ECE 546
Lecture - 23
Jitter Basics

Spring 2024

Jose E. Schutt-Aine
Electrical & Computer Engineering
University of Illinois
jesa@illinois.edu

Probe Further

- D. Derickson and M. Muller, "Digital Communications Test and Measurement", Prentice Hall, 2007.
- Kyung Suk (Dan) Oh and Xingchao (Chuck) Yuan, High-Speed Signaling: Jitter Modeling, Analysis, and Budgeting, Prentice Hall, 2012
- Mike Peng Li, Jitter, Noise and Signal Integrity at High-Speed, Prentice Hall, 2008

Jitter Definition

Jitter is difference *in time* between when an event was ideally to occur and when it actually did occur.

- **Timing uncertainties in digital transmission systems**
- **Utmost importance because timing uncertainties cause bit errors**
- **There are different types of jitter**

Jitter Characteristics

- Jitter is a signal timing deviation referenced to a recovered clock from the recovered bit stream

- Measured in Unit Intervals and captured visually with eye diagrams

- Two types of jitter
  - Deterministic (non Gaussian)
  - Random

- The total jitter (TJ) is the sum of the random (RJ) and deterministic jitter(DJ)

Types of Jitter

- **Deterministic Jitter (DDJ)**
  - Data-Dependent Jitter (DDJ)
  - Periodic Jitter (PJ)
  - Bounded Uncorrelated Jitter (BUJ)
- **Random Jitter (RJ)**
  - Gaussian Jitter
  - $f^{-\alpha}$ Higher-Order Jitter

Jitter Effects

Bandwidth Limitations

- Cause intersymbol interference (ISI)
- ISI occurs if time required by signal to completely charge is longer than bit interval
- Amount of ISI is function of channel and data content of signal

Oscillator Phase Noise

- Present in reference clocks or high-speed clocks
- In PLL based clocks, phase noise can be amplified

Phase Noise & Phase Jitter

- **Phase noise in clock oscillators**
  - **Phase offset term that continually changes timing of signal**

$$S(t) = P\left(t + \phi(t)\right)$$

- $S(t)$: signal waveform with phase noise
- P: undistorted signal
- $\phi(t)$: phase noise

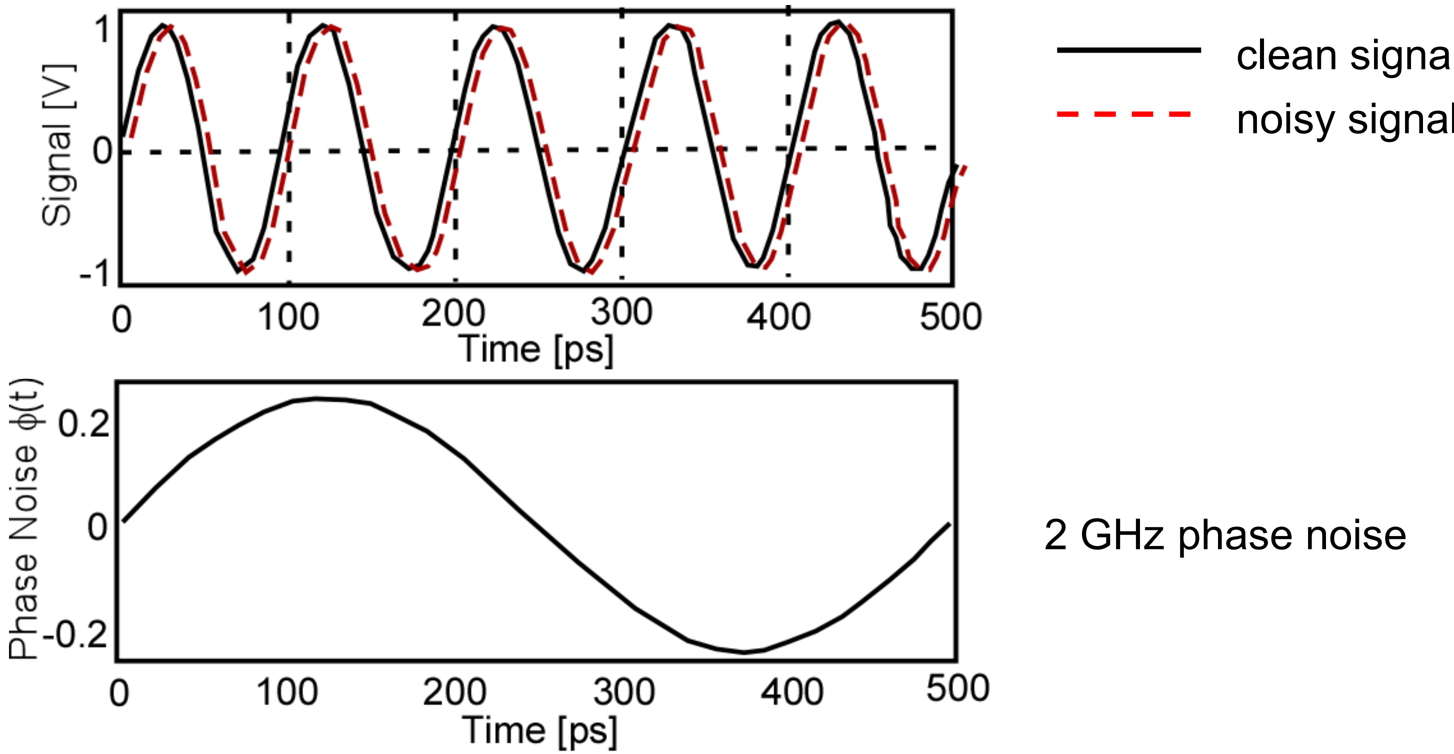
Example:

$$P(t) = \sin\left(10\times10^9\times2\pi t\right)$$

$$\phi(t) = \frac{1}{4}\sin\left(2\times10^9\times2\pi t\right)$$

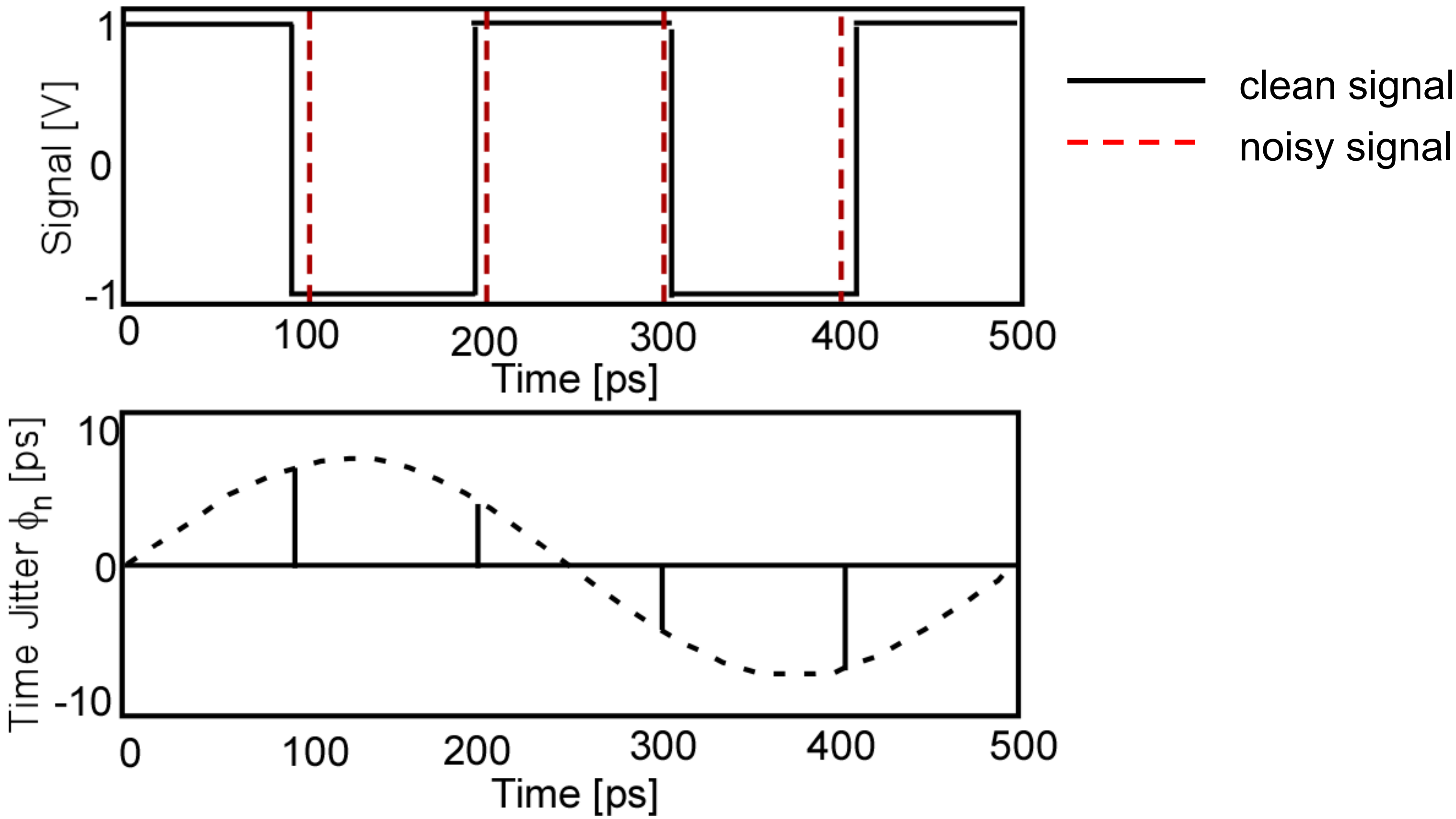
$$S(t) = \sin\left(10\times10^9\times2\pi t + 0.25\sin(2\times10^9\times2\pi t)\right)$$

Phase Noise



Signal [V]
1
0
-1
0
100
200
300
400
500
Time [ps]
clean signal
noisy signal
Phase Noise ϕ(t)
0.2
0
-0.2
0
100
200
300
400
500
Time [ps]
2 GHz phase noise

Phase Jitter



Phase Jitter

- **Phase jitter in digital systems**
  - **Variability in timing of transition in digital systems is called phase jitter**
  - **Phase jitter is digital equivalent of phase noise**
  - **Always defined relative to the ideal position of the transitions**

For a jittered digital signal

$$t_n = T_n - \phi_n$$

t_n is the actual time of the *nth* transition

T_n is the ideal timing value of the *nth* transition

ϕ_n is the time offset of the transition ← phase jitter term

Example: 10 Gbits/s → T_n has bit intervals of 100 ps. Transitions take place at 0, 100, 200 ps

Cycle-to-Cycle Jitter

- Phase jitter causes bit periods to contract and expand
- Actual bit periods are given by the time difference between 2 consecutive transitions

$$P_n = t_{n+1} - t_n = \left(T_{n+1} - \phi_{n+1}\right) - \left(T_n - \phi_n\right)$$

Ideal bit period:

$$TB_n = T_{n+1} - T_n$$

Period jitter:

$$PerJ_n = TB_n - P_n$$

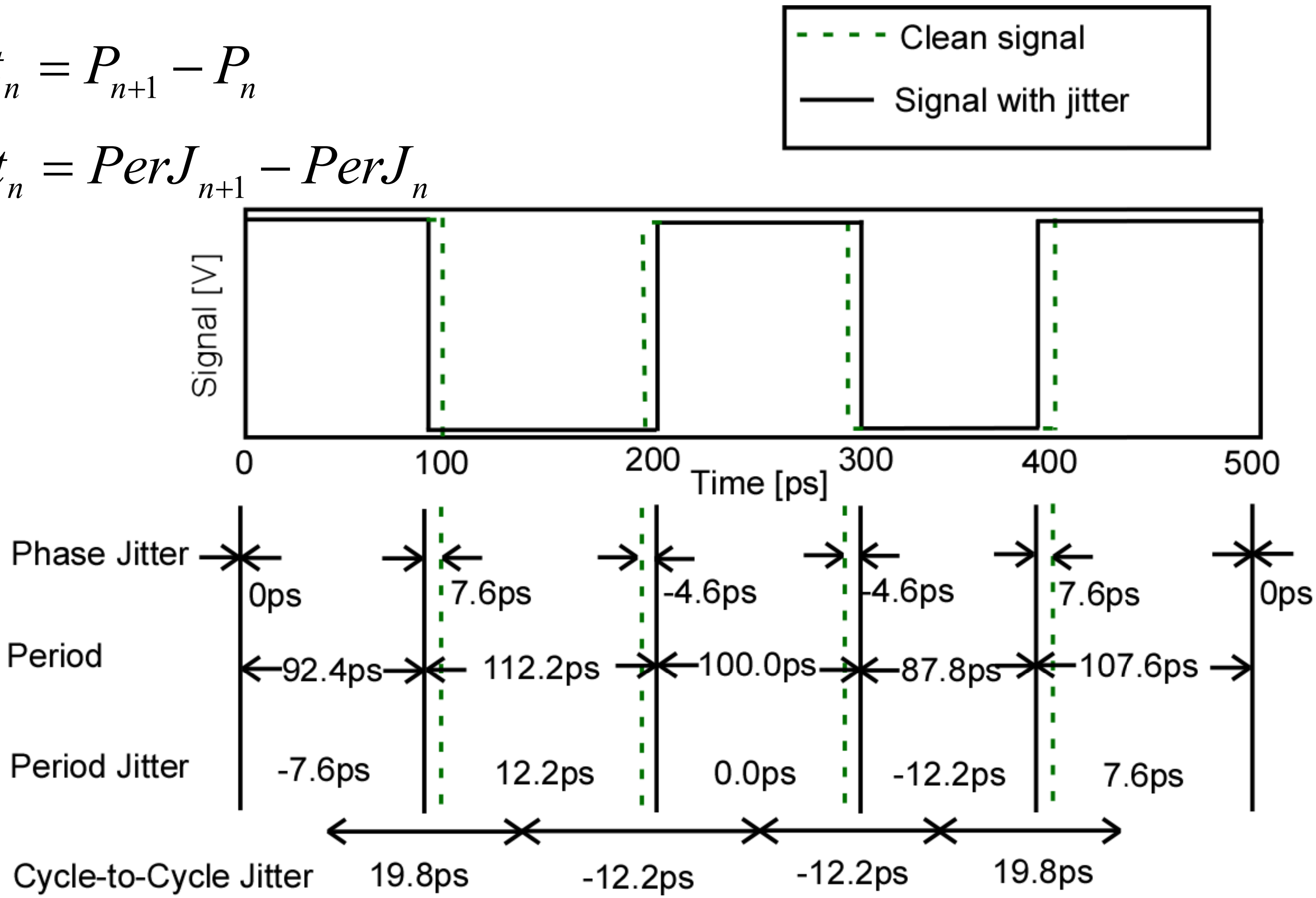
$$PerJ_n = \left(T_{n+1} - T_n\right) - \left(T_{n+1} - T_n + \phi_n - \phi_{n+1}\right) = \phi_{n+1} - \phi_n$$

Cycle-to-Cycle Jitter

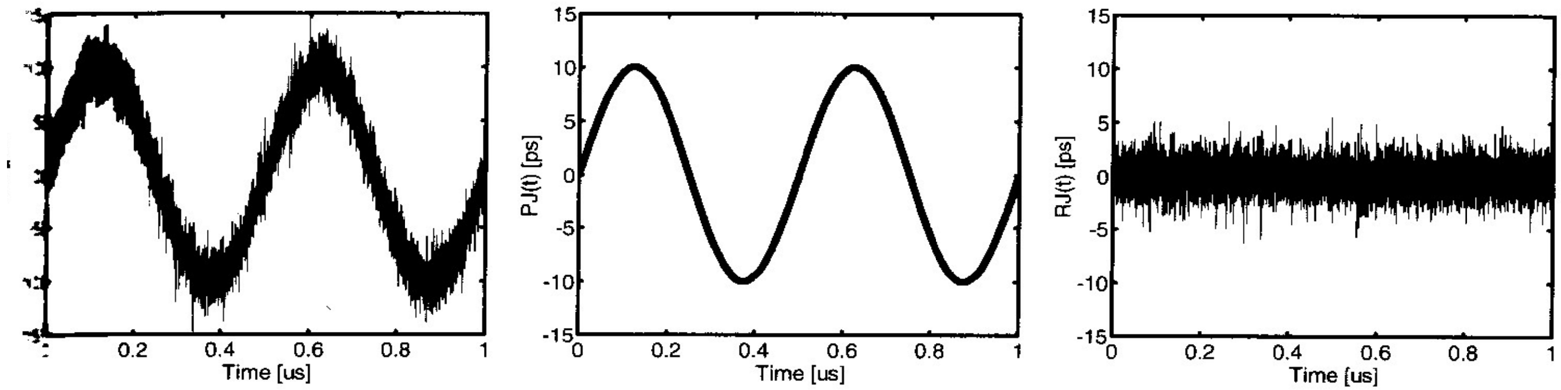
Cycle-to-cycle jitter:

$$CCJit_n = P_{n+1} - P_n$$

$$CCJit_n = PerJ_{n+1} - PerJ_n$$



Total Jitter Time Waveform



TJ(t) = PJ(t) + RJ(t)

The total jitter waveform is the sum of individual components

Jitter Statistics

- Most common way to look at jitter is in statistical domain
- Because one can observe jitter histograms directly on oscilloscopes
- No instruments to measure jitter time waveform or frequency spectrum directly

Jitter Histograms and Probability Density Functions

- Built directly from time waveforms
- Frequency information is lost
- Peak-to-peak value depends on observation time

Note: A jitter histogram does not contain all the information about the jitter

Probability Density Function

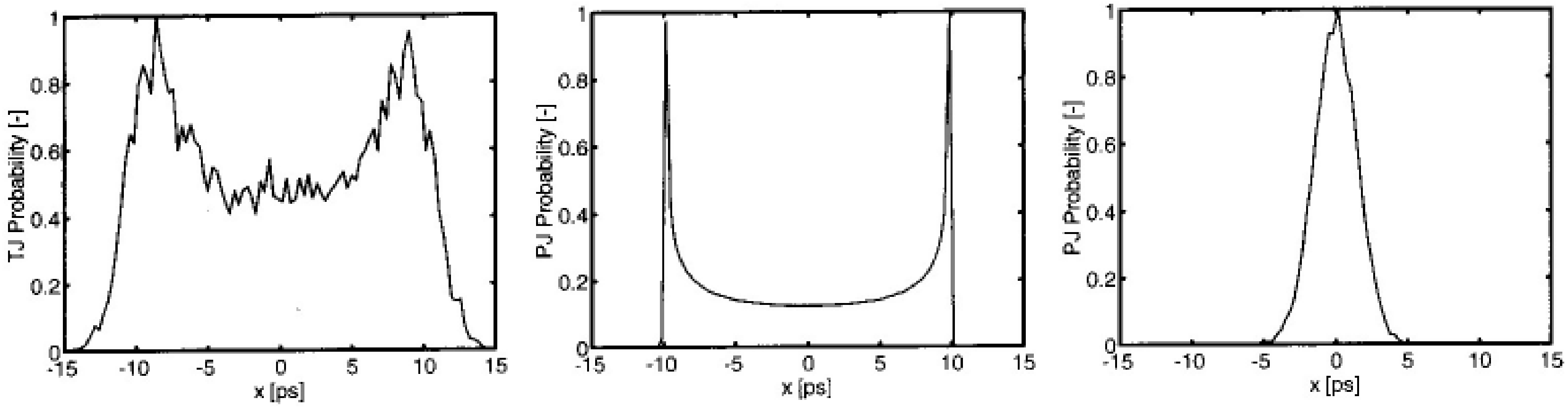
$x \longleftrightarrow pdf_x$

$y \longleftrightarrow pdf_y$

$z = x+y \longleftrightarrow pdf_z=pdf_y*pdf_z$

The PDF of the sum of 2 independent random variable is the convolution of the pdfs of those 2 variables

Jitter Statistics



TJ(x) = PJ(x) * RJ(x)

The total jitter PDF is the convolution of individual components

Jitter Mechanisms

- Transfer of Level Noise into the Time Domain
  - Noise on digital data signals causes jitter because it offsets the threshold crossing point in time
- Bandwidth Limitations
  - Primarily caused by intersymbol interference
- Oscillator Phase Noise
  - Phase noise present in reference clocks especially in systems based on PLL

Jitter Mechanisms

Jitter Mechanisms

- Transfer of noise into time domain
- Bandwidth limitation in channels
- Oscillator phase noise

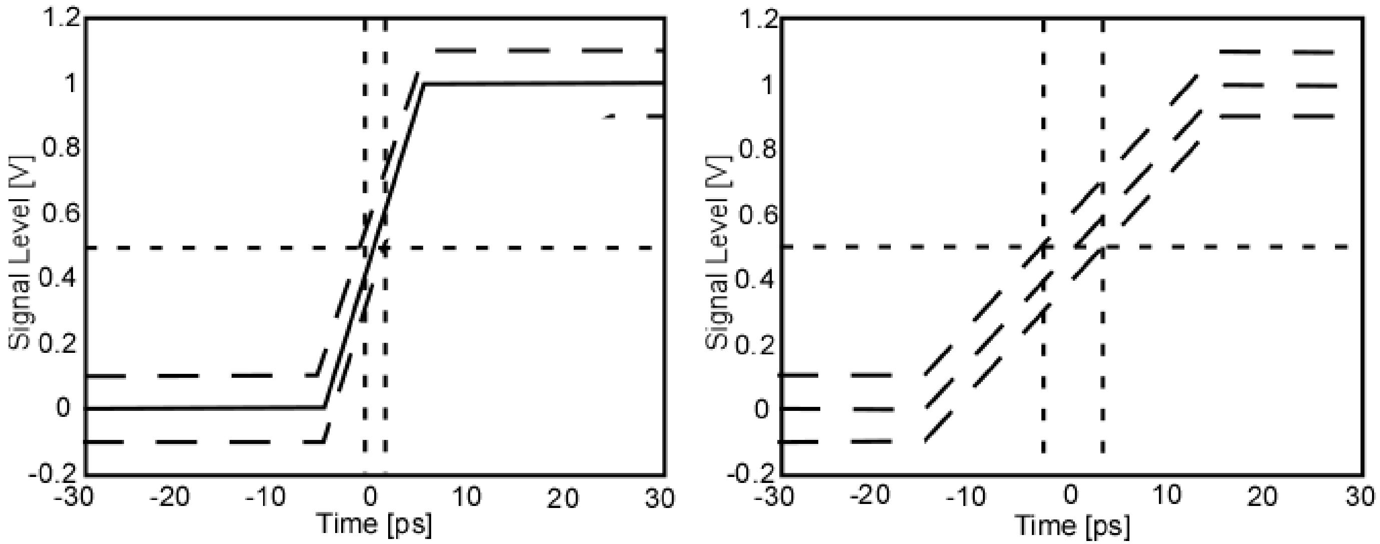
$$NJ_{pk-pk} = t_t \frac{V_{Noise}}{V_H - V_L}$$

t_t	rise time
V_{Noise}	pk-pk noise amplitude
V_H	Hi signal level
V_L	Lo signal level

Jitter Mechanisms

Linear model

$$NJ_{pk-pk} = t_t \frac{V_{Noise}}{V_H - V_L}$$



Jitter ~ 2ps

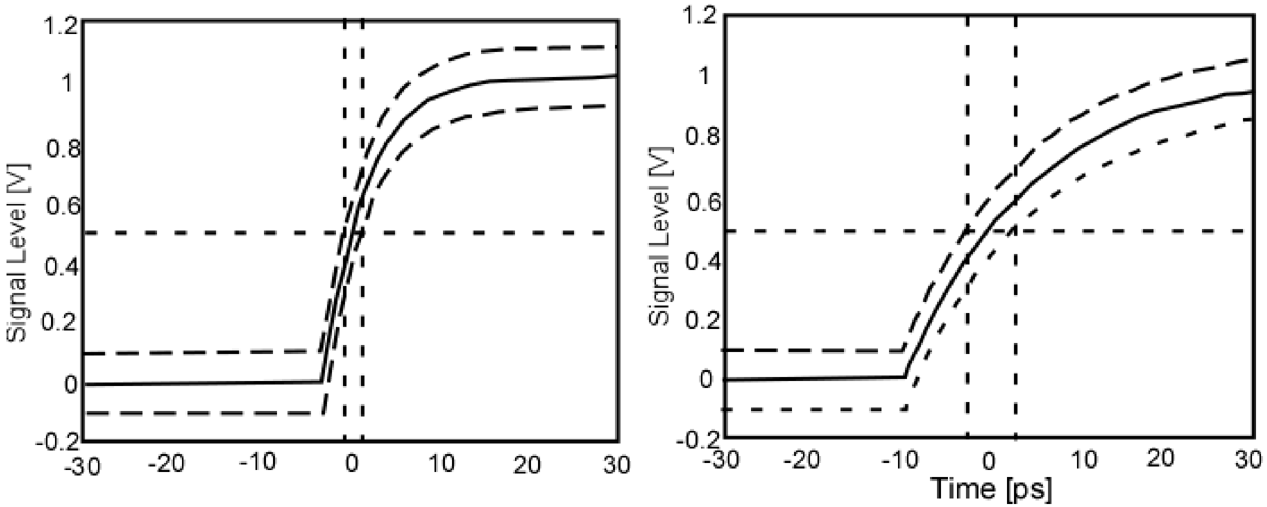
Jitter ~ 6ps

Random noise caused by thermal effects

Jitter Mechanisms

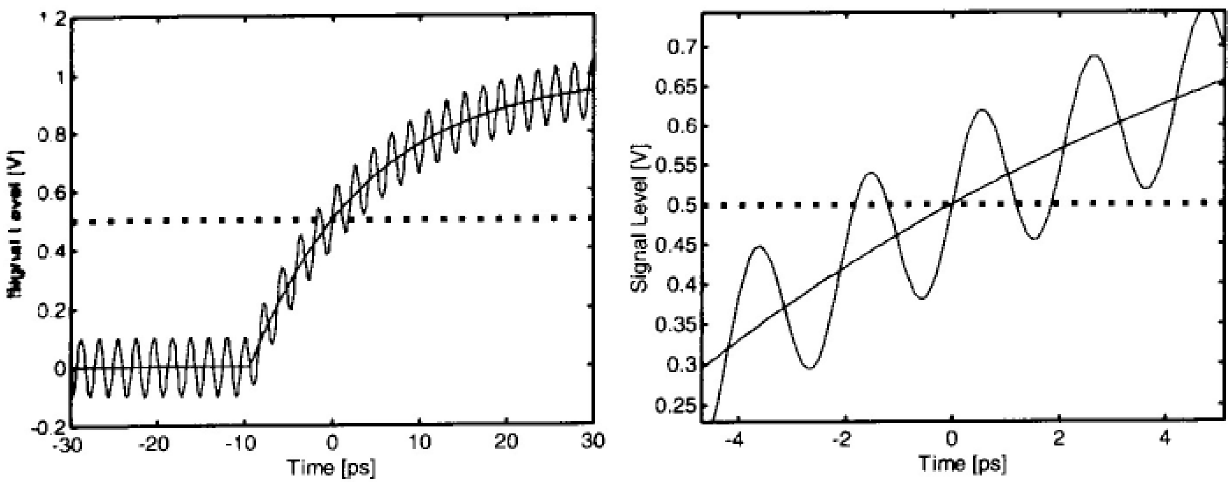
First order model

$$NJ_{pk-pk} = -\tau\left(\ln\left(0.5 - V_{Noise}\right) + \ln\left(0.5 + V_{Noise}\right)\right)$$



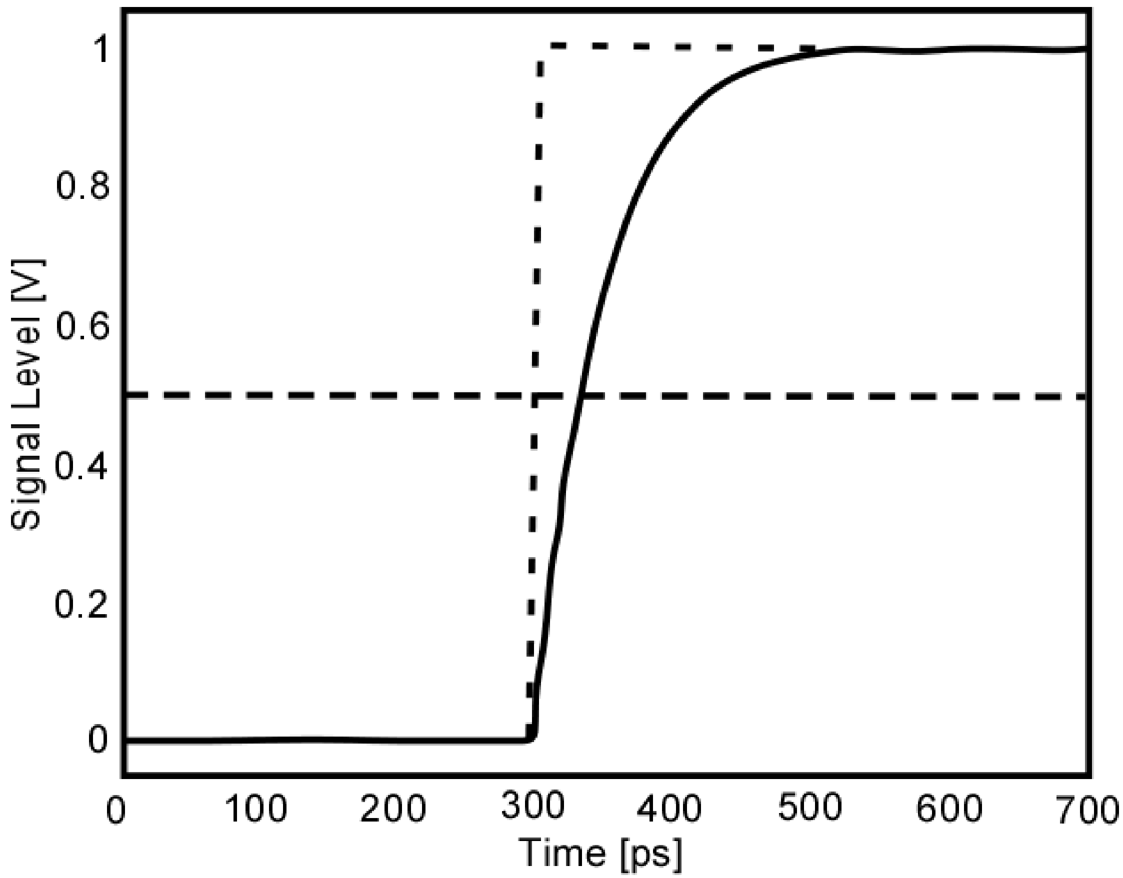
Periodic noise: switching power, crosstalk, etc…

Jitter Mechanisms



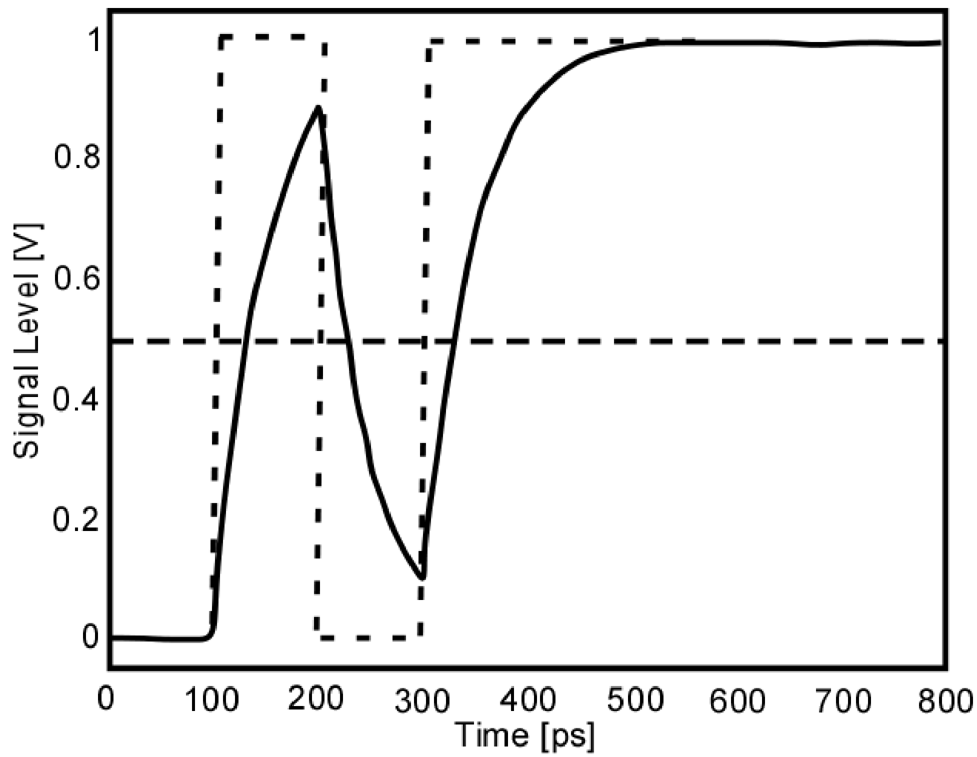
Multiple threshold crossing of a signal with high-frequency level noise

Bandwidth Limitations



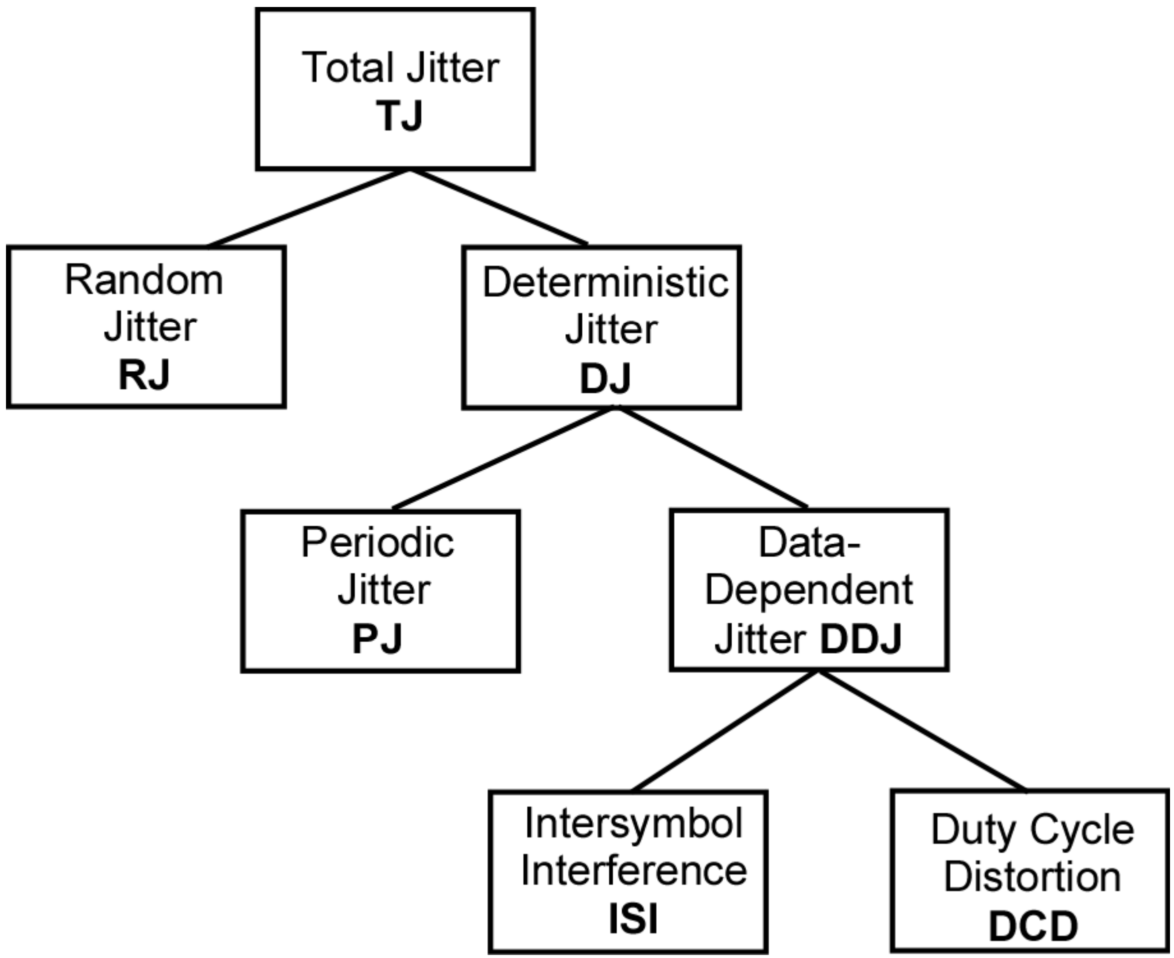
0001111 data pattern

Bandwidth Limitations



0101111 data pattern

Jitter Classification



Gaussian Random Jitter

- **Random jitter can be described by a Gaussian distribution with the following probability density function**

$$PDF_{RJ}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

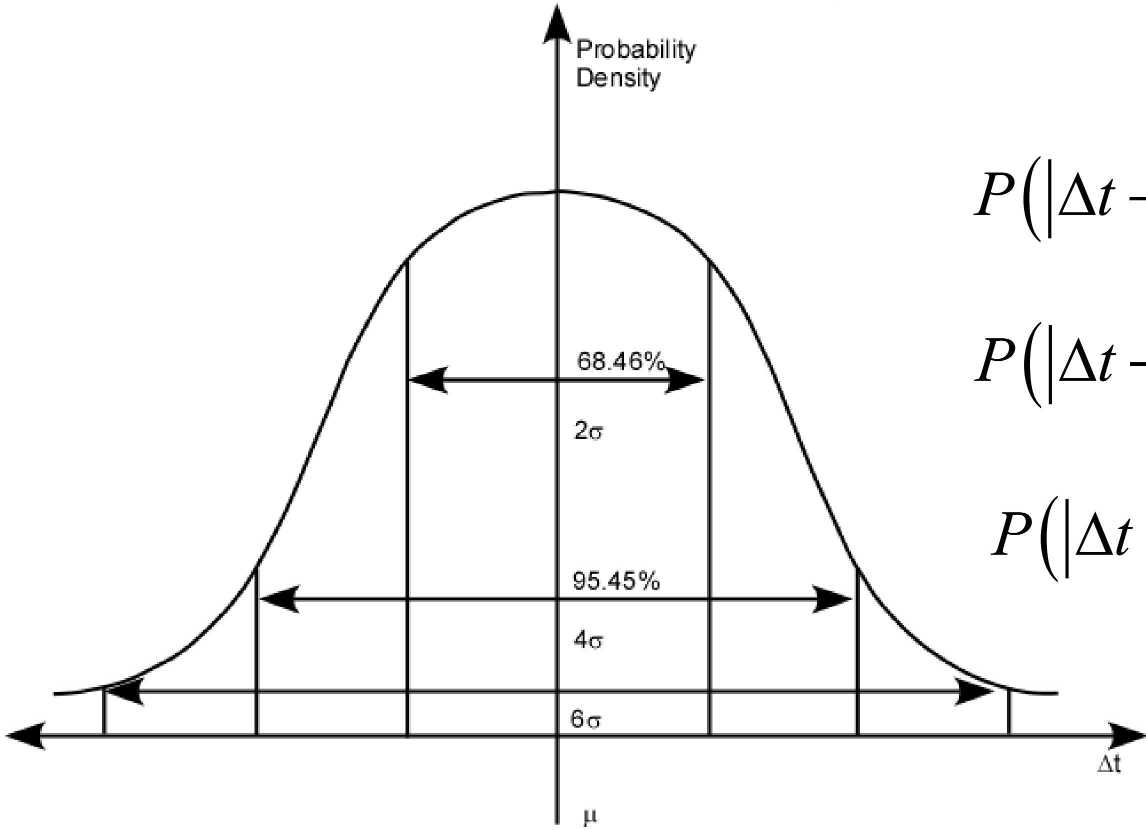
x : independent value

σ : RMS value

μ : mean of distribution (zero by definition)

➔**Note: the PDF of a Gaussian process is unbounded, i.e, its PDF is not zero unless the jitter *Δt* approaches infinity**

Gaussian Jitter PDF



$$P\left(\left|\Delta t-\mu\right|\leq\sigma\right)=0.6826$$

$$P\left(\left|\Delta t-\mu\right|\leq 2\sigma\right)=0.9545$$

$$P\left(\left|\Delta t-\mu\right|\leq 3\sigma\right)=0.9973$$

Can be used to estimate the probability when the deviation of the random jitter variable Δt is within a multiple of its σ value.

Cumulative Density Function

Cumulative density function (CDF) is defined as:

$$CDF(t) = \int_{-\infty}^{t} PDF(x)dx$$

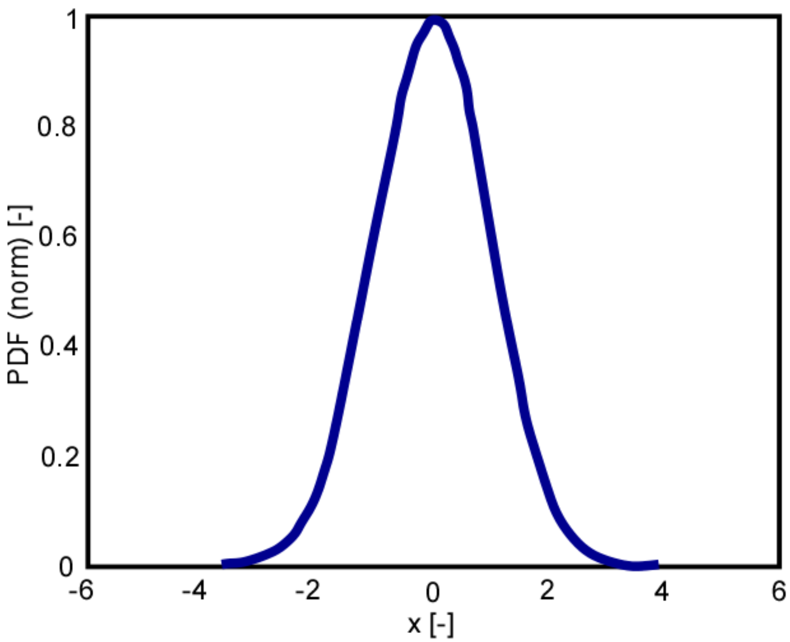
CDF(t) **tells us the probability that the transition occurred earlier than *t*. For random jitter, we get:**

$$CDF_{RJ}(x) = \frac{1}{2} + \frac{1}{2} erf \frac{x}{\sigma\sqrt{2}}$$

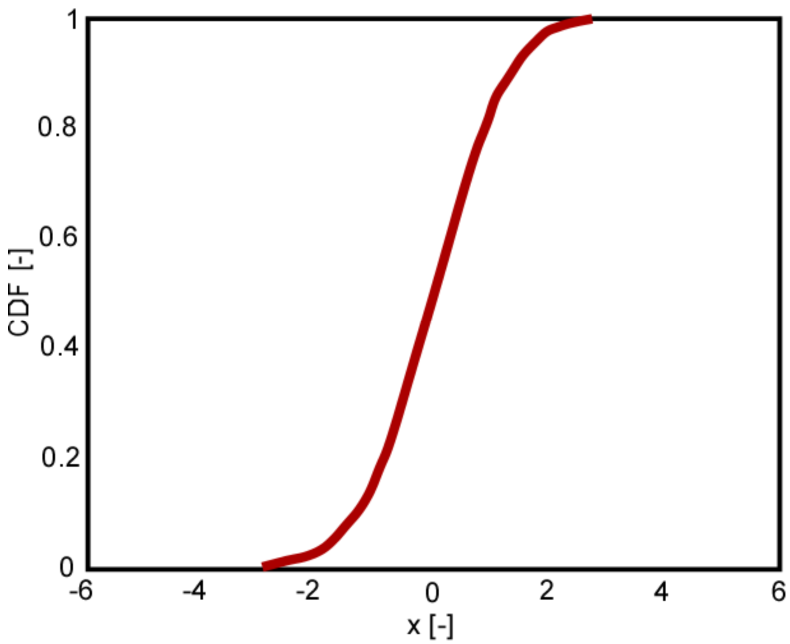
erf **is the error function**

PDF and CDF of Random Jitter

PDF



CDF



Causes of Deterministic Jitter

- **Crosstalk**
  - **Noisy neighboring signals**

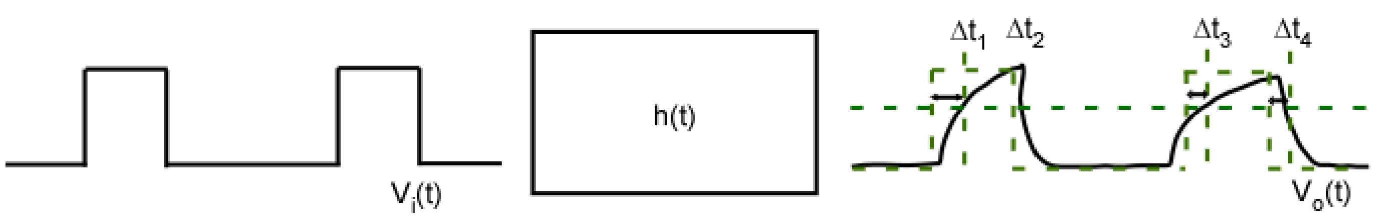
- **Interference**

- **Reflections**
  - **Imperfect terminations**
  - **Discontinuities (e.g. multi-drop buses, stubs)**

- **Simultaneous switching noise (SSN)**
  - **Noisy reference plane or power rail**
  - **Shift in threshold voltages**

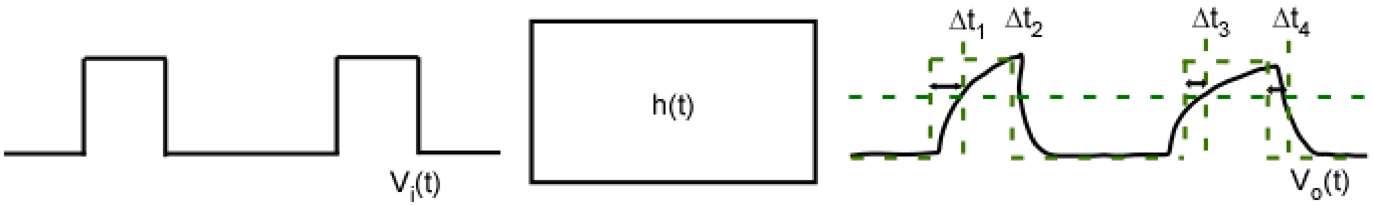
Data-Dependent Jitter

- **Most commonly encountered DJ type**
- **Dominant limiting factor for link channels**
- **Due to *memory* of lossy electrical or optical system**
- **Bit transition of current bit depends on the transition times of the previous bits**



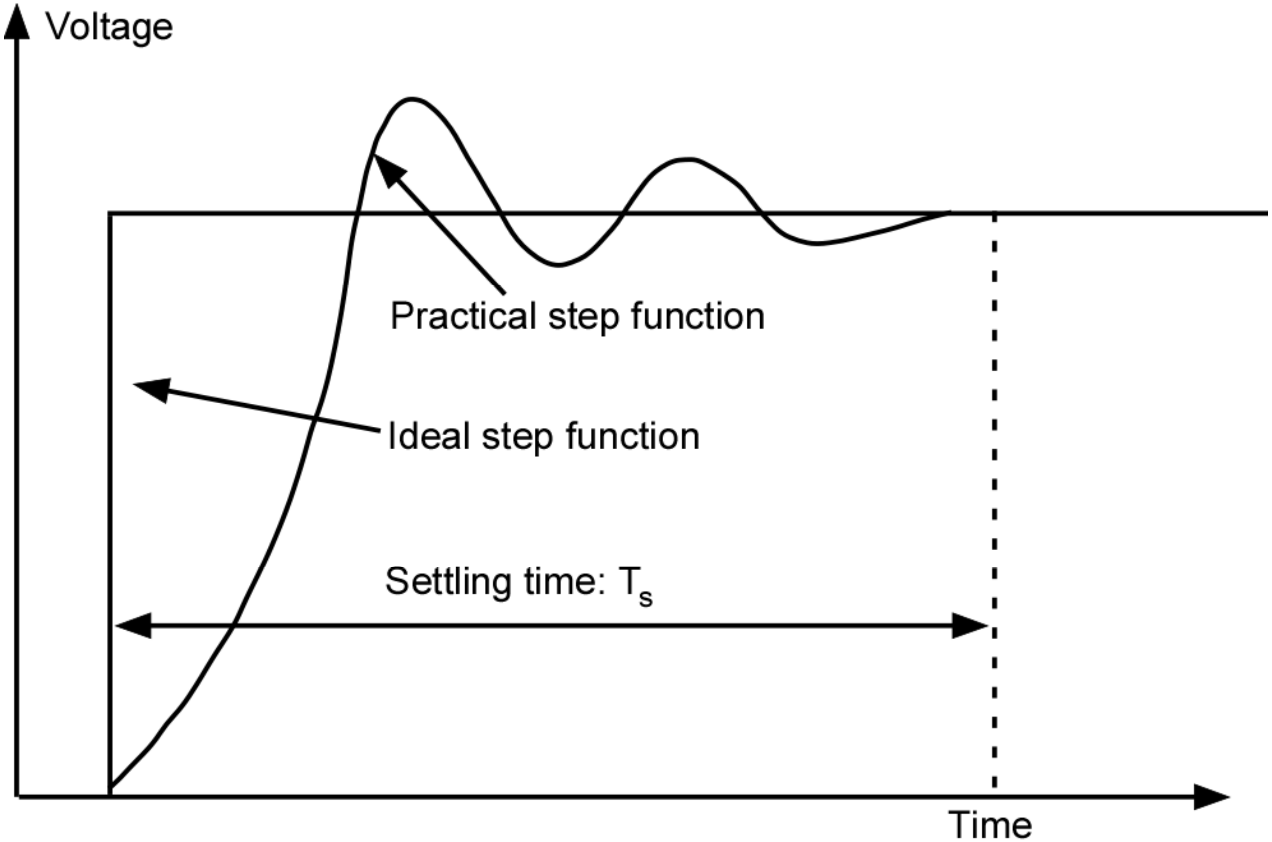
Data-Dependent Jitter

- **DDJ depends on the impulse response of the system that generates the pattern**
- **DDJ depends on the input pattern**
- **DDJ is a distribution with its sample size equal to the number of transitions of the data pattern**



- **Duty cycle distortion (DCD) occurs for clock patterns of repeating bits**

Data-Dependent Jitter



- **Since channel does not have zero-rise time step response or infinite bandwidth, jitter is to be expected**
- **Settling time gives good indication of jitter**

Model for DDJ

The generic form for DDJ PDF is:

$$f_{DDJ}(\Delta t) = \sum_{i=1}^{N} P_i^{DDJ} \delta\left(\Delta t - D_i^{DDJ}\right)$$

P_i^{DDJ} is the probability for the DDJ value of D_i^{DDJ}

P_i^{DDJ} satisfies the condition $\sum_{i=1}^{N} P_i^{DDJ} = 1$

Periodic Jitter

Periodic jitter is a *repeating* jitter signal at a certain period or frequency. It is described by:

$$\Delta t = A\cos\left(\omega t + \phi_o\right)$$

ω: angular frequency

ϕ_o: initial phase

The PDF for the single PJ is given by

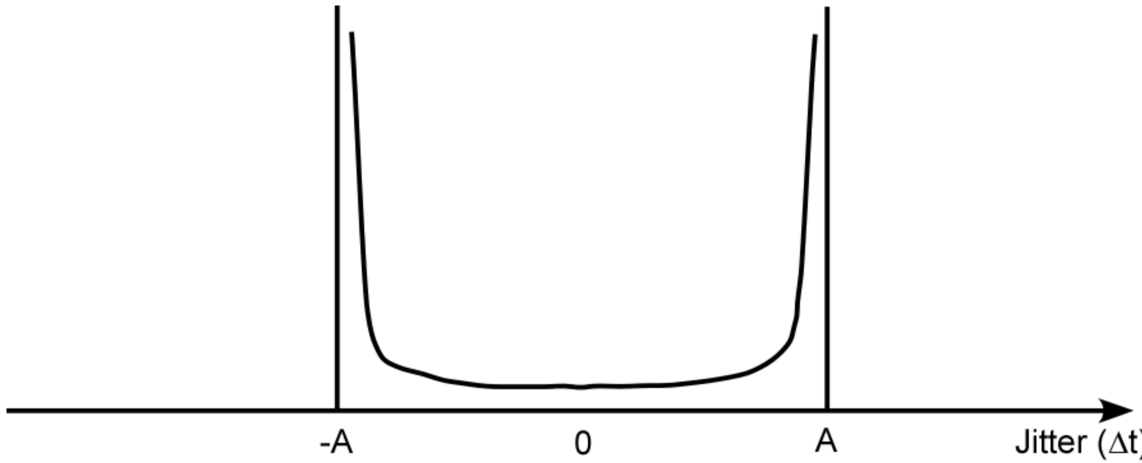
$$f_{PJ}\left(\Delta t\right) = \frac{1}{\pi\sqrt{1-\left(\Delta t / A\right)^2}}, \quad -A \leq \Delta t \leq A$$

Which can be approximated by

$$f_{PJ}\left(\Delta t\right) \approx \frac{1}{2}\left[\delta\left(\Delta t - A\right) + \delta\left(\Delta t + A\right)\right]$$

Periodic Jitter

PDF for single sinusoidal



$$f_{PJ}(\Delta t) = \frac{1}{\pi\sqrt{1-(\Delta t/A)^2}}, \quad -A \le \Delta t \le A$$

Periodic Jitter

There are 3 common waveforms for the theoretical analysis of periodic jitter

Rectangle Periodic Jitter

$$PDF_{PJ-rect}(x) = \frac{1}{2}\delta\left(-\frac{m}{2}\right) + \frac{1}{2}\delta\left(\frac{m}{2}\right)$$

Triangle Periodic Jitter

$$PDF_{PJ-triang}(x) = \begin{cases} \dfrac{1}{m} & for\ |x| < \dfrac{m}{2} \\ 0 & otherwise \end{cases}$$

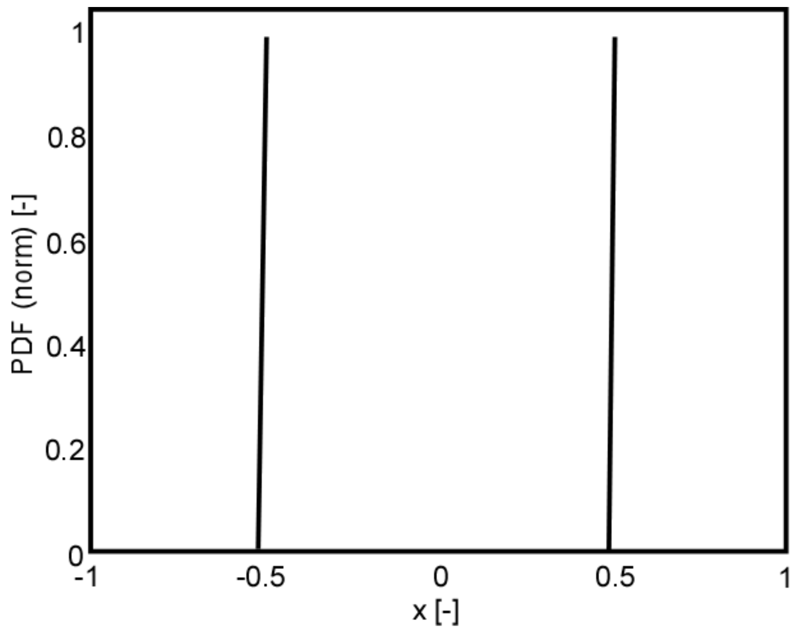
Periodic Jitter

Sinusoidal Periodic Jitter

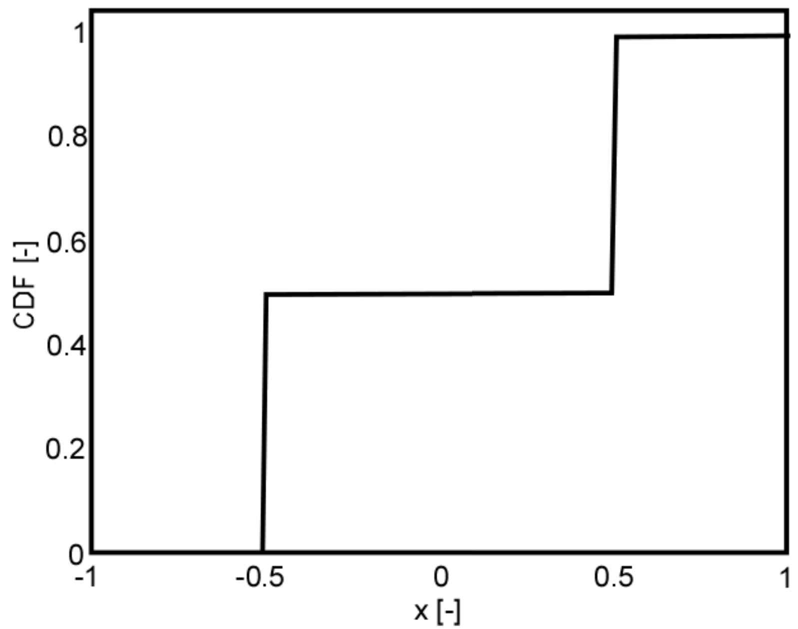
$$PDF_{PJ-line}(x) = \begin{cases} \dfrac{1}{\pi\sqrt{m/2 - \left(\sqrt{\dfrac{2}{m}}\,x\right)^2}} & for\ |x| < \dfrac{m}{2} \\ 0 & otherwise \end{cases}$$

Rectangular Periodic Jitter

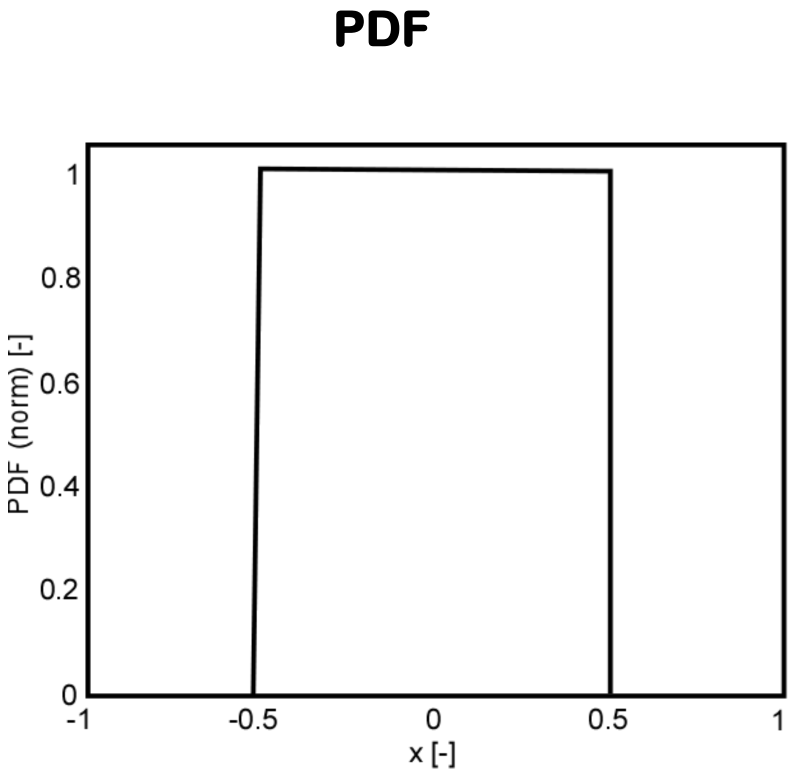
PDF

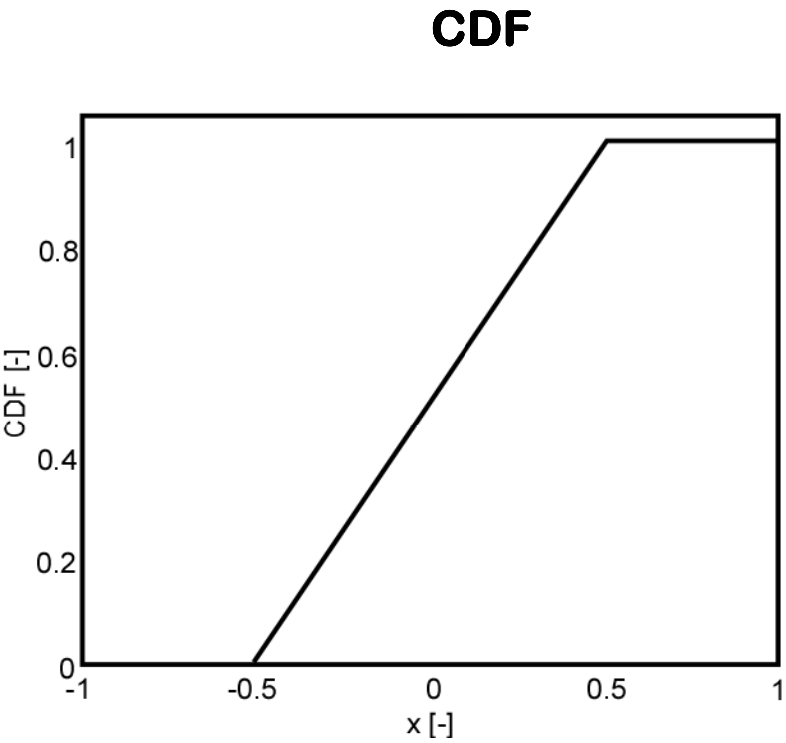


CDF



Triangular Periodic Jitter





Sinusoidal Periodic Jitter