ECE 546
Lecture - 09
Lossy Transmission Lines

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Loss in Transmission Lines

Signal amplitude decreases with distance from the source.
Skin Effect in Lines

Low Frequency  

High Frequency

Very High Frequency

\[ \delta \]
Skin Effect in Microstrip

Skin Effect in Microstrip

Current density varies as

\[ J = J_0 e^{-y/\delta} e^{-jy/\delta} \]

Note that the phase of the current density varies as a function of \( y \)

\[ I = \int_{0}^{\infty} J_0 w e^{-y/\delta} e^{-jy/\delta} dy = \frac{J_0 w\delta}{1+j} \]

\[ \sigma E_o = J_o \Rightarrow E_o = \frac{J_o}{\sigma} \]

The voltage measured over a section of conductor of length \( D \) is:

\[ V = E_o D = \frac{J_0 D}{\sigma} \]
Skin Effect in Microstrip

The skin effect impedance is

\[ Z_{\text{skin}} = \frac{V}{I} = \frac{J_o D (1 + j)}{\sigma J_o w \delta} = \frac{D}{w} (1 + j) \sqrt{\pi f \mu \rho} \]

where \( \rho = \frac{1}{\sigma} \) is the bulk resistivity of the conductor

\[ Z_{\text{skin}} = R_{\text{skin}} + jX_{\text{skin}} \]

with

\[ R_{\text{skin}} = X_{\text{skin}} = \frac{D}{w} \sqrt{\pi f \mu \sigma} \]

- Skin effect has reactive (inductive) component
Lossy Transmission Line

Telegraphers Equation: Time Domain

\[- \frac{\partial V}{\partial z} = RI + L \frac{\partial I}{\partial t}\]

\[- \frac{\partial I}{\partial z} = GV + C \frac{\partial V}{\partial t}\]
Lossy Transmission Line

Telegraphers Equation: Frequency Domain

\[-\frac{\partial V}{\partial z} = (R + j \omega L)I = ZI\]

\[-\frac{\partial I}{\partial z} = (G + j \omega C)V = YV\]
Lossy Transmission Line

- Forward wave
- Backward wave

R, L, G, C,
Lossy Transmission Line

\[ V(z) = A e^{-\alpha z} e^{-j\beta z} + Be^{\alpha z} e^{+j\beta z} \]

\[ I(z) = \frac{1}{Z_0} \left[ A e^{-\alpha z} e^{-j\beta z} - Be^{\alpha z} e^{+j\beta z} \right] \]

\[ Z_0 = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} \]

\[ \gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \]
Effects of Losses

- Signal attenuation

- Dispersion \( \gamma = \alpha(\omega) + j\beta(\omega) = \sqrt{(R + j\omega L)(G + j\omega C)} \)

- Rise time degradation
RC Transmission Line

![Diagram of RC Transmission Line](image)

$Z_{in} = \frac{RL \coth \left( \frac{RL}{2} \sqrt{\frac{C\omega}{R}} \right)}{\sqrt{2} \sqrt{\frac{C\omega}{R}} (1 + j)}$

For very high $\omega$, $\arg(Z_{in}) \approx 45^\circ$

$R$ : series resistance per unit length
$C$ : shunt capacitance per unit length
**RC Transmission Line**

If $$\omega \ll \frac{2}{RCl^2}$$ then

$$Z_{\text{in}} = \frac{Rl}{2} + \frac{1}{jCl\omega} = \frac{R_T}{2} + \frac{1}{jC_T\omega}$$

$$R_T = Rl$$ : total resistance
$$C_T = Cl$$ : total capacitance
RC Transmission Line

**Pulse Characteristics:**
- rise time: 100 ps
- fall time: 100 ps
- pulse width: 4ns

**Line Characteristics**
- length: 3 mm
- near end termination: 50 Ω
- far end termination: 65 Ω
Long Cable

100m Category-5 Cable

Category 5/100-meter

S11 Magnitude

Frequency (GHz)

Category 5/100-meter

S11 Phase (deg)

Frequency (GHz)

Category 5/100-meter

S21 Magnitude

Frequency (GHz)

Category 5/100-meter

S21 Phase (deg)

Frequency (GHz)
Short Cable
1m Category-5 Cable
Category 5 Cable

Resistance and velocity

[Graphs showing resistance and velocity ratio vs. frequency for Category 5/100-meter cable]
Cable Loss Model

\[ R(f) = R_s \cdot f^p \]

\[ \nu_r = \nu_{ro} + \nu_{rs} \cdot f \]

\[ Z = R(f) + j\omega L = R_{\text{skin}} + j(R_{\text{skin}} + \omega L) \]

<table>
<thead>
<tr>
<th>Cable Type</th>
<th>( Z_0 ) (( \Omega ))</th>
<th>( \nu_{ro} ) (m/ns)</th>
<th>( \nu_{rs} ) (m/ns-GHz)</th>
<th>( R_s ) (( \Omega/m \cdot \text{GHz}^p ))</th>
<th>( p )</th>
<th>( f_{\text{max}} ) (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Category 5</td>
<td>100</td>
<td>0.724</td>
<td>-0.165</td>
<td>15.38</td>
<td>0.482</td>
<td>0.2</td>
</tr>
<tr>
<td>24-Ga</td>
<td>100</td>
<td>0.678</td>
<td>1.157</td>
<td>29.03</td>
<td>0.593</td>
<td>0.1</td>
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<tr>
<td>Category 3</td>
<td>100</td>
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<td>11.06</td>
<td>12.31</td>
<td>0.473</td>
<td>0.01</td>
</tr>
<tr>
<td>SMA</td>
<td>50</td>
<td>0.700</td>
<td>0.113</td>
<td>7.94</td>
<td>0.415</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Lossy TL Simulation

- To simulate lossy TL with resistive loads
  - No closed form solution
  - Simplest method is to use IFFT

\[ v(t, z) = \text{IFFT}\left\{ A e^{-\alpha z} e^{-j\beta z} + B e^{+\alpha z} e^{+j\beta z} \right\} \]

\[ i(t, z) = \text{IFFT}\left\{ \frac{1}{Z_o} \left[ A e^{-\alpha z} e^{-j\beta z} + A e^{+\alpha z} e^{+j\beta z} \right] \right\} \]

\[ Z_o = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} \quad \gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \]

\[ T = \frac{Z_o}{Z_1 + Z_o} \]

\[ A = \frac{TV_s(\omega)}{1 - \Gamma_1 \Gamma_2 e^{-2\gamma l}} \quad B = \Gamma_2 e^{-2\gamma l} A \quad \Gamma_2 = \frac{Z_2 - Z_o}{Z_2 + Z_o} \]

\[ \Gamma_1 = \frac{Z_1 - Z_o}{Z_1 + Z_o} \]

Time-Domain Simulations

\[ Z_S = 50 \, \Omega \]

[Diagram: A circuit with a voltage source \( V_s \), a 50 \( \Omega \) impedance, and a cable with an open end at the far end.]
Pulse Propagation (CAT-5)
Pulse Propagation (MP/CM)
Pulse Propagation (RG174)