different types of jitter

Jitter Characteristics

- Jitter is a signal timing deviation referenced to a recovered clock from the recovered bit stream
- Measured in Unit Intervals and captured visually with eye diagrams
- Two types of jitter
  - Deterministic (non Gaussian)
  - Random
- The total jitter (TJ) is the sum of the random (RJ) and deterministic jitter(DJ)

Types of Jitter

- Deterministic Jitter (DDJ)
  - Data-Dependent Jitter (DDJ)
  - Periodic Jitter (PJ)
  - Bounded Uncorrelated Jitter (BUJ)

- Random Jitter (RJ)
  - Gaussian Jitter
  - $f^{-\alpha}$ Higher-Order Jitter

Jitter Effects

Bandwidth Limitations

- Cause intersymbol interference (ISI)
- ISI occurs if time required by signal to completely charge is longer than bit interval
- Amount of ISI is function of channel and data content of signal

Oscillator Phase Noise

- Present in reference clocks or high-speed clocks
- In PLL based clocks, phase noise can be amplified

Phase Noise & Phase Jitter

- **Phase noise in clock oscillators**
  - **Phase offset term that continually changes timing of signal**

$$S(t) = P\big(t + \phi(t)\big)$$

$S(t)$: signal waveform with phase noise

P: undistorted signal

$\phi(t)$: phase noise

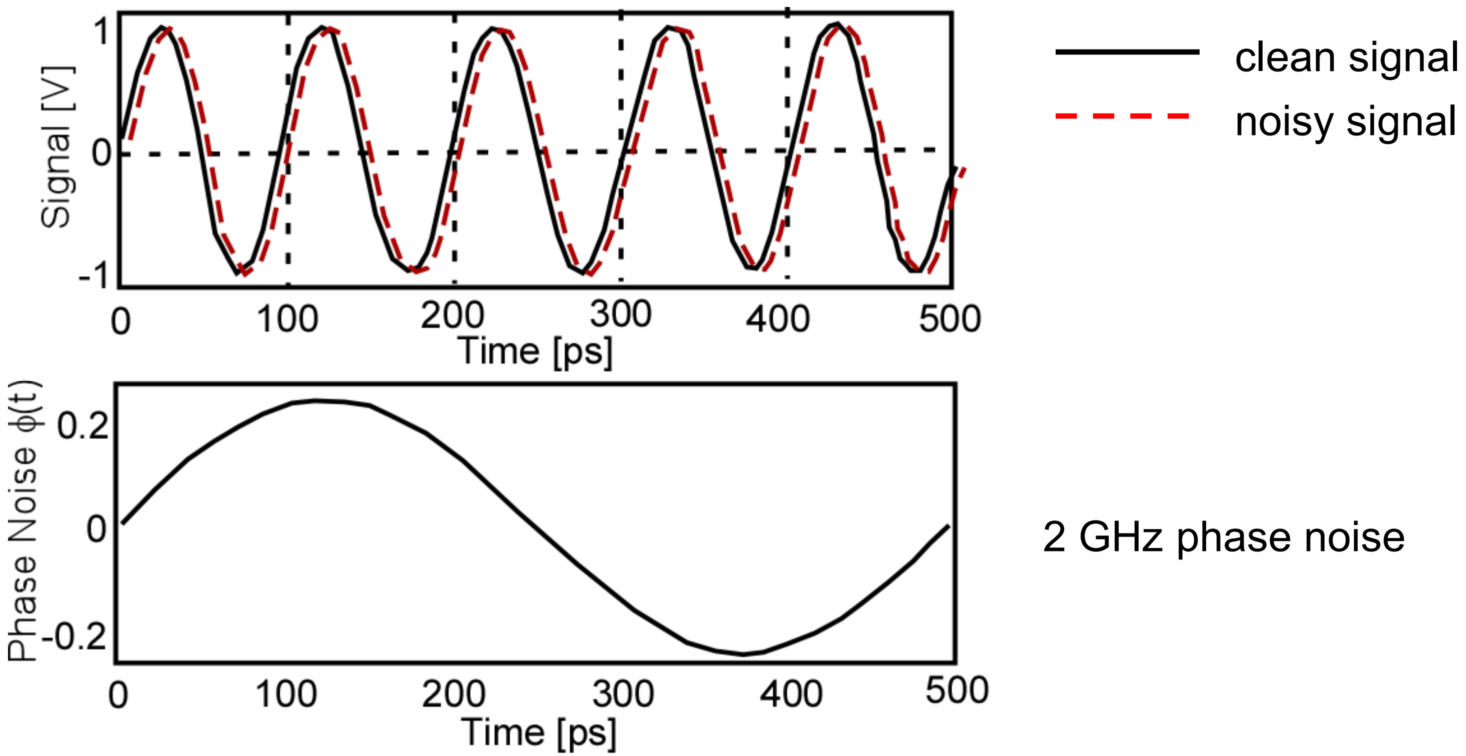
Example:

$$P(t) = \sin\left(10\times10^9 \times 2\pi t\right)$$

$$\phi(t) = \frac{1}{4}\sin\left(2\times10^9 \times 2\pi t\right)$$

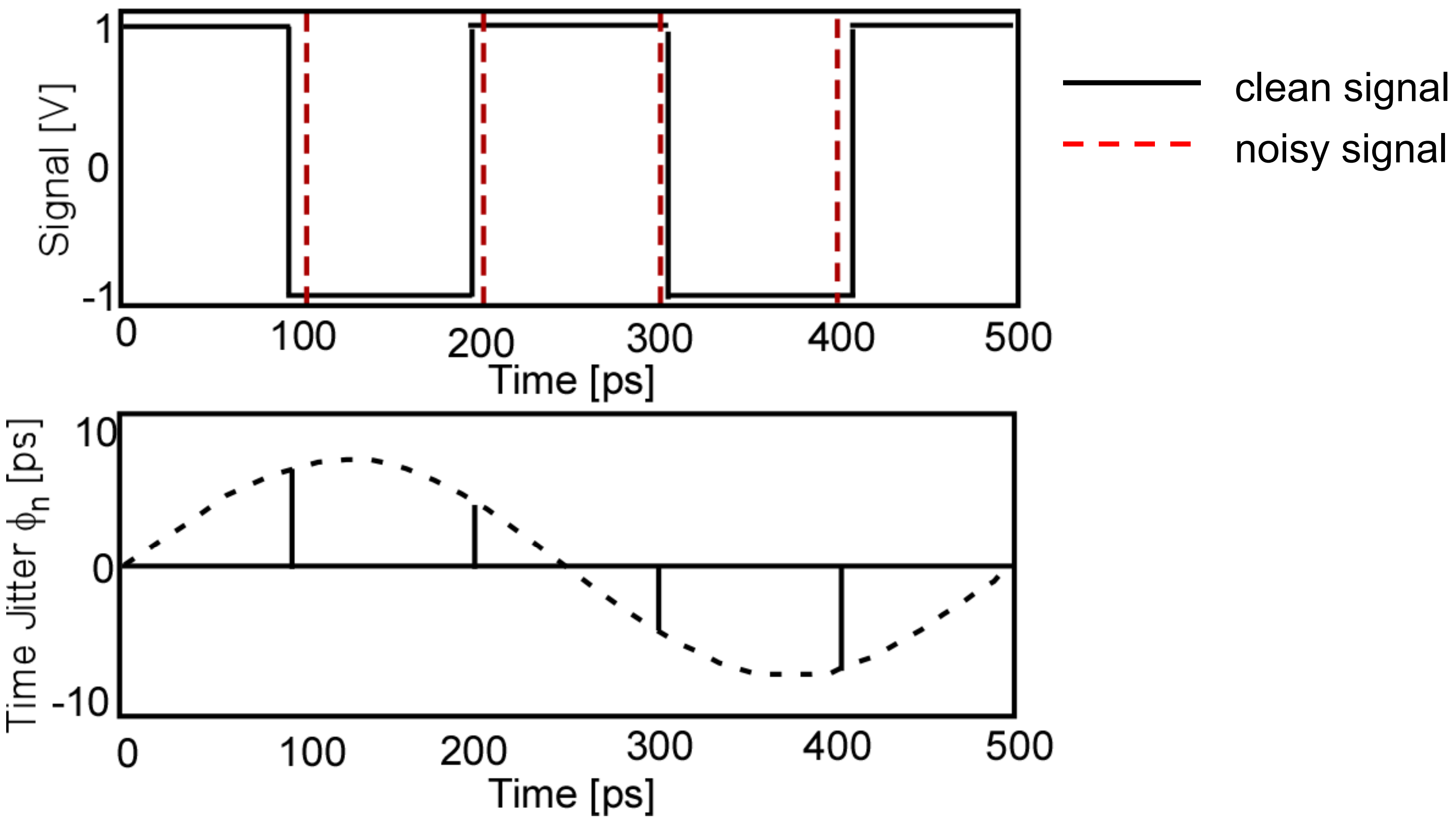
$$S(t) = \sin\left(10\times10^9 \times 2\pi t + 0.25\sin(2\times10^9 \times 2\pi t)\right)$$

Phase Noise



2 GHz phase noise

Phase Jitter



Phase Jitter

- **Phase jitter in digital systems**
  - **Variability in timing of transition in digital systems is called phase jitter**
  - **Phase jitter is digital equivalent of phase noise**
  - **Always defined relative to the ideal position of the transitions**

For a jittered digital signal

$$t_n = T_n - \phi_n$$

t_n is the actual time of the *nth* transition

T_n is the ideal timing value of the *nth* transition

ϕ_n is the time offset of the transition ← phase jitter term

Example: 10 Gbits/s → T_n has bit intervals of 100 ps.
Transitions take place at 0, 100, 200 ps

Cycle-to-Cycle Jitter

- Phase jitter causes bit periods to contract and expand
- Actual bit periods are given by the time difference between 2 consecutive transitions

$$P_n = t_{n+1} - t_n = \left(T_{n+1} - \phi_{n+1}\right) - \left(T_n - \phi_n\right)$$

Ideal bit period:

$$TB_n = T_{n+1} - T_n$$

Period jitter:

$$PerJ_n = TB_n - P_n$$

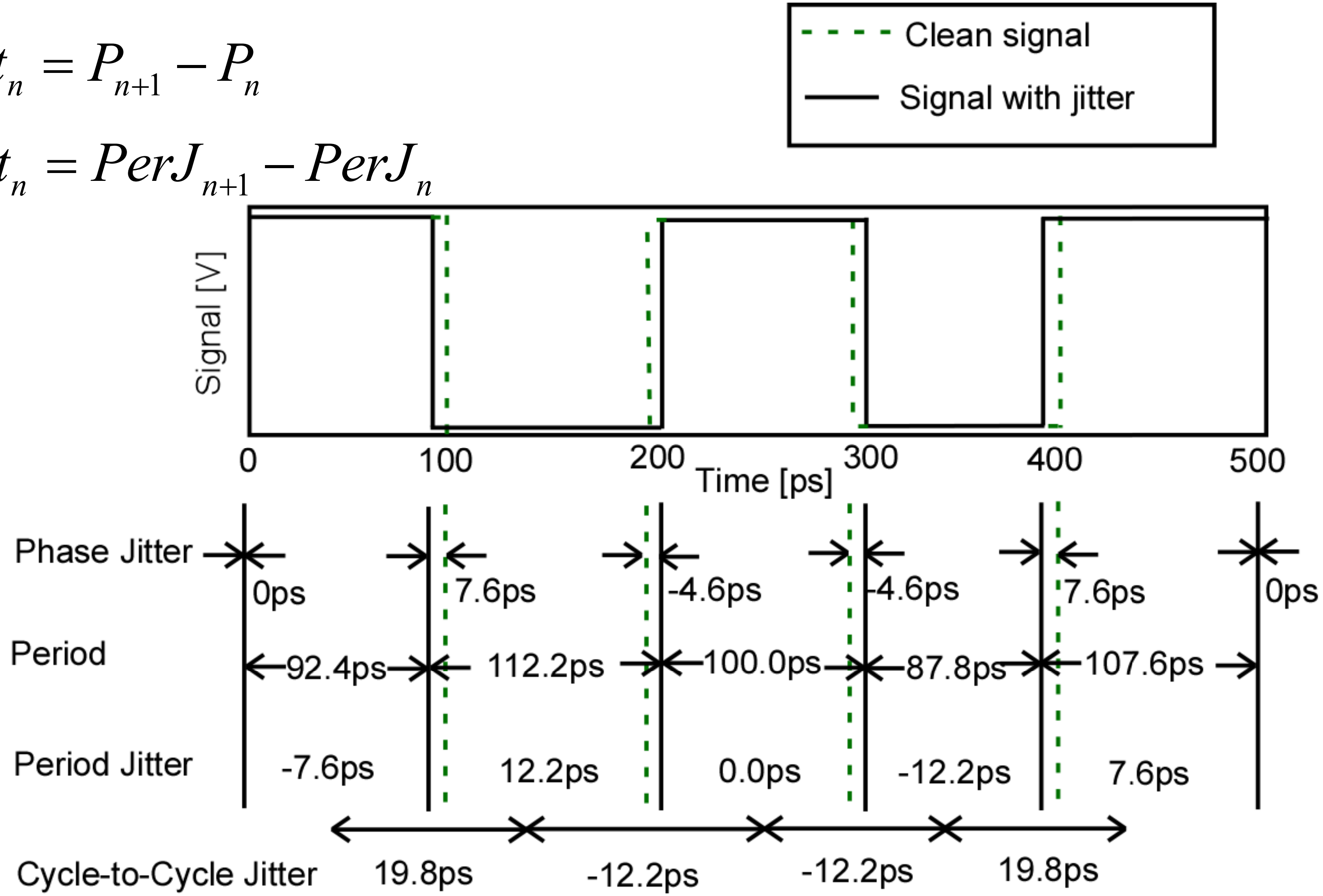
$$PerJ_n = \left(T_{n+1} - T_n\right) - \left(T_{n+1} - T_n + \phi_n - \phi_{n+1}\right) = \phi_{n+1} - \phi_n$$

Cycle-to-Cycle Jitter

Cycle-to-cycle jitter:

$$CCJit_n = P_{n+1} - P_n$$

$$CCJit_n = PerJ_{n+1} - PerJ_n$$

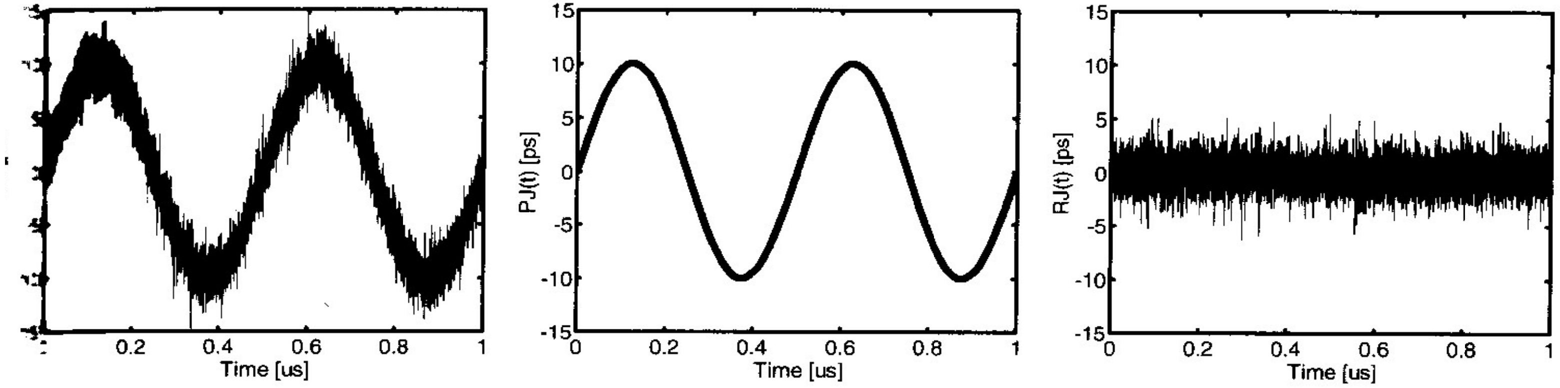


Duty Cycle Distortion (DCD)

- Difference between rise/fall delays
- Causes:
  - Asymmetric drivers
  - Threshold mismatch

$$DCD = t_{rise} - t_{fall}$$

Total Jitter Time Waveform



TJ(t) = PJ(t) + RJ(t)

The total jitter waveform is the sum of individual components

Jitter Statistics

- Most common way to look at jitter is in statistical domain
- Because one can observe jitter histograms directly on oscilloscopes
- No instruments to measure jitter time waveform or frequency spectrum directly

Jitter Histograms and Probability Density Functions

- Built directly from time waveforms
- Frequency information is lost
- Peak-to-peak value depends on observation time

Note: A jitter histogram does not contain all the information about the jitter

Probability Density Function

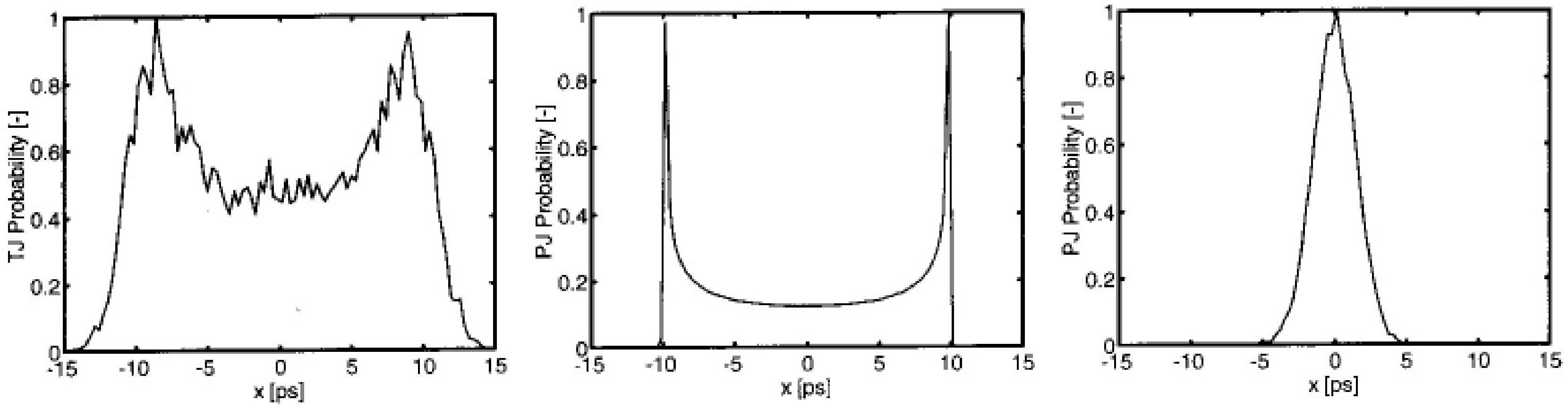
$$x \longleftrightarrow pdf_x$$

$$y \longleftrightarrow pdf_y$$

$$z = x+y \longleftrightarrow pdf_z = pdf_y * pdf_z$$

The PDF of the sum of 2 independent random variable is the convolution of the pdfs of those 2 variables

Jitter Statistics



$$TJ(x) = PJ(x) * RJ(x)$$

The total jitter PDF is the convolution of individual components

Jitter Mechanisms

- Transfer of Level Noise into the Time Domain
  - Noise on digital data signals causes jitter because it offsets the threshold crossing point in time
- Bandwidth Limitations
  - Primarily caused by intersymbol interference
- Oscillator Phase Noise
  - Phase noise present in reference clocks especially in systems based on PLL

Jitter Mechanisms

Jitter Mechanisms

- Transfer of noise into time domain
- Bandwidth limitation in channels
- Oscillator phase noise

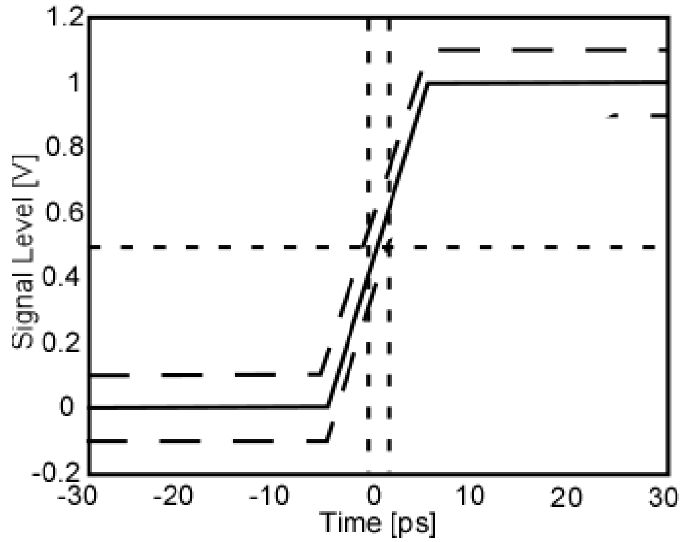
$$NJ_{pk-pk} = t_t \frac{V_{Noise}}{V_H - V_L}$$

t_t	rise time
V_{Noise}	pk-pk noise amplitude
V_H	Hi signal level
V_L	Lo signal level

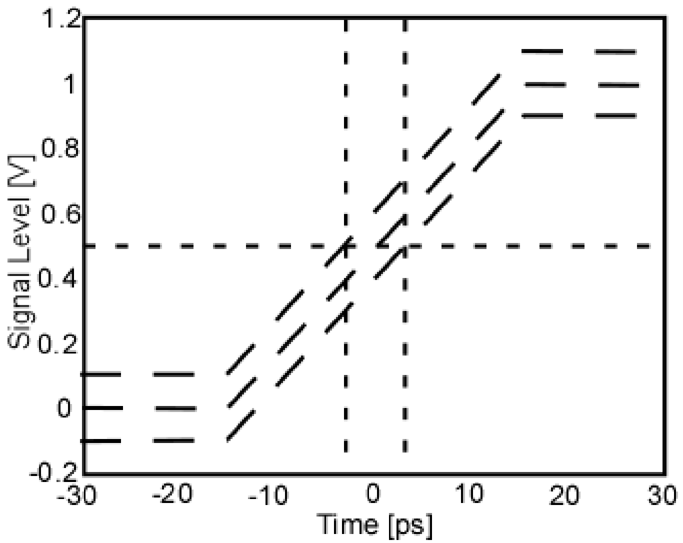
Jitter Mechanisms

Linear model

$$NJ_{pk-pk} = t_t \frac{V_{Noise}}{V_H - V_L}$$



Jitter ~ 2ps



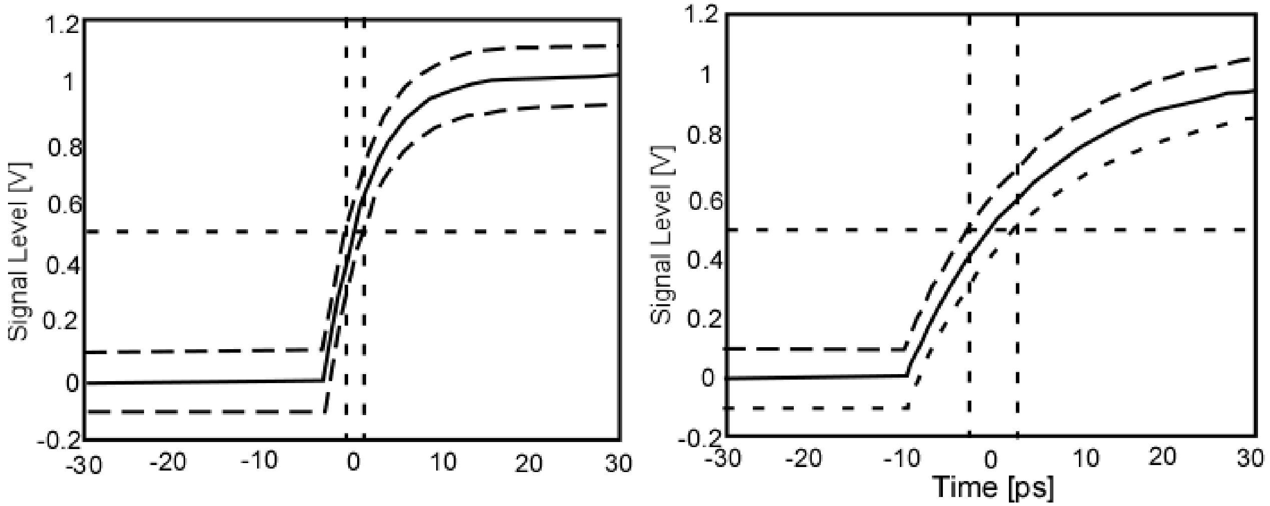
Jitter ~ 6ps

Random noise caused by thermal effects

Jitter Mechanisms

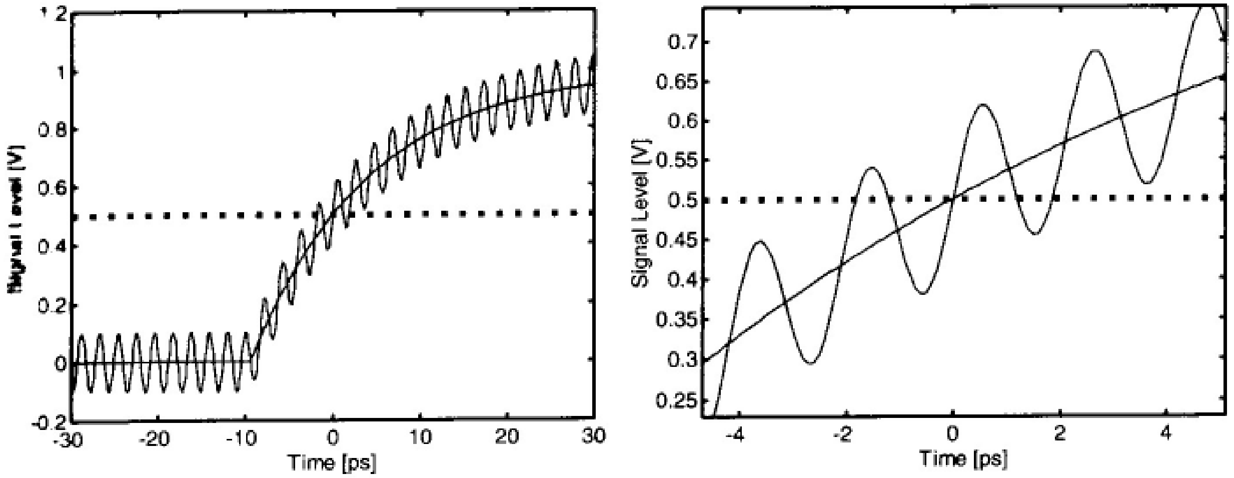
First order model

$$NJ_{pk-pk} = -\tau\left(\ln\left(0.5 - V_{Noise}\right) + \ln\left(0.5 + V_{Noise}\right)\right)$$



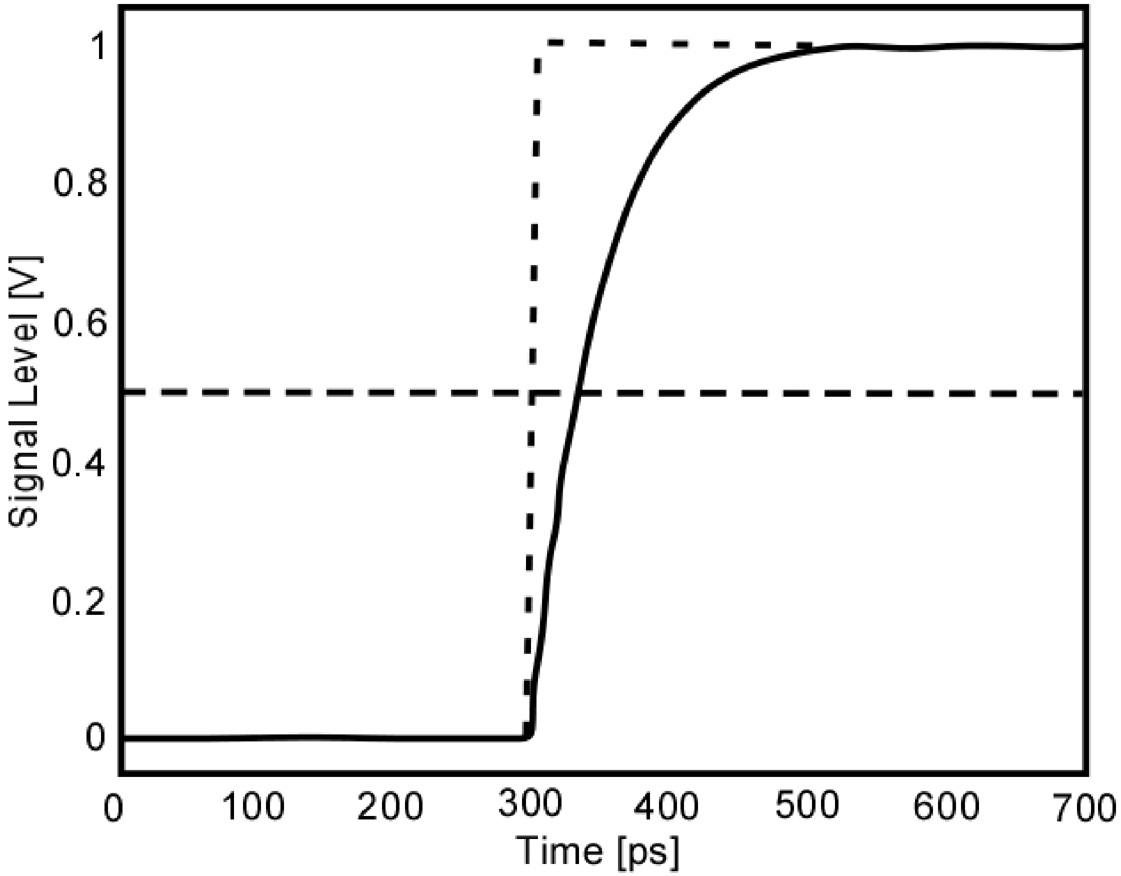
Periodic noise: switching power, crosstalk, etc…

Jitter Mechanisms



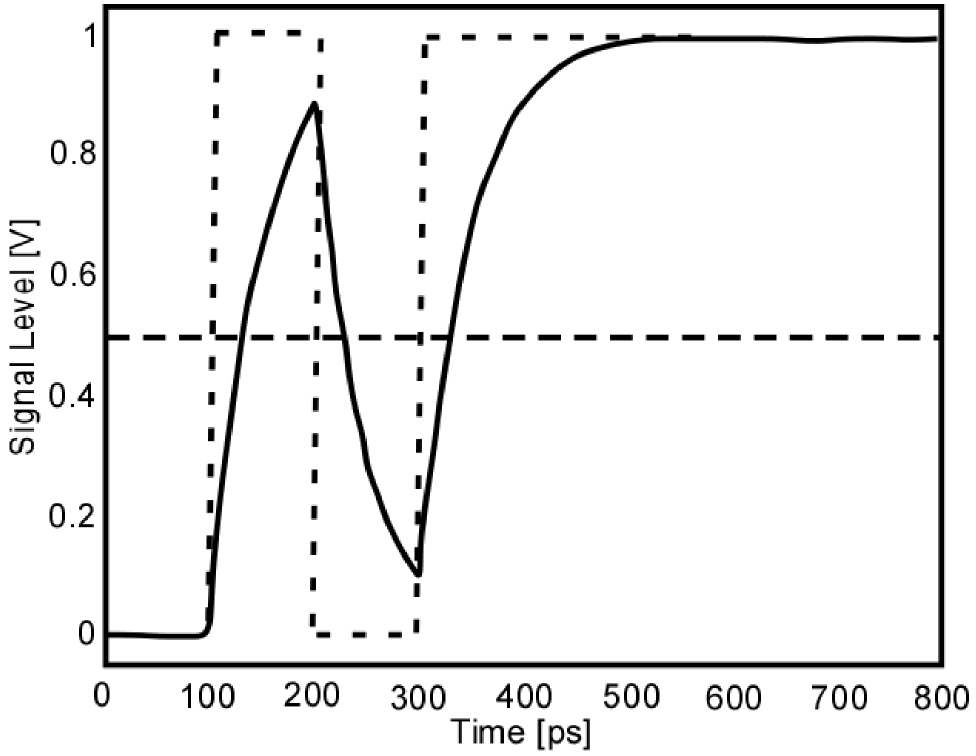
Multiple threshold crossing of a signal with high-frequency level noise

Bandwidth Limitations



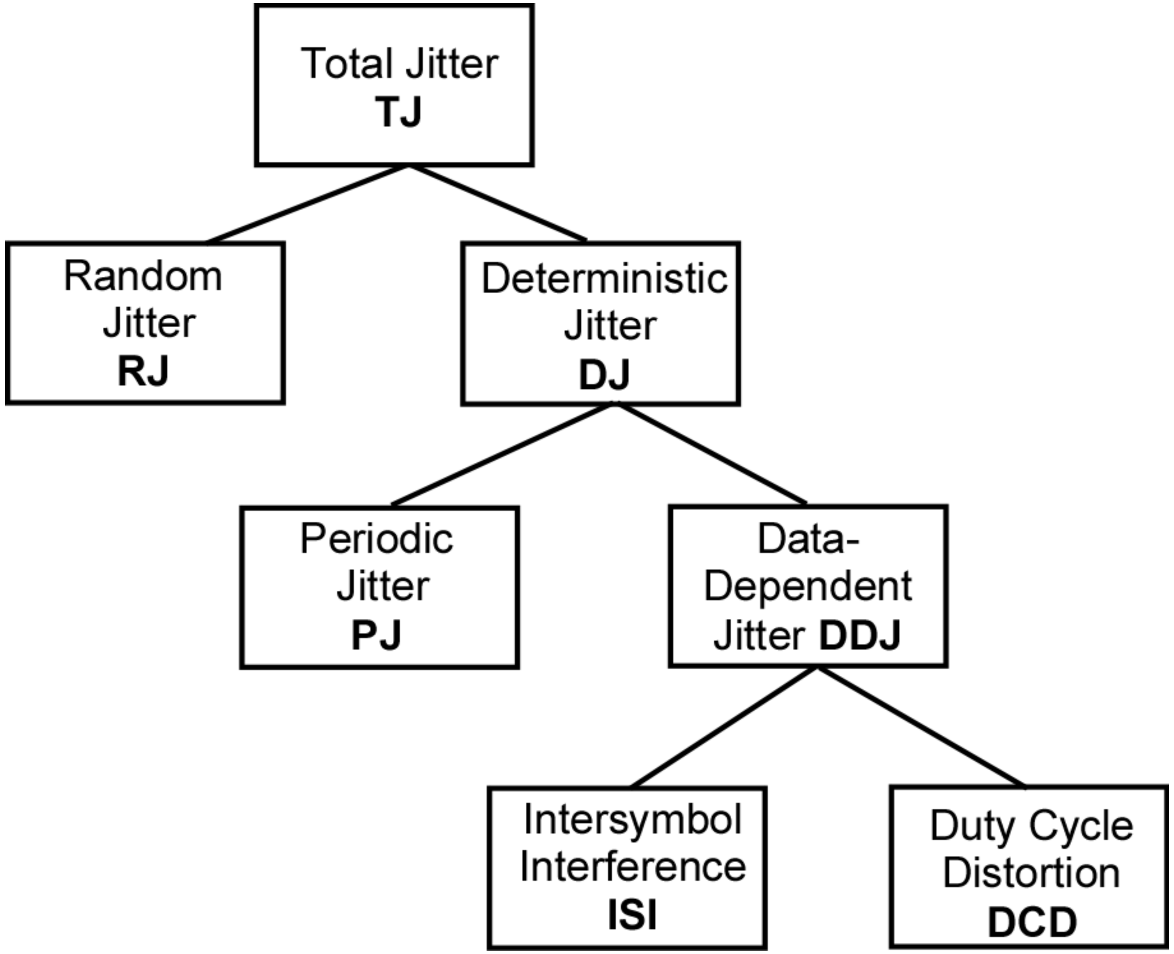
0001111 data pattern

Bandwidth Limitations



0101111 data pattern

Jitter Classification



Gaussian Random Jitter

- **Random jitter can be described by a Gaussian distribution with the following probability density function**

$$PDF_{RJ}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

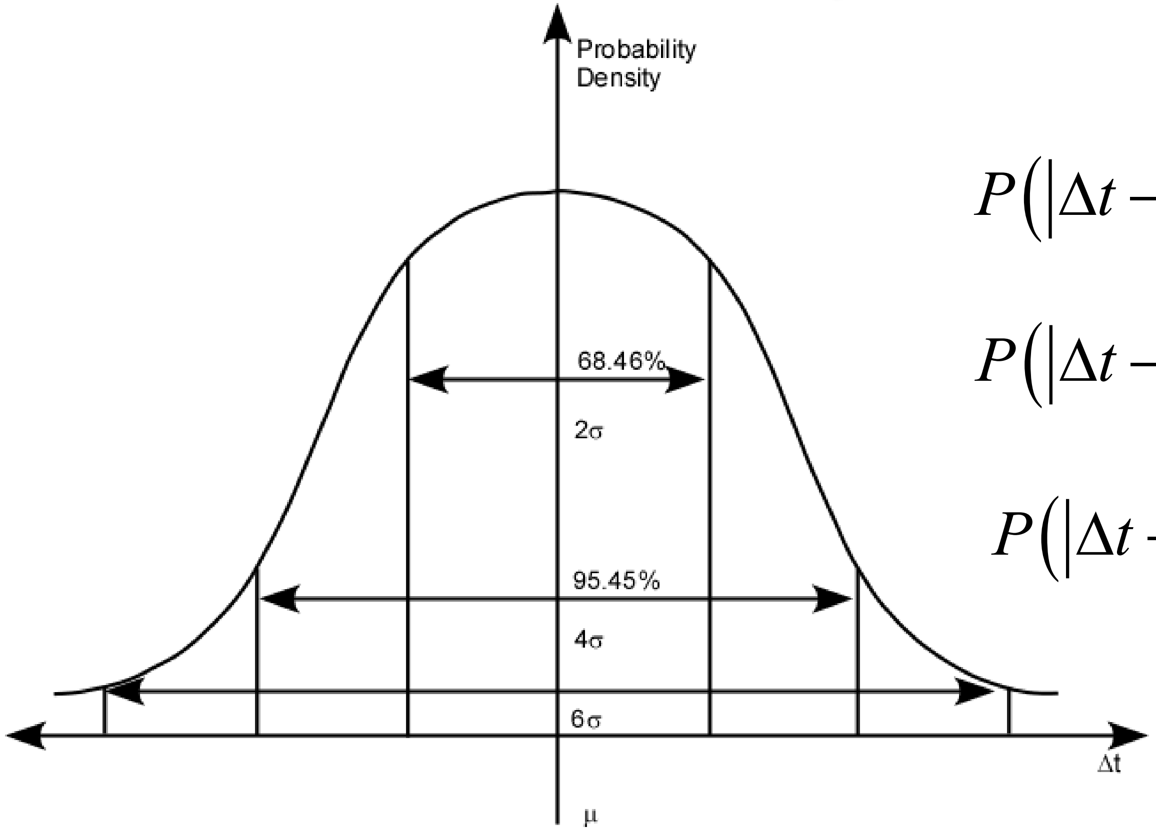
x : independent value

σ : RMS value

μ : mean of distribution (zero by definition)

➔**Note: the PDF of a Gaussian process is unbounded, i.e, its PDF is not zero unless the jitter *Δt* approaches infinity**

Gaussian Jitter PDF



$$P\left(|\Delta t-\mu|\le\sigma\right)=0.6826$$

$$P\left(|\Delta t-\mu|\le 2\sigma\right)=0.9545$$

$$P\left(|\Delta t-\mu|\le 3\sigma\right)=0.9973$$

Can be used to estimate the probability when the deviation of the random jitter variable Δt is within a multiple of its σ value.

Cumulative Density Function

Cumulative density function (CDF) is defined as:

$$CDF(t) = \int_{-\infty}^{t} PDF(x)dx$$

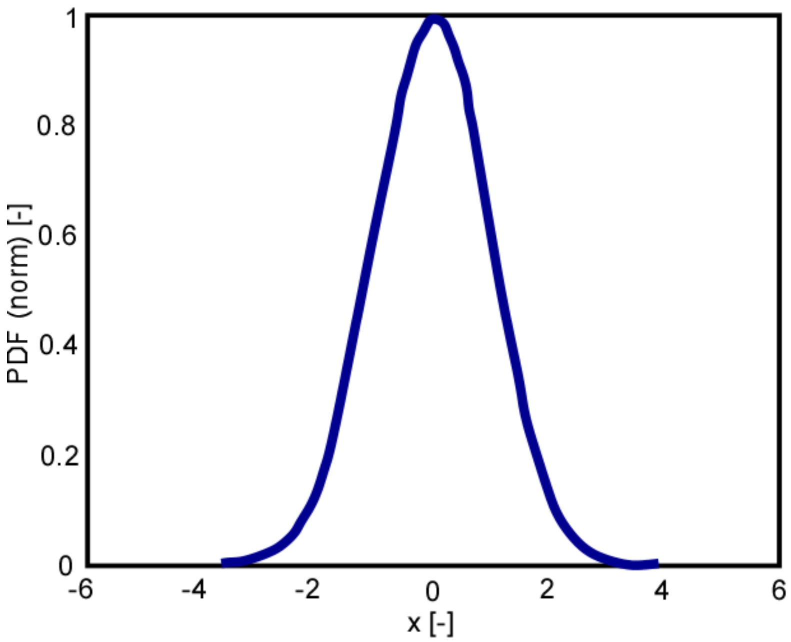
CDF(t) **tells us the probability that the transition occurred earlier than *t*. For random jitter, we get:**

$$CDF_{RJ}(x) = \frac{1}{2} + \frac{1}{2} erf \frac{x}{\sigma\sqrt{2}}$$

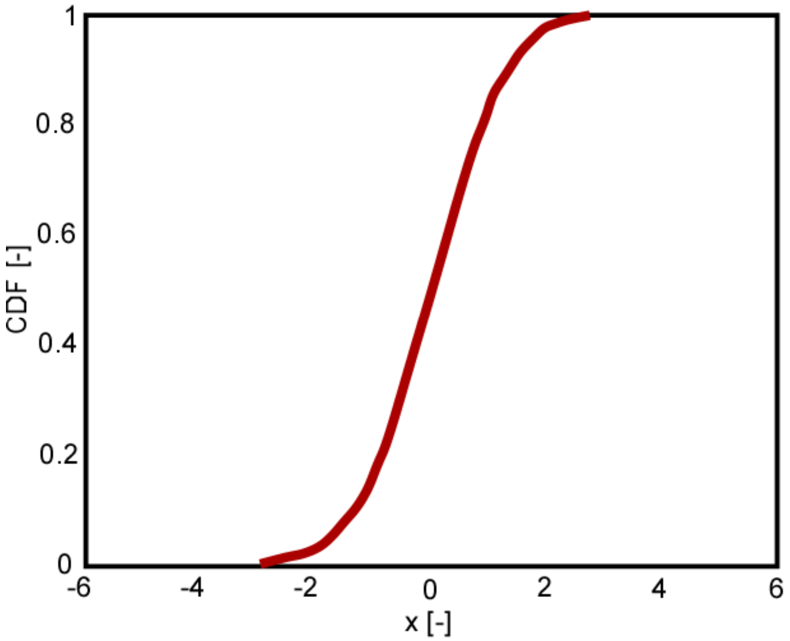
erf **is the error function**

PDF and CDF of Random Jitter

PDF



CDF

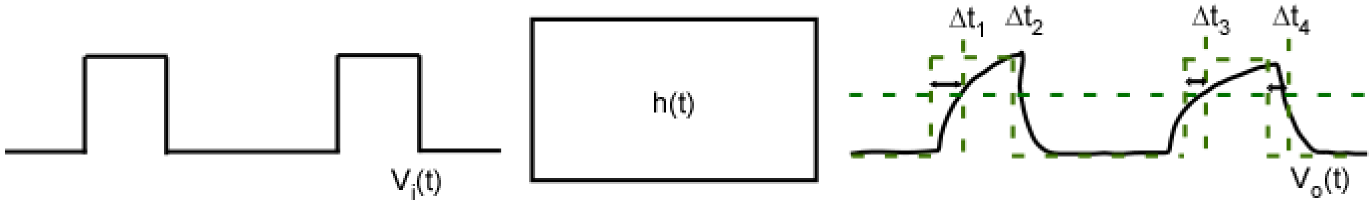


Causes of Deterministic Jitter

- Crosstalk
  - Noisy neighboring signals
- Interference
- Reflections
  - Imperfect terminations
  - Discontinuities (e.g. multi-drop buses, stubs)
- Simultaneous switching noise (SSN)
  - Noisy reference plane or power rail
  - Shift in threshold voltages

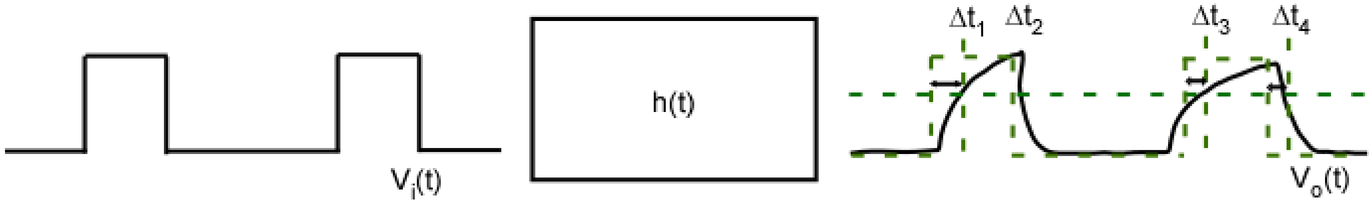
Data-Dependent Jitter

- Most commonly encountered DJ type
- Dominant limiting factor for link channels
- Due to *memory* of lossy electrical or optical system
- Bit transition of current bit depends on the transition times of the previous bits



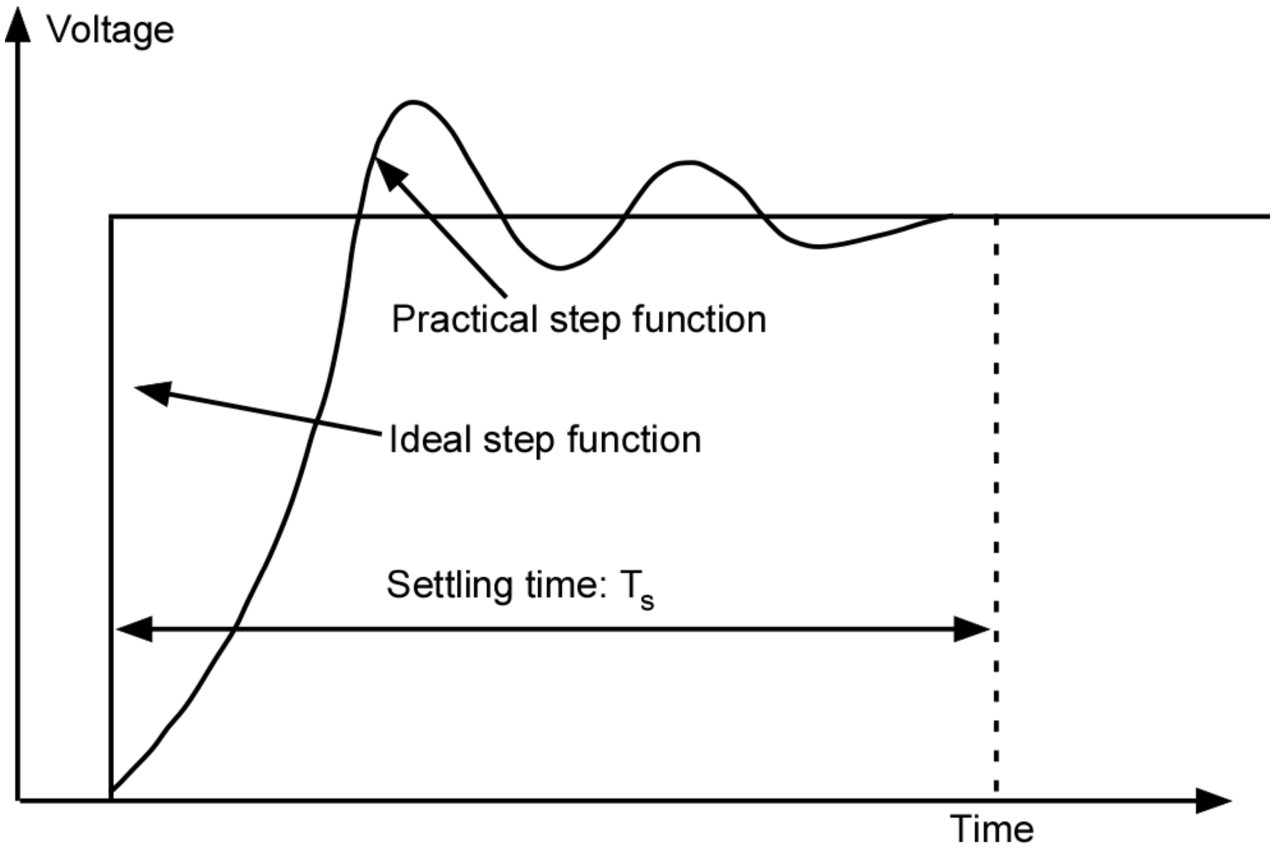
Data-Dependent Jitter

- DDJ depends on the impulse response of the system that generates the pattern
- DDJ depends on the input pattern
- DDJ is a distribution with its sample size equal to the number of transitions of the data pattern



- Duty cycle distortion (DCD) occurs for clock patterns of repeating bits

Data-Dependent Jitter



- **Since channel does not have zero-rise time step response or infinite bandwidth, jitter is to be expected**
- **Settling time gives good indication of jitter**

Model for DDJ

The generic form for DDJ PDF is:

$$f_{DDJ}(\Delta t) = \sum_{i=1}^{N} P_i^{DDJ} \delta\left(\Delta t - D_i^{DDJ}\right)$$

P_i^{DDJ} is the probability for the DDJ value of D_i^{DDJ}

P_i^{DDJ} satisfies the condition $\sum_{i=1}^{N} P_i^{DDJ} = 1$

Periodic Jitter

Periodic jitter is a *repeating* jitter signal at a certain period or frequency. It is described by:

$$\Delta t = A\cos(\omega t + \phi_o)$$

ω: angular frequency

ϕ_o: initial phase

The PDF for the single PJ is given by

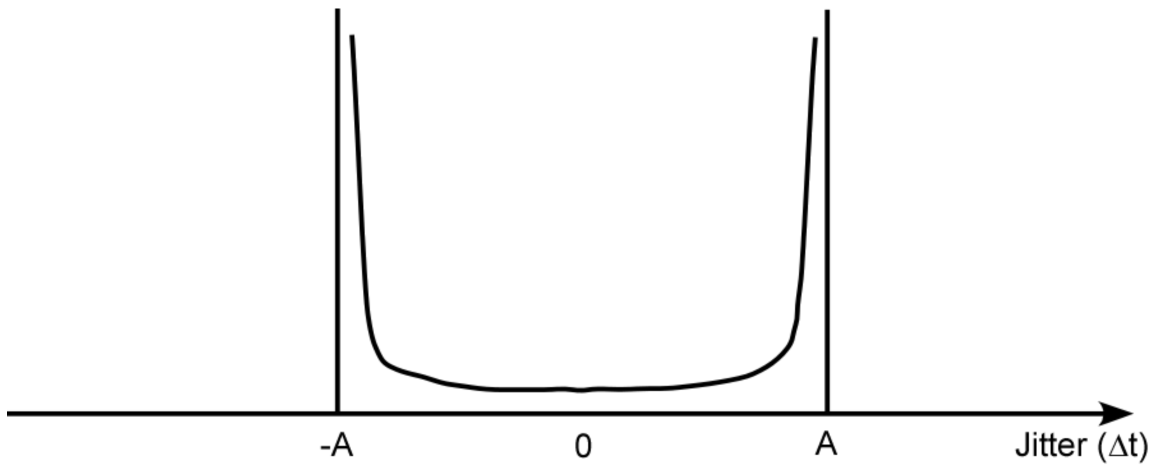
$$f_{PJ}(\Delta t) = \frac{1}{\pi\sqrt{1-(\Delta t / A)^2}}, \quad -A \le \Delta t \le A$$

Which can be approximated by

$$f_{PJ}(\Delta t) \approx \frac{1}{2}\left[\delta(\Delta t - A) + \delta(\Delta t + A)\right]$$

Periodic Jitter

PDF for single sinusoidal



$$f_{PJ}(\Delta t) = \frac{1}{\pi\sqrt{1-(\Delta t / A)^2}}, \quad -A \le \Delta t \le A$$

Periodic Jitter

There are 3 common waveforms for the theoretical analysis of periodic jitter

Rectangle Periodic Jitter

$$PDF_{PJ-rect}(x) = \frac{1}{2}\delta\left(-\frac{m}{2}\right) + \frac{1}{2}\delta\left(\frac{m}{2}\right)$$

Triangle Periodic Jitter

$$PDF_{PJ-triang}(x) = \begin{cases} \frac{1}{m} & for\ |x| < \frac{m}{2} \\ 0 & otherwise \end{cases}$$

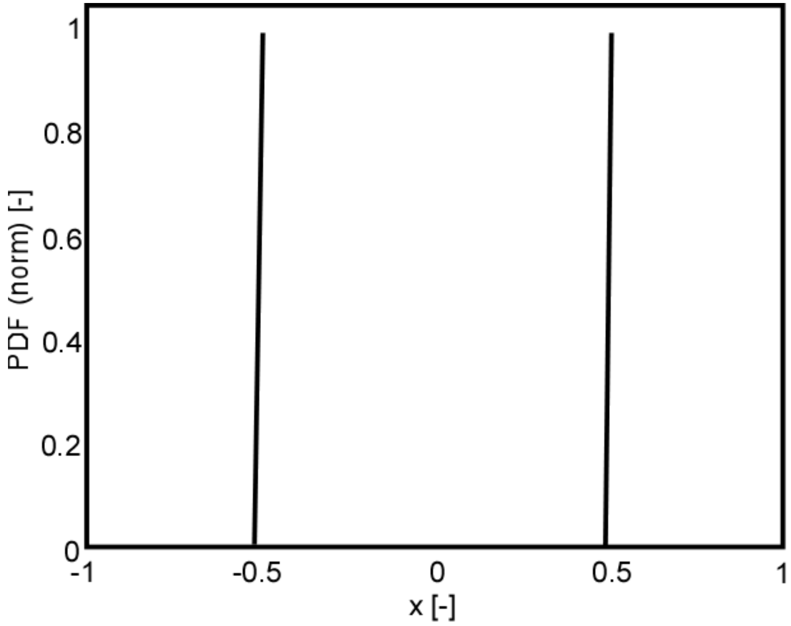
Periodic Jitter

Sinusoidal Periodic Jitter

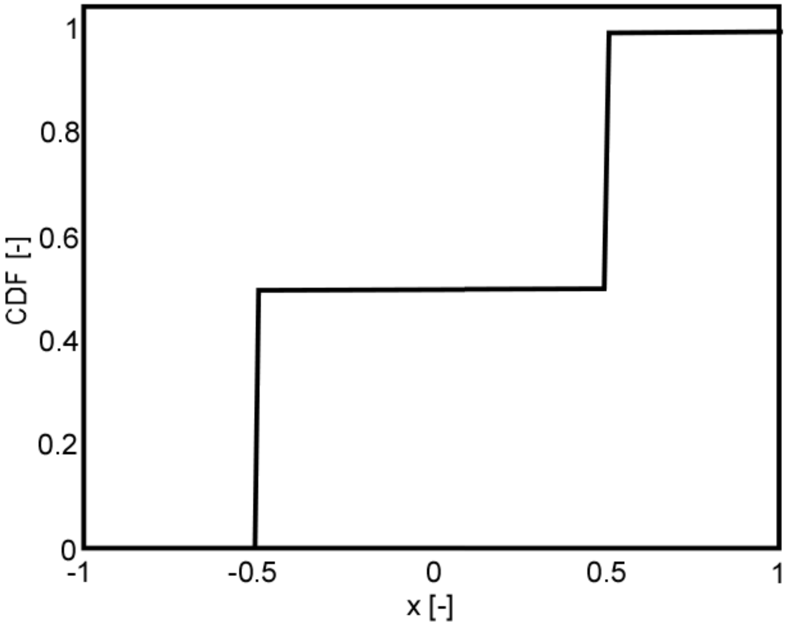
$$PDF_{PJ-line}(x) = \begin{cases} \dfrac{1}{\pi\sqrt{m/2 - \left(\sqrt{\dfrac{2}{m}}\,x\right)^2}} & for\ |x| < \dfrac{m}{2} \\ 0 & otherwise \end{cases}$$

Rectangular Periodic Jitter

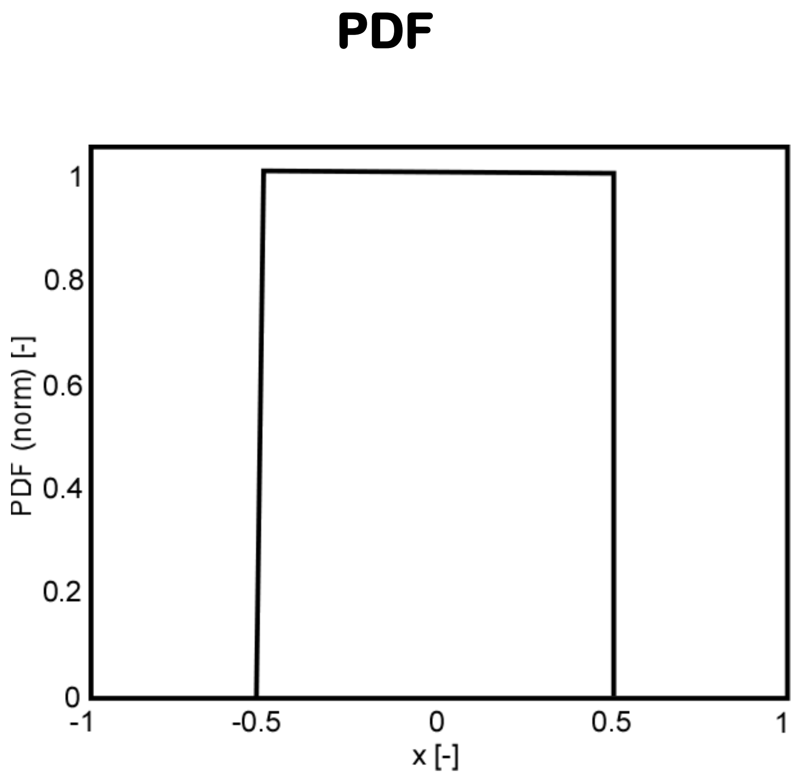
PDF

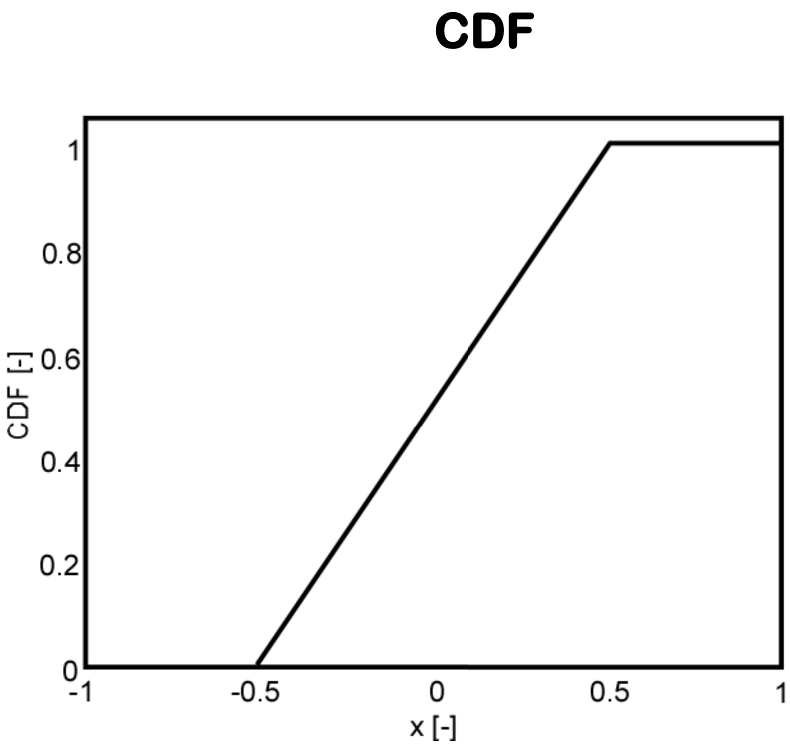


CDF



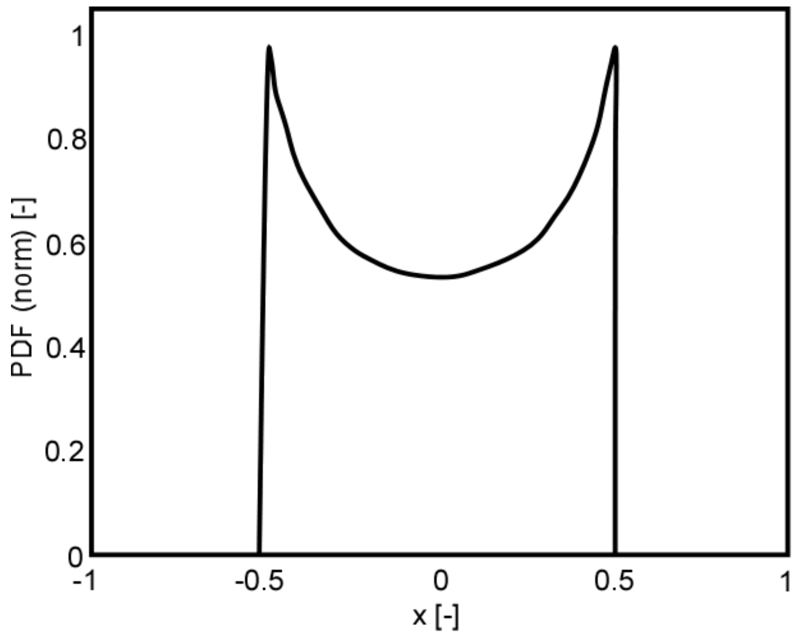
Triangular Periodic Jitter



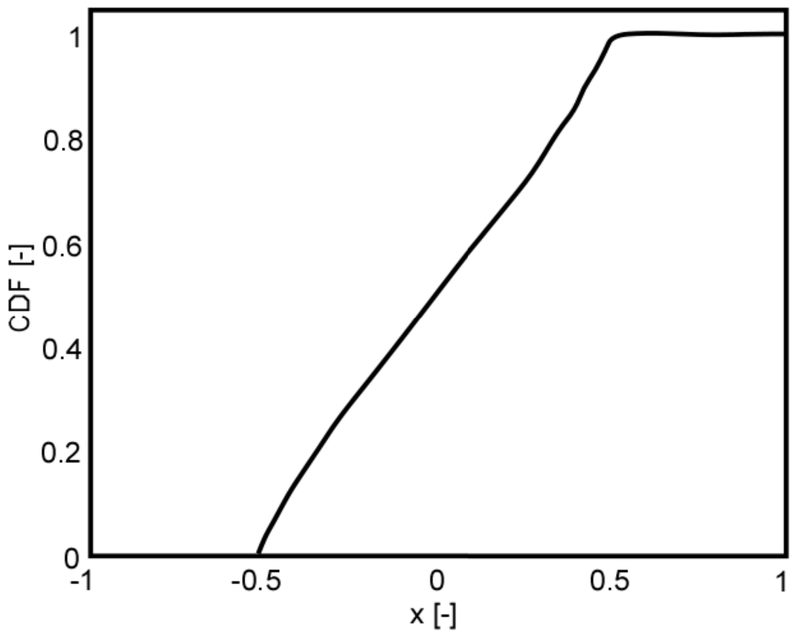


Sinusoidal Periodic Jitter

PDF



CDF



Mitigation

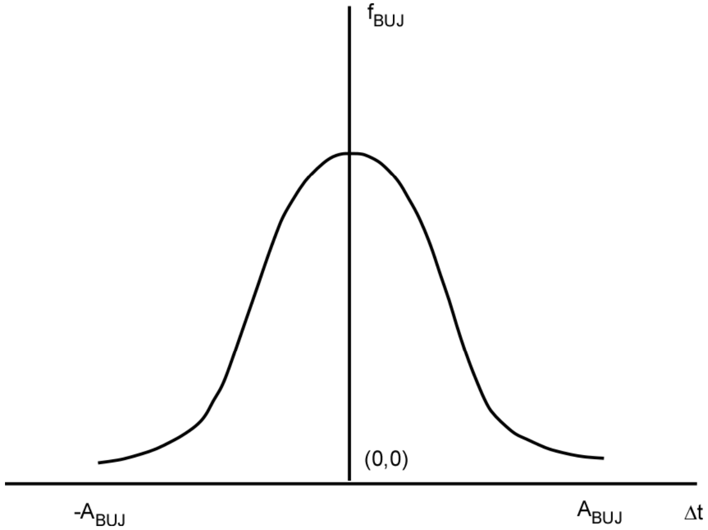
- Circuit Level
  - Increase slew rate
  - Symmetric design
  - Low-noise biasing
- System Level
  - Equalization (FFE, DFE)
  - Shielding
  - Power integrity
- Clocking
  - Clean PLL design
  - Low phase noise oscillators

Bounded Uncorrelated Jitter

BUJ is primarily due to crosstalk

The PDF for BUJ is given by

$$f_{PJ}\left(\Delta t\right)=\begin{cases}\dfrac{p_{BUJ}}{\sqrt{2\pi}\sigma_{BUJ}}e^{-\frac{t^2}{2\sigma_{BUJ}^2}} & for\ \left|\Delta t\right|\leq A_{BUJ}\\ 0 \quad\quad for\ \left|\Delta t\right|\leq A_{BUJ}\end{cases}$$



Mix of Random and Periodic Jitters

Gaussian RJ and Rectangle PJ

➔ Obtain convolution of 2 PDFs

$$RJ * PJ_{rect} = \int_{-\infty}^{+\infty} RJ(t-\tau)\left[\delta\left(-\frac{m}{2}\right)+\delta\left(\frac{m}{2}\right)\right]d\tau$$

$$= \frac{1}{2\sigma\sqrt{2\pi}}\left[e^{-\frac{(t-m/2)^2}{2\sigma^2}} + e^{\frac{(t+m/2)^2}{2\sigma^2}}\right]$$

Result is the sum of 2 Gaussian distributions with equal RMS value offset by the PJ peak-to-peak value . It is called the DUAL DIRAC DISTRIBUTION

Jitter Mixing

- **Problem**
  - In tests, we have measured jitter histograms and need to extract the individual jitter components
  - Ideally, we could use deconvolution into components. However without prior knowledge of deterministic jitter, it is not possible
  - Use dual Dirac distribution model which would yield the worst case deterministic jitter

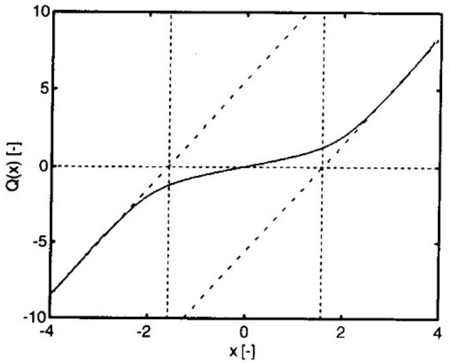
Q-Scale Transformation

Q-scale is defined such that the Gaussian distribution mapped onto the Q-scale is a straight line

Use CDF

$$CDF(x) = \frac{1}{2} + \frac{1}{2} erf\left(\frac{x}{\sigma\sqrt{2}}\right)$$

$$Q(x) = \sqrt{2} erf^{-1}\left(2CDF(x) - 1\right) = \frac{x}{\sigma}$$

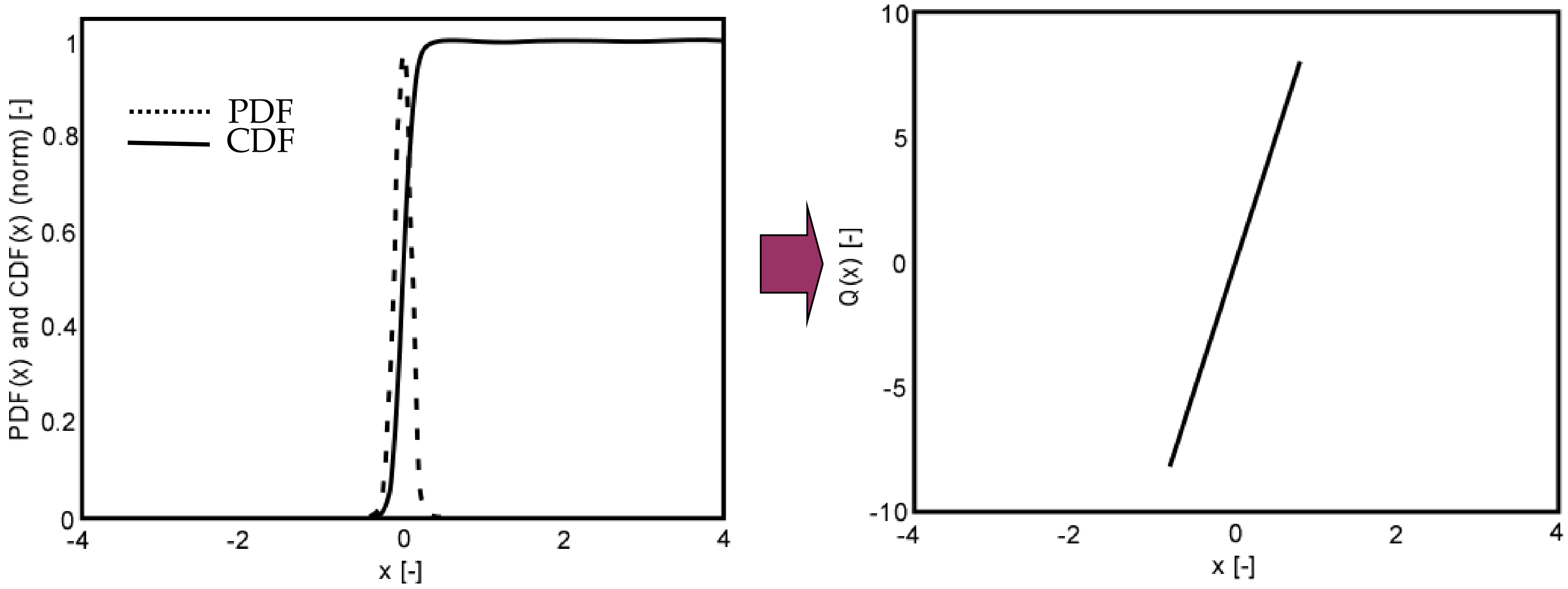


A Gaussian CDF is a straight line in the Q scale with slope 1/σ. DJ is given by distance d

Q-Scale Transformation

Gaussian RJ

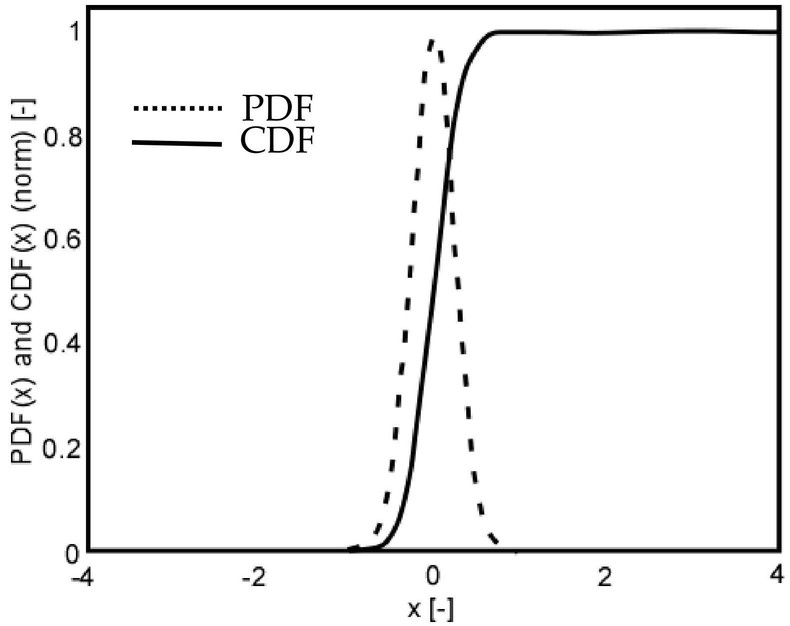
$\sigma = 0.5$

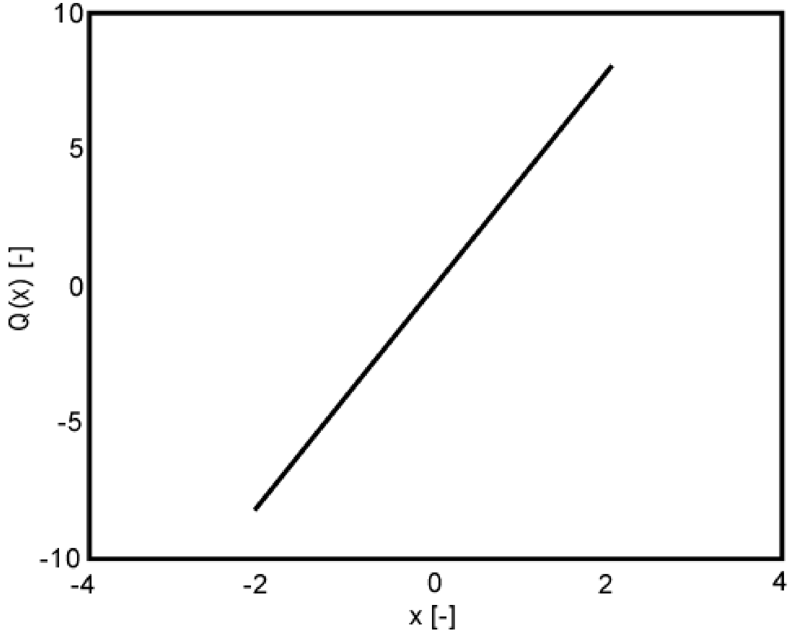


Q-Scale Transformation

Gaussian RJ

$\sigma = 0.25$



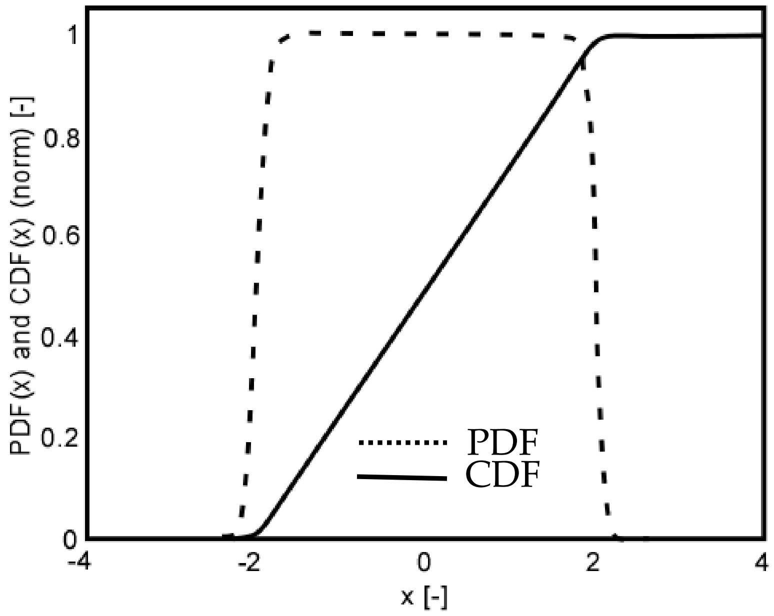


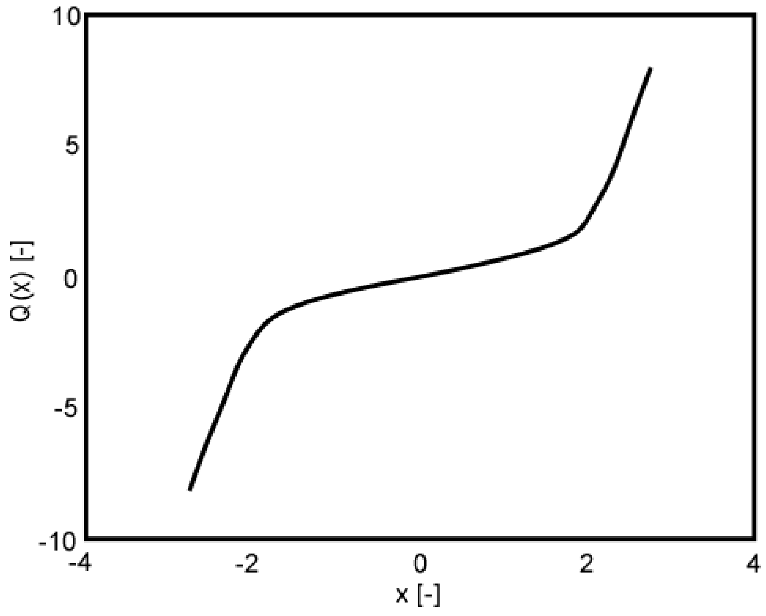
Q-Scale - Generalization

$$Q(x) = \sqrt{2}\,erf^{-1}\left(2CDF(x) - 1\right) = \frac{x}{\sigma}$$

Mixed Gaussian RJ and PJ

$\sigma = 0.1$



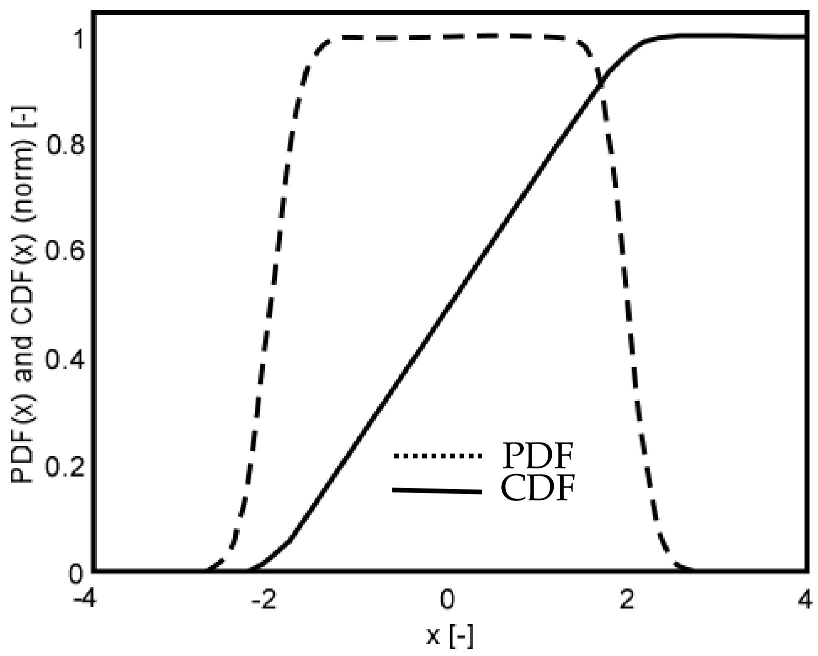


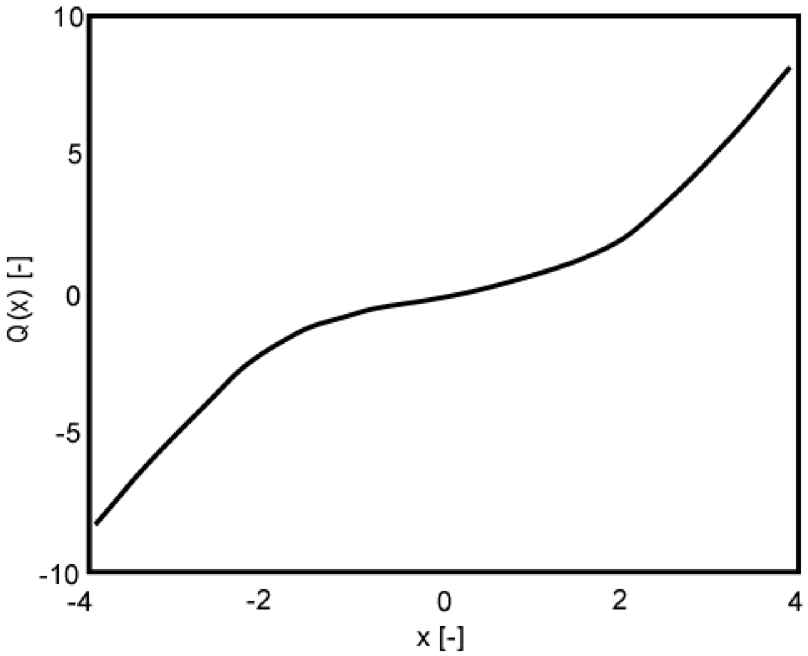
Q-Scale - Generalization

$$Q(x) = \sqrt{2}erf^{-1}\left(2CDF(x) - 1\right) = \frac{x}{\sigma}$$

Mixed Gaussian RJ and PJ

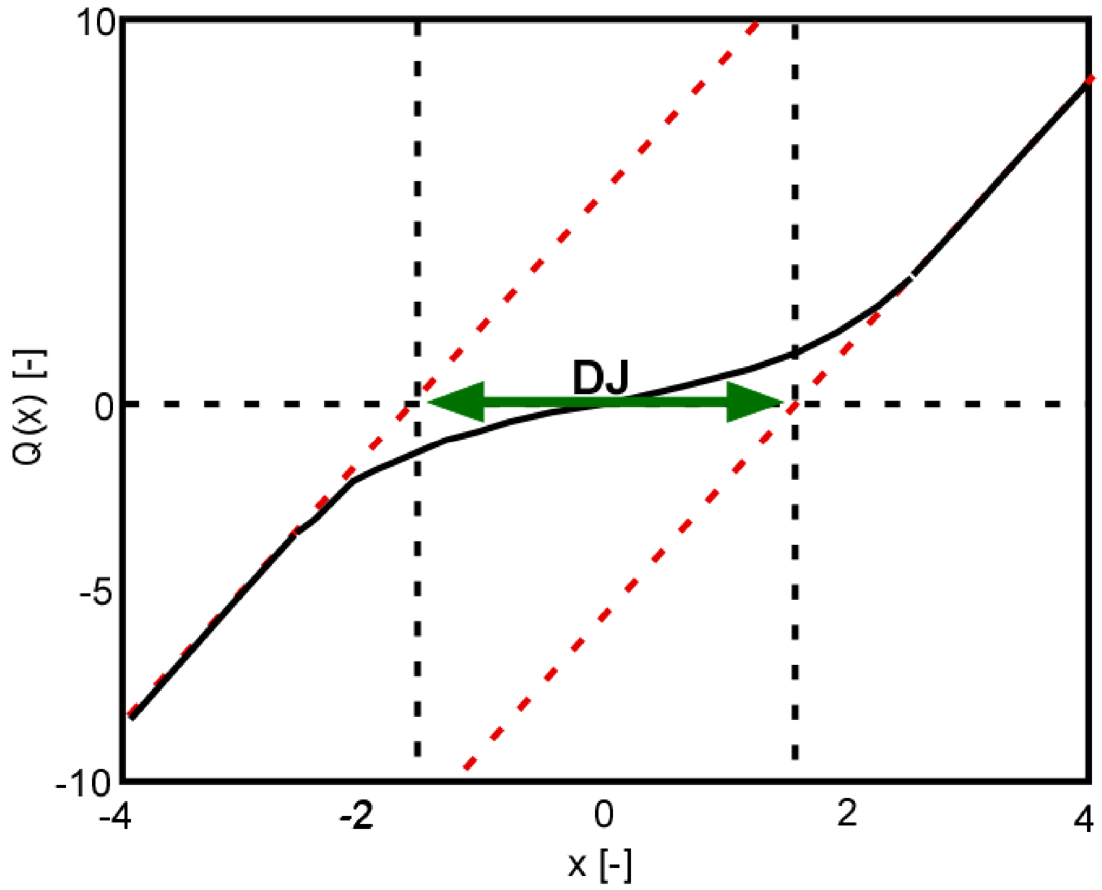
$\sigma = 0.25$





Dual Dirac Model

Mixed Gaussian RJ and Triangular PJ



Jitter Mixing

- Problem
  - In tests, we have measured jitter histograms and need to extract the individual jitter components
  - Ideally, we could use deconvolution into components. However without prior knowledge of deterministic jitter, it is not possible
  - Use dual Dirac distribution model which would yield the worst case deterministic jitter

Random Jitter Extraction

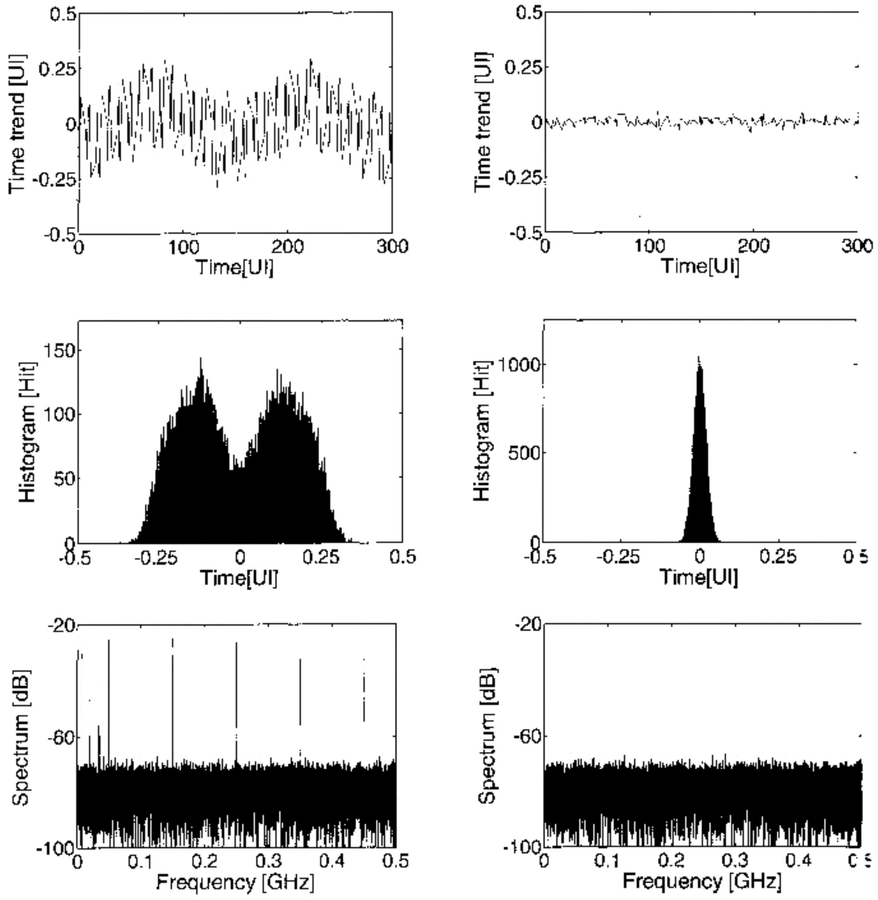
- **Spectrum Analysis**
  - Extract random jitter by using the assumption that it has a piecewise linear spectrum
  - Impulses are attributed to DJ
  - Noise floor is due to RJ

Extracting Random Jitter

Time domain

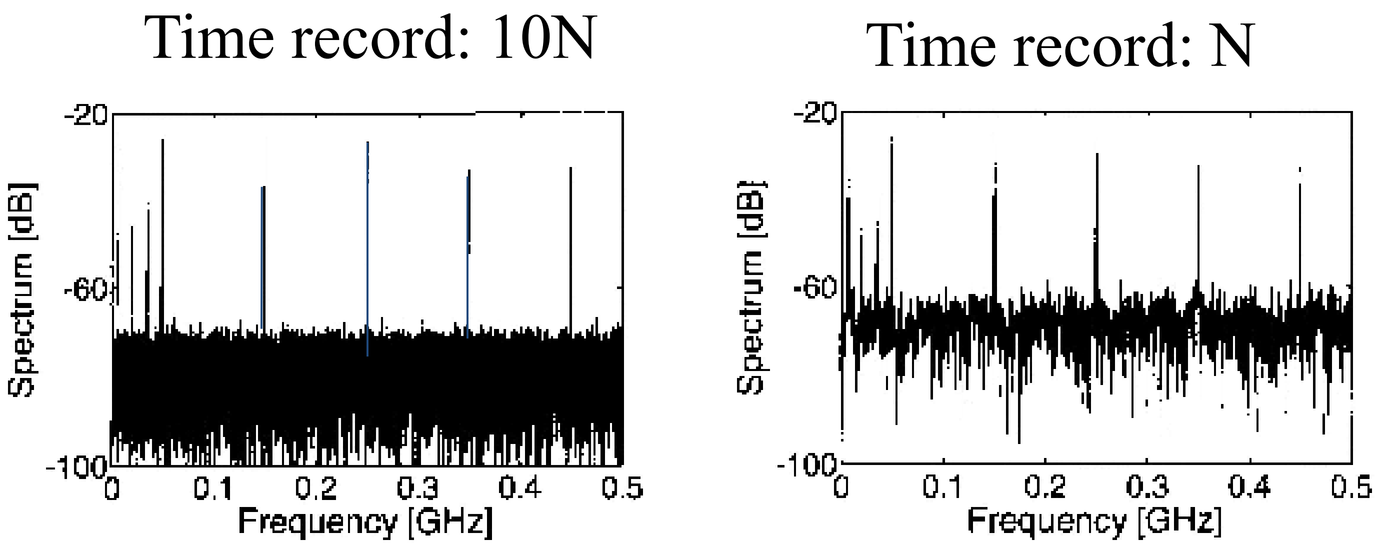
Statistical domain

Spectral domain



Total jitter | **Random jitter**

Jitter Spectrum



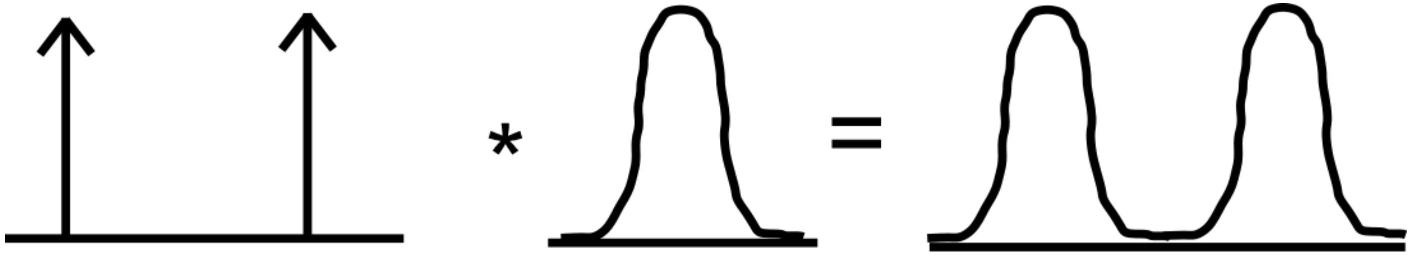
A longer FFT yields a spectrum with greater frequency resolution and lower noise floor.

Random Jitter Extraction

- **Tail-Fit**
  - **Extract random jitter under the assumption that its probability density function follows a Gaussian distribution**
  - **Make use of the Dual-Dirac Model**

Dual Dirac Model

Unknowns
- gap between 2 impulses
- σ for Gaussian distribution

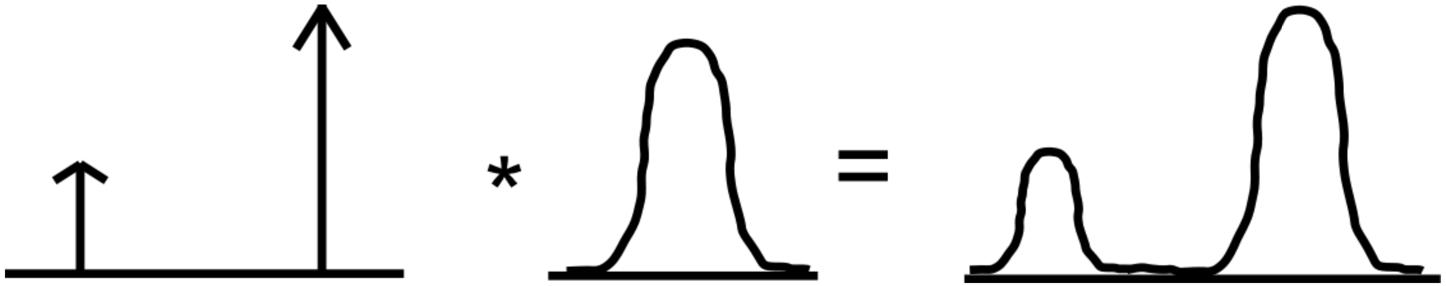


- Equal Amplitudes
  - Two unknown variables
  - Linear Problem
  - Explicit solution

Dual Dirac Model

Unknowns

- gap between 2 impulses
- σ for Gaussian distribution
- ratio of 2 impulse amplitudes



- Unequal Amplitudes
  - Three unknown variables
  - Nonlinear Problem
  - No explicit solution

Dual Dirac Model

Assume Gaussian RJ and Rectangle PJ

➔ Obtain convolution of 2 PDFs

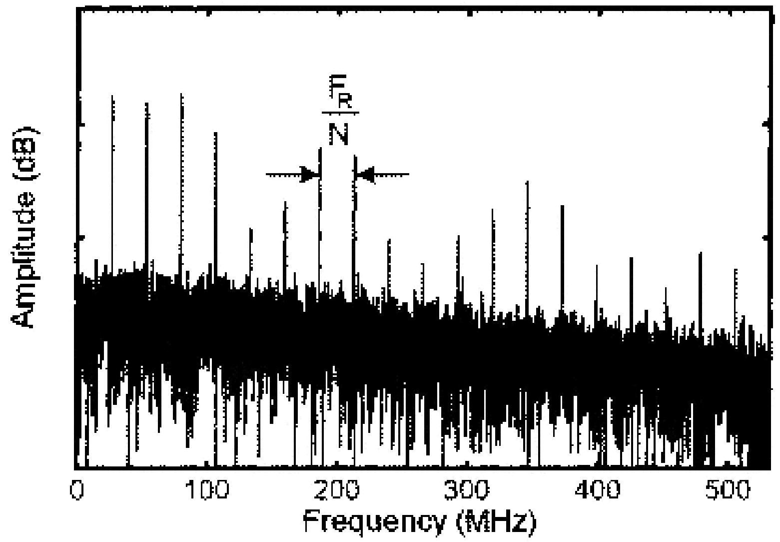
$$RJ * PJ_{rect} = \int_{-\infty}^{+\infty} RJ(t-\tau)\left[\delta\left(-\frac{m}{2}\right)+\delta\left(\frac{m}{2}\right)\right]d\tau$$

$$= \frac{1}{2\sigma\sqrt{2\pi}}\left[e^{-\frac{(t-m/2)^2}{2\sigma^2}} + e^{\frac{(t+m/2)^2}{2\sigma^2}}\right]$$

Result is the sum of 2 Gaussian distributions with equal RMS value offset by the PJ peak-to-peak value . It is called the DUAL DIRAC DISTRIBUTION

DDJ and DC D

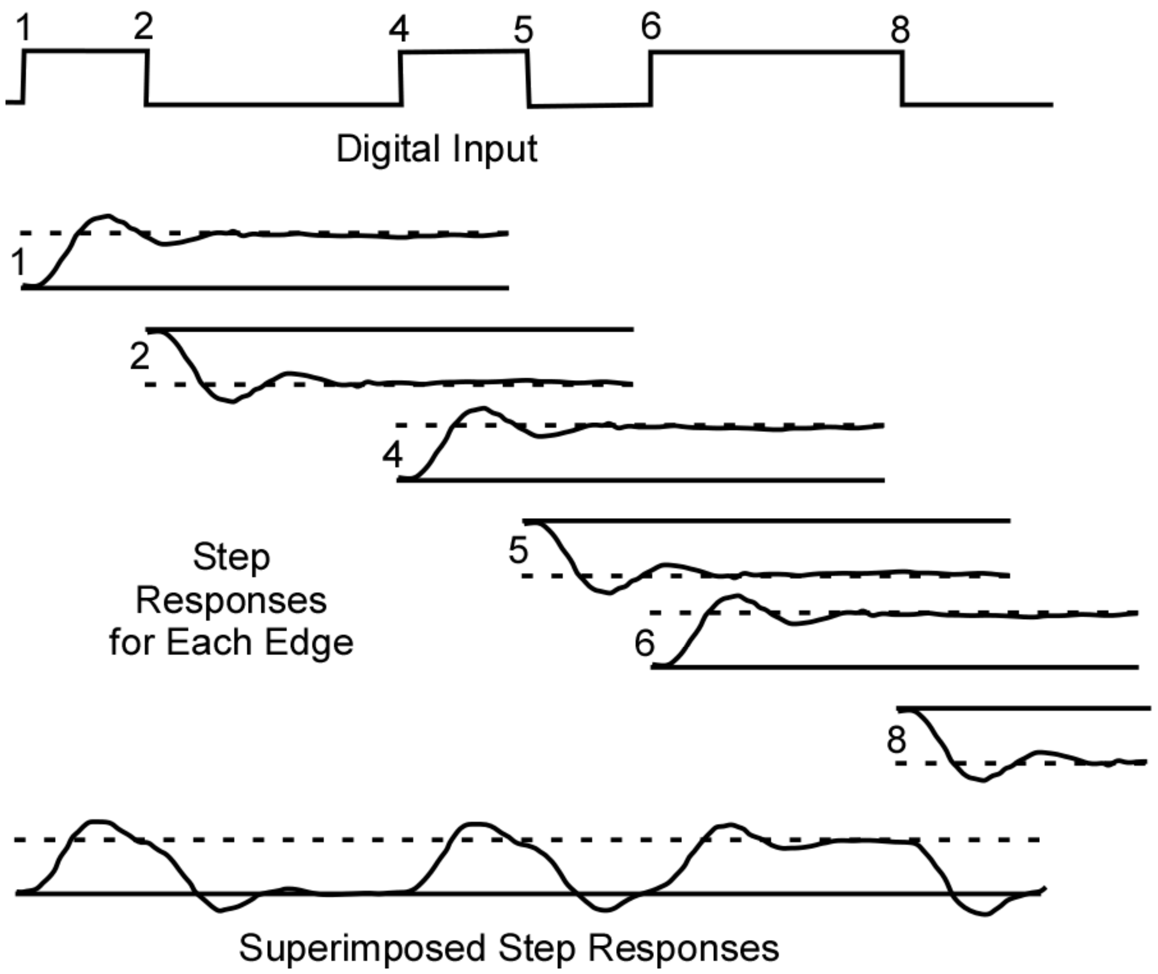
- DDJ and DCD are correlated to the data pattern



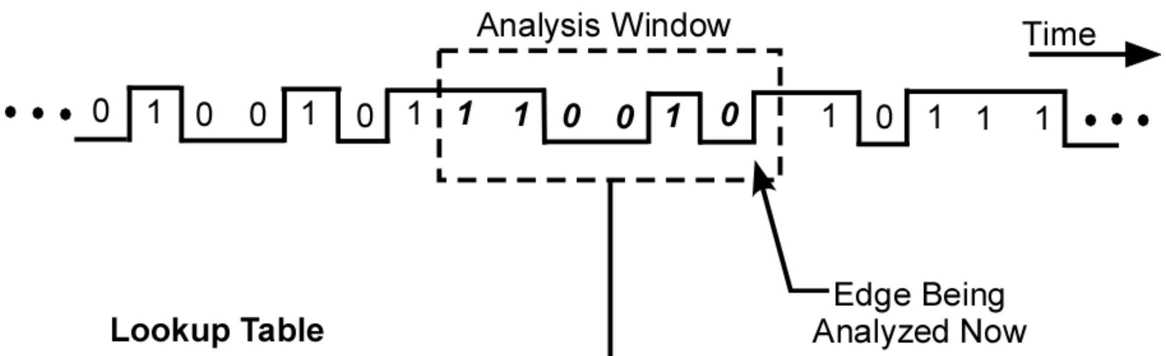
F_R=1.0625 Gbits/s
N=40 bits

For N bits, transmitted at rate F_R, the jitter components due to DDJ and DCD will appear in the spectrum at multiple of F_R/N

Pattern Correlation



Pattern Correlation



Lookup Table

0	0	0	0	0	0	E_0
⋮						
1	1	0	0	0	1	E_{49}
1	*1*	*0*	*0*	*1*	*0*	E_{50}
1	1	0	0	1	1	E_{51}
⋮						
1	1	1	1	1	0	E_{62}
1	1	1	1	1	1	E_{63}

$$E_{50} = \frac{\sum \{TIE_k\}}{K}$$

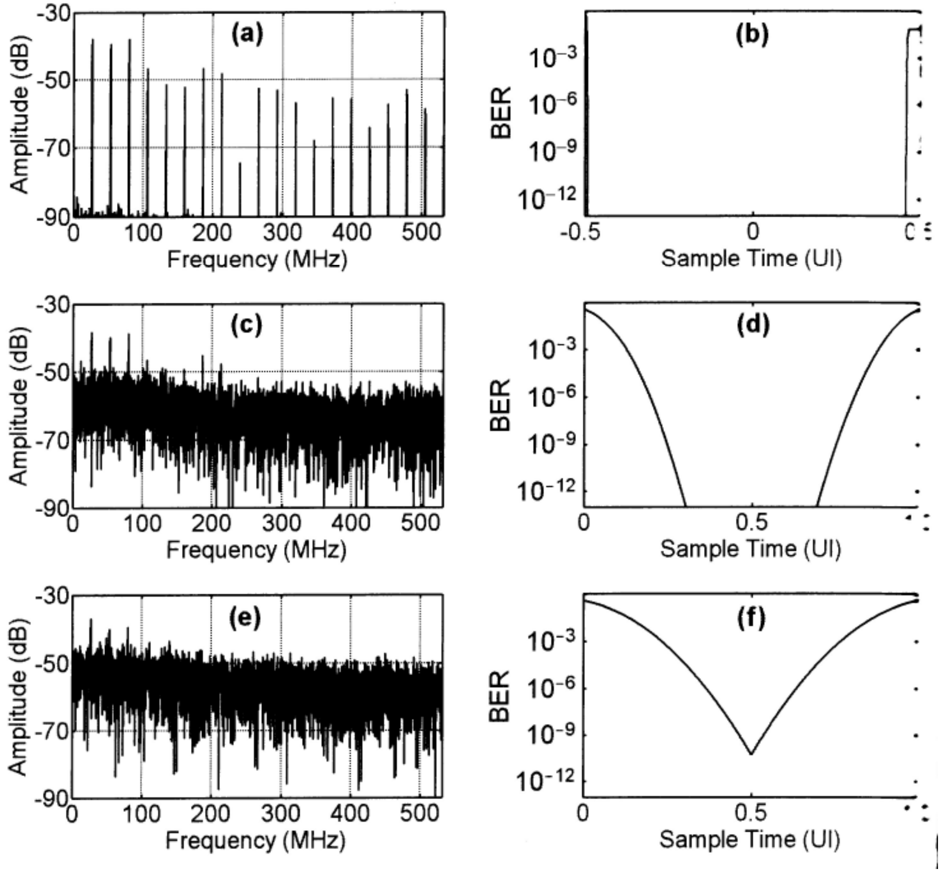
The phase errors from all occurrences of each M-bit patterns are averaged together to estimate the phase error due to that M-bit pattern

Extracting DDJ

DDJ Dominant

DDJ & RJ

RJ Dominant



Spectral domain

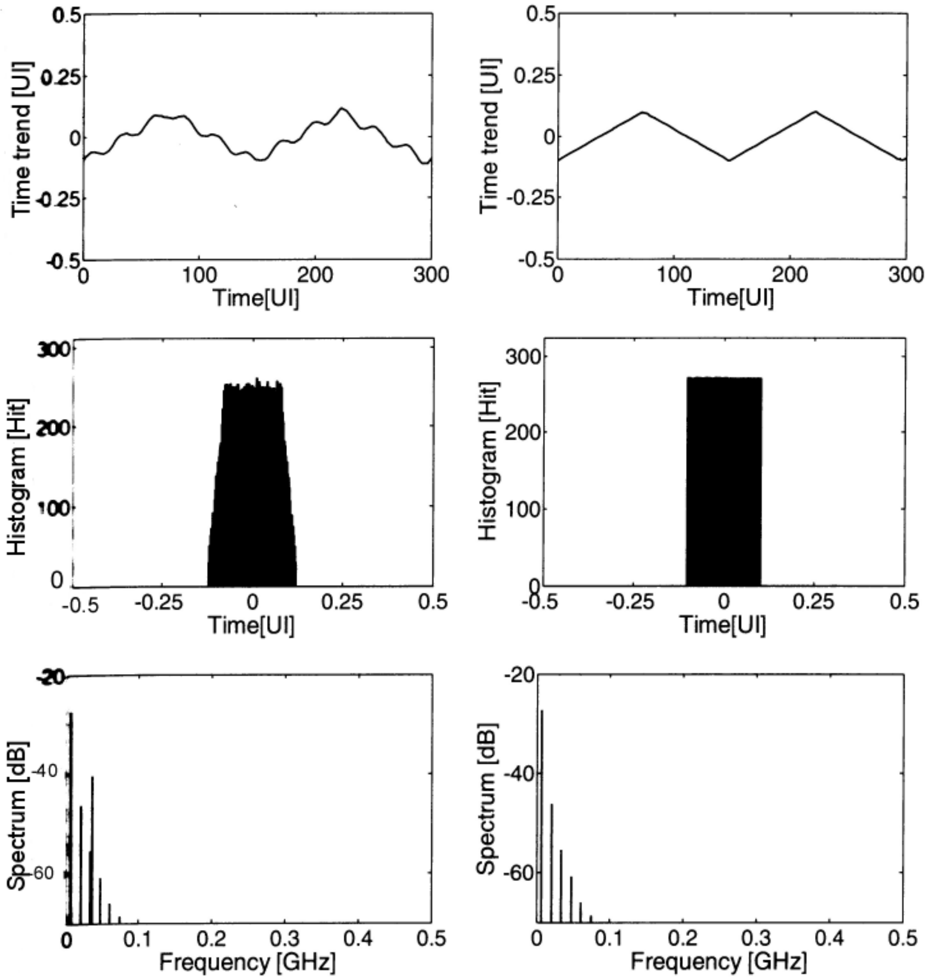
Eye

Periodic Jitter

Time domain

Statistical domain

Spectral domain



PJ **PJ subcomponent**

Clock Jitter

In a computer system, the clock is used to provide timing or synchronization for the system.

In a communication system, the clock is used to specify when a data switch or bit transaction should be transmitted and received

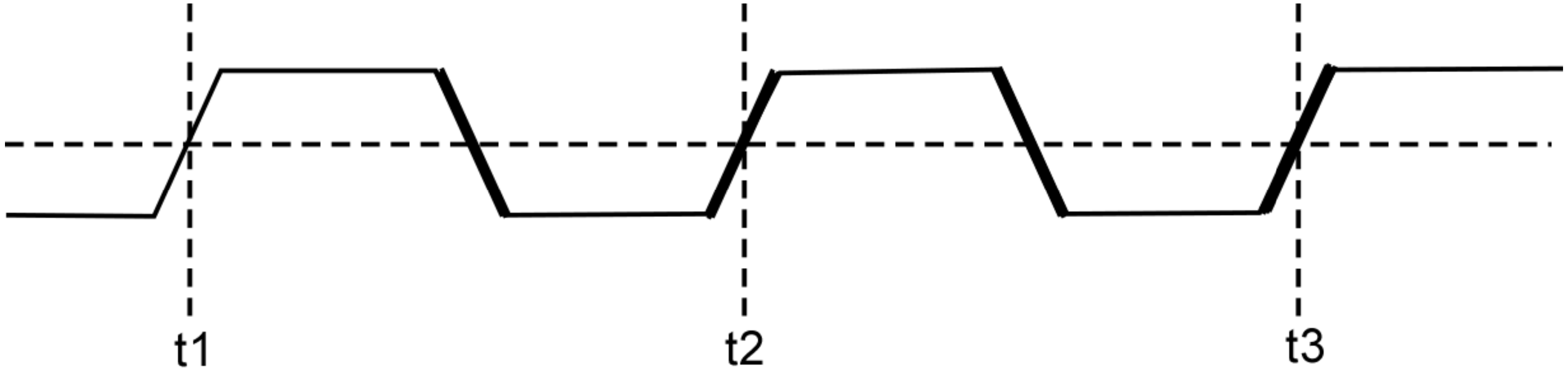
In a synchronized system, a central global clock is distributed to its subsystem

Clock jitter is the single most important degrader of clock performance

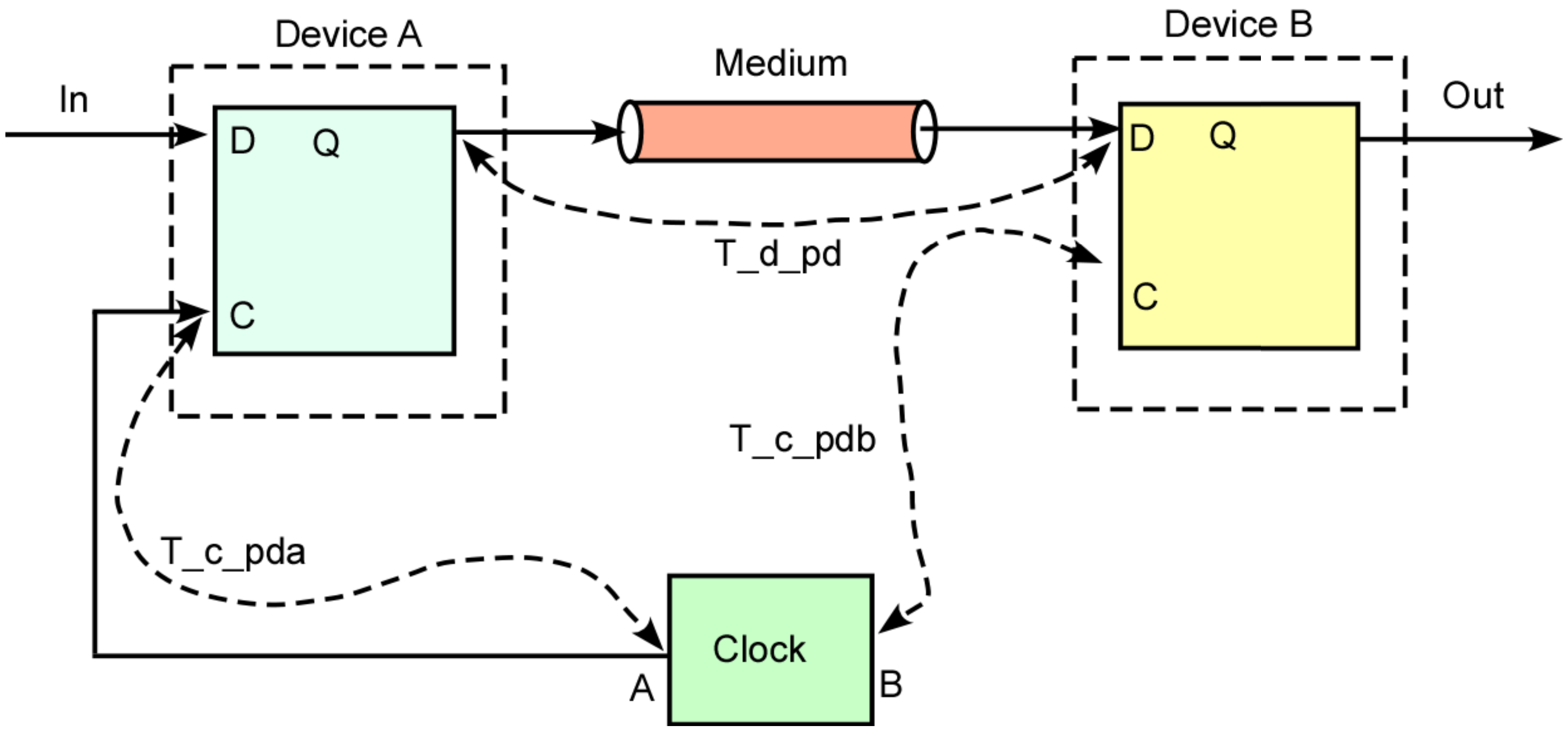
Definition

- Most of the definitions of data jitter (DJ, Rj,…) apply to clock jitter

- ISI does not apply to clock jitter

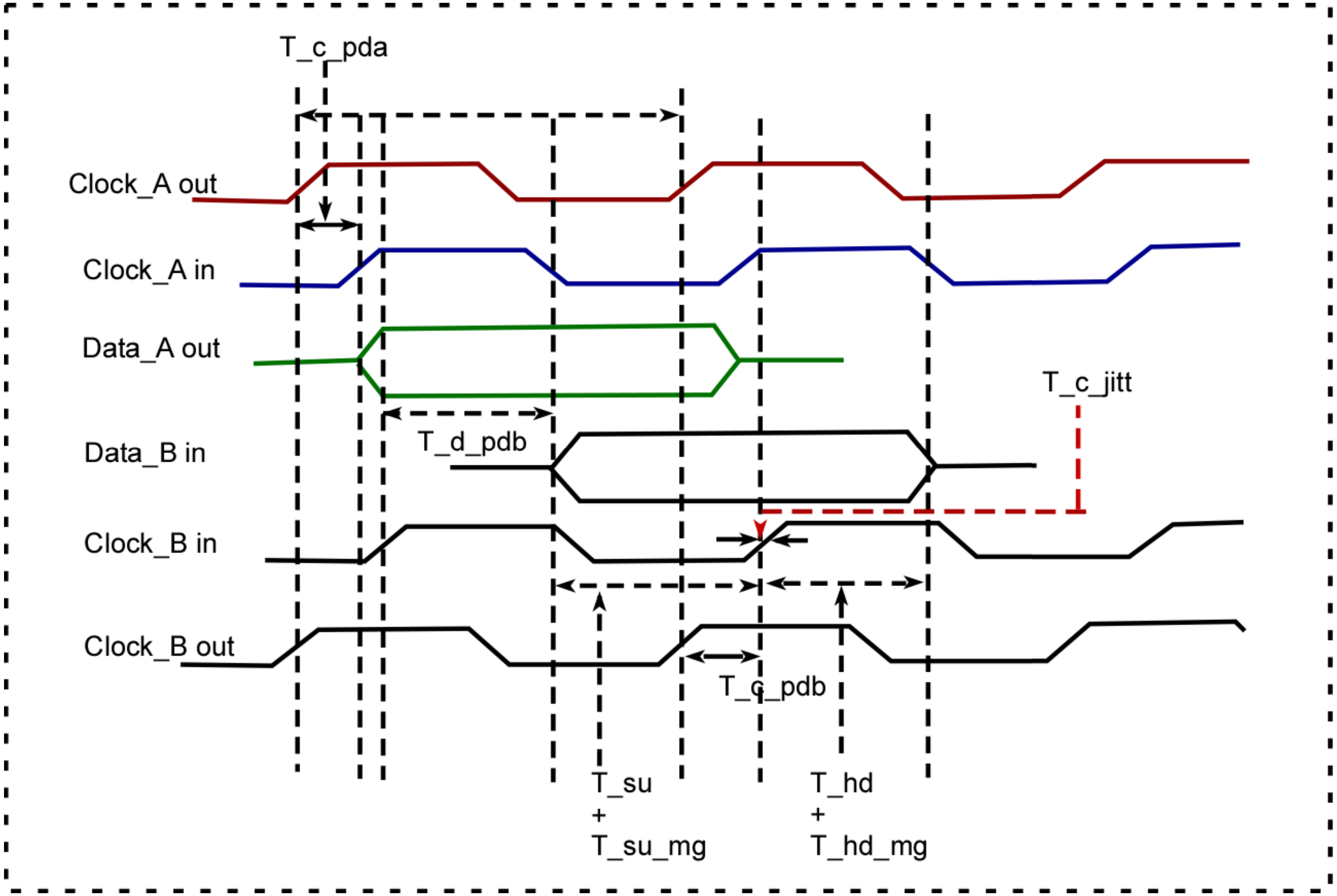


Synchronized System



- Initial clock pulse causes A to latch data from input and launch it into channel
- Second clock causes B to latch the incoming data

Timing Parameters



Timing Conditions

The minimum conditions are that both setup time and hold time margin should be larger than 0

$$T_0 \geq -T_{c\_jitt} + T_{c\_skew} + T_{d\_pd} + T_{su}$$

$$T_{hd} \leq T_{d\_pd} + T_{c\_skew} - T_{c\_jitt}$$

These give a quantitative description of how clock jitter and clock skew affect the performance of the synchronized system in which a common or global clock for both driver and receiver is used

Skew Impact

- $T_{c\_jitter}=0$, $T_{c\_skew}>0$
  - The minimum clock period increases. The maximum hold time increases ➔hold time condition easier to meet
- $T_{c\_jitter}=0$, $T_{c\_skew}<0$
  - The minimum clock period decreases. The maximum hold time decreases ➔hold time condition harder to meet (race condition)

Jitter Impact

- $T_{c\_skew}=0$, $T_{c\_jitter}>0$ (longer cycle)
  - The minimum clock period increases. The maximum hold time decreases →hold time condition harder to meet

- $T_{c\_skew}=0$, $T_{c\_jitter}<0$ (shorter cycle)
  - The minimum clock period decreases. The maximum hold time increases →hold time condition easier to meet

System Performance

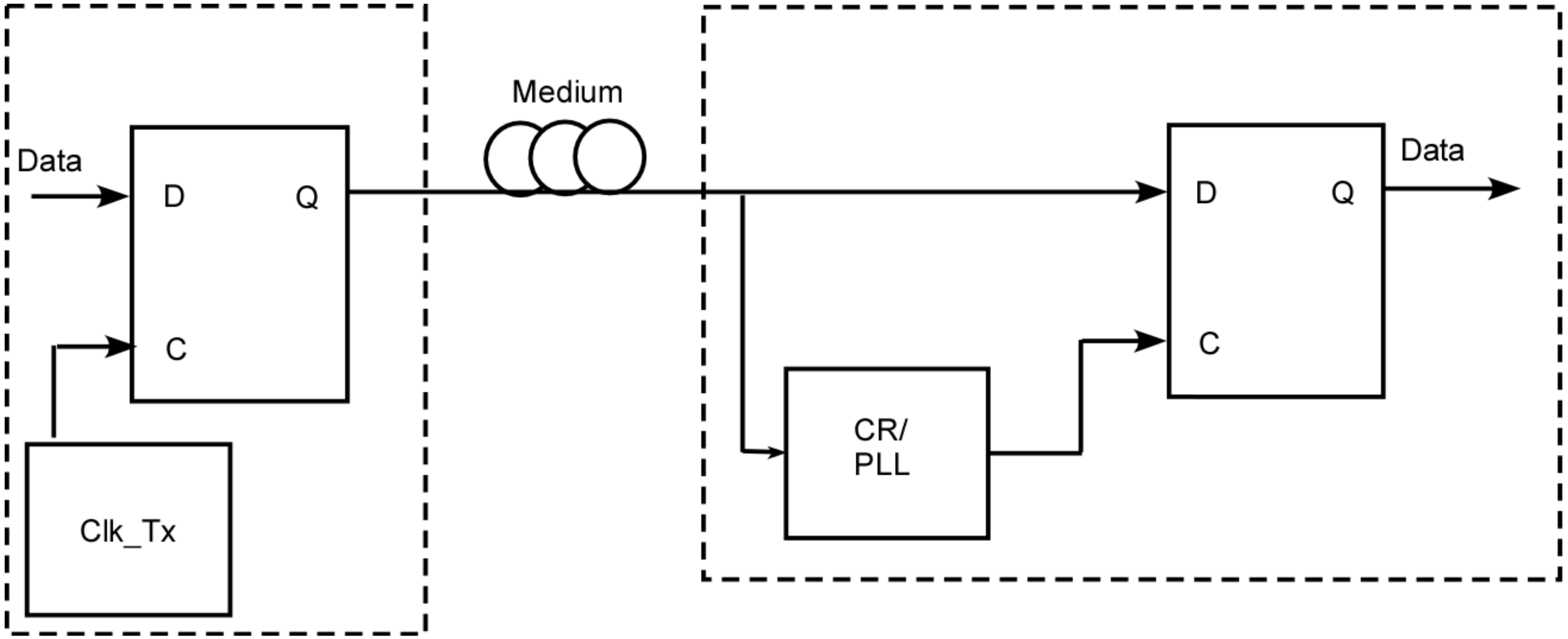
1. Positive jitter over one clock period makes both clock period and hold time hard to meet

2. A longer cycle does more harm to system performance

3. When both skew and jitter are present, system performance can be any of the four scenarios just discussed

Asynchronized System

The skew of a synchronized system becomes hard to manage when the data rate increases(~1 Gb/s). At multiple Gb/s data rates, an asynchronized system is commonly used.



Clock Types

- Synchronized System
  - Global clock is used to update and determine bits

- Asynchronized System
  - Only data is sent
  - Clock is embedded in data
  - Clock recovery unit (CRU) recovers clock at receiver

Asysnchronized Link

$$DJ_{clk\_tot} = DJ_{clk\_tx} + DJ_{clk\_rx}$$

$$\sigma^2_{clk\_tot} = \sigma^2_{clk\_tx} + \sigma^2_{clk\_rx}$$

Low-frequency jitter from the transmitter clock can be tracked or attenuated by the clock recovery function if it has a high enough corner frequency. A low phase noise oscillator within a PLL clock recovery also provides smaller random jitter generations.

Phase Jitter

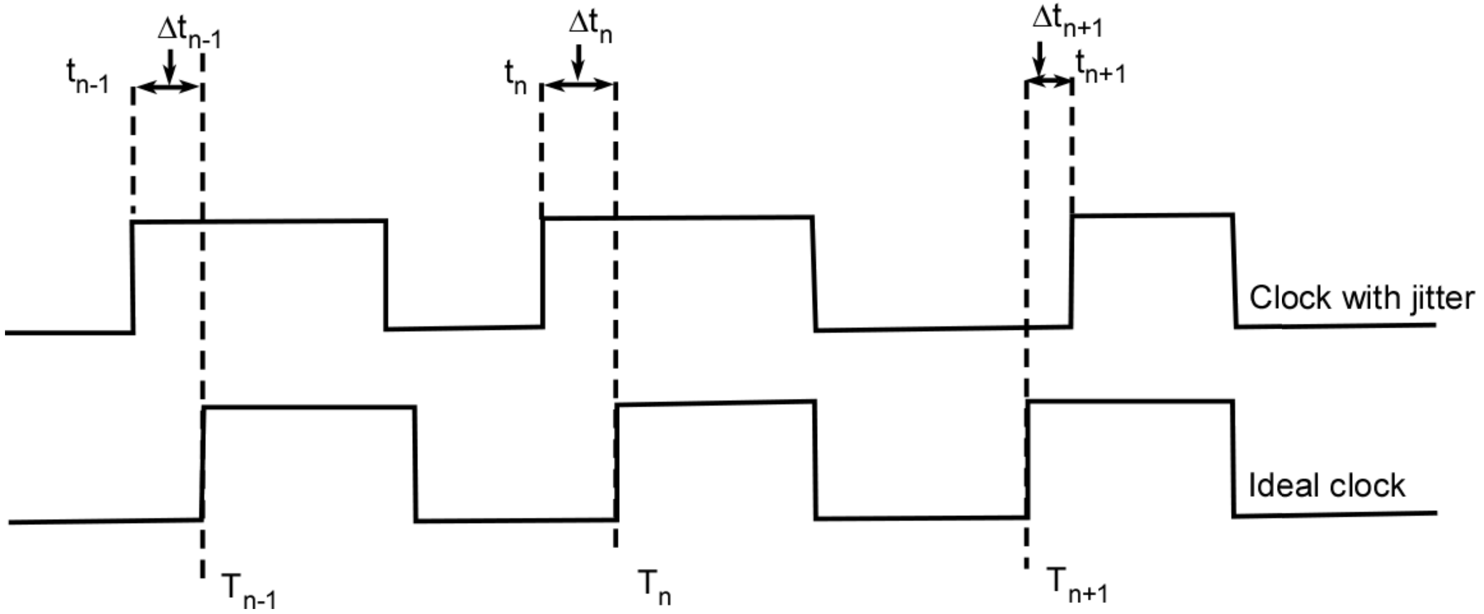
t_n : timing for nth edge for **jittery** clock

T_n : timing for nth edge for **ideal** clock

T_o : ideal clock period

$$\Delta t_n = t_n - T_n$$

$$T_n = nT_o$$



Phase Jitter

Phase jitter captures the instance timing deviation from the ideal for each transition. Jitter measured with phase jitter is absolute and accumulates over time.

In frequency domain

$$\phi_n = \frac{t_n}{T_o} 2\pi$$

Period Jitter

Period jitter is defined as the period deviation from the ideal period.

$$\Delta t_{pn} = \left(t_n - t_{n-1}\right) - T_o$$

using previous relations

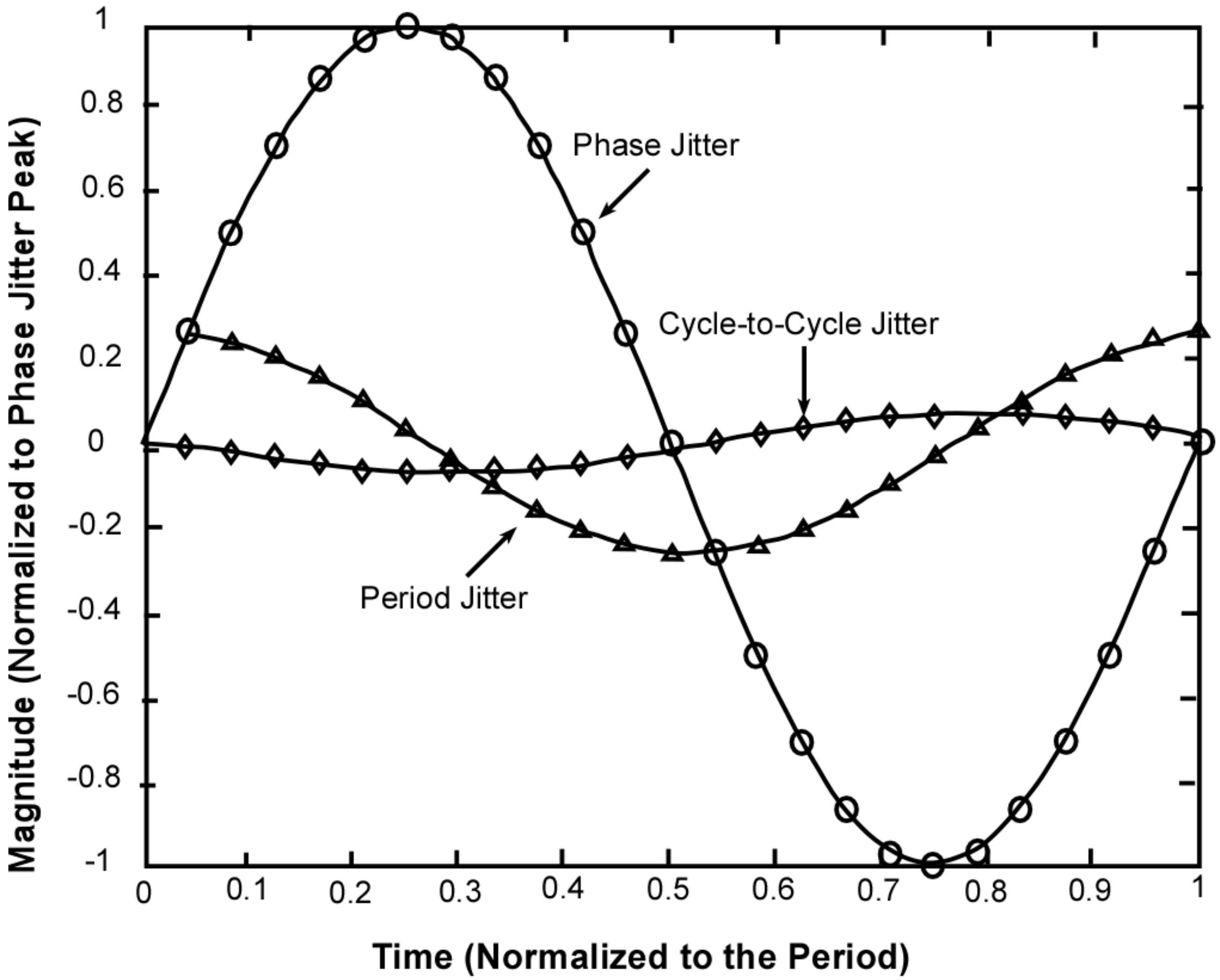
$$\Delta t_{pn} = \Delta t_n - \Delta t_{n-1}$$

in terms of phase units

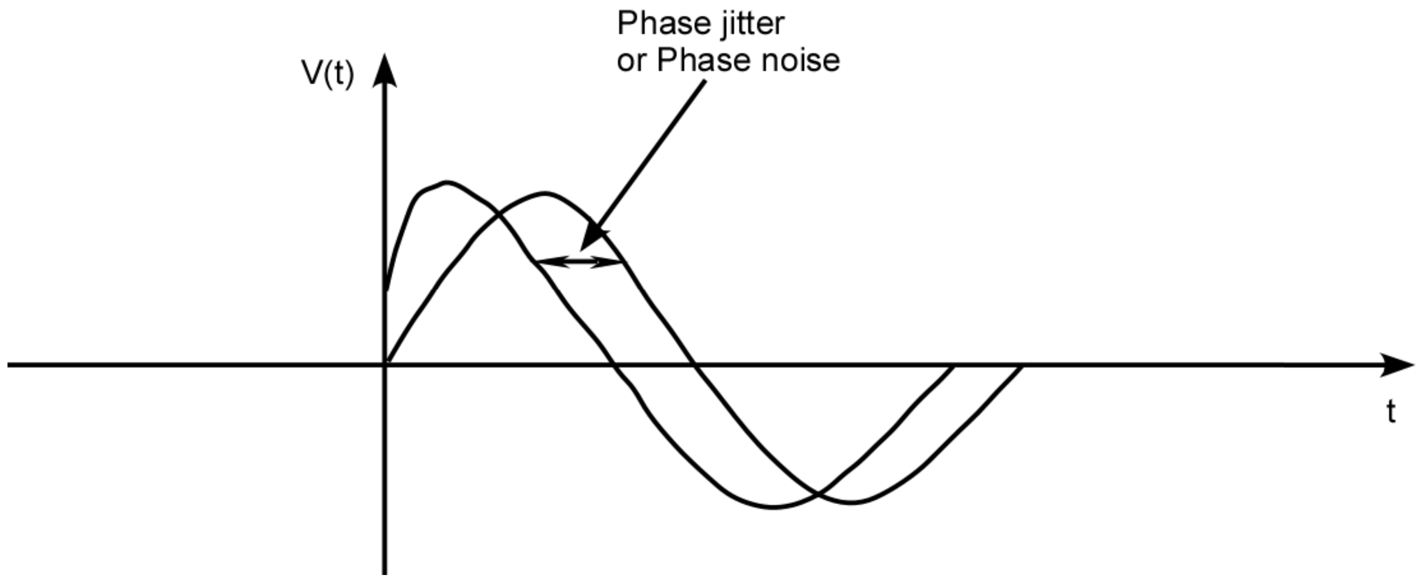
$$\phi_n^{'} = \Phi_n - \Phi_{n-1}$$

Period jitter and phase jitter are not independent ➔we can derive one from the other.

Phase, Period and CTC Jitter

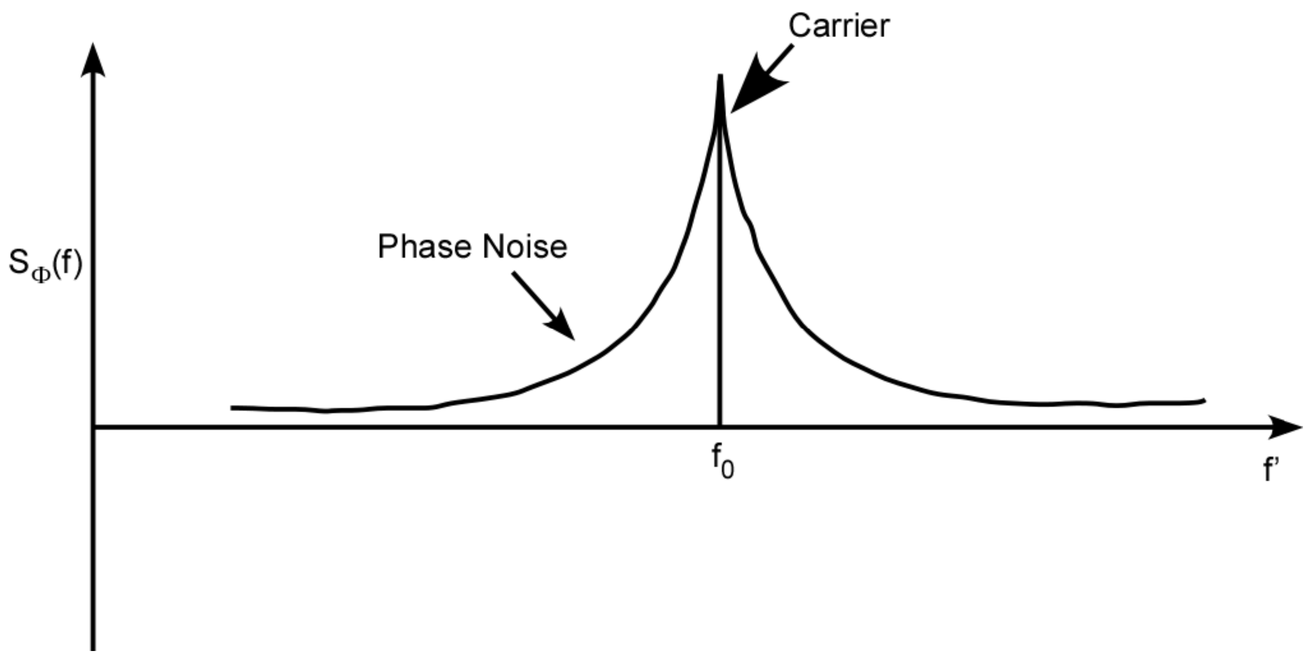


Phase Jitter in Time Domain



If the phase varies, the waveform V(t) shifts back and forth along the time axis and this creates phase jitter

Phase Jitter in Spectral Domain



Phase noise appears as sidebands centered around the carrier frequency

Phase Jitter

Phase noise magnitude is specified relative to the carrier's power on a per-hertz basis

$$L(f) = \frac{P_n(f)}{P_o \Delta f}$$

$P_n(f)$: phase noise power (in watts)

P_o : carrier's power (in watts)

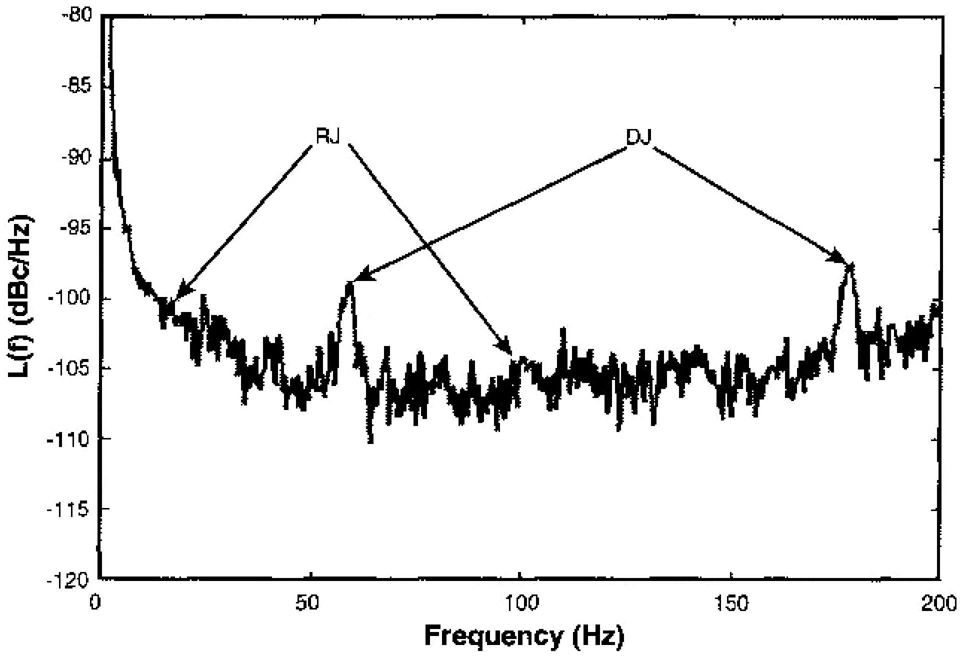
Δf : phase noise bandwidth (in hertz)

$$L(f) = \frac{1}{2} S_\Phi(f) \quad \text{or} \quad L(f) = 10\log_{10}\left(\frac{S_\Phi(f)}{2}\right)$$

$S_\Phi(f)$: PSD of phase noise

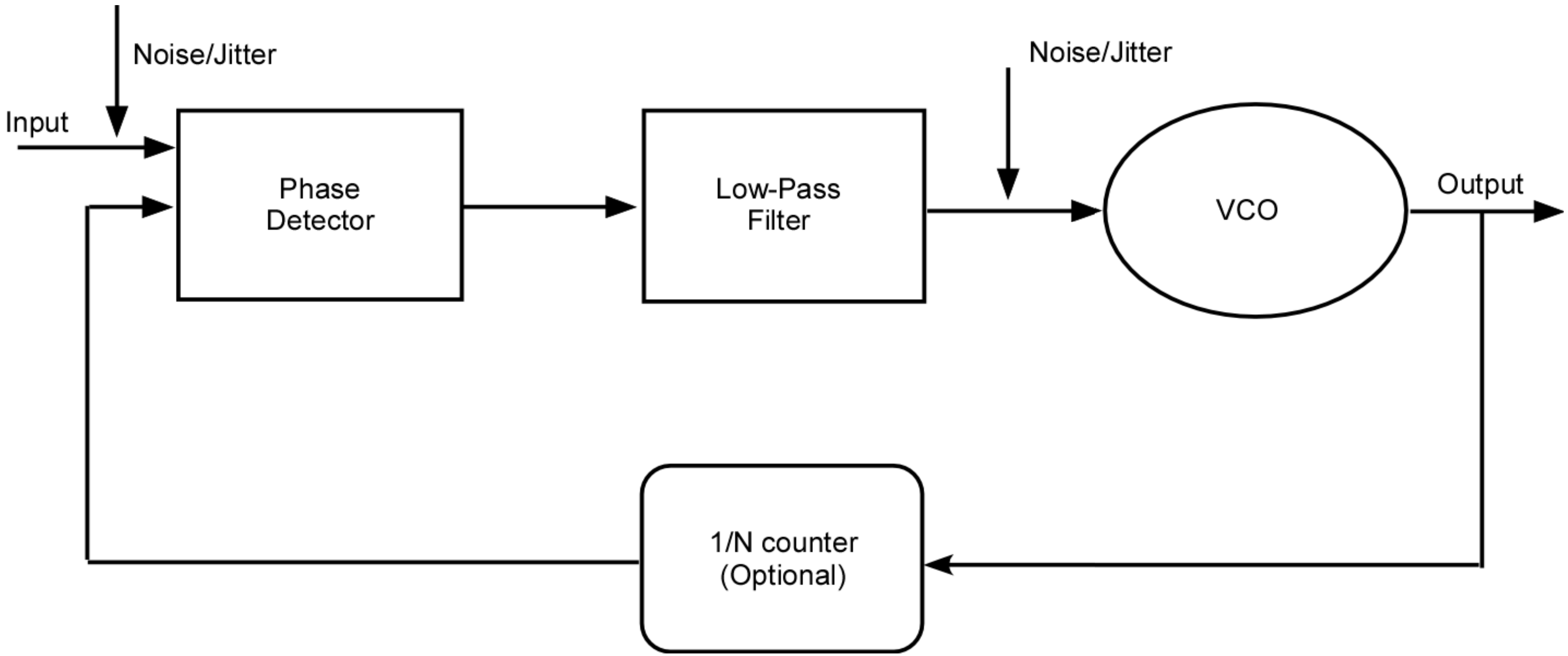
Phase Noise to Phase Jitter

Need: convert phase noise measured in the frequency domain to phase jitter for PLLs, clocks and oscillators



From the phase noise PSD, random jitter and deterministic jitter can be identified

Phase Lock Loop



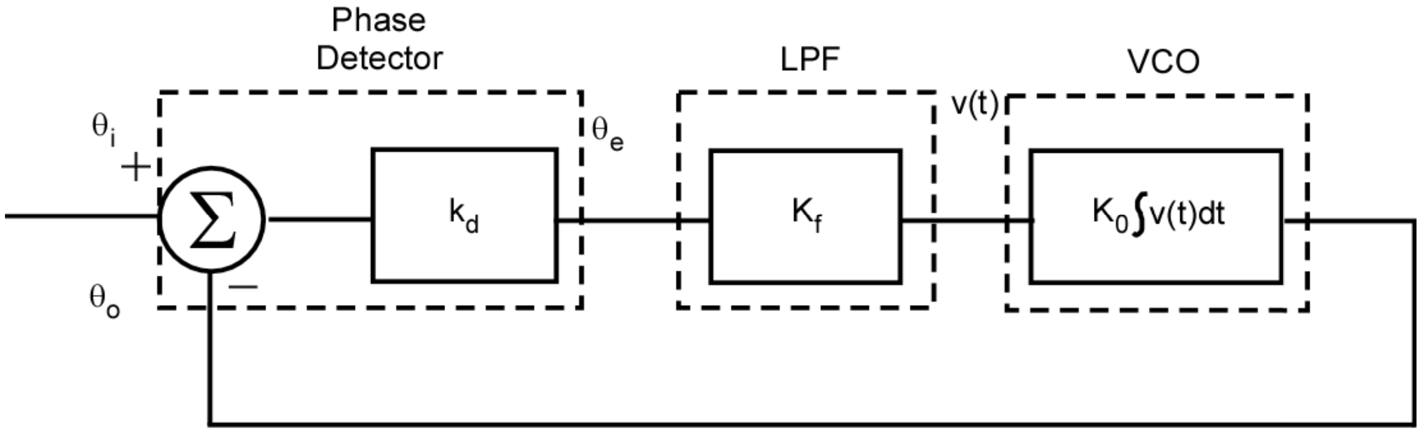
Phase noise or jitter is the key metric for evaluating the performance of a PLL system

Jitter in PLLs

- External Source
  - Reference clock input

- Internal Source
  - Voltage controlled oscillator (VCO)

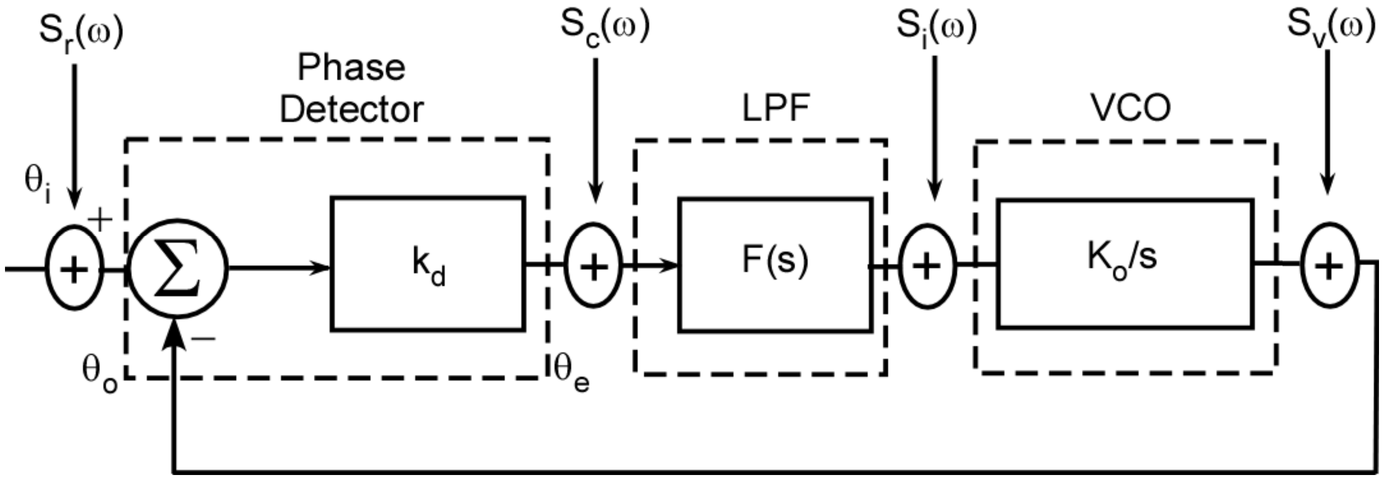
Time Domain PLL Analysis



- When PLL is a first-order system, it can be modeled by a closed-form solution

- It is not straightforward to model jitter/noise process with loop components in the time domain

Frequency- Domain PLL Analysis

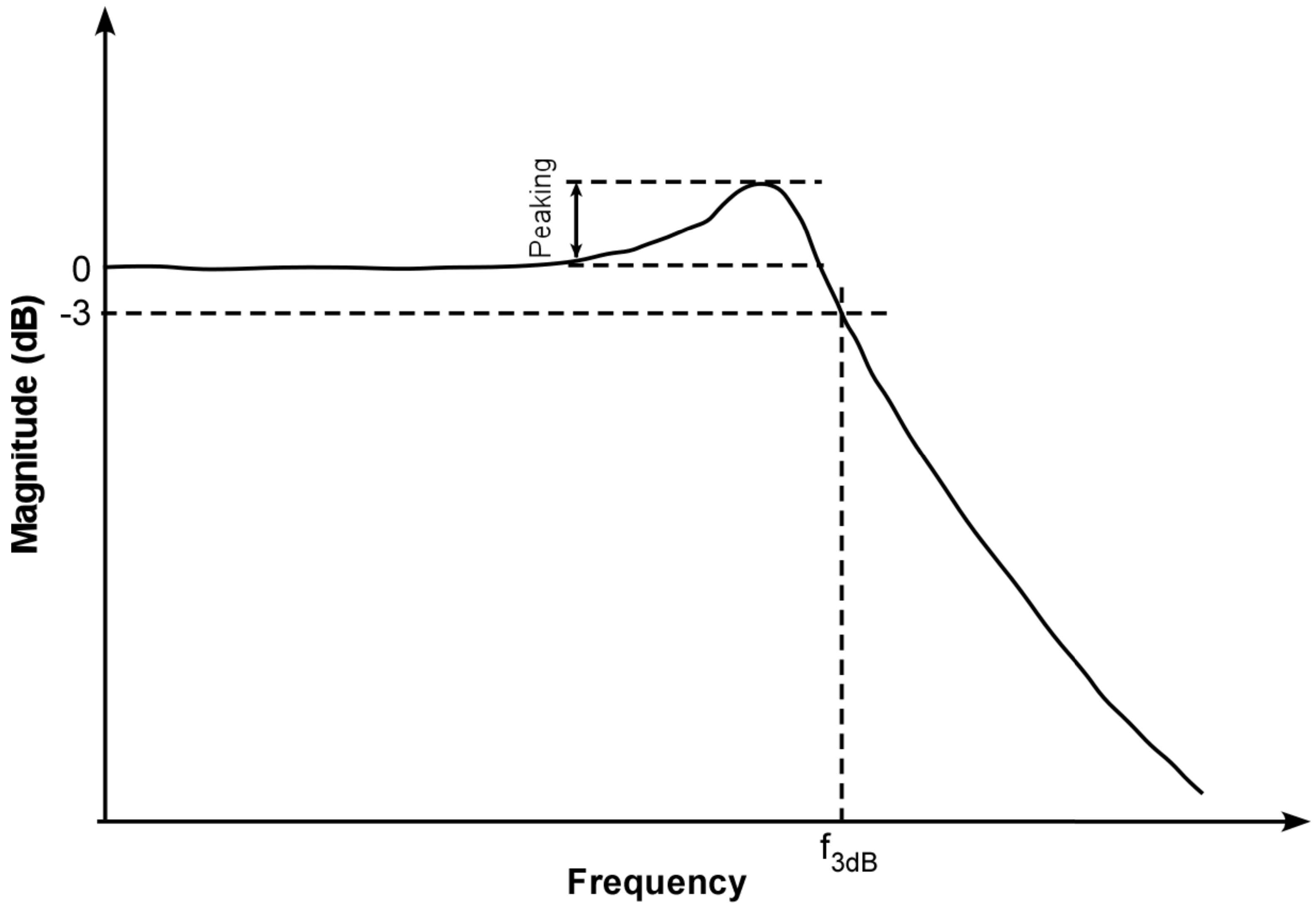


$$H_o(s) = \frac{\theta_o(s)}{\theta_i(s)} = \frac{K_d K_o F(s)}{s + K_d K_o F(s)}$$

The error transfer function is:

$$H_e(s) = \frac{\theta_e(s)}{\theta_i(s)} = 1 - H_o(s)$$

PLL Transfer Function



PLL Frequency Response

- Large peaking causes PLL to be unstable
- Larger 3dB frequency → faster PLL tracking
- Larger peaking → jitter amplification→ bit error

For PLL stability, Barkhausen condition must be satisfied

$$\left| K_d K_o \frac{F(s)}{s} \right| = 1$$

$$Arg\left[K_d K_o \frac{F(s)}{s} \right] = 180^\circ$$

PLL Frequency Response