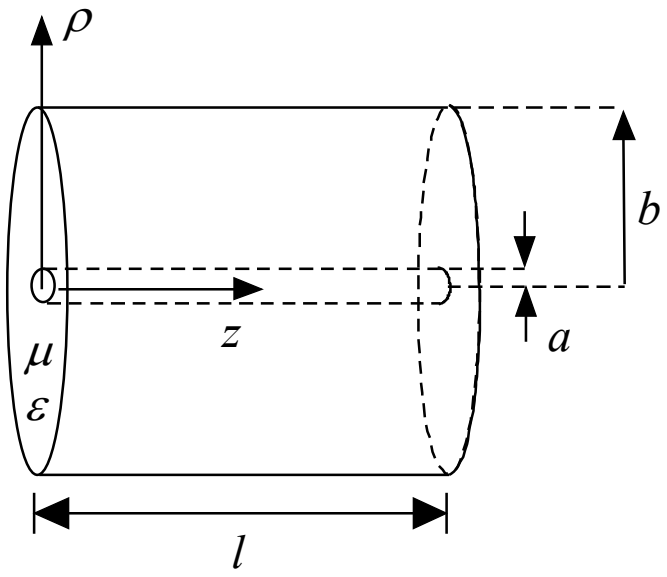
ECE 546 HOMEWORK No 1

Solutions

1. Consider the coaxial system shown below. The solid inner conductor has radius a. The outer conductor has radius b. The medium between the two conductors has permittivity ε and permeability μ.



(a) Show that the inductance per-unit-length is given by:

$$L = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right)$$

(b) Show that the capacitance per-unit-length is given by:

$$C = \frac{2\pi\varepsilon}{\ln(b/a)}$$

Solution

(a) In the dielectric between the conductors, Ampere's law requires

$$\oint_C \mathbf{H}\cdot\mathbf{dl} = \iint_S \mathbf{J}\cdot\mathbf{ds}$$

$$2\pi r H_\phi = i \Rightarrow H_\phi = \frac{i}{2\pi r}$$

Since, $\mathbf{B} = \mu\mathbf{H}$, then $B_\phi = \dfrac{\mu i}{2\pi r}$

Using $L = \dfrac{1}{i^2}\iiint_V \mathbf{B}\cdot\mathbf{H}dV$

$$L = \frac{1}{i^2} l \int_a^b 2\pi r \frac{i}{2\pi r}\frac{\mu i}{2\pi r} dr$$

The integration takes place only over the space filled with dielectric. Completing the integration yields $L = \dfrac{\mu}{2\pi}\ln\left(\dfrac{b}{a}\right)$

(b)

From Gauss' law $\oint_S \mathbf{D}\cdot d\mathbf{s} = Q_{encl} \Rightarrow 2\pi r l \varepsilon E = Q_{encl}$

$$\mathbf{E} = \frac{Q_{encl}}{2\pi r l \varepsilon}\hat{r}$$

$$V = -\int_{r=b}^{a}\left(\hat{r}\frac{\sigma}{2\pi\varepsilon r}\right)\cdot\left(\hat{r}dr\right)$$

$$V = -\frac{Q_{encl}}{2\pi\varepsilon l}\int_{r=b}^{a}\frac{dr}{r} = +\frac{Q_{encl}}{2\pi\varepsilon l}\int_{r=a}^{b}\frac{dr}{r} = \frac{Q_{encl}}{2\pi\varepsilon l}\ln\left(\frac{b}{a}\right)$$

$$C_{total} = \frac{Q_{encl}}{V} = -\frac{Q_{encl}}{\dfrac{Q_{encl}}{2\pi\varepsilon l}\ln\left(\dfrac{b}{a}\right)} = \frac{2\pi\varepsilon l}{\ln\left(b/a\right)}$$

$$C = \frac{C_{total}}{l} = \frac{2\pi\varepsilon}{\ln\left(b/a\right)}$$