Fundamentals of a 3-D "Snowball" Model for Surface Roughness Power Losses

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Abstract

SEM photographs of a typical copper conductors prepared by the PCB industry exhibit a 3-D "snowball" structure of copper surface distortions [1]. We have developed an analytical basis for the electromagnetic scattering by the copper "snowballs" to predict additional power losses to those presented by the propagating medium [2] that compare well with the observed measurements for a set of rough microstrip lines. In this paper we describe the fundamental concepts involved with the 3-D scattering theory of our analysis.

I. Introduction

If an electromagnetic pulse is caused by a voltage $V(0,t)$ on one end of a copper microstrip trace as shown in figure 1, the electric field intensity below the trace will propagate down the transmission line (in the *z*-direction) at a phase velocity $c_2 = c/\sqrt{\varepsilon_r}$ and it will fringe and disperse as it propagates.

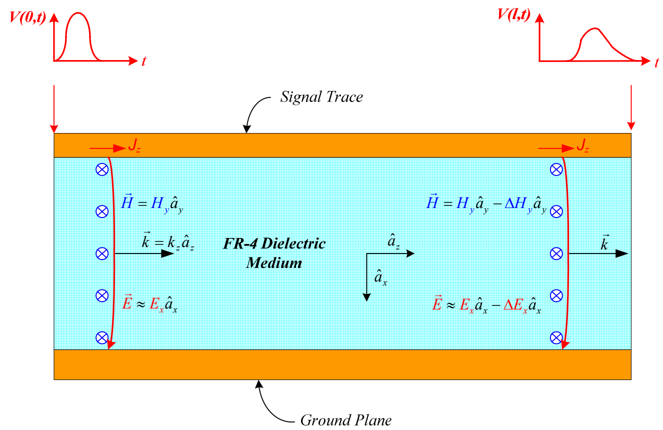

Fig. 1 Electric and Magnetic field intensities as they propagate down the geometric center of a micro-strip waveguide.

On the bottom side of the trace there will be a surface charge density, $\sigma_{e,S} = \varepsilon_2 E_{2,x}$, and a complementary negative charge density on the ground plane. As the charge density moves at speed c_2 down the trace it creates a magnetic field intensity, $\vec{H}_2 = \sigma_{e,S} c_2\, \hat{a}_y = \varepsilon_2 E_{2,x} c_2 \hat{a}_y = \hat{a}_y\, E_{2,x}/\eta_2$, in the propagating medium.

An SEM photograph of the underside of the micro-strip trace is shown in figure 2 and the corresponding "snowball" model of the distribution of spheres are shown in figure 3.

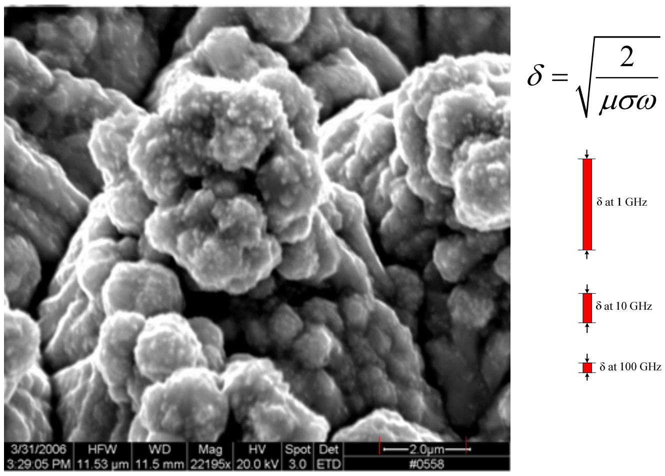

Fig 2 SEM Photograph of surface distortions for a rough copper surface. Copper skin depths for three frequencies are shown for relative scale.

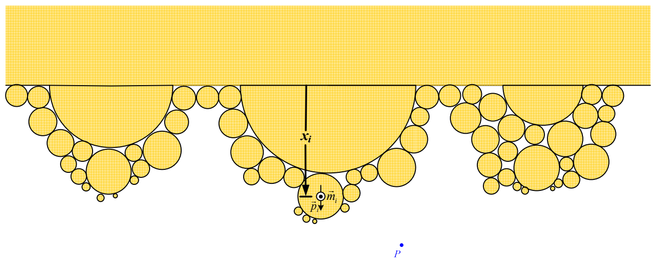

Fig. 3 Model cross-section of a distribution of spheres for analyzing power losses due to surface irregularities.

A periodic field intensity will behave in a similar manner as shown in figure 4, where we have added a single exaggerated copper sphere (a "snowball") of radius a_i in the path of a propagating electromagnetic wave.

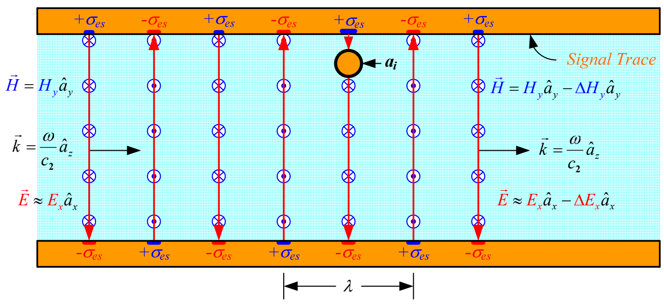

Fig. 4 Cross-section of periodic electromagnetic waves as they impinge upon an isolated "snowball" below a copper trace.

We show an exaggerated view of the charge density required to support the periodic fields in figure 5.

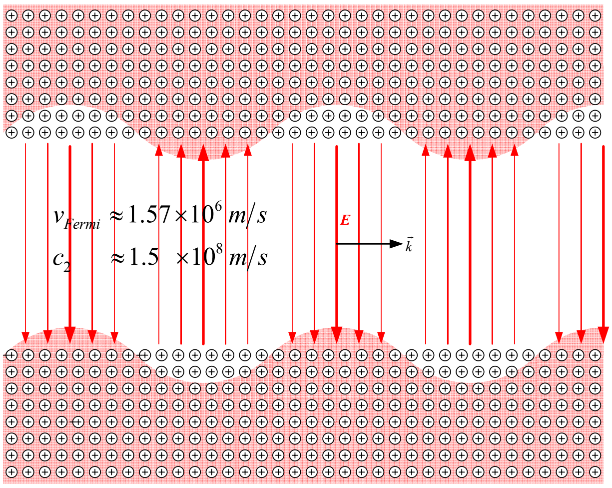

Fig. 5 Transverse displacement of conduction electrons relative to copper ion cores as they produce a surface charge density to support the propagating electric field intensity near the medium interface. Magnetic field intensity lines have been omitted in this view for clarity.

In figure 5 we have also indicated the large difference between the velocity of conduction electrons at the Fermi surface, v_F, to the speed of the propagating wave in the medium, c_2, to show that the charge density must be a transverse charge wave that propagates on the metal surface. The displacement of the conduction electron cloud needed to create the requisite charge density for a fcc copper cubic lattice is less than a nuclear dimension.

In figure 6 we show how the electric and magnetic field intensity impinges upon an isolated copper "snowball" to induce an electric dipole moment, $\vec{p}_i$, and a magnetic dipole moment, $\vec{m}_i$, to produce scattering of the incident electromagnetic field outside the sphere.

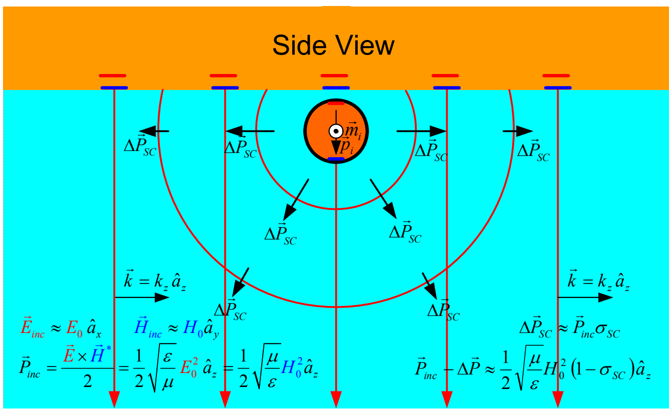

Fig. 6 Electric and magnetic dipole moment induced by an electromagnetic wave as it propagates past a copper "snowball."

II. Perfect Electric Conductors

We show in figure 7 the electric dipole moment induced in a copper sphere by the incident electric field intensity. The charge density on the surface is similar to that of a PEC due to the fact that the normal component of the electric field does not penetrate the surface (as we show below for a good plane conductor). The periodic oscillation of the dipole moment causes radiation to be scattered as we previously showed in figure 4 and we can describe the scattered power in terms of a scattering cross-section, σ_{sc}.

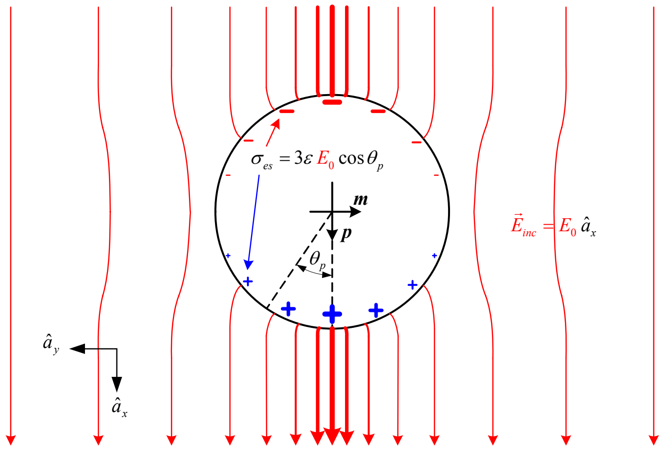

Fig. 7 Rear view of the electric field intensity as it induces an electric dipole moment, $\vec{p}_i = 4\pi\varepsilon a_i^3 E_0 \hat{a}_x$, in a good conducting sphere.

The incident magnetic field intensity also induces a magnetic dipole moment in a perfectly conducting sphere as shown in figure 8.

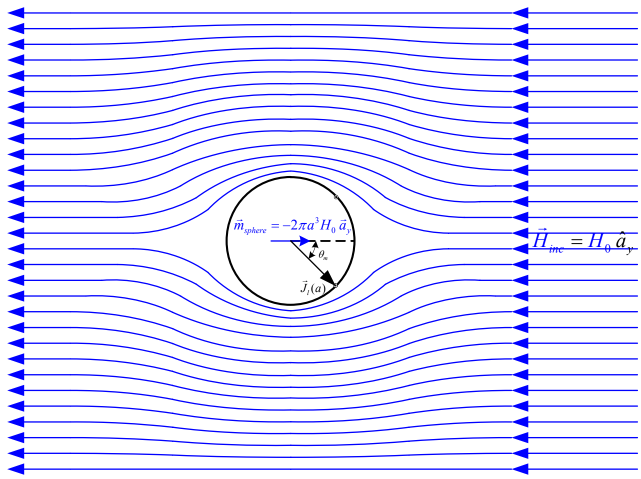

Fig. 8 Rear view of the magnetic field intensity as it induces a magnetic dipole moment, $\vec{m}_i = -2\pi a_i^3 H_0 \hat{a}_y$, in a perfect conducting sphere.

The scattered field in the far field region, $r>>a_i$, and the differential scattering cross section can be expressed [3] as

$$\vec{E}_{sc} = \frac{1}{4\pi\varepsilon_0} k^2 \frac{e^{jkr}}{r} \left[\left(\hat{a}_r \times \vec{p}_i \right) \times \hat{a}_r - \hat{a}_r \times \vec{m}_i / c \right]$$
$$\frac{d\sigma}{d\Omega} = \frac{k^4}{\left(4\pi\varepsilon_0 E_0\right)^2} \left| \hat{a}_x \cdot \vec{p}_i + \left(\hat{a}_r \times \hat{a}_x \right) \cdot \vec{m}_i / c \right|^2 \qquad (1)$$

If all of the scattered radiation from a PEC is lost from the incident wave then power lost, ΔP_i is:

$$\sigma_{sc} = \int_0^{2\pi} \int_0^{\pi} \frac{d\sigma_{sc}}{d\Omega} \sin\theta \, d\theta \, d\phi$$
$$\frac{\Delta P_i}{P_{inc}} = \sigma_{sc} = \frac{15}{8\pi} \left(\frac{\omega^4}{c_2^4} \right) \left(\frac{4\pi a_i^3}{3} \right)^2 \qquad (2)$$

III. Good Conductors

In a flat good conductor, such as a perfectly flat trace, the magnetic field intensity penetrates the surface according to a skin depth formulation as shown in figure 9.

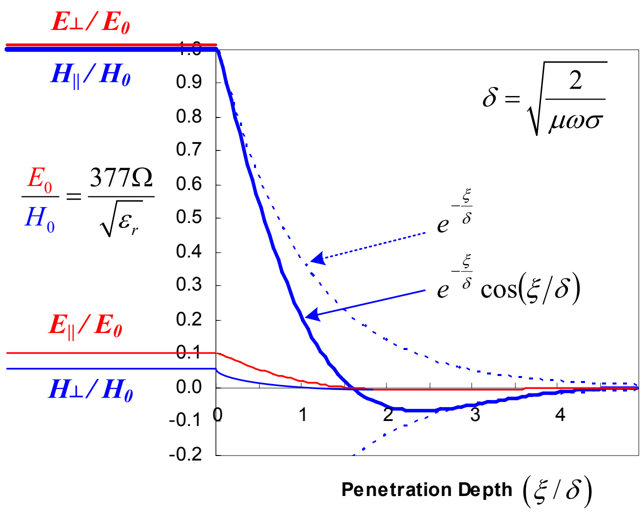

Fig. 9 Tangential Component of the magnetic field intensity as a function of depth, ξ/δ, inside a good conductor.

In figure 9, the periodic magnetic field intensity on the surface is attenuated by conduction losses as it moves slowly into the conductor as indicated by an exponential envelope (blue dotted lines). The magnetic field intensity inside the conductor propagates in the positive ξ direction at phase velocity

$$u_p = \omega\delta = \frac{c}{\left(\frac{1}{\sqrt{2}} \sqrt{\frac{\sigma}{\omega\varepsilon_0}} \right)} \qquad (3)$$

where the quantity inside the square root is a figure of merit of the conductivity of a good conductor. For copper

$$\frac{\sigma}{\omega\varepsilon_0} = \begin{Bmatrix} 1.04\times10^9 & \text{at} & 1\text{ GHz} \\ 1.04\times10^8 & \text{at} & 10\text{ GHz} \\ 1.04\times10^7 & \text{at} & 100\text{ GHz} \end{Bmatrix} \qquad (4)$$

The magnetic field intensity inside the conductor is oscillating in time with the external (incident) field so that by the time it reaches $\xi/\delta = \pi/2$ the field was that caused by the surface (external) field when it had zero magnitude. For greater depths the field was caused by a surface field that had the opposite sign so it has a negative value due to the "retarded" field at the surface [4].

For a copper "snowball" we must find the solution to the Helmholtz equation

$$\vec{\nabla}^2 \vec{H}_s + k^2 \vec{H}_s = 0 \text{ with } k^2 = \omega^2 \mu\varepsilon \left(1 + i \frac{\sigma}{\omega\varepsilon_0} \right) \qquad (5)$$

for the geometry shown in figure 10.

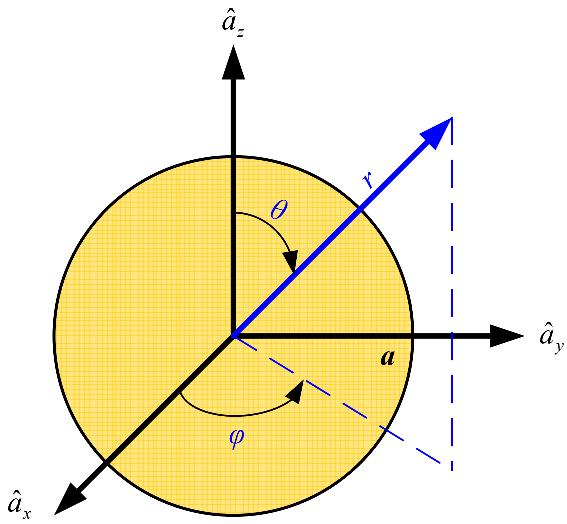

Fig. 10 Helmholtz regions for the magnetic field intensity inside ($r \le a$) and outside ($r \ge a$) a good conducting sphere.

The solutions inside the sphere are given by

$$H(r,\theta,\phi) = \frac{3}{2} H_0 \sin\theta_m \left\{ \frac{i_l\left((1+j)\frac{r}{\delta} \right)}{i_l\left((1+j)\frac{a}{\delta} \right)} \right\}$$
$$= \frac{3}{2} H_0 \sin\theta_m \left\{ \frac{ber_1\left(\frac{r}{\delta}\right) + jbei_1\left(\frac{r}{\delta}\right)}{ber_1\left(\frac{a}{\delta}\right) + jbei_1\left(\frac{a}{\delta}\right)} \right\} \qquad (6)$$

Because of the spherical symmetry, the radial functions that satisfy the Helmholtz equation with a purely imaginary k^2 as given in equation 5 are modified spherical Bessel functions with a complex argument. Those functions have a real and an imaginary part called the ber and the bei functions. In equation 6 we have retained only the $l=1$ coefficients that correspond to dipole boundary conditions as these are the largest terms for this value of in the long wavelength limit ($ka \ll 1$) and higher scattering coefficients become small very rapidly as l increases [3].

With the value of $\vec{H}_c(r,\theta_m)$ inside the conductor, we can use Ampere's Law and the quasi-static approximation to find the current distribution inside the isolated "snowball" as:

$$\vec{J}(r,\theta_m) \approx \vec{\nabla} \times \vec{H}_c(r,\theta_m)$$

$$\vec{J} \approx \frac{\frac{3}{2} H_0 \sin\theta_m \left[(1+j)\frac{1}{\delta}\right]}{i_1\left[(1+j)\frac{a}{\delta}\right]} \left\{ \frac{i_1(x)}{x} + \frac{1}{3} i_0(x) + \frac{2}{3} i_2(x) \right\}_{x=(1+j)\frac{r}{\delta}} \hat{a}_{\phi_m}$$

A plot of the current density and the magnetic field intensity interior to the snowball is shown in figure 11. There is azimuthal symmetry in this plot (no dependence on $\hat{a}_\phi$).

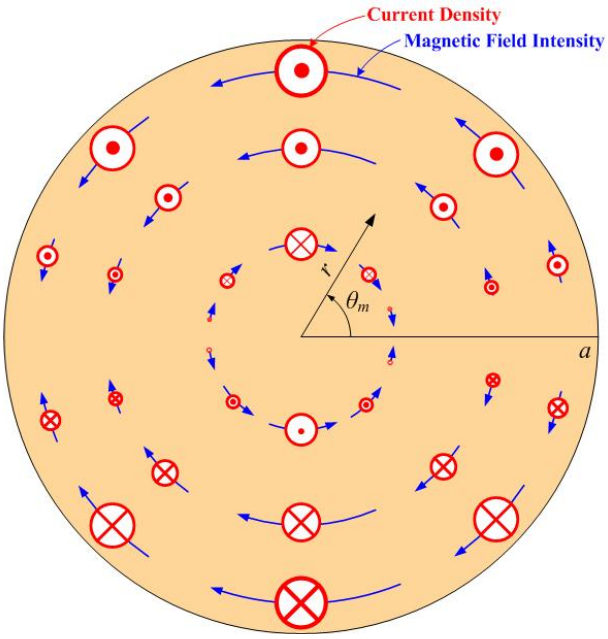

Fig. 11 Current density and magnetic field intensity as a function of radius and angle interior to a sphere of radius ***a***.

We have indicated a turn-around in the interior current density due to the fact of retarded field at $(\xi/\delta) = \pi/2$ but this assumes the sphere has a radius of more than 1.57 δ. The field penetration will not turn-around if the sphere has a smaller radius and the net penetration is a function of the radius of the sphere compared to the skin depth as shown in figure 12.

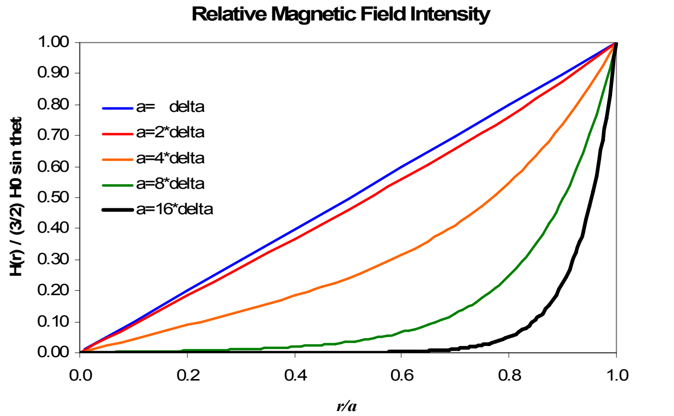

Fig. 12. Envelope of the magnetic field intensity (equation 6) as a function of radius inside a sphere for several size spheres.

IV. CONCLUSIONS

From these fundamental solutions for an isolated "snowball", we have been able to calculate the effective magnetic dipole moment of an arbitrarily large good conducting sphere and its phase relative to a PEC sphere. In addition, we have calculated the power lost when an incident wave induces these dipoles. We have also calculated the additional loss on the nearby flat conducting trace due to the dipole and its image and we have evaluated the reduction in field at a "snowball" due to its first few neighbors. Finally, a distribution of the snowballs in figure 3 have been chosen to match the observed SEM pictures in figure 2 and we have found these to match the losses measured for a rough surface to frequencies up to 50 GHz.

References

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