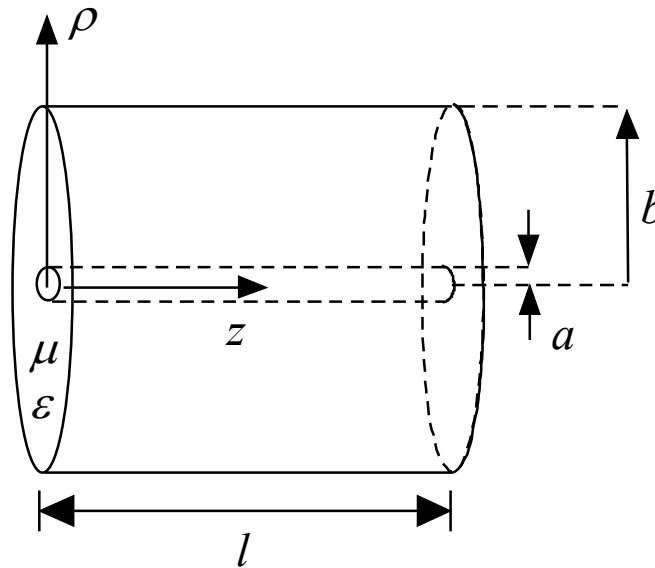


ECE 546 HOMEWORK No 1 Due Friday, February 13, 2026

Solutions

1. Consider the coaxial system shown below. The solid inner conductor has radius a . The outer conductor has radius b . The medium between the two conductors has permittivity ϵ and permeability μ .



(a) Show that the inductance per-unit-length is given by:

$$L = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right)$$

(b) Show that the capacitance per-unit-length is given by:

$$C = \frac{2\pi\epsilon}{\ln(b/a)}$$

(c) Assuming you have a connector with dimensions $a = 1\text{mm}$, $b = 2\text{mm}$ and a dielectric constant $\epsilon_r = 2.1$, what is the maximum frequency at which this connector can be used?

Solution

(a) In the dielectric between the conductors, Ampere's law requires

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \iint_S \mathbf{J} \cdot d\mathbf{s}$$

$$2\pi r H_\phi = i \Rightarrow H_\phi = \frac{i}{2\pi r}$$

Since, $\mathbf{B} = \mu\mathbf{H}$, then $B_\phi = \frac{\mu i}{2\pi r}$

Using $L = \frac{1}{i^2} \iiint_V \mathbf{B} \cdot \mathbf{H} dV$

$$L = \frac{1}{i^2} l \int_a^b 2\pi r \frac{i}{2\pi r} \frac{\mu i}{2\pi r} dr$$

The integration takes place only over the space filled with dielectric. Completing the integration yields $L = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right)$

(b)

From Gauss' law $\oint_S \mathbf{D} \cdot d\mathbf{s} = Q_{encl} \Rightarrow 2\pi r l \epsilon E = Q_{encl}$

$$\mathbf{E} = \frac{Q_{encl}}{2\pi r l \epsilon} \hat{r}$$

$$V = - \int_{r=b}^a \left(\hat{r} \frac{\sigma}{2\pi \epsilon r} \right) \cdot (\hat{r} dr)$$

$$V = - \frac{Q_{encl}}{2\pi \epsilon l} \int_{r=b}^a \frac{dr}{r} = + \frac{Q_{encl}}{2\pi \epsilon l} \int_{r=a}^b \frac{dr}{r} = \frac{Q_{encl}}{2\pi \epsilon l} \ln\left(\frac{b}{a}\right)$$

$$C_{total} = \frac{Q_{encl}}{V} = - \frac{Q_{encl}}{\frac{Q_{encl}}{2\pi \epsilon l} \ln\left(\frac{b}{a}\right)} = \frac{2\pi \epsilon l}{\ln(b/a)}$$

$$C = \frac{C_{total}}{l} = \frac{2\pi \epsilon}{\ln(b/a)}$$

(c)

The cutoff frequency of the TE₁₁ mode in a coaxial waveguide is given by:

$$f_{c11} = \frac{1}{\pi(a+b)\sqrt{\mu\epsilon}}$$

Using the above numbers, $f_{c11} = \frac{0.3 \times 10^9}{\pi(1+2) \times 10^{-3} \sqrt{2.1}} = 21.965 \times 10^9 \text{ Hz} = 21.9 \text{ GHz}$.