ECE 451
Advanced Microwave Measurements

Special Transmission Lines

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Parallel-Plate Waveguide

\[ \nabla^2 E + \omega^2 \mu \varepsilon E = 0 \]

**TE\textsubscript{mn} modes:** \( E_z = 0, H_z \neq 0, H_x \neq 0 \)

\[ E_y = E_o \sin(\beta_x x) e^{-jkz} \quad \beta_x = \frac{m\pi}{a} \]

Wave will propagate if \( f > f_c = \frac{m}{2a\sqrt{\mu \varepsilon}} \)

**TM\textsubscript{mn} modes:** \( H_z = 0, E_z \neq 0, E_x \neq 0 \)

\[ H_y = H_o \cos(\beta_x x) e^{-jkz} \]

Same dispersion relation as TE\textsubscript{mn} modes
TEM Mode

Special case $m=0 \Rightarrow \text{TM}_0$ or TEM mode

$E_z = 0, H_z = 0, E_x \neq 0, H_y \neq 0$

$E_x = E_0 e^{-j k z} = \sqrt{\frac{\mu}{\varepsilon}} H_0 e^{-j k z}$

$H_y = H_0 e^{-j k z}$

$k = \beta_z = \omega \sqrt{\mu \varepsilon}$

$Z_0 = \sqrt{\frac{\mu}{\varepsilon}}$
Parallel-Plate in TEM

\[ L = \frac{\mu a}{w} \]

\[ C = \frac{\varepsilon w}{a} \]
Coaxial Cables

Laplace’s Equation for potential $\psi$

\[
\nabla^2 \psi = 0 \Rightarrow \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \psi}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 \psi}{\partial \phi^2} = 0
\]

Solution: $\psi(\rho, \phi) = \frac{V_o \ln(b/\rho)}{\ln(b/a)}$

For a TEM mode of propagation

\[
L = \frac{\mu}{2\pi} \ln(b/a)
\]

\[
C = \frac{2\pi \varepsilon}{\ln(b/a)}
\]

\[
\beta = \omega \sqrt{LC} = \omega \sqrt{\mu \varepsilon}
\]

\[
Z_o = \sqrt{L/C} = \sqrt{\frac{\mu}{\varepsilon}} \frac{2\pi}{\ln(b/a)}
\]
Coaxial Cables

Higher order modes: TE modes: \[ H_z = h_z(\rho, \phi) e^{-j\beta z} \]

\[ \left( \frac{\partial^2}{\partial \rho^2} \frac{1}{\rho} \frac{\partial^2}{\partial \phi^2} + k^2 - \beta^2 = 0 \right) h_z = 0 \]

The first higher order mode is the TE_{11} mode

Approximate solution for \( k_c \) is: \[ k_c = \frac{2}{a+b} \]

From \( k_c \), find cutoff frequency \( f_c \):

\[ f_c = \frac{ck_c}{2\pi \sqrt{\varepsilon_r}} = \frac{vk_c}{2\pi} \]

\[ f_c = \frac{2c}{(a+b)2\pi \sqrt{\varepsilon_r}} = \frac{c}{\sqrt{\varepsilon_r} \pi (a+b)} \]
Coaxial Line with Losses

Infinite Conductivity

\[ Z_o = \frac{\sqrt{\mu / \varepsilon}}{2\pi} \ln(b/a) \]

Finite Conductivity

\[ Z_o = \frac{\sqrt{\mu / \varepsilon}}{2\pi} \ln(b/a) \left[ 1 + \frac{(1/a + 1/b)}{4\sqrt{\pi f \mu \sigma} \ln(b/a)} (1 - j) \right] \]
Types of Transmission Lines

- Coaxial line
- Microstrip
- Waveguide
- Stripline
- Coplanar line
- Slot line
Microstrip
Microstrip and Stripline

Wave propagation in stripline is closer to the TEM mode of propagation and the propagation of velocity is approximately $c/\sqrt{\varepsilon_r}$.