ECE 546
Lecture - 23
Jitter Basics

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Probe Further

• Kyung Suk (Dan) Oh and Xingchao (Chuck) Yuan, High-Speed Signaling: Jitter Modeling, Analysis, and Budgeting, Prentice Hall, 2012
• Mike Peng Li, Jitter, Noise and Signal Integrity at High-Speed, Prentice Hall, 2008
Jitter Definition

Jitter is difference *in time* between when an event was ideally to occur and when it actually did occur.

- Timing uncertainties in digital transmission systems
- Utmost importance because timing uncertainties cause bit errors
- There are different types of jitter
Jitter Characteristics

• Jitter is a signal timing deviation referenced to a recovered clock from the recovered bit stream

• Measured in Unit Intervals and captured visually with eye diagrams

• Two types of jitter
  – Deterministic (non Gaussian)
  – Random

• The total jitter (TJ) is the sum of the random (RJ) and deterministic jitter (DJ)
Types of Jitter

• Deterministic Jitter (DDJ)
  - Data-Dependent Jitter (DDJ)
  - Periodic Jitter (PJ)
  - Bounded Uncorrelated Jitter (BUJ)

• Random Jitter (RJ)
  - Gaussian Jitter
  - $f^{-\alpha}$ Higher-Order Jitter
Jitter Effects

Bandwidth Limitations

- Cause intersymbol interference (ISI)
- ISI occurs if time required by signal to completely charge is longer than bit interval
- Amount of ISI is function of channel and data content of signal

Oscillator Phase Noise

- Present in reference clocks or high-speed clocks
- In PLL based clocks, phase noise can be amplified
Phase Noise & Phase Jitter

- **Phase noise in clock oscillators**
  - Phase offset term that continually changes timing of signal

\[
S(t) = P\left(t + \phi(t)\right)
\]

Signal waveform with phase noise
Undistorted signal
Phase noise

Example:

\[
P(t) = \sin\left(10 \times 10^9 \times 2\pi t\right)
\]

\[
\phi(t) = \frac{1}{4}\sin\left(2 \times 10^9 \times 2\pi t\right)
\]

\[
S(t) = \sin\left(10 \times 10^9 \times 2\pi t + 0.25 \sin(2 \times 10^9 \times 2\pi t)\right)
\]
Phase Noise

- Clean signal
- Noisy signal

2 GHz phase noise
Phase Jitter

<table>
<thead>
<tr>
<th>Signal [V]</th>
<th>Time [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>-1</td>
<td>200</td>
</tr>
<tr>
<td>0</td>
<td>300</td>
</tr>
<tr>
<td>1</td>
<td>400</td>
</tr>
<tr>
<td>0</td>
<td>500</td>
</tr>
</tbody>
</table>

- clean signal
- noisy signal

<table>
<thead>
<tr>
<th>Time Jitter $\phi_n$ [ps]</th>
<th>Time [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
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<td>-10</td>
<td>500</td>
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</table>
Phase Jitter

• Phase jitter in digital systems
  ➢ Variability in timing of transition in digital systems is called phase jitter
  ➢ Phase jitter is digital equivalent of phase noise
  ➢ Always defined relative to the ideal position of the transitions

For a jittered digital signal

\[ t_n = T_n - \phi_n \]

\( t_n \) is the actual time of the \( nth \) transition

\( T_n \) is the ideal timing value of the \( nth \) transition

\( \phi_n \) is the time offset of the transition \( \leftarrow \) phase jitter term

Example: 10 Gbits/s \( \Rightarrow \) \( T_n \) has bit intervals of 100 ps.
Transitions take place at 0, 100, 200 ps
Cycle-to-Cycle Jitter

• Phase jitter causes bit periods to contract and expand
• Actual bit periods are given by the time difference between 2 consecutive transitions

\[ P_n = t_{n+1} - t_n = (T_{n+1} - \phi_{n+1}) - (T_n - \phi_n) \]

Ideal bit period:

\[ TB_n = T_{n+1} - T_n \]

Period jitter:

\[ PerJ_n = TB_n - P_n \]

\[ PerJ_n = (T_{n+1} - T_n) - (T_{n+1} - T_n + \phi_n - \phi_{n+1}) = \phi_{n+1} - \phi_n \]
Cycle-to-Cycle Jitter

**Cycle-to-cycle jitter:**

\[
CCJit_n = P_{n+1} - P_n
\]

\[
CCJit_n = PerJ_{n+1} - PerJ_n
\]
Total Jitter Time Waveform

\[ TJ(t) = PJ(t) + RJ(t) \]

The total jitter waveform is the sum of individual components.
Jitter Statistics

- Most common way to look at jitter is in statistical domain
- Because one can observe jitter histograms directly on oscilloscopes
- No instruments to measure jitter time waveform or frequency spectrum directly

Jitter Histograms and Probability Density Functions

- Built directly from time waveforms
- Frequency information is lost
- Peak-to-peak value depends on observation time

Note: A jitter histogram does not contain all the information about the jitter
Probability Density Function

The PDF of the sum of 2 independent random variables is the convolution of the pdfs of those 2 variables.

\[ z = x + y \quad \Rightarrow \quad pdf_z = pdf_x * pdf_y \]
Jitter Statistics

\[ TJ(x) = PJ(x) \ast RJ(x) \]

The total jitter PDF is the convolution of individual components.
Jitter Mechanisms

• **Transfer of Level Noise into the Time Domain**
  ➢ Noise on digital data signals causes jitter because it offsets the threshold crossing point in time

• **Bandwidth Limitations**
  ➢ Primarily caused by intersymbol interference

• **Oscillator Phase Noise**
  ➢ Phase noise present in reference clocks especially in systems based on PLL
Jitter Mechanisms

- Transfer of noise into time domain
- Bandwidth limitation in channels
- Oscillator phase noise

\[ NJ_{pk-pk} = t_t \frac{V_{Noise}}{V_H - V_L} \]

- \( t_t \): rise time
- \( V_{Noise} \): pk-pk noise amplitude
- \( V_H \): Hi signal level
- \( V_L \): Lo signal level
Jitter Mechanisms

Linear model

\[ NJ_{pk-pk} = t_t \frac{V_{Noise}}{V_H - V_L} \]

Jitter \( \sim 2\text{ps} \)  
Jitter \( \sim 6\text{ps} \)

Random noise caused by thermal effects
Jitter Mechanisms

First order model

\[ NJ_{pk-pk} = -\tau \left( \ln(0.5 - V_{\text{Noise}}) + \ln(0.5 + V_{\text{Noise}}) \right) \]

Periodic noise: switching power, crosstalk, etc…
Jitter Mechanisms

Multiple threshold crossing of a signal with high-frequency level noise
Bandwidth Limitations

00011111 data pattern
Bandwidth Limitations

0101111 data pattern
Jitter Classification

Total Jitter (TJ)
- Random Jitter (RJ)
- Deterministic Jitter (DJ)
  - Periodic Jitter (PJ)
  - Data-Dependent Jitter (DDJ)
    - Intersymbol Interference (ISI)
    - Duty Cycle Distortion (DCD)
Gaussian Random Jitter

- Random jitter can be described by a Gaussian distribution with the following probability density function

\[ PDF_{RJ}(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]

- \( x \): independent value
- \( \sigma \): RMS value
- \( \mu \): mean of distribution (zero by definition)

=> Note: the PDF of a Gaussian process is unbounded, i.e., its PDF is not zero unless the jitter \( \Delta t \) approaches infinity
Gaussian Jitter PDF

\[
P\left(\left| \Delta t - \mu \right| \leq \sigma \right) = 0.6826
\]

\[
P\left(\left| \Delta t - \mu \right| \leq 2\sigma \right) = 0.9545
\]

\[
P\left(\left| \Delta t - \mu \right| \leq 3\sigma \right) = 0.9973
\]

Can be used to estimate the probability when the deviation of the random jitter variable \( \Delta t \) is within a multiple of its \( \sigma \) value.
Cumulative Density Function

Cumulative density function (CDF) is defined as:

\[ CDF(t) = \int_{-\infty}^{t} PDF(x)dx \]

\( CDF(t) \) tells us the probability that the transition occurred earlier than \( t \). For random jitter, we get:

\[ CDF_{RJ}(x) = \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{x}{\sigma \sqrt{2}} \right) \]

\( \text{erf} \) is the error function
PDF and CDF of Random Jitter

PDF

CDF
Causes of Deterministic Jitter

• **Crosstalk**
  – Noisy neighboring signals

• **Interference**

• **Reflections**
  – Imperfect terminations
  – Discontinuities (e.g. multi-drop buses, stubs)

• **Simultaneous switching noise (SSN)**
  – Noisy reference plane or power rail
  – Shift in threshold voltages
Data-Dependent Jitter

- Most commonly encountered DJ type
- Dominant limiting factor for link channels
- Due to memory of lossy electrical or optical system
- Bit transition of current bit depends on the transition times of the previous bits
Data-Dependent Jitter

- DDJ depends on the impulse response of the system that generates the pattern
- DDJ depends on the input pattern
- DDJ is a distribution with its sample size equal to the number of transitions of the data pattern

- Duty cycle distortion (DCD) occurs for clock patterns of repeating bits
Data-Dependent Jitter

• Since channel does not have zero-rise time step response or infinite bandwidth, jitter is to be expected
• Settling time gives good indication of jitter
Model for DDJ

The generic form for DDJ PDF is:

$$f_{DDJ}(\Delta t) = \sum_{i=1}^{N} P_i^{DDJ} \delta(\Delta t - D_i^{DDJ})$$

$P_i^{DDJ}$ is the probability for the DDJ value of $D_i^{DDJ}$

$P_i^{DDJ}$ satisfies the condition $\sum_{i=1}^{N} P_i^{DDJ} = 1$
Periodic Jitter

Periodic jitter is a *repeating* jitter signal at a certain period or frequency. It is described by:

$$\Delta t = A \cos(\omega t + \phi_o)$$

$\omega$: angular frequency

$\phi_o$: initial phase

The PDF for the single PJ is given by

$$f_{PJ}(\Delta t) = \frac{1}{\pi \sqrt{1 - (\Delta t / A)^2}}, \quad -A \leq \Delta t \leq A$$

Which can be approximated by

$$f_{PJ}(\Delta t) \approx \frac{1}{2} \left[ \delta(\Delta t - A) + \delta(\Delta t + A) \right]$$
Periodic Jitter

PDF for single sinusoidal

\[ f_{PJ}(\Delta t) = \frac{1}{\pi \sqrt{1 - (\Delta t/A)^2}}, \quad -A \leq \Delta t \leq A \]
Periodic Jitter

There are 3 common waveforms for the theoretical analysis of periodic jitter

**Rectangle Periodic Jitter**

\[
PDF_{PJ-rect}(x) = \frac{1}{2} \delta \left( -\frac{m}{2} \right) + \frac{1}{2} \delta \left( \frac{m}{2} \right)
\]

**Triangle Periodic Jitter**

\[
PDF_{PJ-triang}(x) = \begin{cases} 
\frac{1}{m} & \text{for } |x| < \frac{m}{2} \\
0 & \text{otherwise}
\end{cases}
\]
Periodic Jitter

Sinusoidal Periodic Jitter

\[
PDF_{PJ-line}(x) = \begin{cases} 
\frac{1}{\pi \sqrt{m/2} - \left(\sqrt{\frac{2}{m}}x\right)^2} & \text{for } |x| < \frac{m}{2} \\
0 & \text{otherwise}
\end{cases}
\]
Rectangular Periodic Jitter

PDF

CDF
Triangular Periodic Jitter

PDF

CDF
Sinusoidal Periodic Jitter

PDF

CDF